

# MODELS OF POLYURETHANE FOAM FORMATION

3M

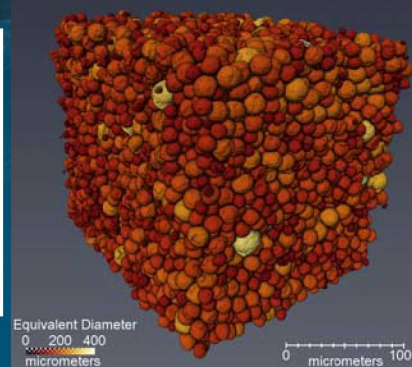
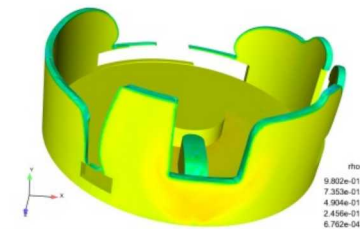
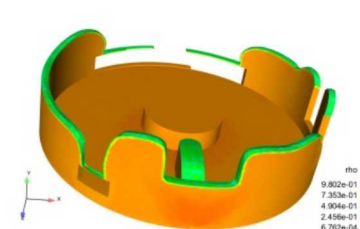
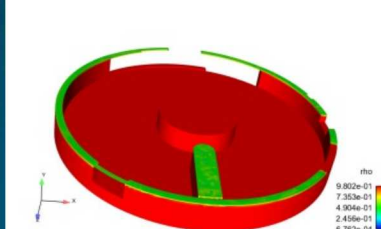
St. Paul, Minnesota  
November 13<sup>th</sup>, 2018



Time = 24.531

Time = 29.315

Time = 32.136



Rekha Rao

Lisa Mondy, Christine Roberts, Melissa Soehnel, Kevin Long, Victor Brunini, David Noble, Tyler Voskuilen, Jamie Kropka, Mathew Celina (SNL)

James Tinsley (KCNCS)

PRESENTED BY

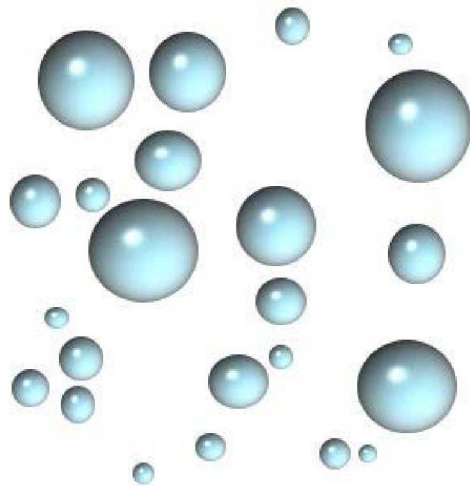
SAND2018-????

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# What is a Foam?

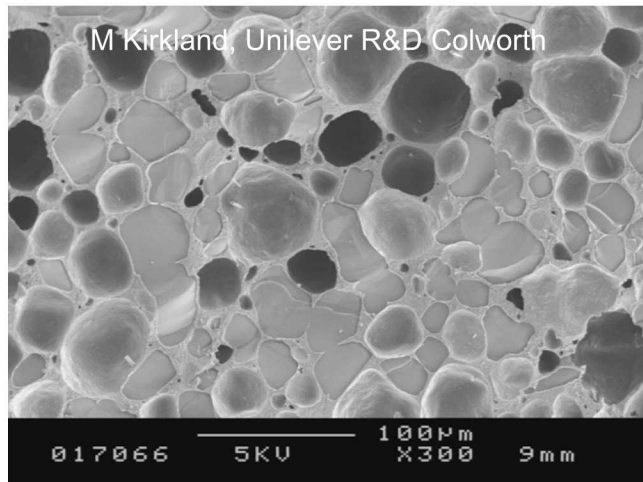


Bubbles

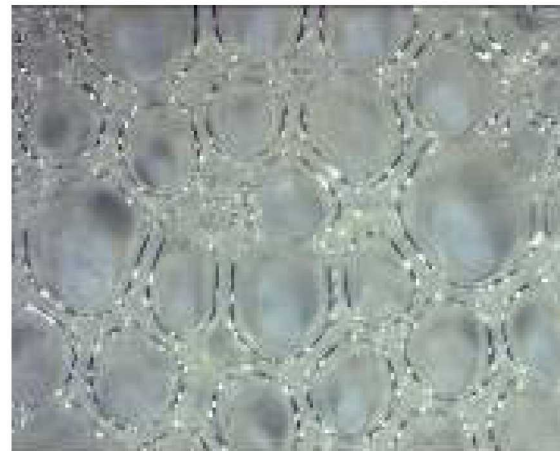


Whipped cream

- A multiphase material of gas bubbles in a liquid or solid matrix
- How do you make a foam?
  - Generate bubbles in a liquid
  - Stabilize them with particles, fat globules, or surfactant
  - Solidify liquid -freezing, polymerization, or phase change – if desired



Ice cream is a foam – that's why it is so much work to make



Epoxy foam is a collection of bubbles in polymer

Foams need enough bubbles to jam, e.g. bubbles are touching or it is just a bubbly liquid



# Some Foam Projects at Sandia



## Explosion Suppression

no foam



foam



Aubert et al. *Scientific American* 254 74 (1986)  
Courtesy of P.B. Rand

## Decontamination



Courtesy of J.B. Kelley

## Encapsulation

Intruders/Unruly Crowds



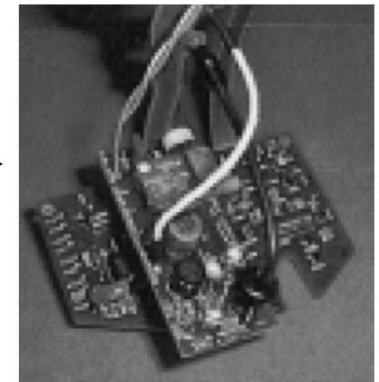
Scott SAND096-2495C; Russick SAND2002-1103P

Jamie Kropka (SNL)

Electronics—removable foam



reversible  
chemistry  
→  
90°C

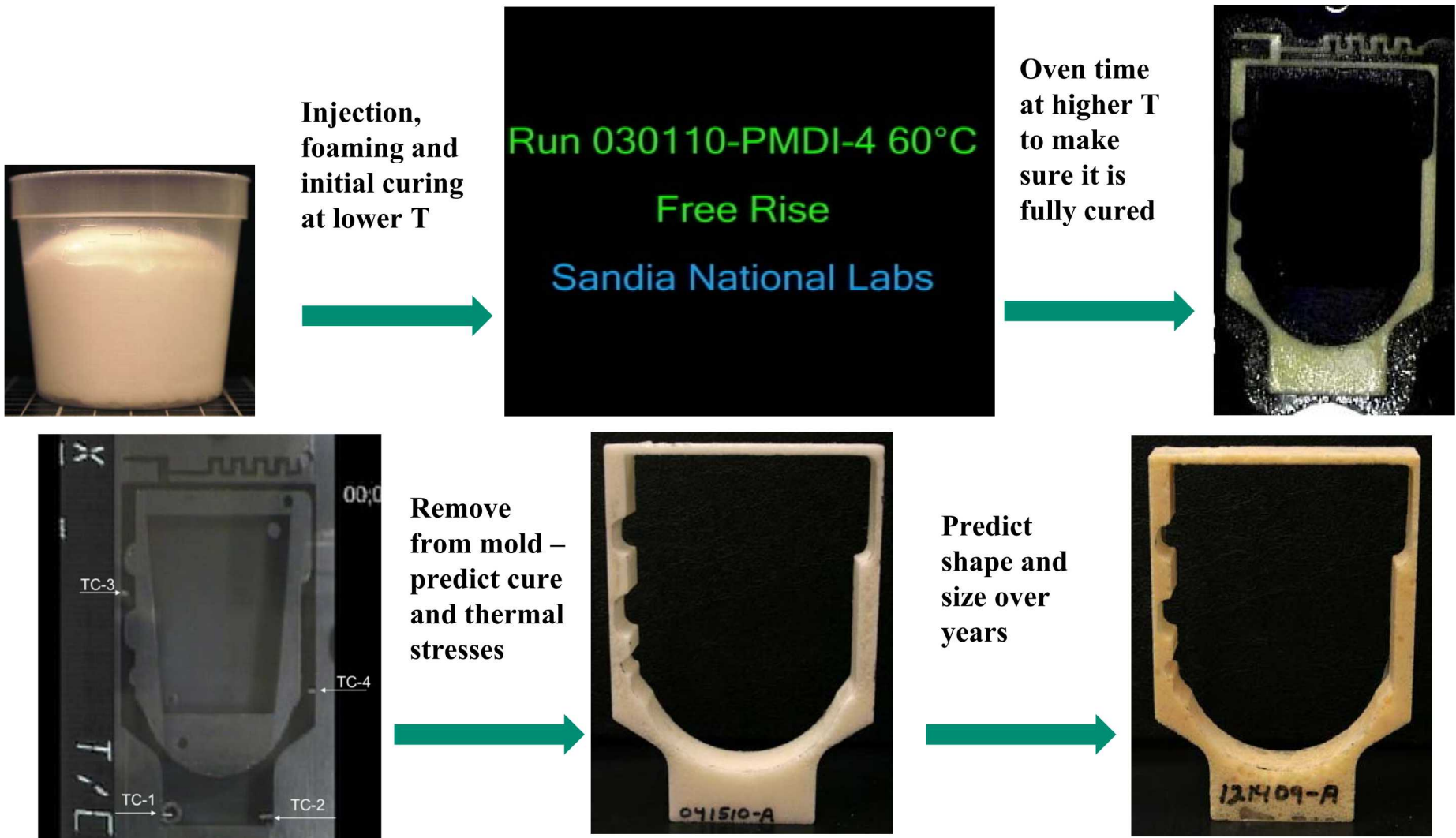


McElhanon et al. *J. Appl. Polym. Sci.* 85 1496 (2002)

# Introduction



**Overarching Goal:** Cradle-to-grave model for foaming, vitrification, cure, aging  
Focus on moderate density PMDI foams







## Stage I

Fluid

**Pre-Gel**  
(0-10<sup>3</sup> seconds)

Chemistry results in both gas production (foaming) and matrix polymerization (curing)

Foaming liquid rises to fill the mold until polymer matrix gelation

Heat, pressure generated

Gelation

## Stage II

Soft-Solid

**Post-Gel Cure**  
(10<sup>3</sup>– 10<sup>4</sup> seconds)

Variations in temperature cause variations in density and extent of cure

Solid polymer matrix locks in density gradients

Further gas production causes bubble pressurization with minimal volume increase

Vitrification

## Stage III

Solid

**Vitrified and Released**  
(10<sup>4</sup> + seconds)

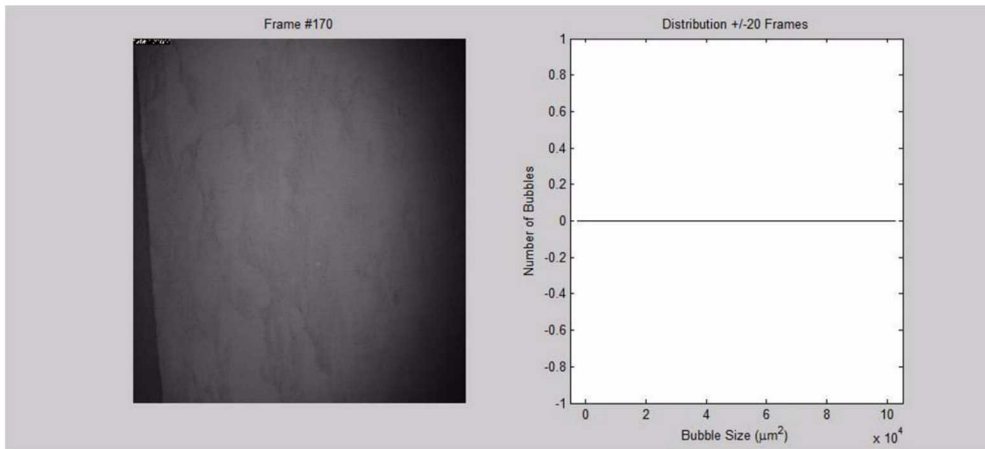
Residual stresses, density, and properties vary spatially

Both long and short term shape change is possible as different parts of the foam relax at different rates

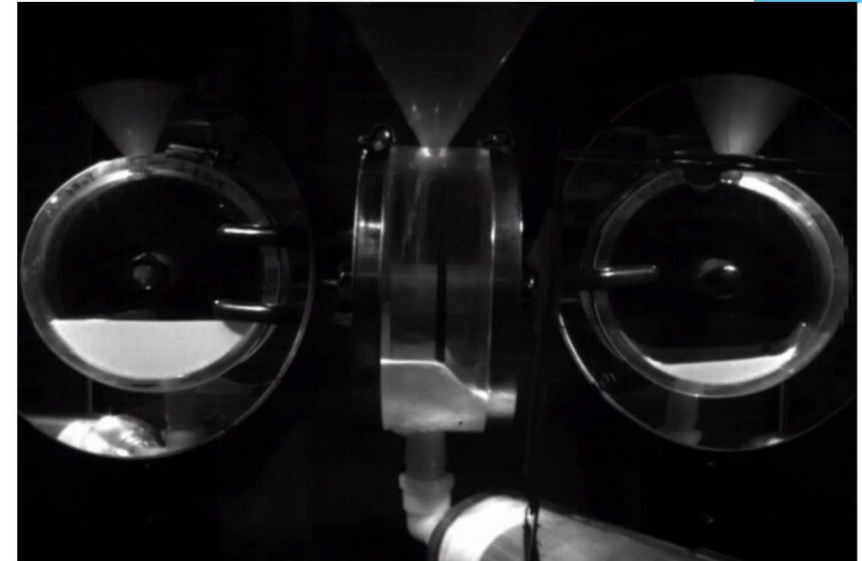
Boundary conditions strongly influence residual stresses

- Processing parameters at earlier stages will affect quality of part at later stages

# Foam Filling is Complex



Foam front moving past camera, with bubble sizes at transparent wall determined with image processing.



3 views of foam filling a mock AFS with several plates spaced unevenly. Vent location is critical to keep from trapping air.

- PMDI is used as an encapsulant for electronic components and lightweight structural parts, to mitigate against shock and vibration.
- **We would like to develop a computational model to help us understand foam expansion for manufacturing applications and how inhomogeneities effect the structural response of the final part, including long term shape stability.**
- Gas generation drives the foam expansion, changing the material from a viscous liquid to a multiphase material.
- Continuous phase is time- and temperature-dependent and eventually vitrifies to a solid.



# Polyurethane (PMDI): Model Development



We use a variety of physically and chemically blown foams. PMDI is used as an encapsulant for electronic components, to mitigate against shock and vibration, and to make light-weight structural parts.

We would like to develop a computational model to help us understand foam expansion for manufacturing applications.

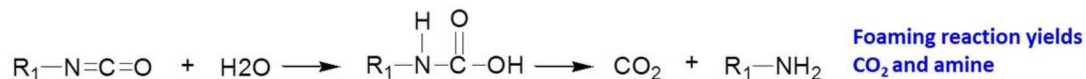
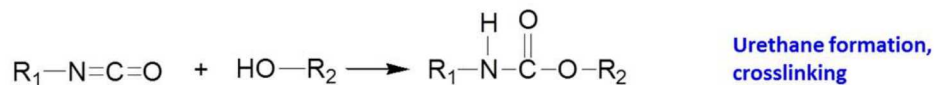
Polyurethane is a chemically blown foam having two primary, competing simultaneous reactions: CO<sub>2</sub> production and polymerization. Separating these reactions can be difficult.

We use IR spectroscopy to track polymerization. IR does not provide a clear signal for the foaming reaction: Tracked with volume generation.

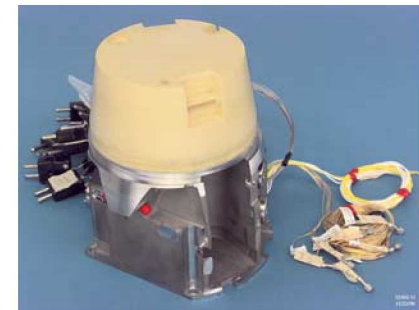
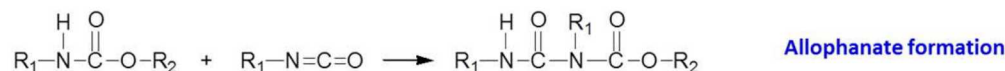
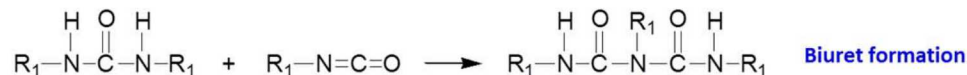
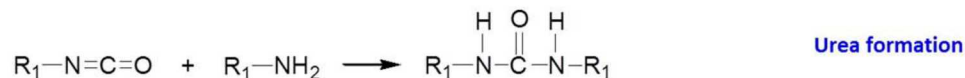


PU has a short pot-life: models can help reduce defects and improve filling process

**Two key reactions:** Isocyanate reaction with polyols and water



**Various follow up reactions:** Isocyanate reaction with amine, urea and urethane



Mock component encapsulated with PMDI from “KCP Encapsulation Design Guide” (Mike Gerding, UUR)

# Kinetic Model Must Include CO<sub>2</sub> Generation and Polymerization Reaction



$$rate_1 = k_1 e^{-\Delta E_1 / RT} [isocyanate]^a [polyol]^b \quad \text{Polymerization}$$

$$rate_2 = k_2 e^{-\Delta E_2 / RT} [isocyanate]^c [H_2O]^d \quad \text{CO}_2 \text{ generation}$$

- Must track five species: water, polyol, polymer, carbon dioxide, and isocyanate, since we have competing primary reaction
- Use experiments to determine Arrhenius rate coefficients

$$\frac{D[CO_2]}{Dt} = +rate_2$$

$$\frac{D[H_2O]}{Dt} = -rate_2$$

$$\frac{D[isocyanate]}{Dt} = -rate_1 - rate_2$$

$$\frac{D[polyol]}{Dt} = -rate_1$$

$$\frac{D[polymer]}{Dt} = +rate_1$$

- Must provide initial conditions for all species
- Integrate rate equations as part of the simulation
- Density predicted from gas generation
- Our kinetics are unique because our formulation is different from literature polyurethanes

$$\rho_{gas} = \frac{PM_{CO_2}}{RT}$$

$$v = \frac{V_{gas}}{V_{liq}} = \frac{M_{CO_2} C_{CO_2}}{\rho_{gas}} \quad \phi_v = \frac{v}{1+v}$$

$$\rho_{foam} = \rho_{gas} \phi_v + \rho_{liq} (1 - \phi_v)$$



# Equations of Motion Include Evolving Material Models



Momentum equation and continuity have variable density, shear viscosity, and bulk viscosity

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho \mathbf{v} \cdot \nabla \mathbf{v} - \nabla p + \nabla \cdot (\mu_f (\nabla \mathbf{v} + \nabla \mathbf{v}^t)) - \nabla \cdot \lambda (\nabla \cdot \mathbf{v}) \mathbf{I} + \rho \mathbf{g}$$

$$\frac{D\rho_f}{Dt} + \rho_f \nabla \cdot \mathbf{v} = 0$$

Energy equation has variable heat capacity and thermal conductivity including a source term for heat of reaction for foaming and curing reactions

$$\rho C_{pf} \frac{\partial T}{\partial t} + \rho C_{pf} \mathbf{v} \cdot \nabla T = \nabla \cdot (k \nabla T) + \rho \phi_e \Delta H_{rxn} \frac{\partial \xi}{\partial t}$$

Extent of reaction equation for polymerization: condensation chemistry

$$\frac{\partial \xi}{\partial t} = \left( \frac{1}{(1+wa)^\beta} \right) \left( k_0 \exp\left(-\frac{E}{RT}\right) \right) (b + \xi^m)(1 - \xi)^n$$

Molar concentration equations for water and carbon dioxide

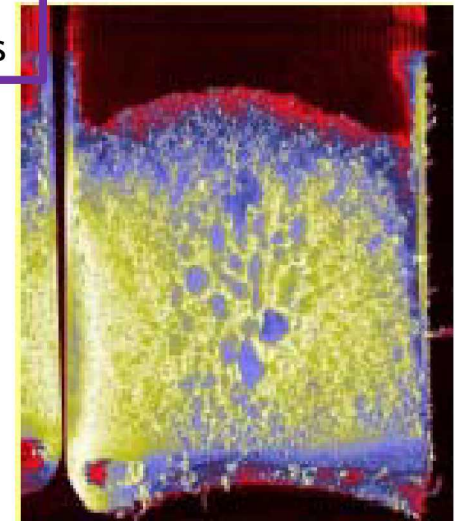
$$\frac{dC_{H_2O}}{dt} = -k_{H_2O} C_{H_2O}^n$$

$$C_{H_2O} = \frac{\rho_{foam} x_{H_2O}}{M_{H_2O}}$$

$$\frac{dC_{CO_2}}{dt} = +k_{H_2O} C_{H_2O}^n$$

$$C_{CO_2} = \frac{\rho_{foam} x_{CO_2}}{M_{CO_2}}$$

$$k_{H_2O} = A_{H_2O} \exp(-E_{H_2O} / RT)$$



NMR imaging shows coarse microstructure (Altobelli, 2006)

# Complex Material Models Vary with Cure, Temperature, and Gas Fraction



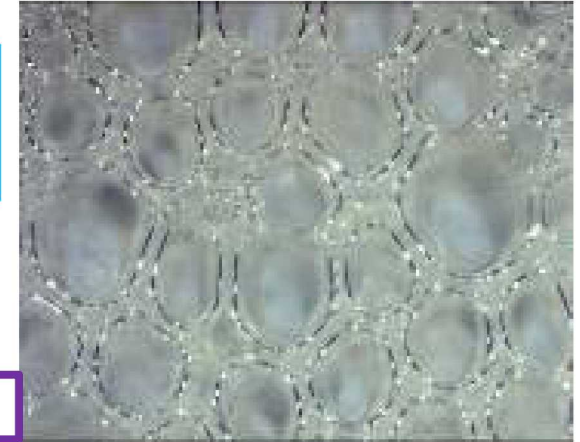
Foaming reaction predicts moles of gas from which we can calculate density

$$\rho_{gas} = \frac{PM_{CO_2}}{RT}$$

$$v = \frac{V_{gas}}{V_{liq}} = \frac{M_{CO_2} C_{CO_2}}{\rho_{gas}} \quad \phi_v = \frac{v}{1+v}$$

$$\rho_{foam} = \rho_{gas} \phi_v + \rho_{liq} (1 - \phi_v)$$

Compressibility built into this model via the ideal gas law for gas density



Thermal properties depend on gas volume fraction and polymer properties

$$k = \frac{2}{3} \left( \frac{\rho}{\rho_e} \right) k_e + \left( 1 - \frac{\rho}{\rho_e} \right) k_v$$

$$C_{pf} = C_{pl} \phi_l + C_{pv} \phi_v + C_{pe} \phi_e$$

Foam is a collection of bubbles in curing polymer

Shear and bulk viscosity depends on gas volume fraction, temperature and degree of cure

$$\mu = \mu_0 \exp\left(\frac{\phi_v}{1-\phi_v}\right) \quad \mu_0 = \mu_0^0 \exp\left(\frac{E_\mu}{RT}\right) \left(\frac{\xi_c^p - \xi^p}{\xi_c^p}\right)^{-q}$$

$$\lambda = \frac{4}{3} \mu_0 \frac{(\phi_v - 1)}{\phi_v}$$

- Experiments to determine foaming and curing kinetics as well as parameters for model
- Equations solved with the finite element method using a level set to determine the location of the free surface (Rao et al., IJNMF, 2012)



# Extent of Reaction for Polymerization



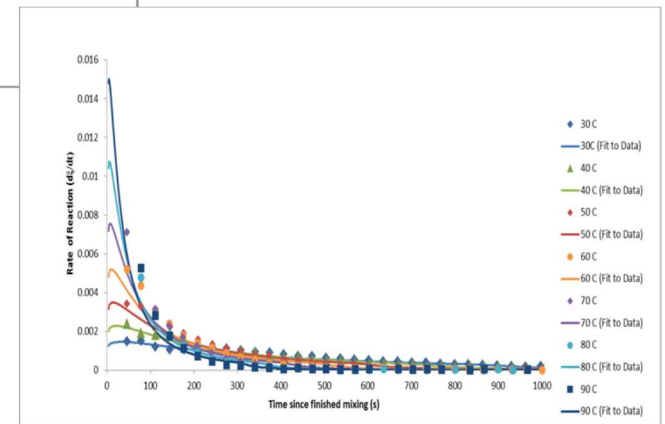
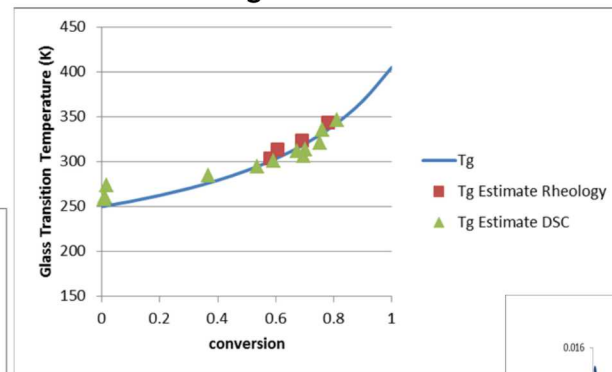
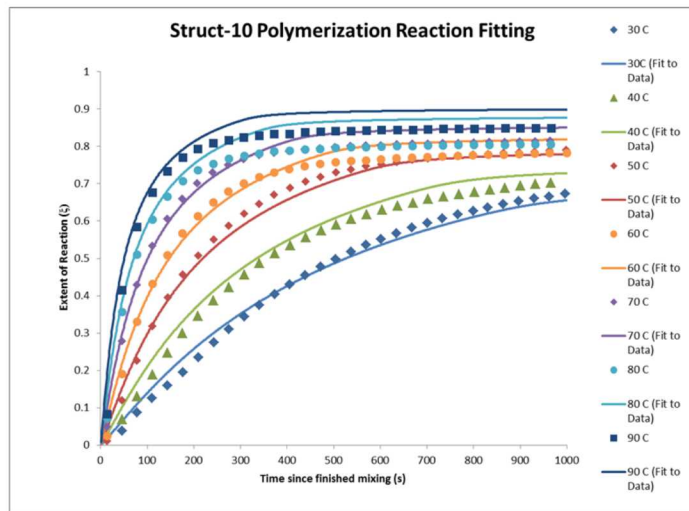
- Fit the rate and the extent of reaction to IR data to a standard equation form
- Fit  $T_g$  to both rheology and DSC data:  $T_g$  changes as cure progresses making this complex

$$\frac{\partial \xi}{\partial t} = \left( \frac{1}{(1 + wa)^\beta} \right) \left( k_0 \exp\left(-\frac{E}{RT}\right) \right) (b + \xi^m)(1 - \xi)^n$$

$$\log_{10} a = \frac{-C_1(T - T_g)}{C_2 + T - T_g}$$

$$T_g = \frac{T_{g0}(1 - \xi) + A\xi T_{g\infty}}{(1 - \xi + A\xi)}$$

- New form captures arrest of reaction below the glass transition temperature ( $T_g$  evolves with extent of reaction)

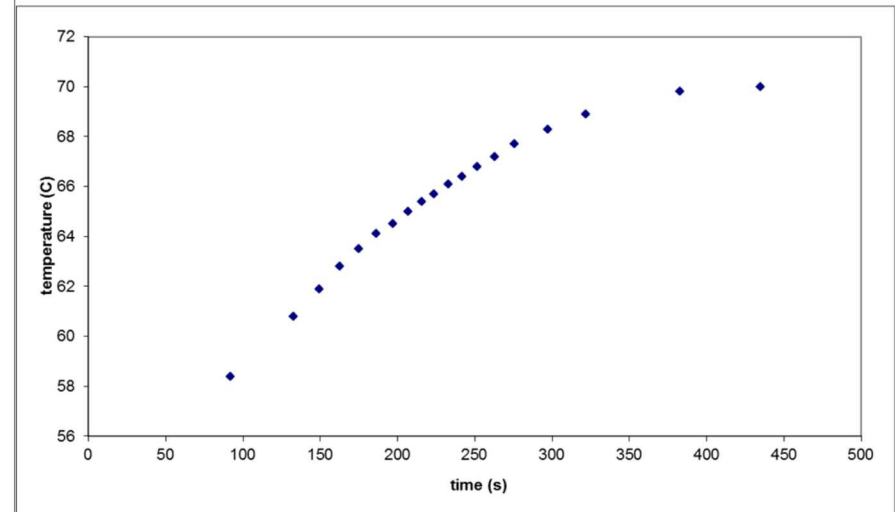
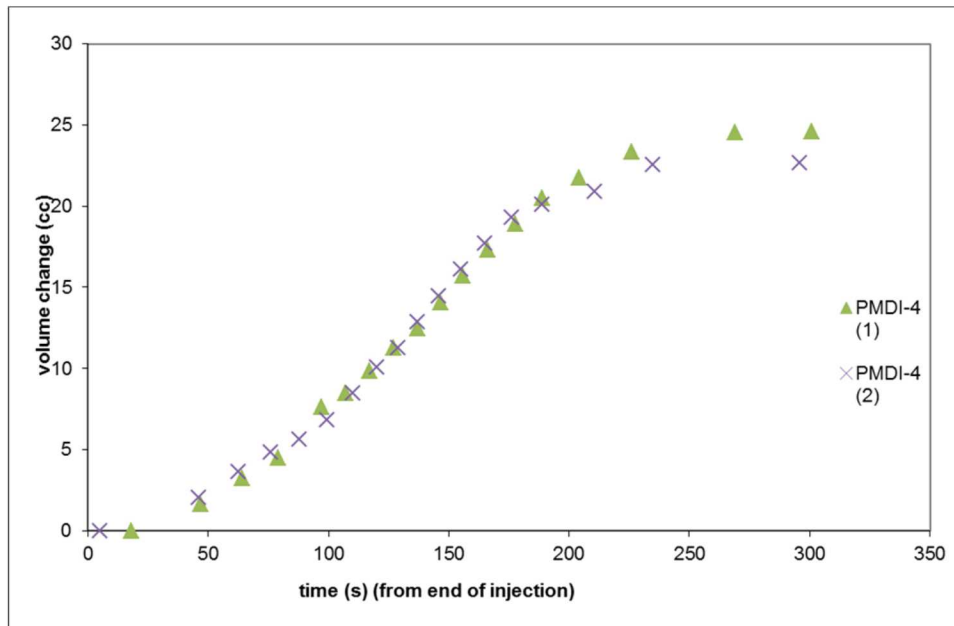
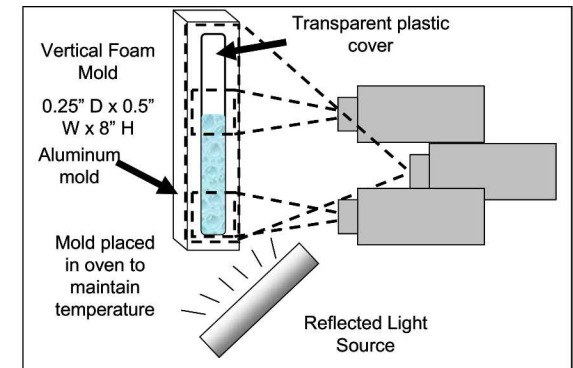


Rate and extent of reaction fit to data, where parameters of the model, including  $T_g$  are optimized for lower temperatures expected in the process. The apparent time-to-gel from rheology is correlated with extent to give a  $T_g$  with conversion. Similar analysis can be done with DSC and results are consistent.

# Measure Height Change in Simple Geometry to Quantify Foaming Reaction



- Data have most uncertainty at early times because reaction is occurring during mixing and injections, but bubbles are being destroyed in these processes, too.
- We can only measure height change after these processes.
- CO<sub>2</sub> loss from bubble breakage at top surface? BUT bottom line: engineering model to predict volume change
- The foam cannot be preheated, so during the foam rise the temperature is not steady.

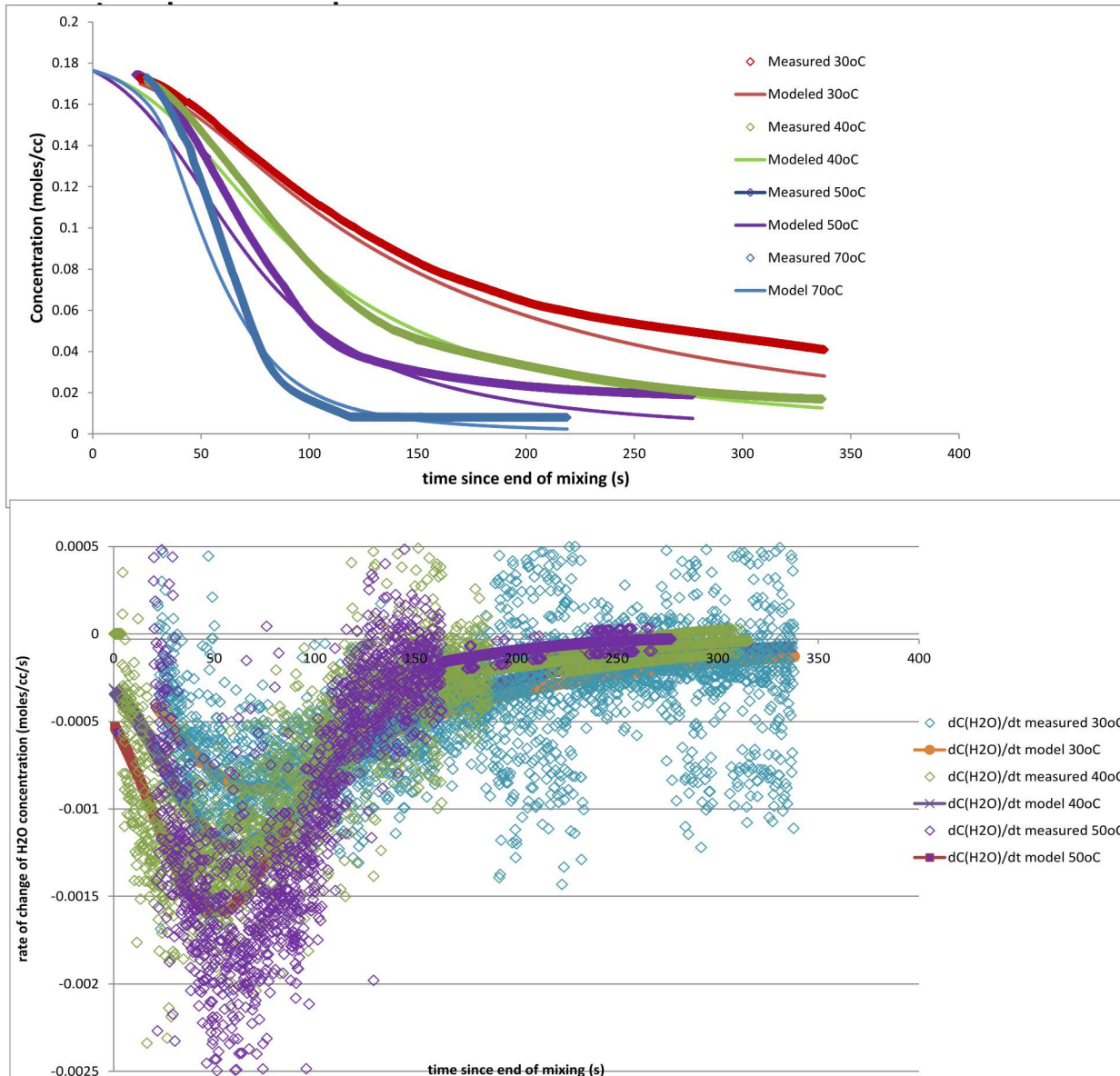




# Kinetics of CO<sub>2</sub> Generation



- Fit the concentration of water and its rate of disappearance



$$\frac{dC_{H_2O}}{dt} = -Nk_{H_2O}C_{H_2O}^n$$

$$\frac{dC_{CO_2}}{dt} = +Nk_{H_2O}C_{H_2O}^n$$

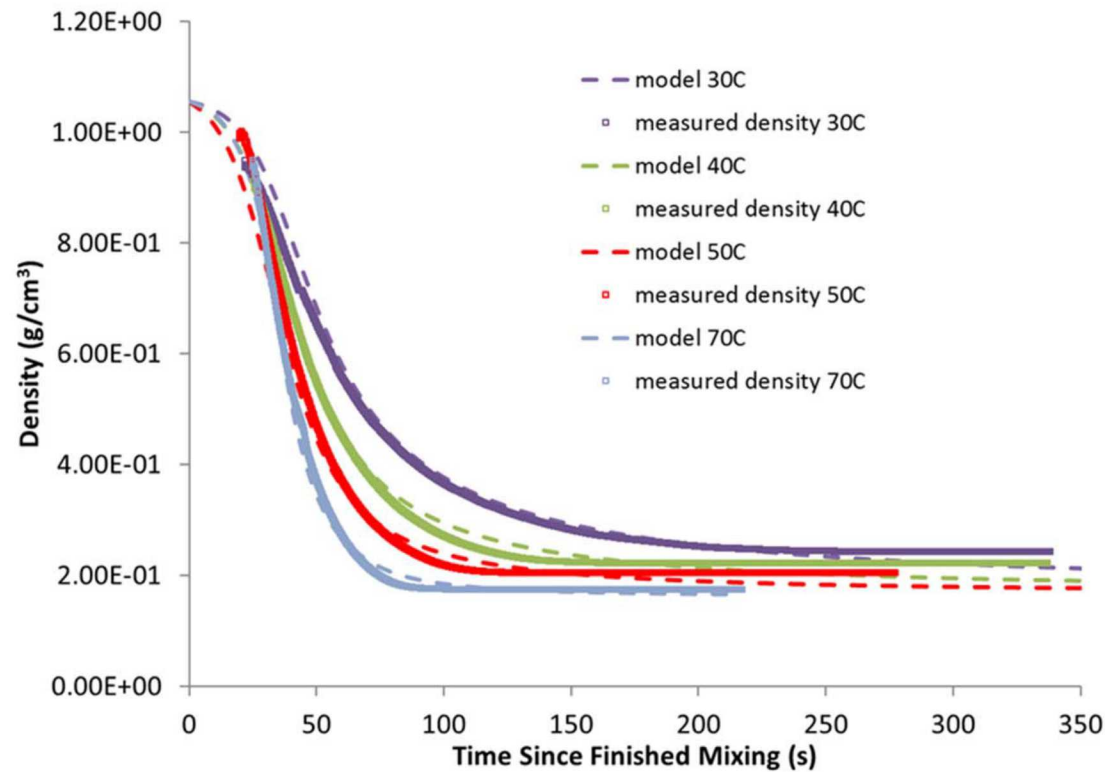
$$N = 0.5 \left\{ 1 + \tanh \left( \frac{t - t_{nucleation}}{t_{scale}} \right) \right\}$$

- Apparent water concentration shows a change in slope
- Model must capture this
- Physically it relates to the solubility of the carbon dioxide in the polymer
- Must super saturate before nucleation and growth

# Kinetics of CO<sub>2</sub> Generation



Predictions of density using a nucleation time of 40s and a time scale of 20s compared to measured density with time in the channel for various temperatures.

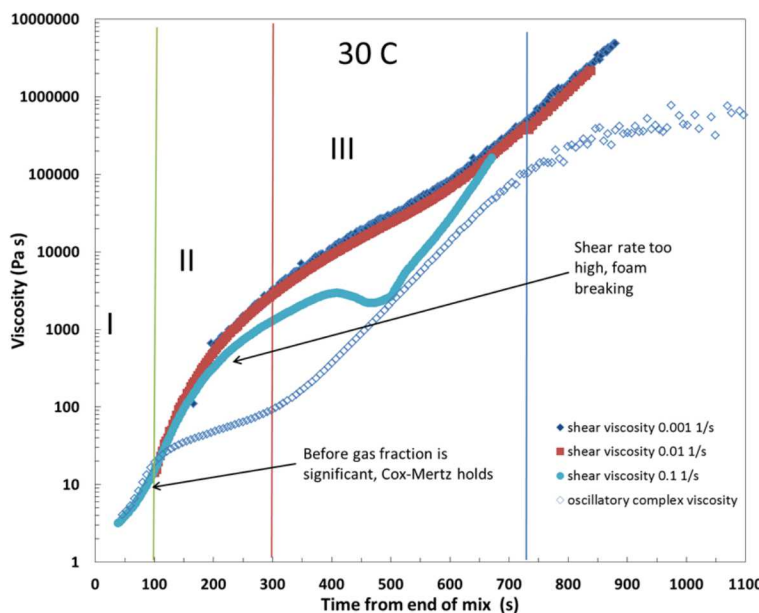
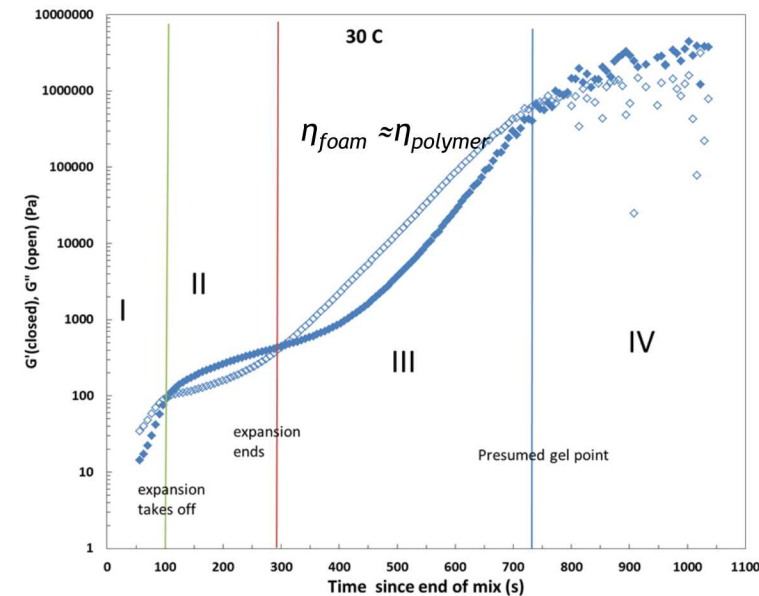


$$\frac{dC_{H_2O}}{dt} = -Nk_{H_2O}C_{H_2O}^n$$
$$\frac{dC_{CO_2}}{dt} = +Nk_{H_2O}C_{H_2O}^n$$

- Experiments give us average density
- Hard to determine evolving density gradients
- Measure density gradients from post-test experiments

Rao et al., "Polyurethane kinetics, for foaming and polymerization," *AIChE Journal*, 2017

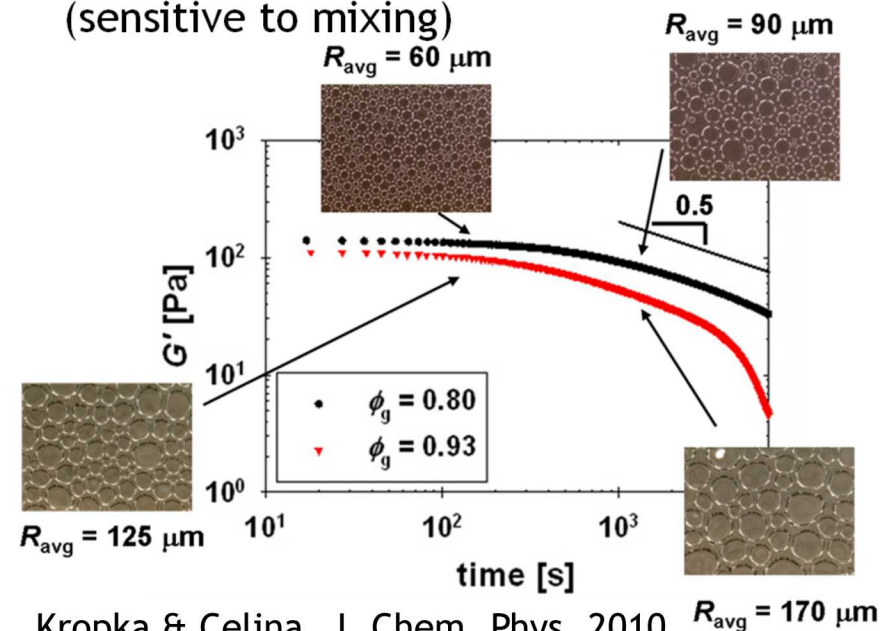
# Viscosity of Foam is Complex



- Foam rheology evolves as gas fraction and polymerization increase

$$\eta_{foam} = \eta_{polymer} \eta_{\phi}$$

- After Bouayad et al. Int J. Mater Form (2009), plot foam rheology as function of distinct phenomenological characteristic times
- Test foam viscosity with steady shear at low shear rates
- Be aware of slip
- Effect of bubble size & coarsening (sensitive to mixing)



Kropka & Celina, J. Chem. Phys. 2010



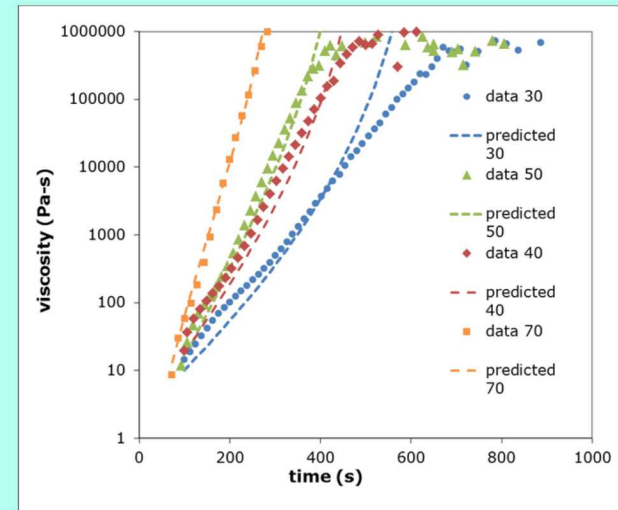
# Model Foam Viscosity as $f(\xi, \varphi)$

## Start with continuous phase viscosity only

- IR kinetics + dry formulation rheology (two sets of experiments) give an approximation of the curing continuous phase rheology
- Relate time of gel point to  $\xi$  to find  $\xi_c$ .

$$\mu_{polymer} = \mu_0^0 \left( \frac{\xi_c - \xi}{\xi_c} \right)^{-6} \quad \xi_c = 0.86$$

$$\mu_0^0 = 600 e^{-1549/RT} \text{ Pa-s}$$

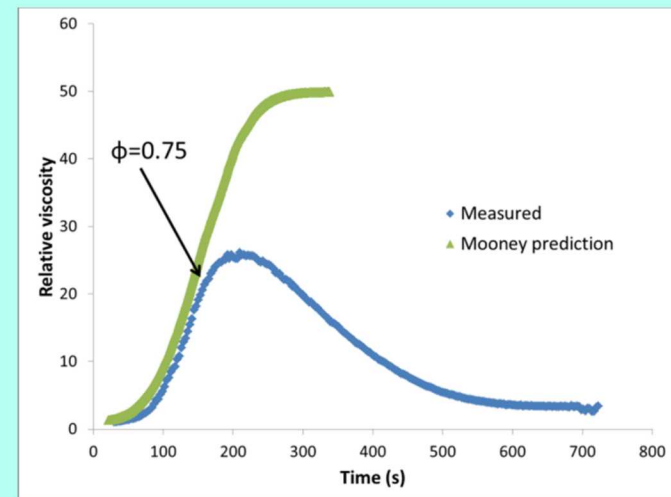


## Relate foam viscosity to continuous phase viscosity

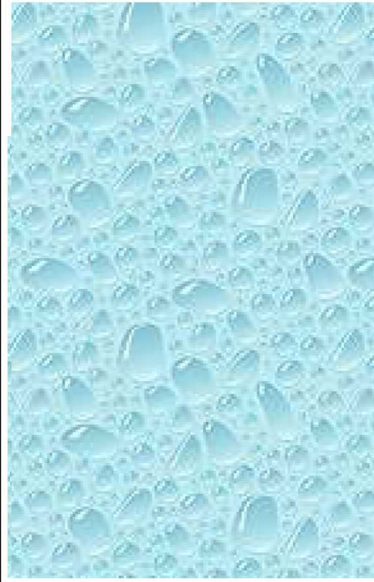
- Foam rise + wet formulation rheology (two sets of experiments) give an approximation of the rheology as a function of gas fraction
- Mooney prediction (for  $\varphi_{gas} < 0.5$ )

$$\mu_{\varphi} = \mu_{polymer} \exp\left(\frac{\varphi_g}{1 - \varphi_g}\right)$$

- For  $\varphi_{gas} > 0.75$  estimate  $\mu_{foam} = \mu_{cure} * f(\xi)$



# Coupled Finite Element Method/Level Set to Solve Foam Dynamics



- Given fluid velocity field,  $u(x,y,z)$ , evolution on a fixed mesh is according to:

$$\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = 0$$

- Purely hyperbolic equation ... fluid particles on  $\phi(x,y,z) = 0$  should stay on this contour indefinitely
  - Does not preserve  $\phi(x,y,z)$  as a distance function
  - Introduces renormalization step.

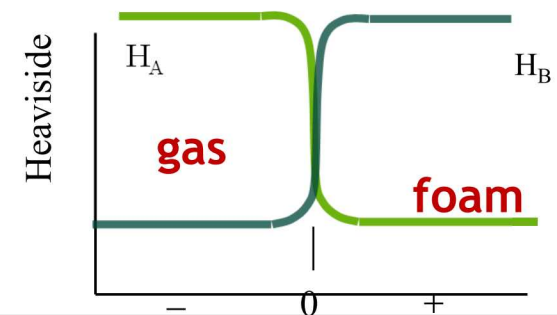
- Equations of motion, kinetics and energy balance averaged based on level set,  $\phi$

$$H_A \rho_A \frac{Du}{Dt} + H_B \rho_B \frac{Du}{Dt} = -\nabla P + H_A \nabla \cdot (\mu_A \dot{\gamma}) + H_B \nabla \cdot (\mu_B \dot{\gamma}) + (H_A \rho_A + H_B \rho_B)g + I.T.,$$

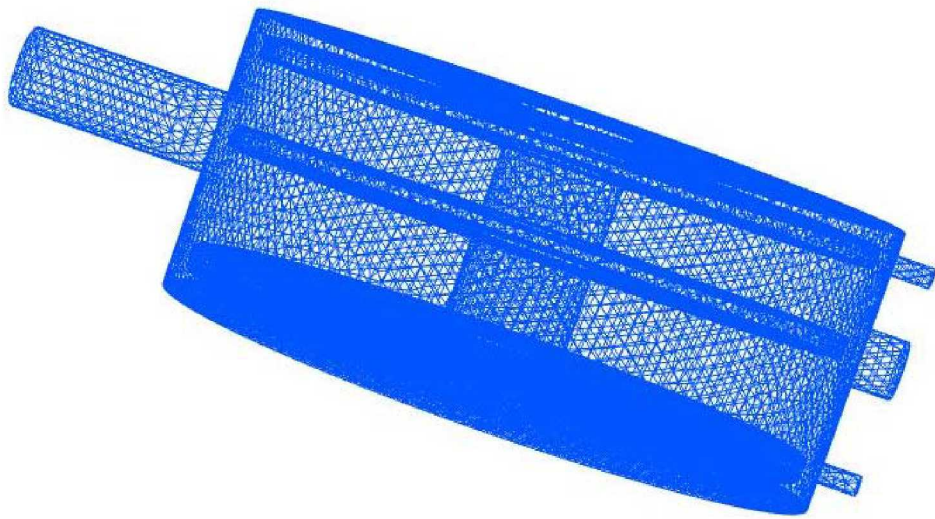
$$H_A \frac{D\rho_A}{Dt} + H_B \frac{D\rho_B}{Dt} + (H_A \rho_A + H_B \rho_B) \nabla \cdot u = 0$$

$$H_A + H_B = 1$$

Rao et al, IJNMF, 2012



# Idealized Foam Encapsulation Part: Board Would Contain Electronics in Real Part



Mold is preheated to ten degrees hotter than the foam

Inflow is asymmetric and fills thinner area first

Boards have different thicknesses of foam

Three vents are used to improve filling

Foam slips at the wall using a Navier slip condition with  $\text{Beta} = .001$

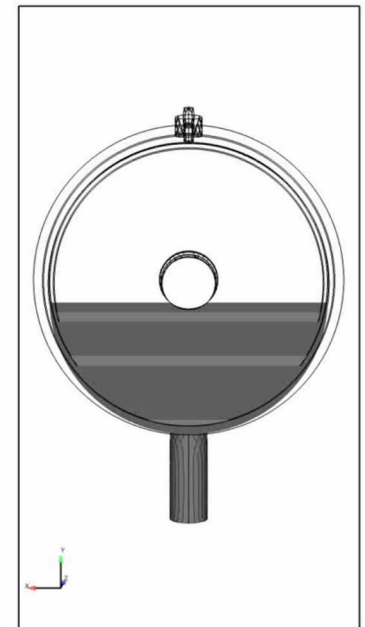
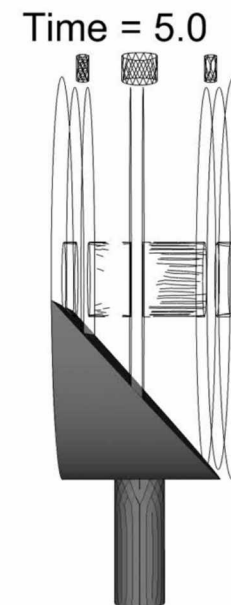
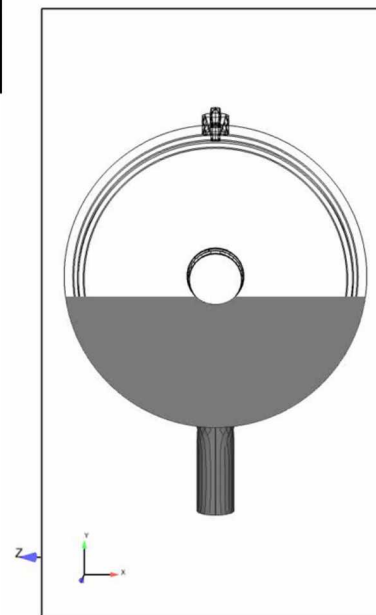
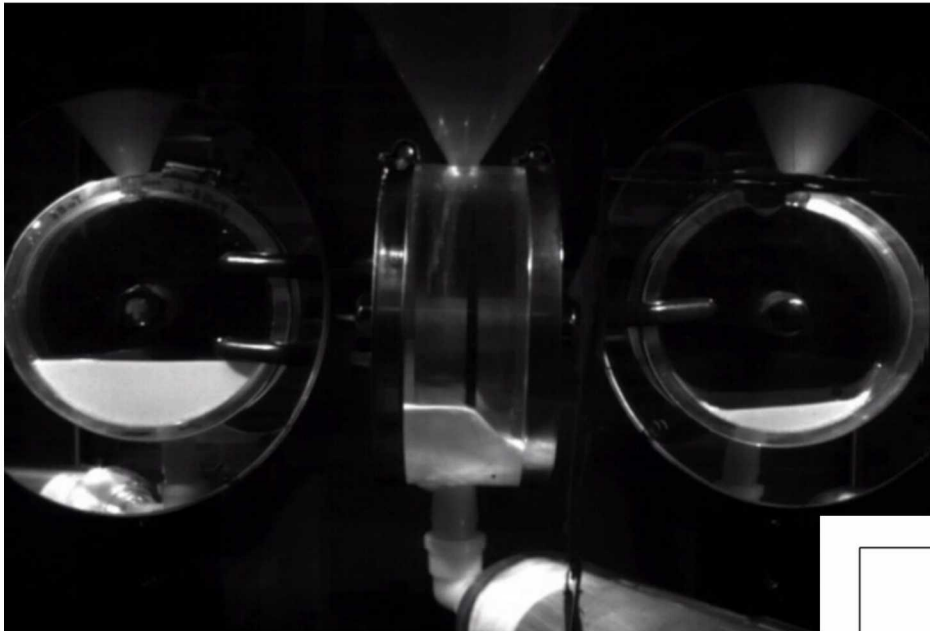
Gas slips ten times more than the foam

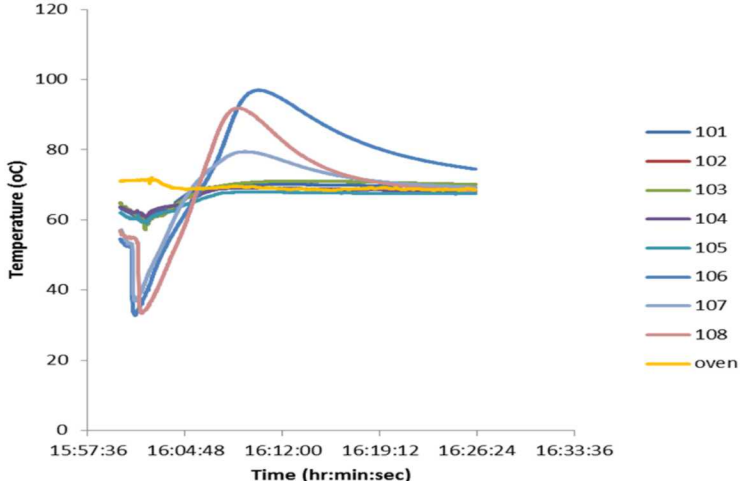
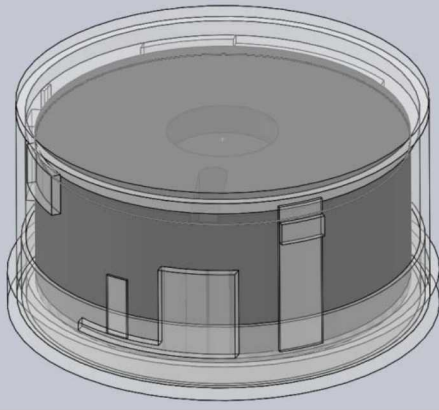
$$\underline{n \cdot \tau \cdot n} = \frac{1}{\beta} (v - v_s) \cdot n$$

$$\beta = (\beta_{\text{gas}} - \beta_{\text{foam}}) H(\phi) + \beta_{\text{foam}}$$



# Foam Filling Simulation of Complex Part with Plates

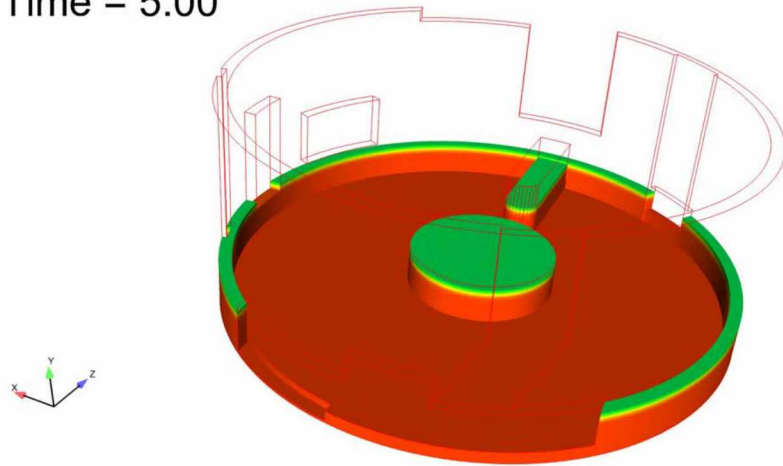




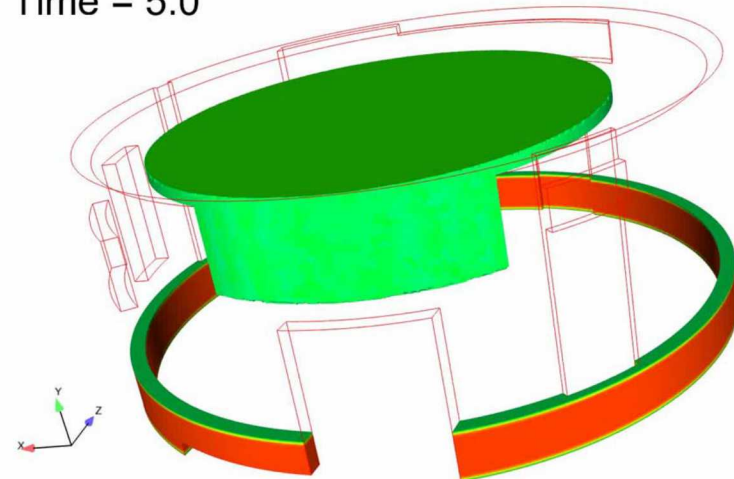
# Expansion Can Help Design a Mold Filling Process



Time = 5.00



Time = 5.0



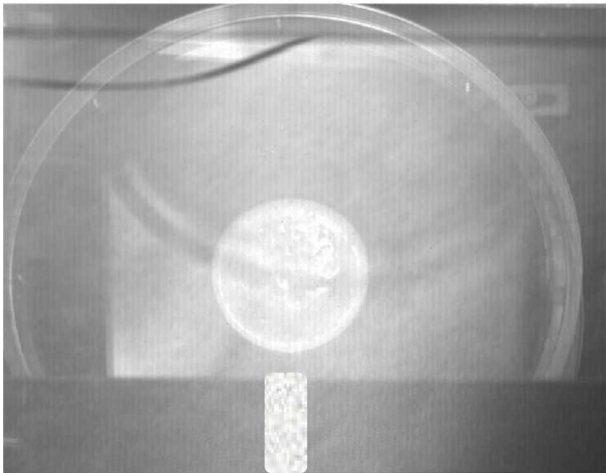
rho  
1.000e+00  
7.750e-01  
5.500e-01  
3.250e-01  
1.000e-01

rho  
1.000e+00  
7.750e-01  
5.500e-01  
3.250e-01  
1.000e-01

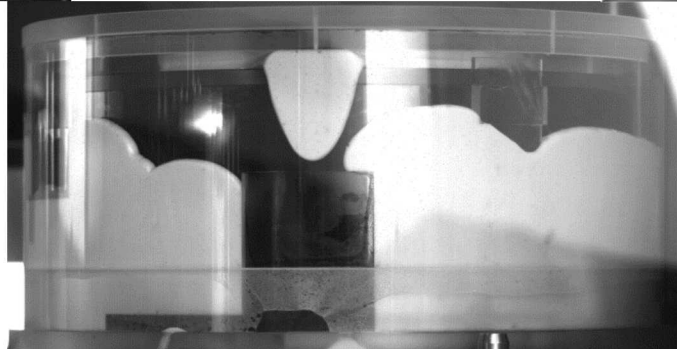
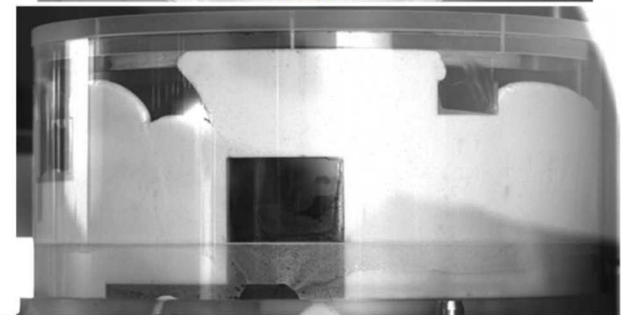
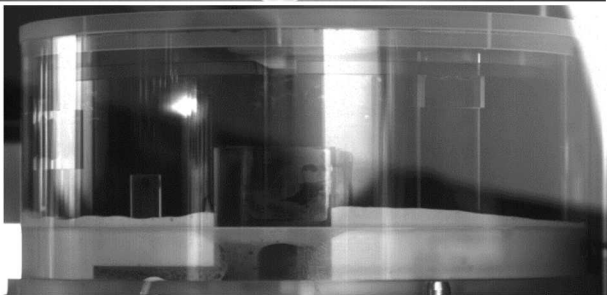
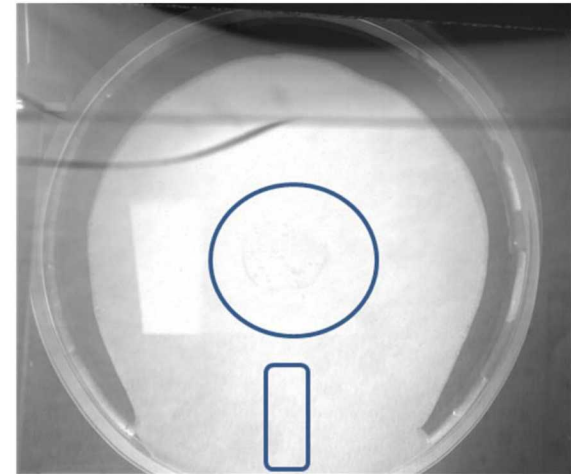




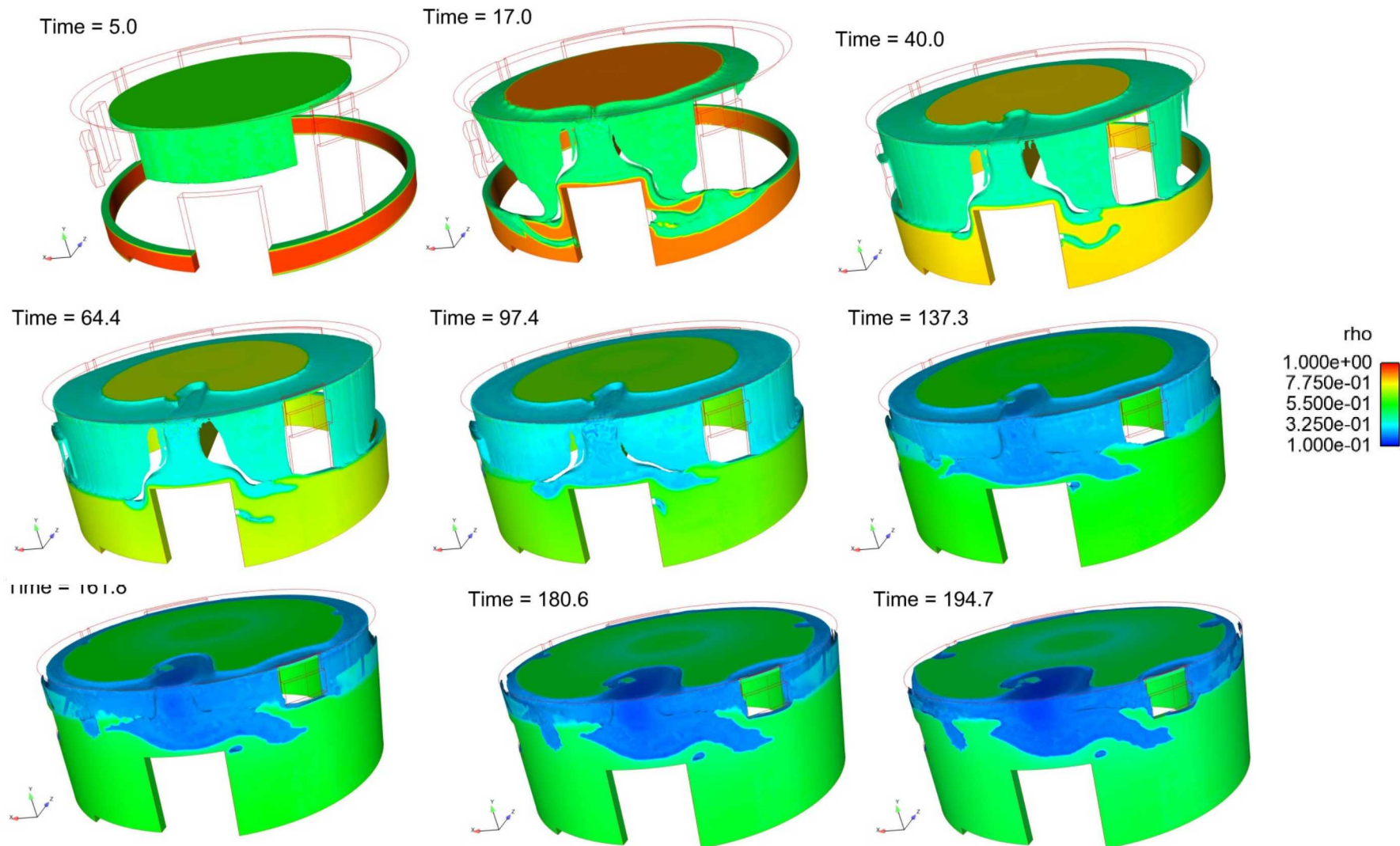
# Filling Method Creates Knit Lines



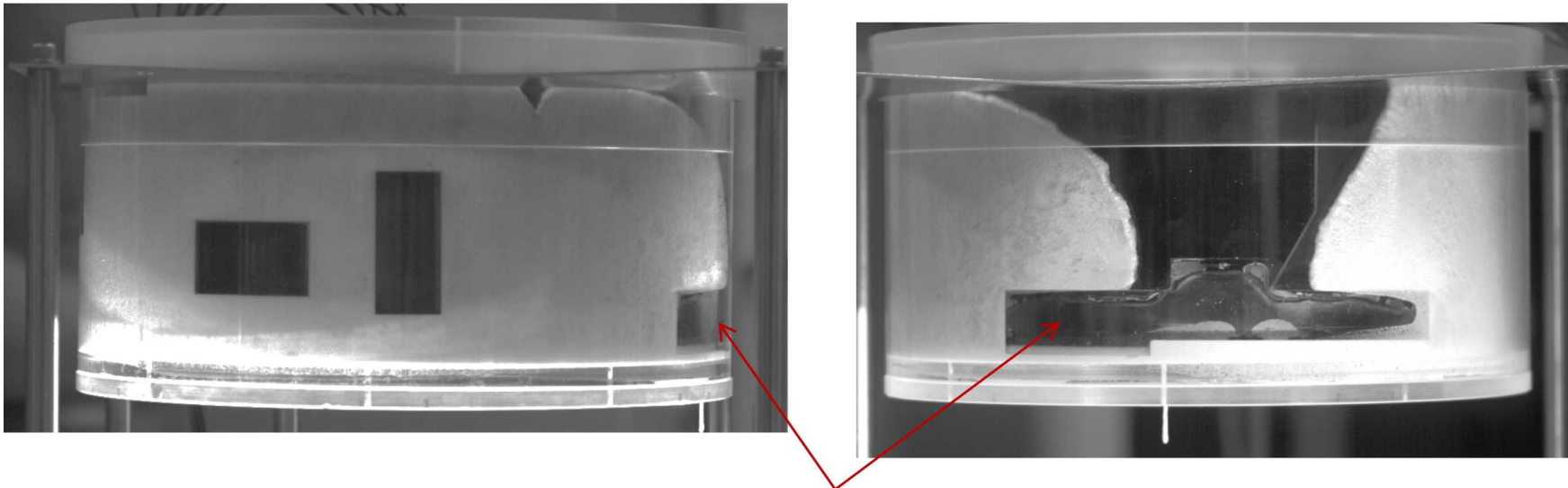
Foaming material is originally placed in top rectangular and cylindrical reservoirs and in bottom rim reservoir, to simulate legacy KC filling method



# Computational Modeling of Foam Expansion Can Help Design a Mold Filling Process



# Last Place to Fill Now on Other Side of Largest Feature

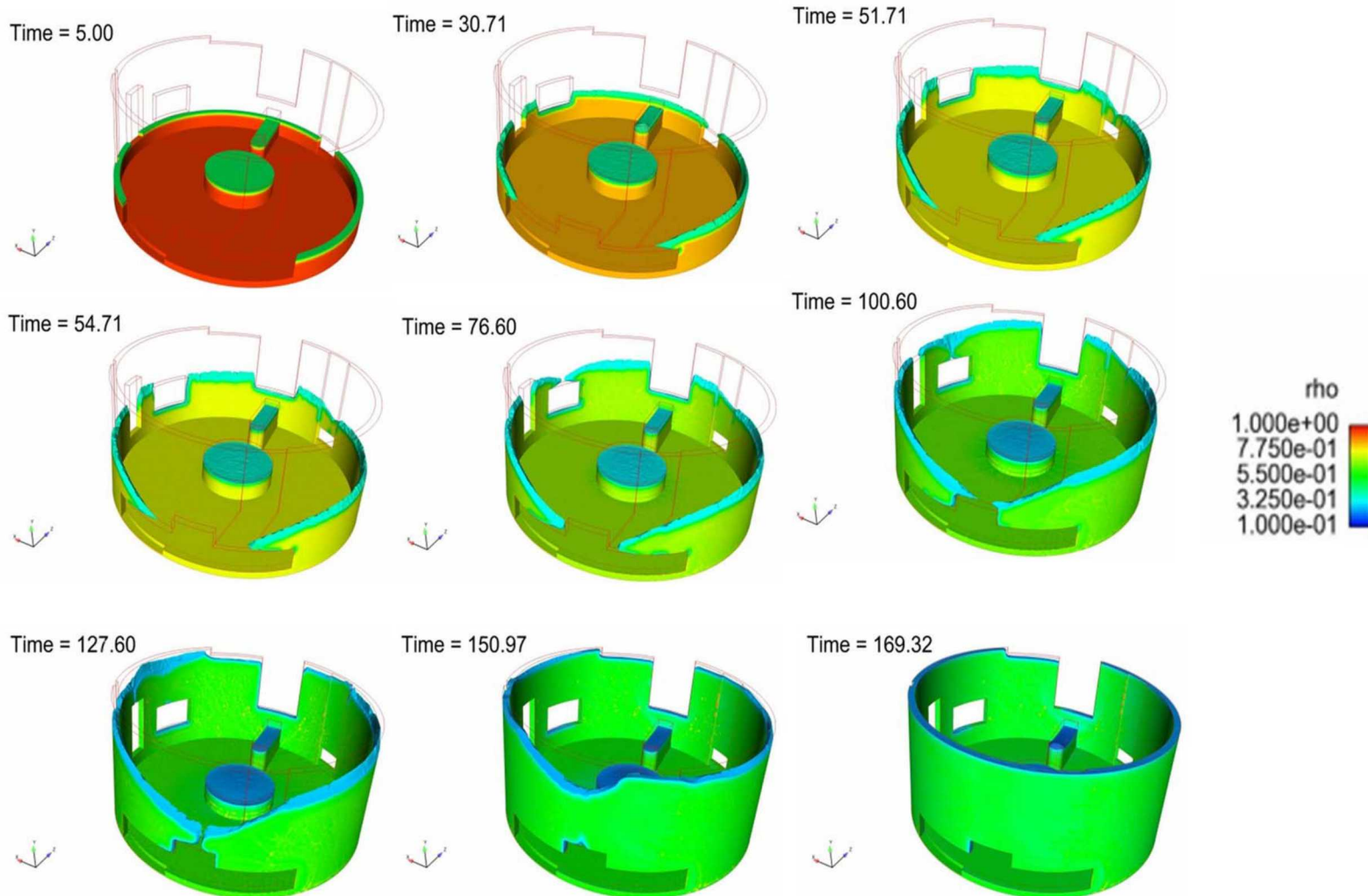


Largest feature

Short shot: less foam than encapsulation test 1, to see where last places to fill would occur. Reaction proceeded faster gelling foam before could finish rising.



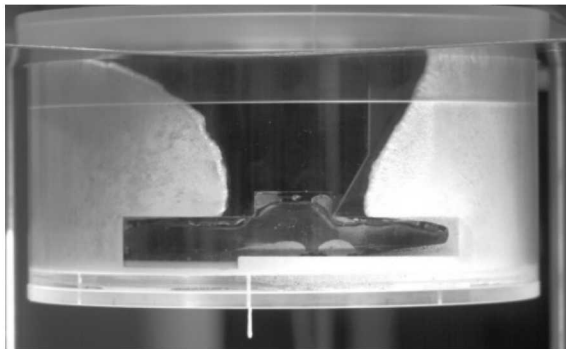
# Computational Modeling of Foam Expansion Can Help Design a Mold Filling Process



# Models Give More Physics than Just the Filling Locations

Models developed for foam filling and curing => density/cure

- The model allows us to look inside the mold
- New kinetics show water depletion and CO<sub>2</sub> variations
- Density variations are seen in the mold
- Foam exotherms significantly even and

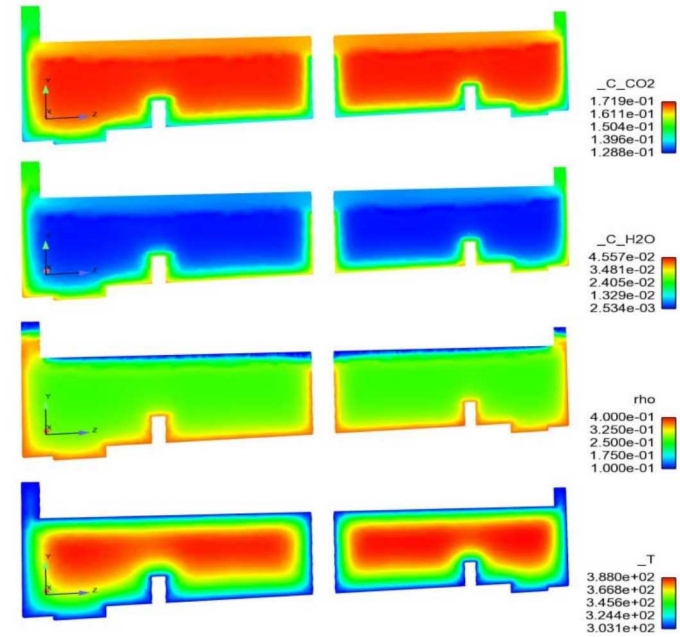
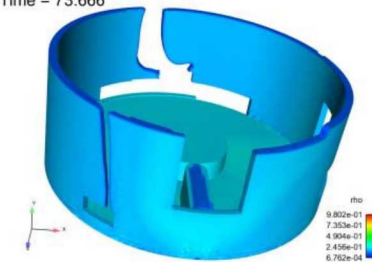
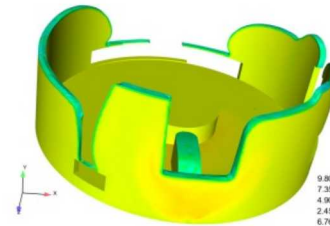
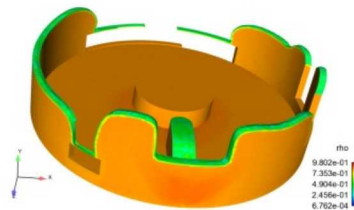
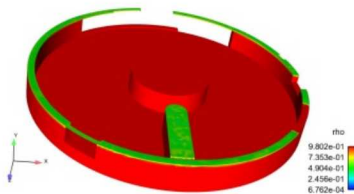


Time = 24.531

Time = 29.315

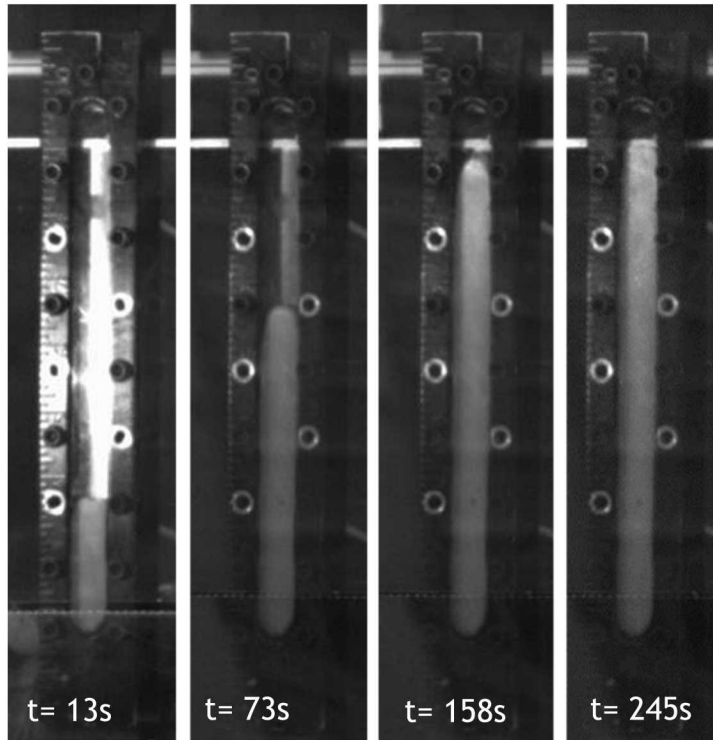
Time = 32.136

Time = 73.666

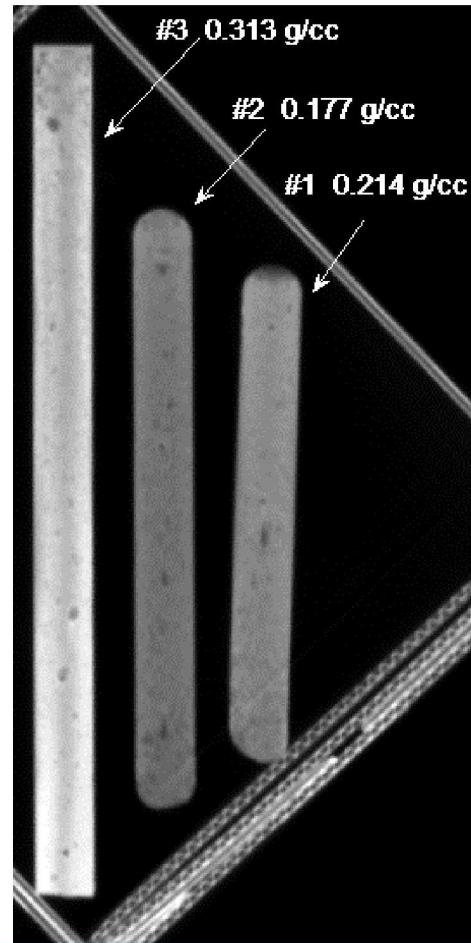


Rao et al., "Density Predictions for polyurethane foam using a finite element/level set method," submitted

# Density Study for Structural Foam PMDI-10



Foam expanding in a mold at 30°C. Time shown on frames is after the end of mixing the resin and the curative together for 45 seconds.



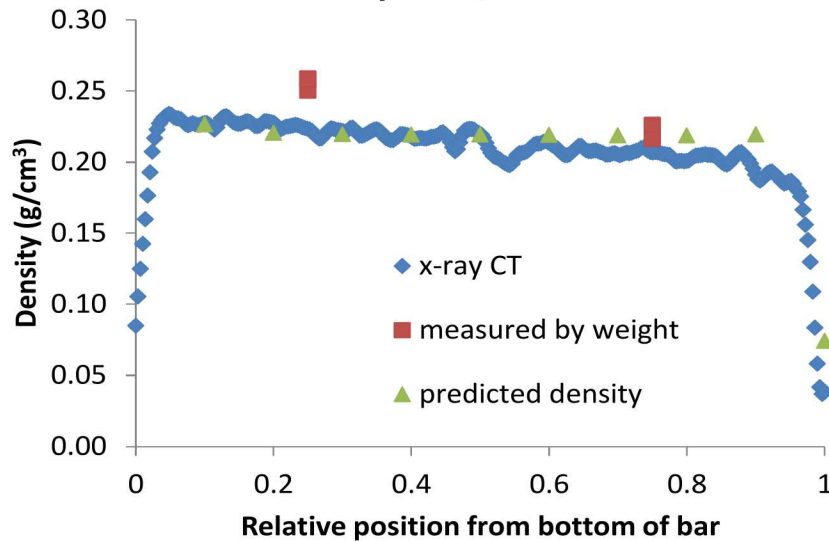
X-ray image of PMDI-10 foam bars:  
1) free rise at 30°C, 2) free rise at 50°C, 3) over packed (1.5) at 30°C

- Can the model predict the effects of over packing seen experimentally?
- Over-packed sample shows higher density and greater density variation
- 17% for free rise and 31% for over-packed foam bars

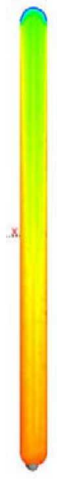


# Density Study for Structural Foam PMDI-10

Sample #1, 30°C

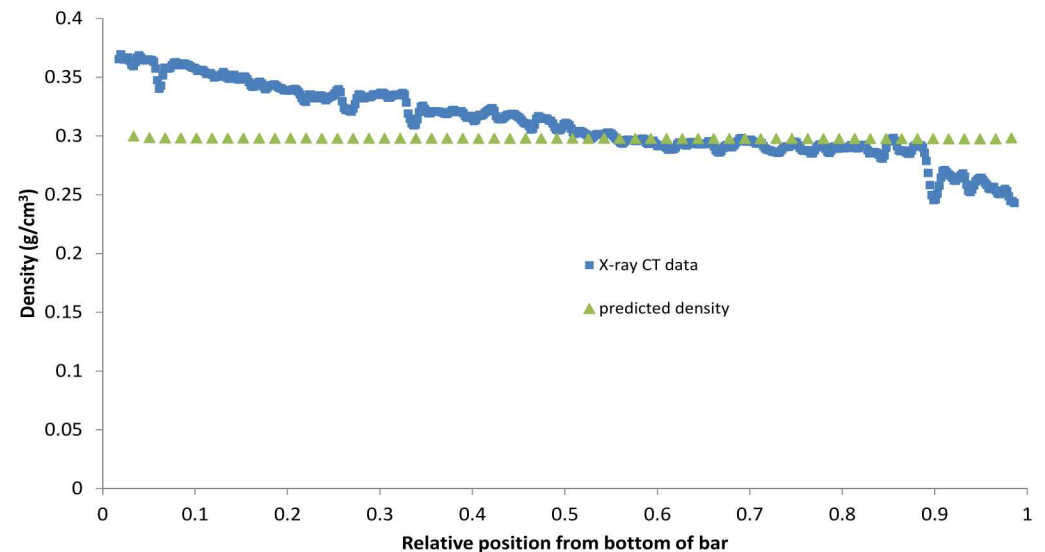


- Free rise foam density gradients. Plots are shown at the centerline of the foam cylinder
- Cylinder is under filled to give the free rise density



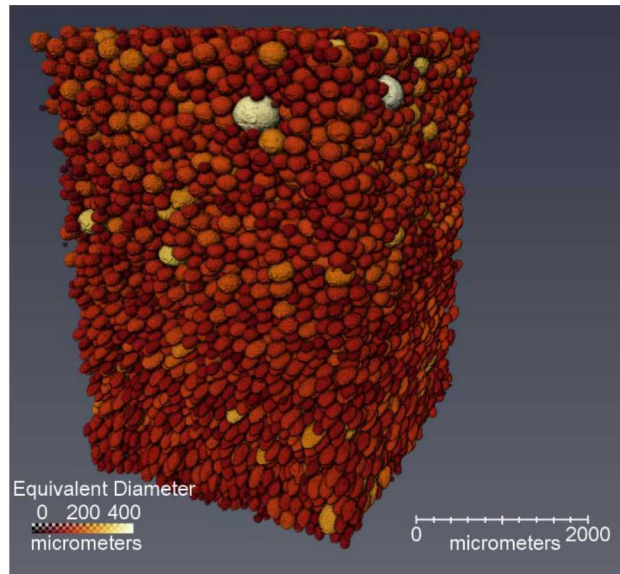
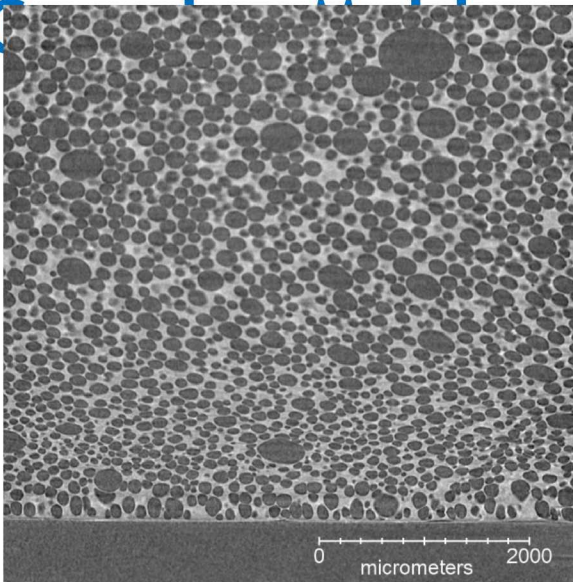
- Over packed (1.5) foam density gradients. Plots are shown at the centerline of the foam cylinder
- Self-closing vent lets air out, but keeps foam in for pressurization

Sample #3, overpacked x 1.5,  
30°C





# CT Microstructure of Bubbles from Large

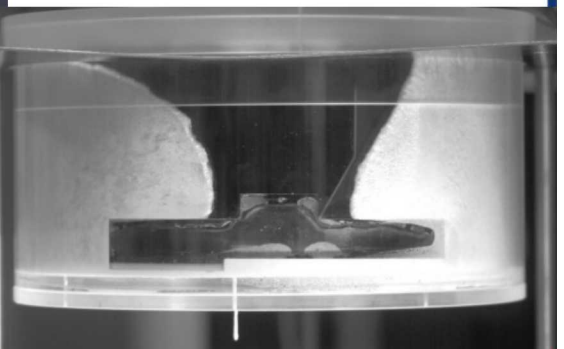
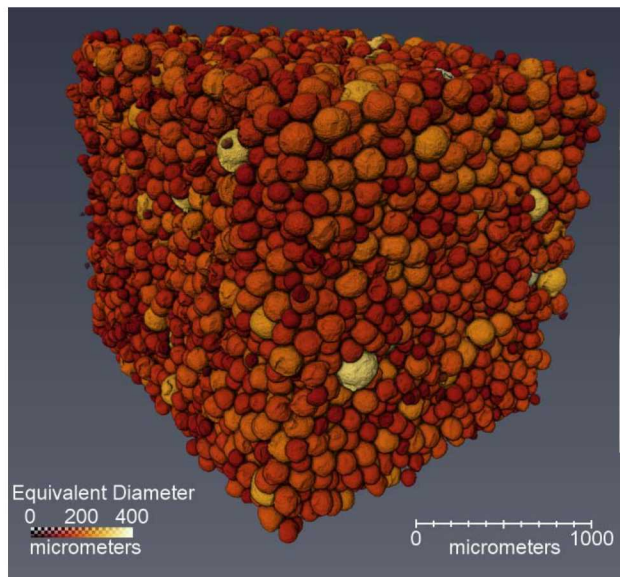
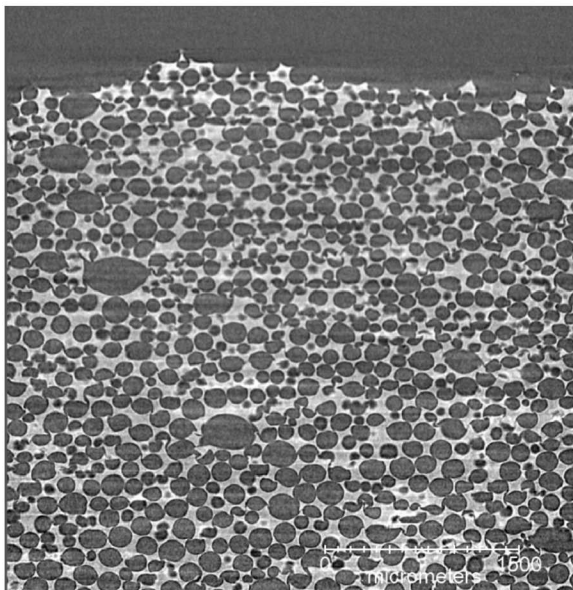


Sample 1 top

## Foam

### microstructure

- Polydisperse bubble sizes
- Shear near boundaries cause elongated ellipsoidal bubbles

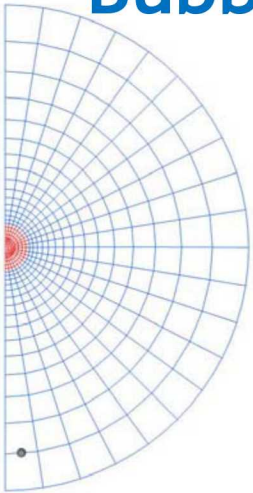


Sample 1 bottom

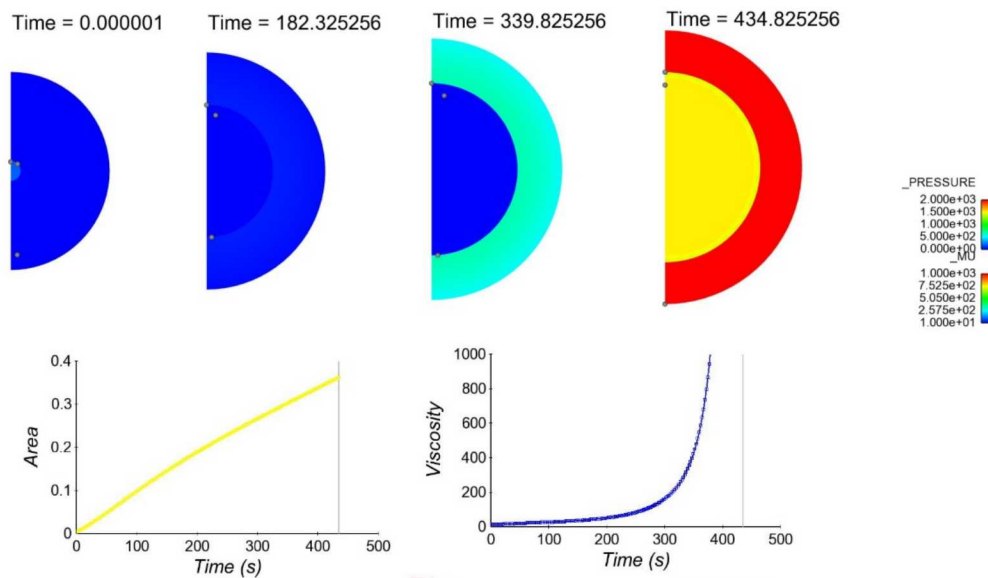
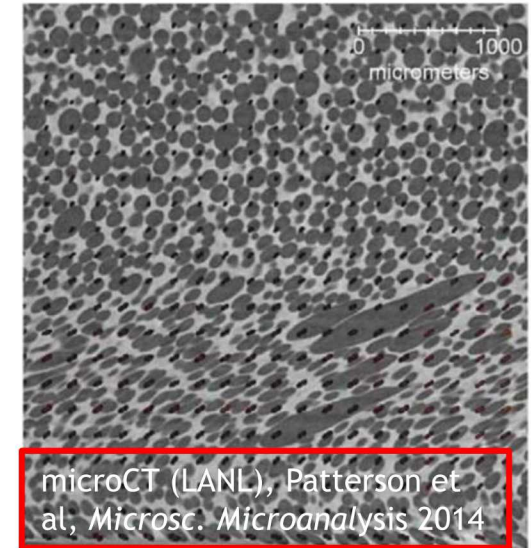




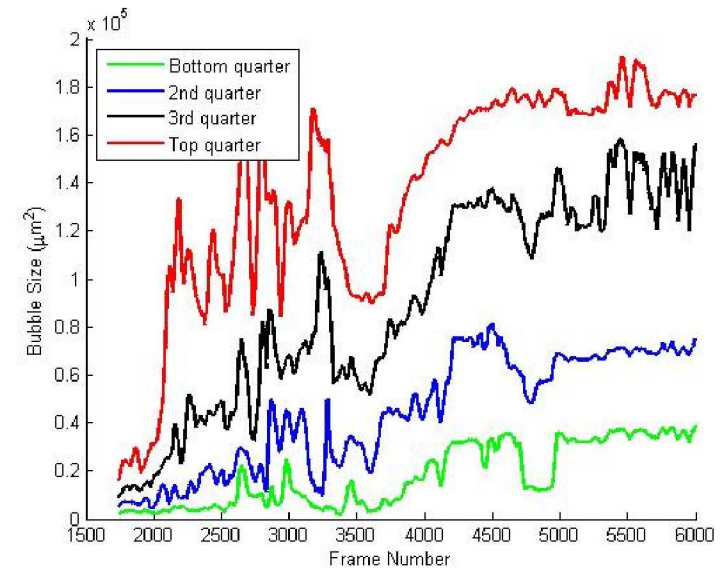
# Bubble Expansion in a Polymerizing Fluid



- Bubble grows as CO<sub>2</sub> enters the bubble (VLE model)
- Growth is halted abruptly once the polymer reaches the gel point and the viscosity diverges
- Post-gelation, bubble pressurization is observed
- ALE mesh is robust over shape change
- Data shows the correct trends when compared to experiment



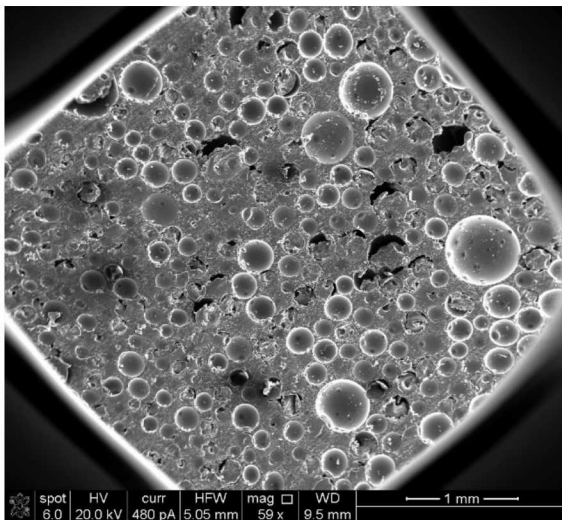
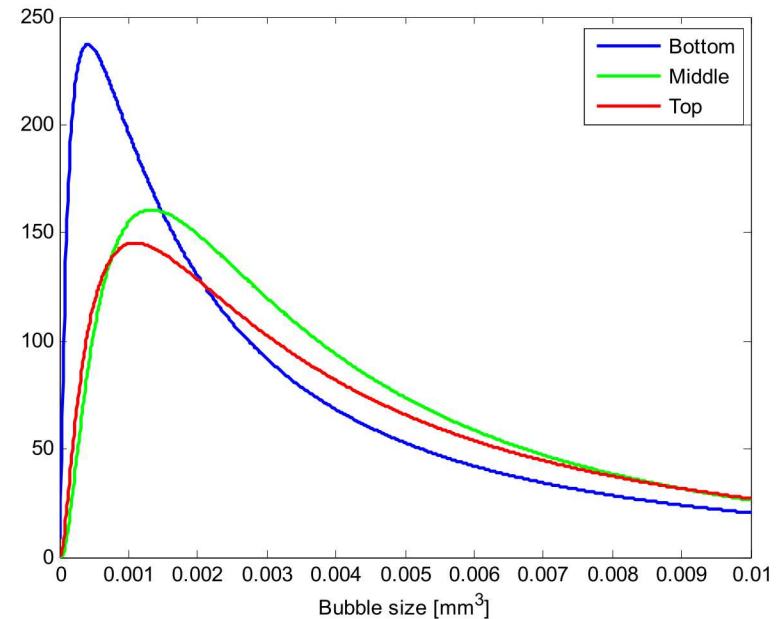
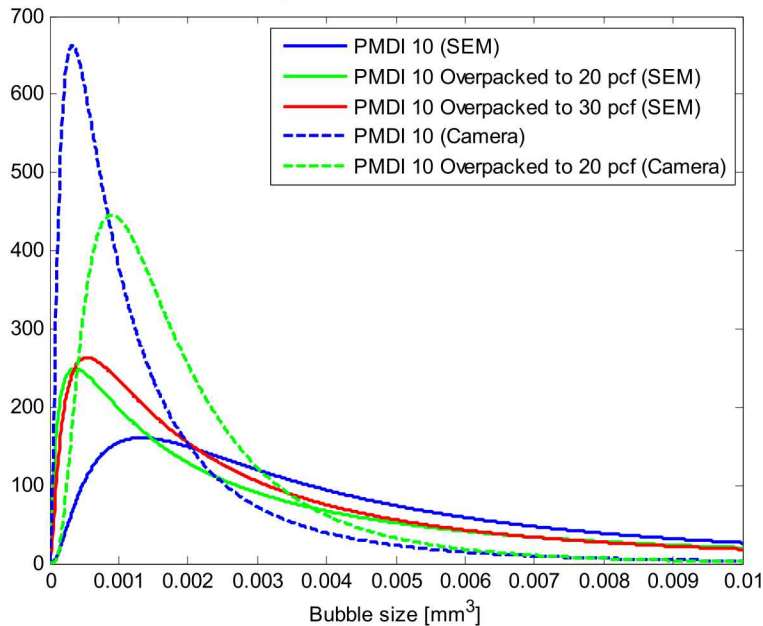
$$\rho \left( \frac{3}{2} \dot{R}^2 + R \ddot{R} \right) = p_{gas} - p_{liq} - 2 \frac{\sigma}{R} - 4 \eta_{polymer} \frac{\dot{R}}{R}$$





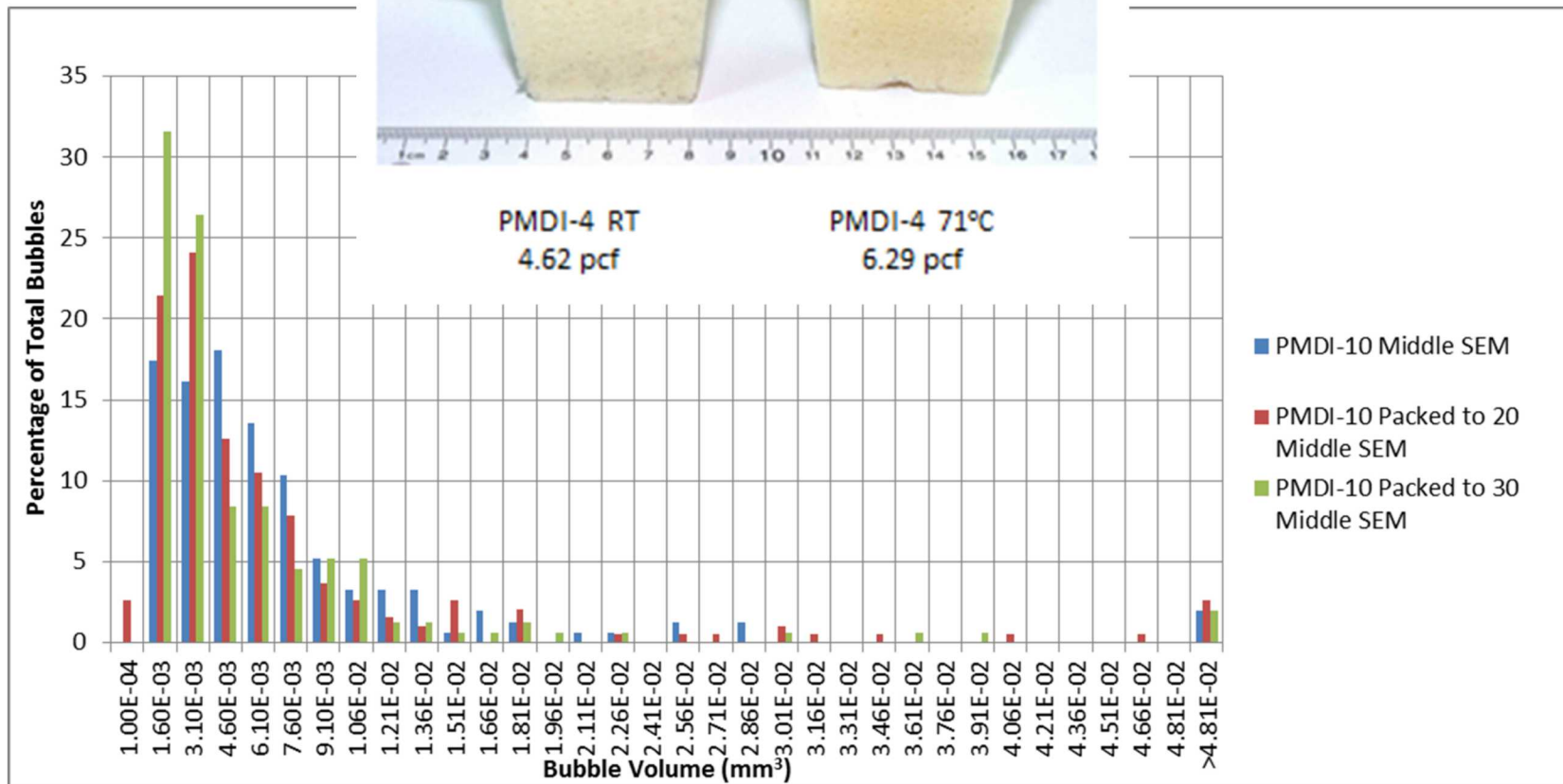


# Bubble Size Data for PMDI-10 and Various Processing Conditions



Log-normal fits to bubble size measurements for (left) overpacked PMDI 10 foam and (right) PMDI 10 free rise foam of various channel height

# Processing Conditions Change Bubble-Size and Final Density



# Equations of Motion Include Evolving Material Models



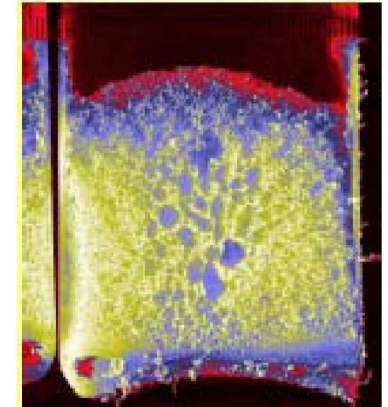
Momentum equation and continuity have variable density, shear viscosity, and bulk viscosity

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho \mathbf{v} \cdot \nabla \mathbf{v} - \nabla p + \nabla \cdot (\mu_f (\nabla \mathbf{v} + \nabla \mathbf{v}^t)) - \nabla \cdot \lambda (\nabla \cdot \mathbf{v}) \mathbf{I} + \rho \mathbf{g}$$

$$\frac{D\rho_f}{Dt} + \rho_f \nabla \cdot \mathbf{v} = 0$$

Energy equation has variable heat capacity and thermal conductivity including a source term for heat of reaction for foaming and curing reactions

$$\rho C_{pf} \frac{\partial T}{\partial t} + \rho C_{pf} \mathbf{v} \cdot \nabla T = \nabla \cdot (k \nabla T) + \rho \phi_e \Delta H_{rxn} \frac{\partial \xi}{\partial t}$$



NMR imaging shows coarse microstructure (Altobelli, 2006)

Extent of reaction equation for polymerization: condensation chemistry with  $T_g$  evolution

$$\frac{\partial \xi}{\partial t} = \left( \frac{1}{(1+wa)^\beta} \right) \left( k_0 \exp\left(-\frac{E}{RT}\right) \right) (b + \xi^m)(1-\xi)^n$$

$$\log_{10} a = \frac{-C_1(T-T_g)}{C_2 + T - T_g}$$

$$T_g = \frac{T_{g0}(1-\xi) + A\xi T_{g\infty}}{(1-\xi + A\xi)}$$

New molar concentration equations for water and carbon dioxide on the next page: kinetics stay the same

$$k_{H_2O} = A_{H_2O} \exp(-E_{H_2O} / RT)$$



# Influence Volume Approach (IVA)

Interchange between bubbles and liquid phase occurs at interface

$$S_{pg} = 3 \frac{\phi}{R} D_g \frac{\partial C}{\partial R} \Big|_{r=R}$$

$$R_{av} = \left[ \frac{3}{4\pi} \frac{\phi}{n} \right]^{1/3}, \quad S_{av} = \left[ \frac{3}{4\pi} \frac{1}{n} \right]^{1/3}$$

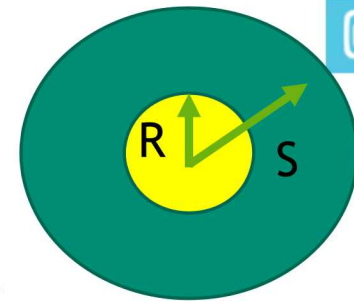
IVA approach assumes a linear profile of CO<sub>2</sub> in the fluid (blue):

$$\frac{\partial C}{\partial R} \Big|_{r=R} = \frac{C_{CO_2} - C(R)}{\Delta r}, \quad C(R) = K_H p_{gas}$$

$$p_{gas} = \rho_{gas} \mathcal{R} T / M_{CO_2}$$

$$S_{pg} = 3 \frac{\phi}{R_{av}} D_g \frac{C_{CO_2} - K_H p_{gas}}{\Delta r}$$

$$\Delta r = \frac{(S_{av}^4 - R_{av}^4) - R_{av}(S_{av}^3 - R_{av}^3)}{S_{av}^3 - R_{av}^3}$$



Advection of Number Density Equation:

- We can either solve an advection equation (more accurate and expensive) or

$$\frac{\partial n}{\partial t} = \nabla \cdot (\vec{v} n)$$

## Nomenclature

- $n$  = nucleation sites/total volume (the number,  $N$ , is constant but the density changes over time (#/cc)
- $m_{in}$  = initial mass injected (g)
- $K_H$  = Henry's law coefficient

# Newer Foam Expansion: Two-phase Carbon Dioxide Models



Water balance in the liquid phase (mol H<sub>2</sub>O/volume total):

$$\frac{\partial C_{H_2O}}{\partial t} + \nabla \cdot \vec{v} C_{H_2O} = D_{H_2O} \nabla^2 C_{H_2O} - (1 - \phi) k_{H_2O} C_{H_2O}^n$$

Carbon dioxide balance in the liquid phase (mol CO<sub>2</sub>/volume total):

$$\frac{\partial C_{CO_2}}{\partial t} + \nabla \cdot \vec{v} C_{CO_2} = D_{CO_2} \nabla^2 C_{CO_2} + (1 - \phi) k_{H_2O} C_{H_2O}^n - S_{pg}$$

Bubble conservation equation: it advects

Carbon dioxide balance in the gas phase (mol CO<sub>2</sub>/volume total):

$$\frac{\partial C_{CO_2}^g}{\partial t} + \nabla \cdot \vec{v} C_{CO_2}^g = S_{pg}$$

$$S_v = \frac{3}{R} \dot{R} \approx \frac{1}{4\eta_{polymer}} \left( (p_{gas} - p_{liq}) - \frac{2\sigma}{R_{av}} \right)$$

$$S_v = \frac{1}{4\eta_{polymer}} \left( (\rho_{gas} \Re T / M_{CO_2} - p_{liq}) - \frac{2\sigma}{R_{av}} \right)$$

Carbon dioxide balance in the gas phase (mass CO<sub>2</sub>/volume bubbles):

$$\frac{\partial \rho_{gas}}{\partial t} + \vec{v} \cdot \nabla \rho_{gas} = -\rho_{gas} S_v + M_{CO_2} S_{pg}$$

This term couples to the subscale. It is the added volume from the bubble size increase during a time step.  $S_v$  has unit of continuity.  $S_{pg}$  is the added mass from reactions.

# Newer Foam Expansion: Two-phase Carbon Dioxide models



Continuity equation is foam density balance (g total/volume foam):

$$\frac{\partial \rho_f}{\partial t} + \vec{v} \cdot \nabla \rho_f + \rho_f \nabla \cdot \vec{v} = 0$$

Gas Volume Fraction (volume foam/volume total):

$$\phi(t) = \frac{\rho_{foam} Y_g}{\rho_{gas}} = \frac{M_{CO_2} C_{CO_2}^g}{\rho_{gas}}$$

Foam Density relationship is the same as before:

$$\rho_f = (\rho_{gas} - \rho_{liq}) \phi(t) + \rho_{liq}$$



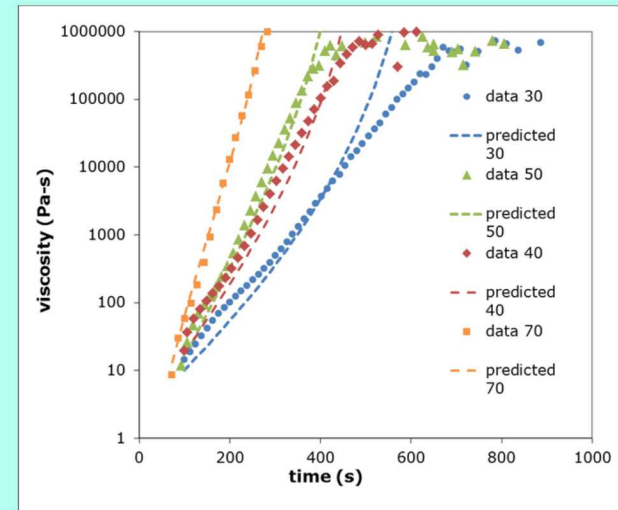
# Model Foam Viscosity as $f(\xi, \varphi)$

## Start with continuous phase viscosity only

- IR kinetics + dry formulation rheology (two sets of experiments) give an approximation of the curing continuous phase rheology
- Relate time of gel point to  $\xi$  to find  $\xi_c$ .

$$\mu_{polymer} = \mu_0^0 \left( \frac{\xi_c - \xi}{\xi_c} \right)^{-6} \quad \xi_c = 0.86$$

$$\mu_0^0 = 600 e^{-1549/RT} \text{ Pa-s}$$

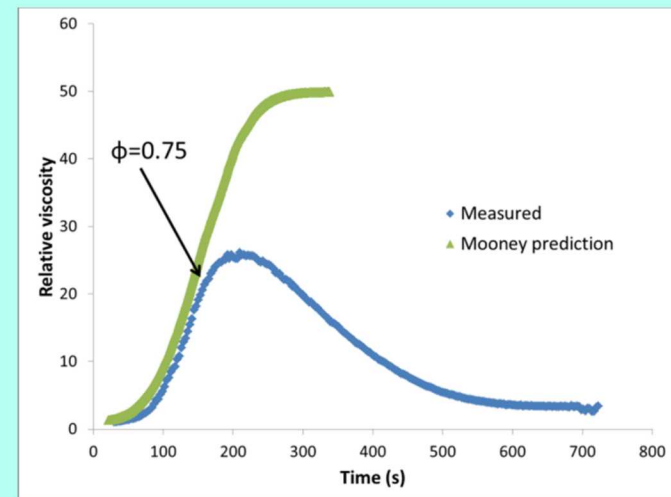


## Relate foam viscosity to continuous phase viscosity

- Foam rise + wet formulation rheology (two sets of experiments) give an approximation of the rheology as a function of gas fraction
- Mooney prediction (for  $\varphi_{gas} < 0.5$ )

$$\mu_{\varphi} = \mu_{polymer} \exp\left(\frac{\varphi_g}{1 - \varphi_g}\right)$$

- For  $\varphi_{gas} > 0.75$  estimate  $\mu_{foam} = \mu_{cure} * f(\xi)$



# Including Bubble-Scale Effects



$$\frac{\partial C_{H_2O}}{\partial t} + \nabla \cdot \vec{v} C_{H_2O} = D_{H_2O} \nabla^2 C_{H_2O} - (1 - \phi) k_{H_2O} C_{H_2O}^n$$

Existing equation with minor mods

$$\frac{\partial C_{CO_2}}{\partial t} + \nabla \cdot \vec{v} C_{CO_2} = D_{CO_2} \nabla^2 C_{CO_2} + (1 - \phi) k_{H_2O} C_{H_2O}^n - S_{pg}$$

Existing equation with mods including source

$$\frac{\partial C_{CO_2}^g}{\partial t} + \nabla \cdot \vec{v} C_{CO_2}^g = S_{pg}$$

New equation similar to liquid

$$\frac{\partial \rho_{gas}}{\partial t} + \vec{v} \cdot \nabla \rho_{gas} = -\rho_{gas} S_v + M_{CO_2} S_{pg}$$

New equation for bubble gas density

$$\frac{\partial n}{\partial t} = \nabla \cdot (\vec{v} n)$$

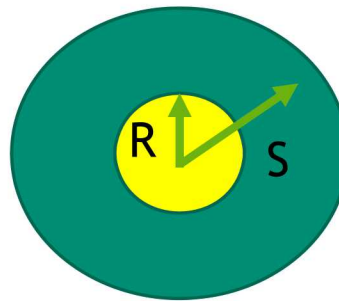
New equation for bubble number density

$$\phi(t) = \frac{\rho_{foam} Y_g}{\rho_{gas}} = \frac{M_{CO_2} C_{CO_2}^g}{\rho_{gas}} \quad \rho_f = (\rho_{gas} - \rho_{liq}) \phi(t) + \rho_{liq}$$

Source terms from bubble scale:

$$S_v = \frac{1}{4\eta_{polymer}} \left( (\rho_{gas} \mathfrak{R}T / M_{CO_2} - p) - \frac{2\sigma}{R_{av}} \right)$$

$$S_{pg} = 3 \frac{\phi}{R_{av}} D_{CO_2} \frac{C_{CO_2} - K_H \rho_{gas} \mathfrak{R}T / M_{CO_2}}{\Delta r}$$



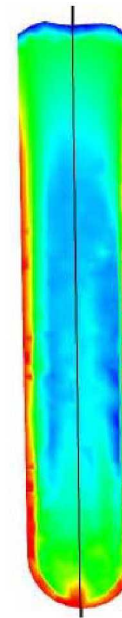
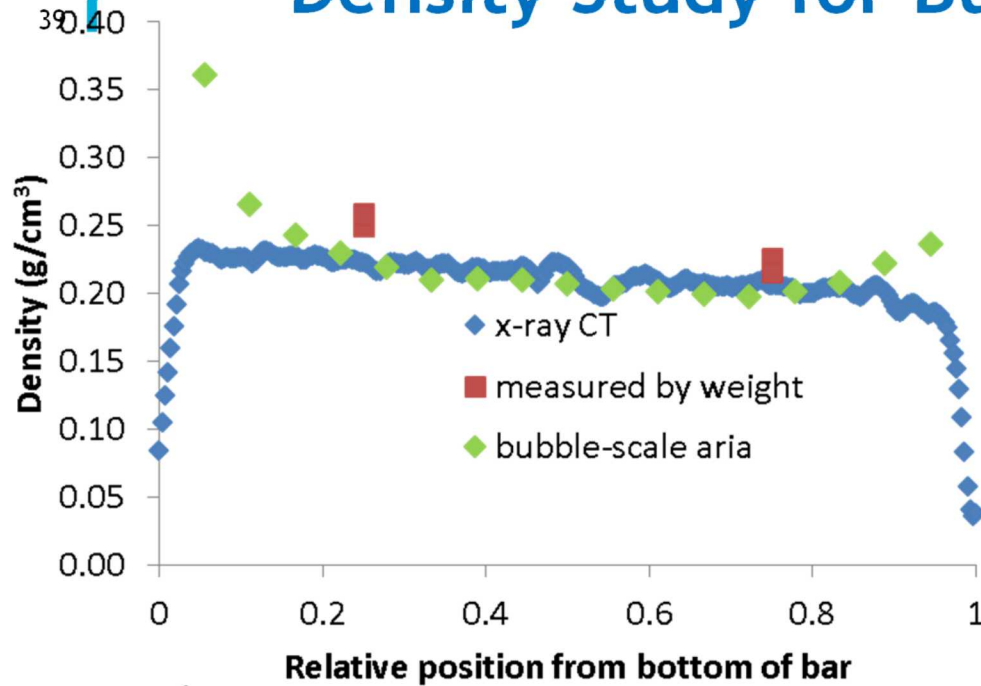
Influence volume approach

$$R_{av} = \left[ \frac{3}{4\pi} \frac{\phi}{n} \right]^{1/3}, \quad S_{av} = \left[ \frac{3}{4\pi} \frac{1}{n} \right]^{1/3}$$

$$\Delta r = \frac{(S_{av}^4 - R_{av}^4) - R_{av} (S_{av}^3 - R_{av}^3)}{S_{av}^3 - R_{av}^3}$$

$$\eta_{polymer} = \eta_0^0 \exp\left(\frac{E_\mu}{RT}\right) \left(\frac{\xi_c^p - \xi^p}{\xi_c^p}\right)^{-q}$$

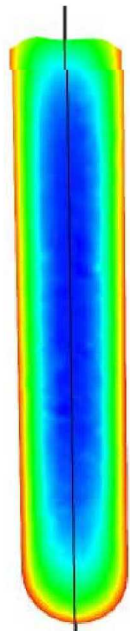
# Density Study for Bubble-Scale Model



rho

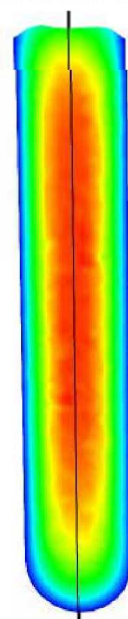
4.000e-01  
3.375e-01  
2.750e-01  
2.125e-01  
1.500e-01

- Free rise foam density gradients. Plots are shown at the centerline of the foam cylinder



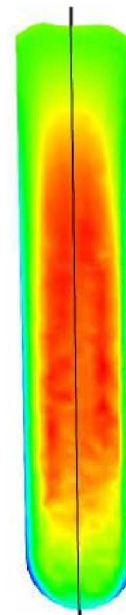
rhog

1.941e-03  
1.924e-03  
1.906e-03  
1.889e-03  
1.871e-03



\_T

3.132e+02  
3.107e+02  
3.082e+02  
3.057e+02  
3.031e+02



R1\_av

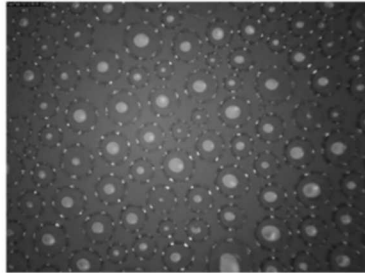
2.300e-01  
2.050e-01  
1.800e-01  
1.550e-01  
1.300e-01



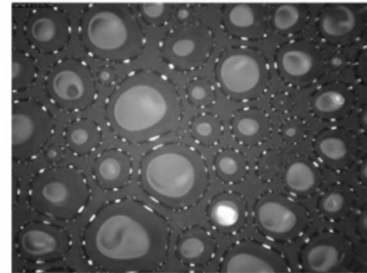
# Density Study for Bubble-Scale Model



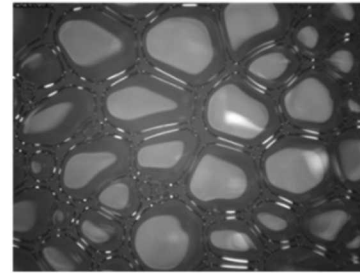
PMDI-4 free rise (bottom camera)



Time = 79.5s

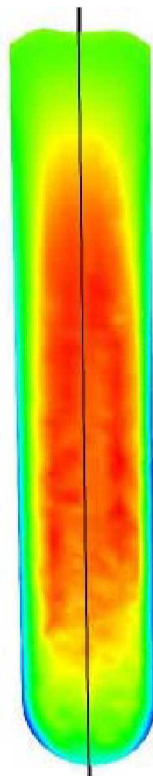


Time = 152s



Time = 266s

- Free rise foam density gradients. Plots are shown at the centerline of the foam cylinder



R1\_av

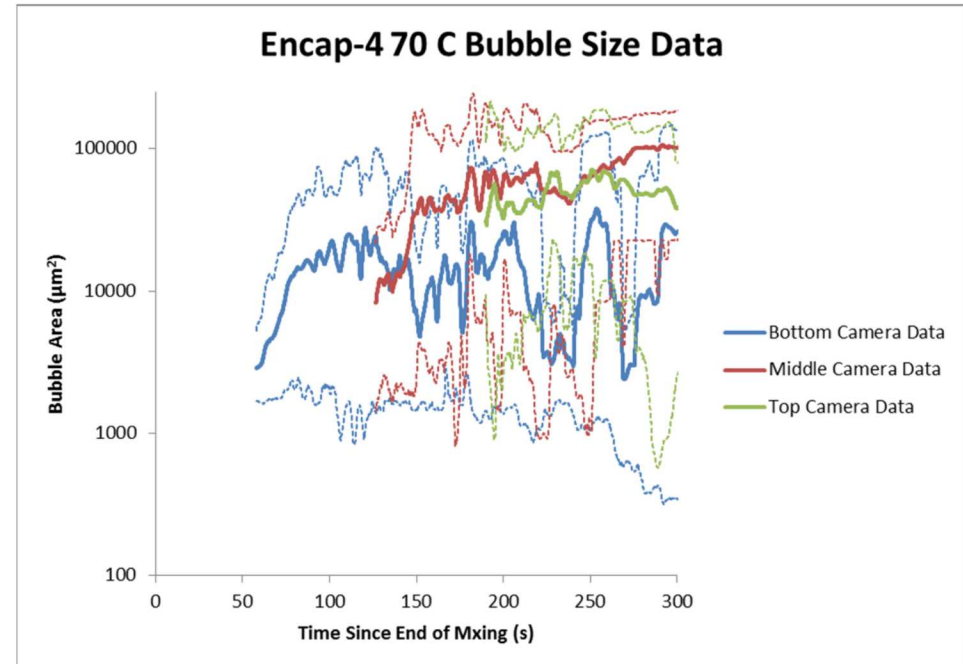
2.300e-01

2.050e-01

1.800e-01

1.550e-01

1.300e-01



# Population Balance Equation (PBE)



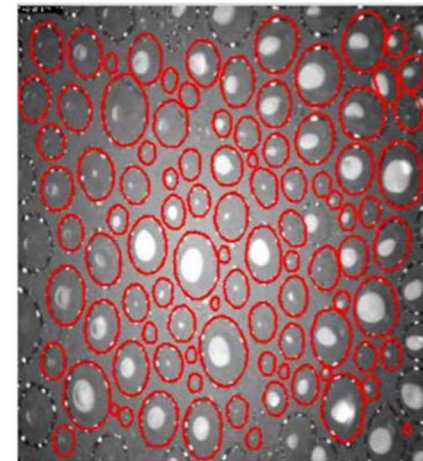
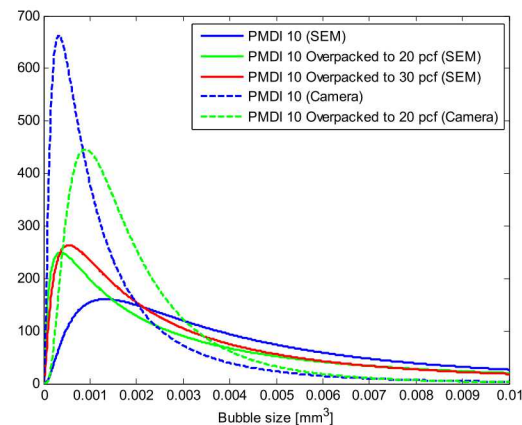
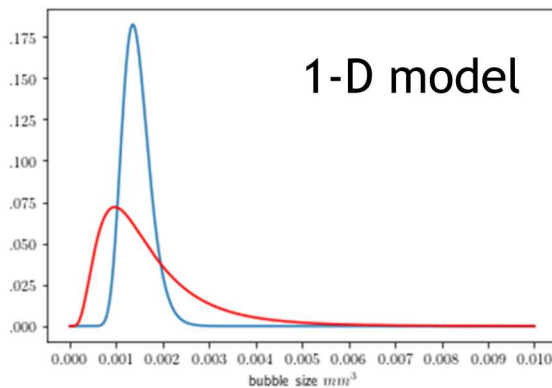
Bubble size distribution (BSD) is described by a number density function,  $n(v)$ , representing the number of bubbles per unit volume of liquid in volume between the range  $v$  and  $v + dv$  (Karimi et al. 2017)

Evolution of the BSD is governed by the following Population Balance Equation (Karimi et al. 2016, 2017):

$$\frac{\partial n(v)}{\partial t} + \nabla \cdot (n(v)\mathbf{u}) + \frac{\partial}{\partial v} [n(v)G(v)] = \frac{1}{2} \int_0^v \beta(v', v-v') n(v') n(v-v') dv' - \int_0^\infty \beta(v, v') n(v) n(v') dv'$$

Where  $\beta(v', v)$  represents the coalescence kernel, and  $G(v)$  represents the growth rate of bubbles.

References: Marchisio, Fox 2005, Karimi et al 2016, 2017



Bottom  
camera  
example  
free rise  
PMDI-10

# Extending Kinetics Formulation to Include PBE



Equation for concentration  $H_2O$  remains unchanged

$$\frac{\partial C_{H_2O}}{\partial t} + \nabla \cdot (C_{H_2O} \mathbf{u}) - D_{H_2O} \nabla^2 C_{H_2O} = -Nk_{H_2O} C_{H_2O}^n$$

But now to account for growth rates of bubbles we have equations for both concentrations of liquid  $CO_2$  and gaseous  $CO_2$  and relate these based on growth rate determined by the QMOM

$$\frac{\partial C_{CO_2}^{liq}}{\partial t} + \nabla \cdot (C_{CO_2}^{liq} \mathbf{u}) - D_{CO_2}^{liq} \nabla^2 C_{CO_2}^{liq} = Nk_{H_2O} C_{H_2O}^n - \bar{G}_1 \frac{P}{RT}$$

$$\frac{\partial C_{CO_2}^{gas}}{\partial t} + \nabla \cdot (C_{CO_2}^{gas} \mathbf{u}) - D_{CO_2}^{gas} \nabla^2 C_{CO_2}^{gas} = \bar{G}_1 \frac{P}{RT}$$

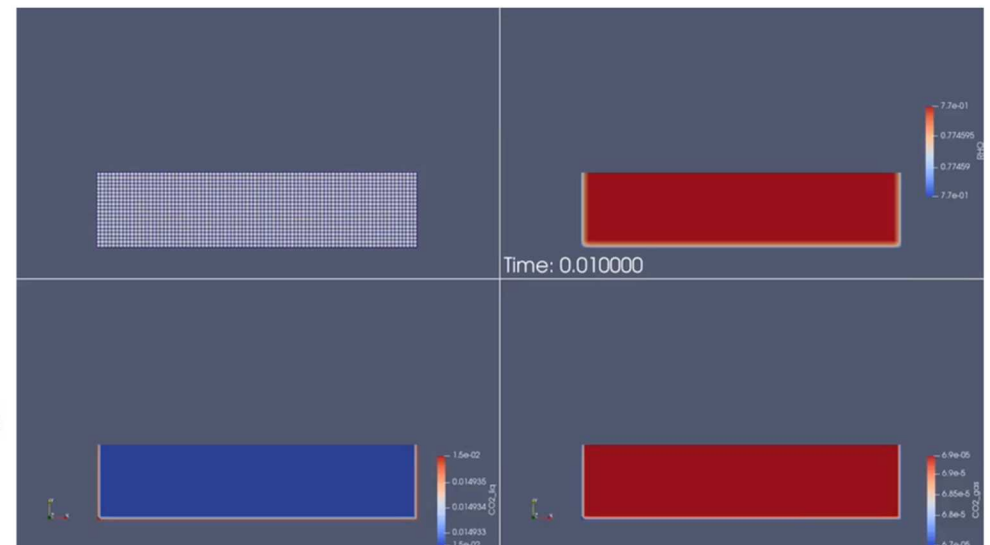
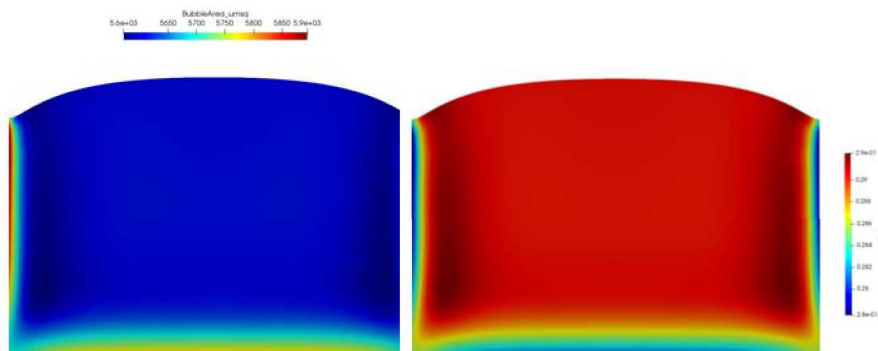
$$\bar{G}_k \cong \sum_{i=1}^N w_i G(v_i) v_i^{k-1}$$

$$G(v_i) = G_0 (w_{CO_2} - w_{max}) / w_{max}$$

Where  $w_{CO_2}$  and  $w_{max}$  are mass fraction of liquid  $CO_2$  and mass fraction related to the maximum solubility of liquid  $CO_2$

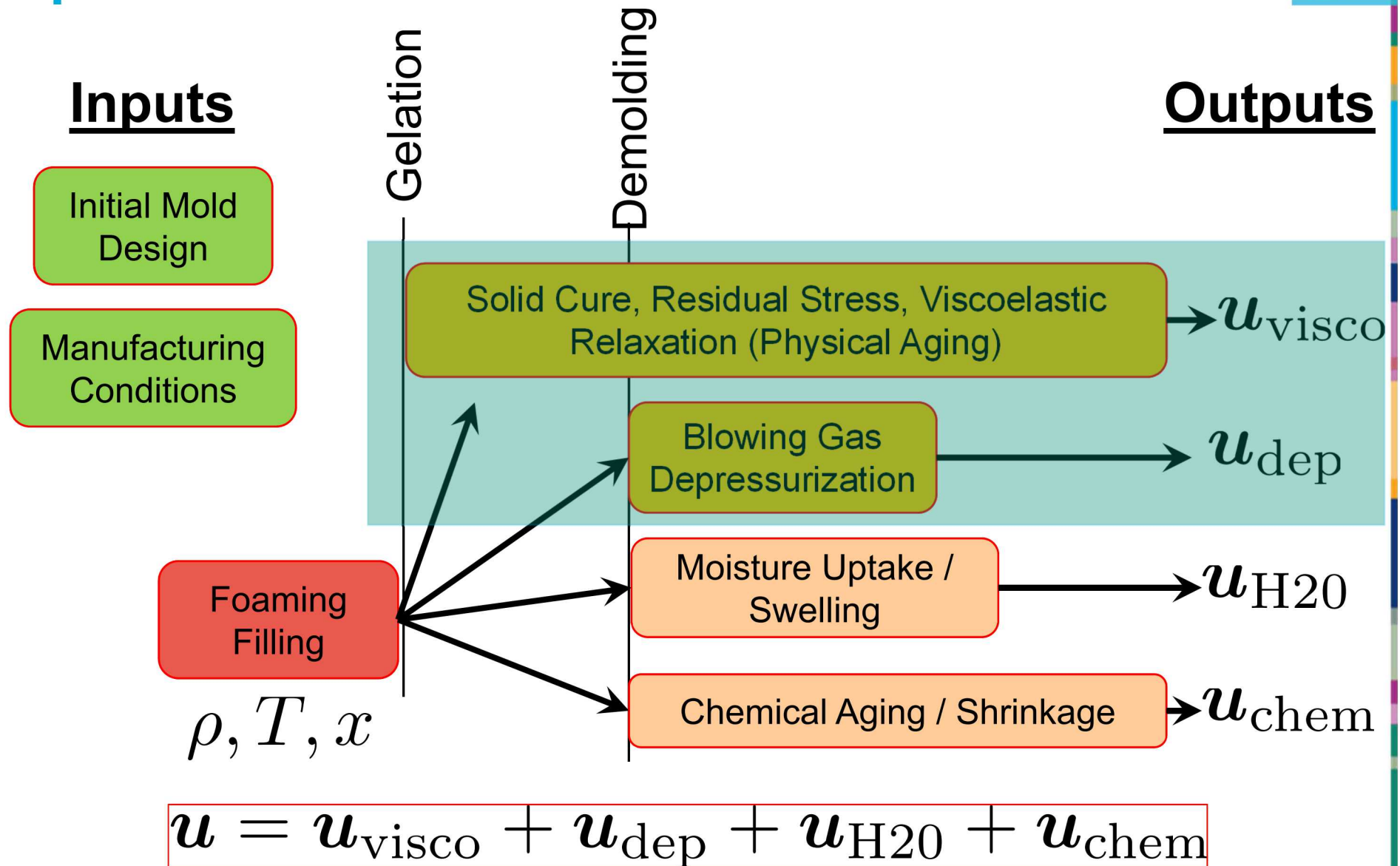
$$G_0 = \frac{\partial V}{\partial t} \approx 5 \times 10^{-8} \frac{cm^3}{s}$$

Following Karimi et al. 2017 our growth rate currently is a simple constant growth rate based on the mass fraction of liquid  $CO_2$





# Cradle-to-Grave Simulation Process



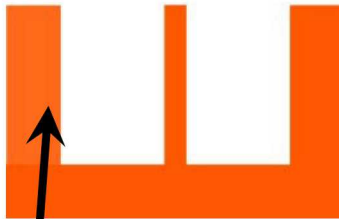
# Inverse Mold Design Process



## Inputs

Manufacturing Conditions

Initial Mold Design



$X_0$

Cradle-to-Grave Simulation

$$X[t] = X_0 + u_{\text{visco}} + u_{\text{dep}} + u_{\text{H2O}} + u_{\text{chem}}$$



## Output

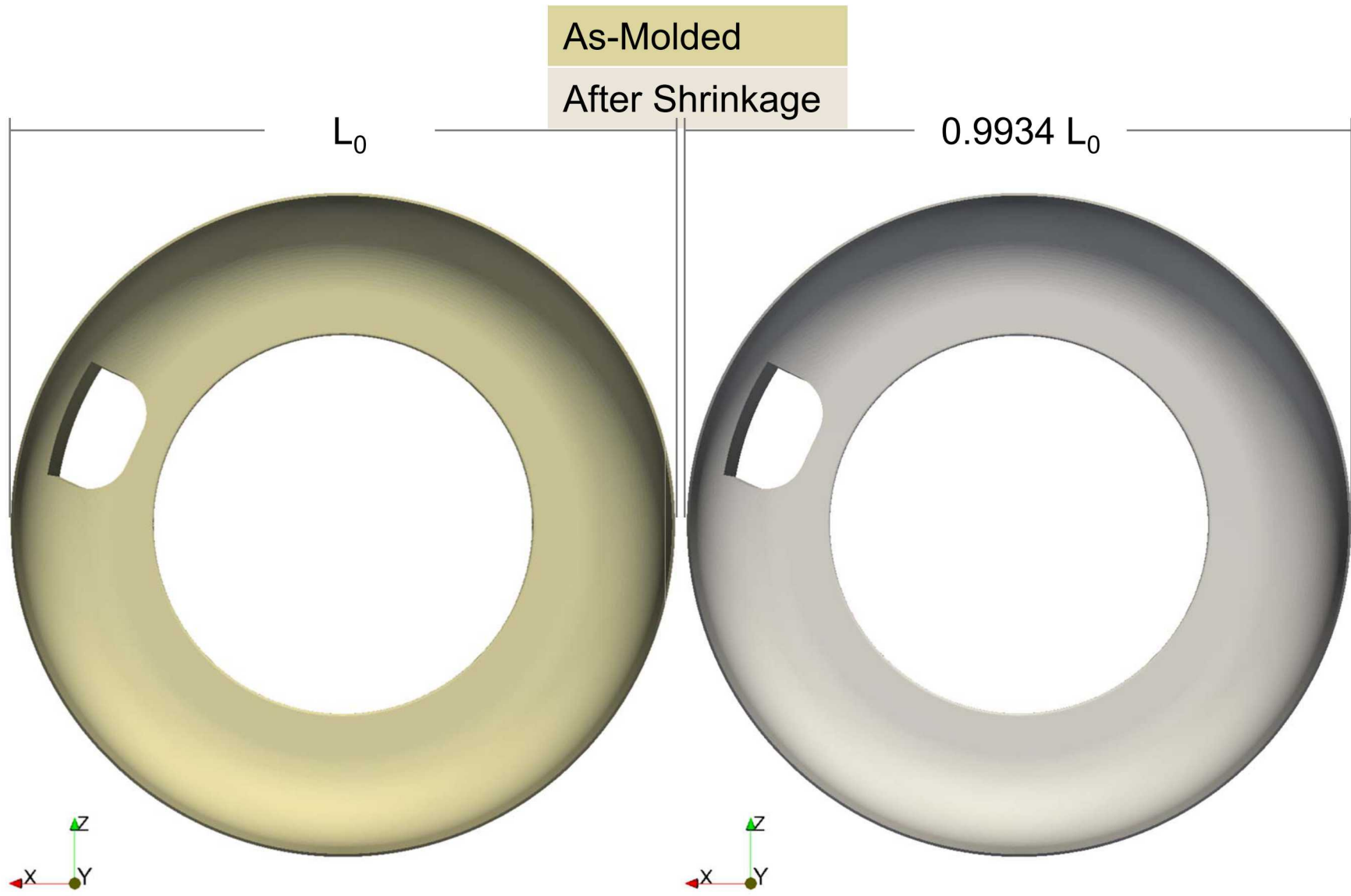
Final Mold Shape



$$X_{\text{new}} = X_0 - u_{\text{visco}} - u_{\text{dep}} - u_{\text{H2O}} - u_{\text{chem}}$$

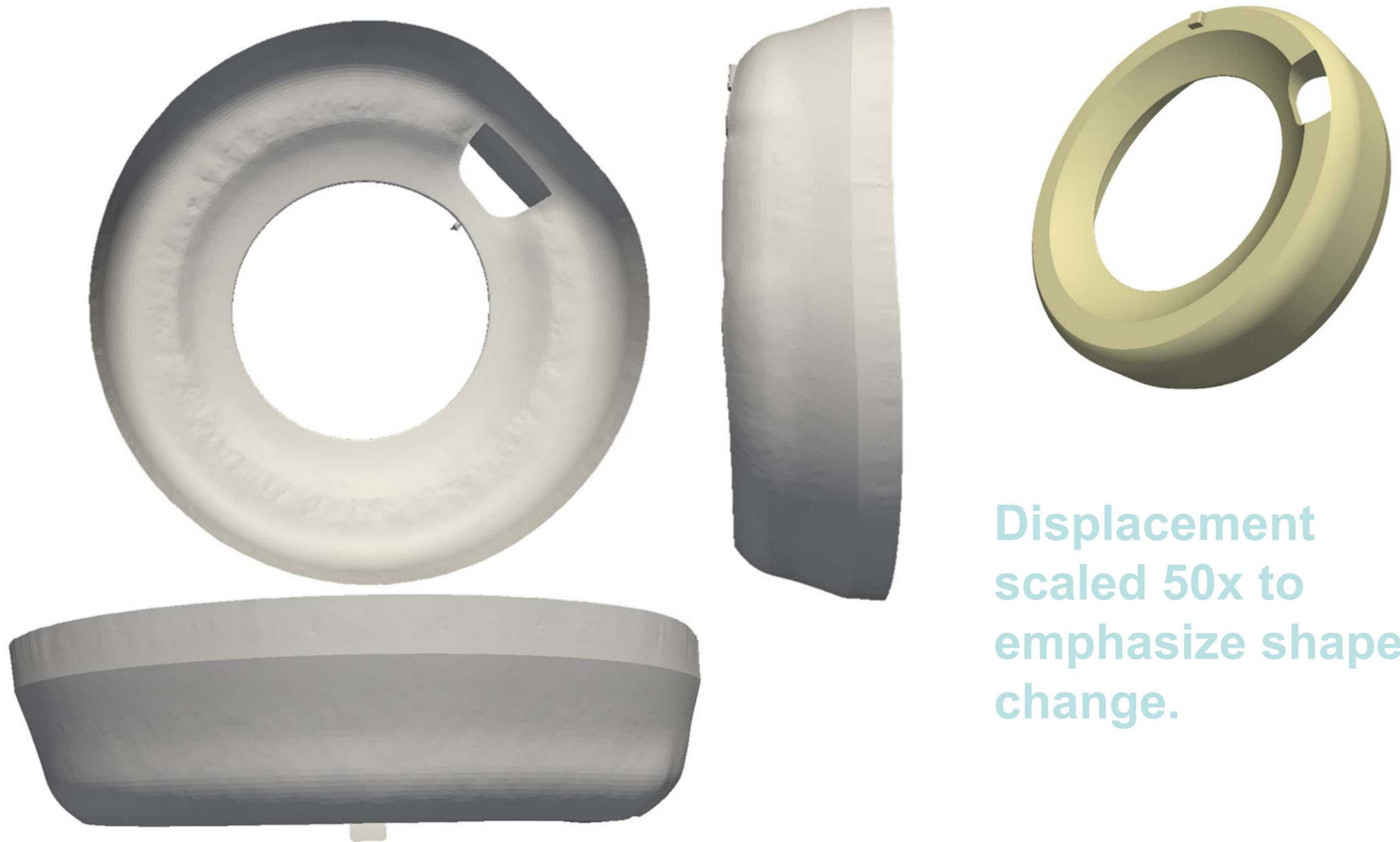
Superposition is employed to combine displacements from different mechanisms and then to “inverse warp” the initial mold design

# Exemplar Part With Featured Regions





# Warpage accentuated near holes and slender regions

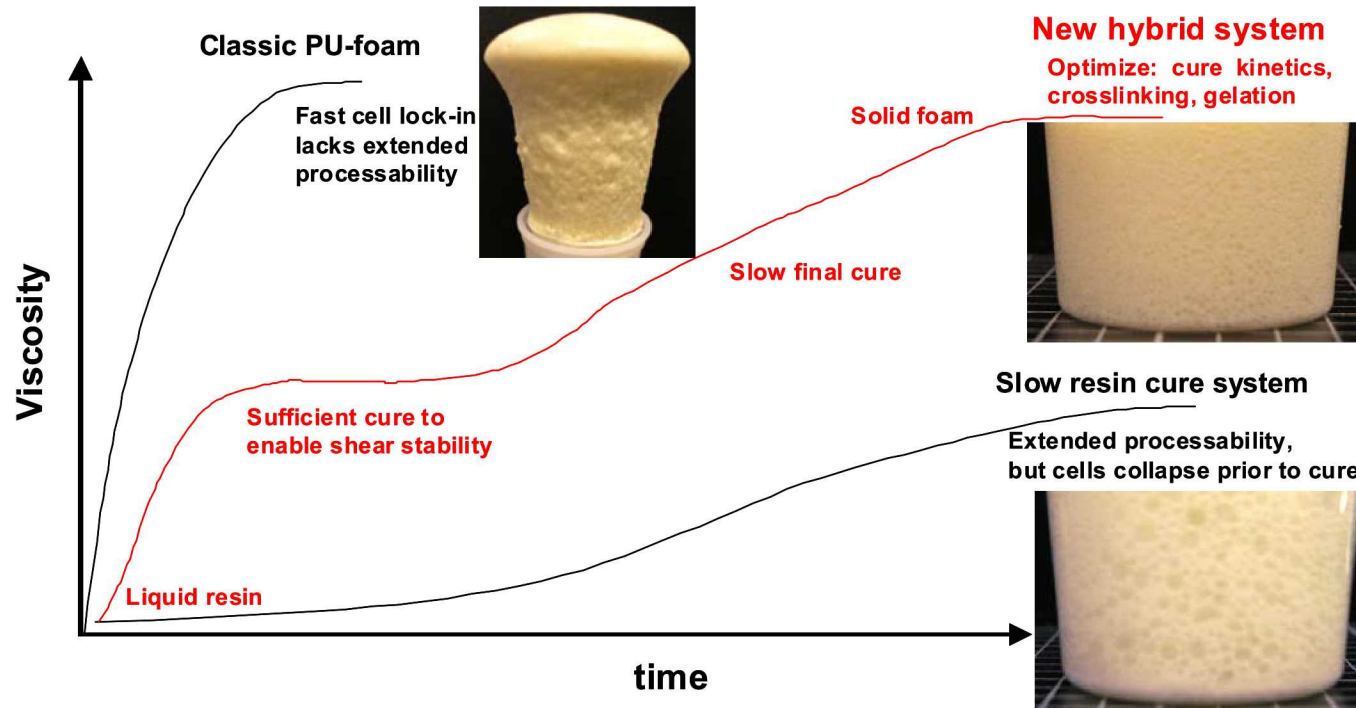


Displacement scaled 50x to emphasize shape change.

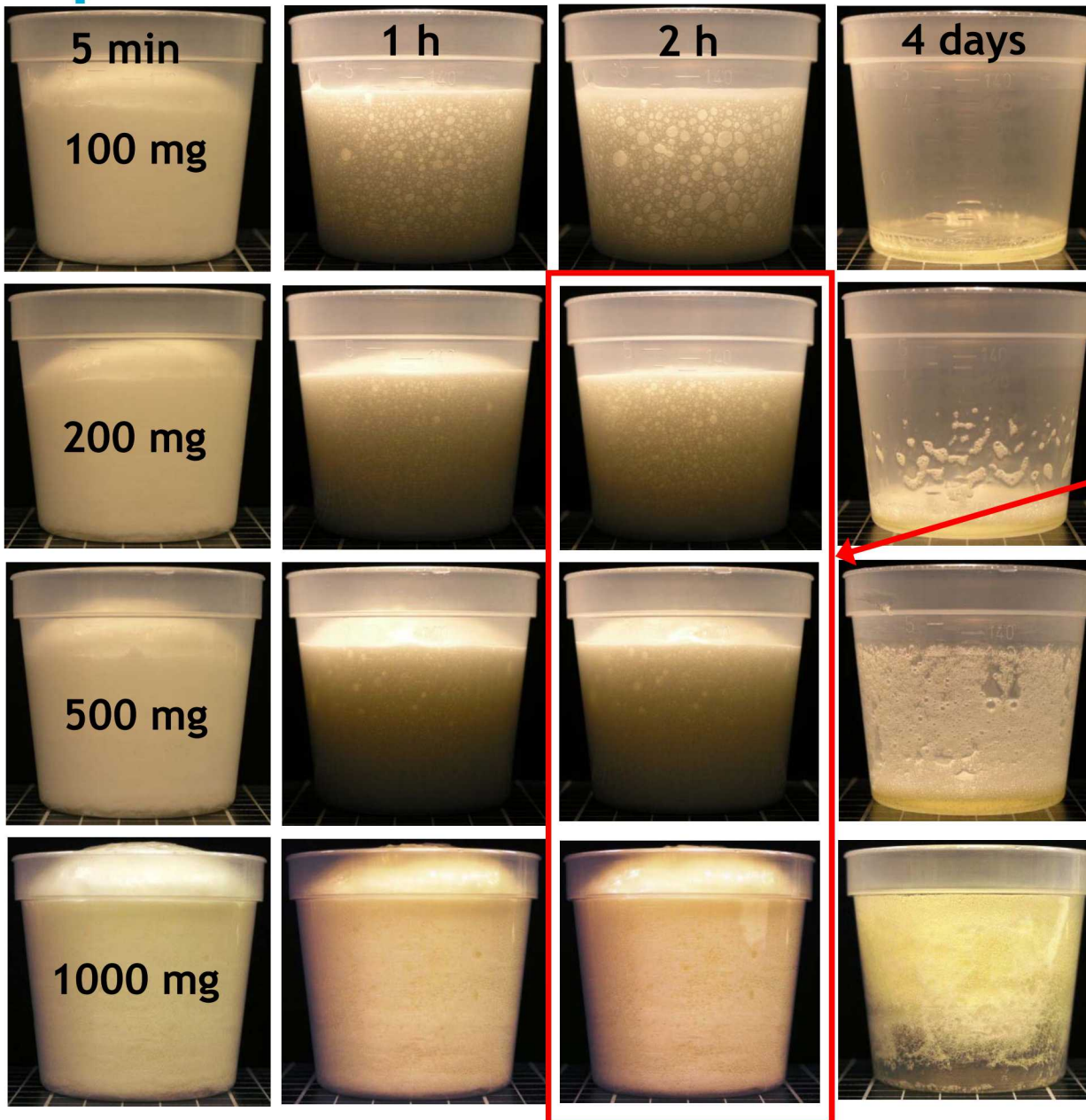
# Efforts to Develop Chemically Blown Curing Epoxy Foam to Eliminate Problems in Physically Blown Foams



- Chemically blown foam less sensitive to mixing, so easier to control
- Strategy: Develop shear-stable foam system that has sufficient polymer network support, but is still amenable to flow and processing (slow penetration of gaps) AND minimize exotherm



## Example: Enhanced persistence with additional crosslinking



- Addition of low conc. fast reacting p-MDI
- Assists forming an early weak support network, beginning gelation and higher Mw
- **Can support long-term fine cell structure**

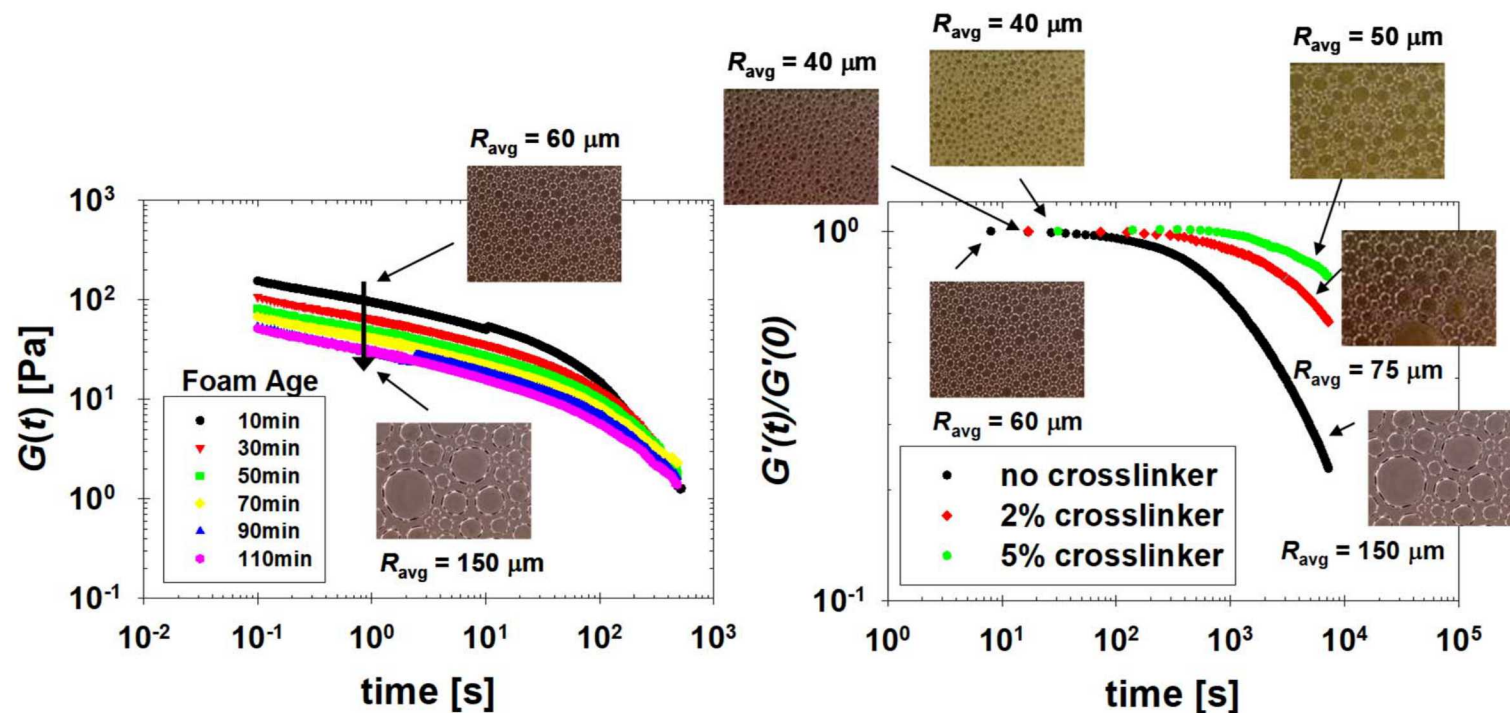
- Network chemistry
- Crosslinking
- Gelation
- Enhanced viscosity, elasticity



# Stability Studies Using Rheology and Observation



- Small amounts of PMDI yields additional stabilization to prevent coarsening
- Rheological tests correlated with bubble size measurements



**Early Gelation Through Light Cross-linking Stabilizes Foam Cells While Foam Remains Processable**

# Approach towards more stable foams

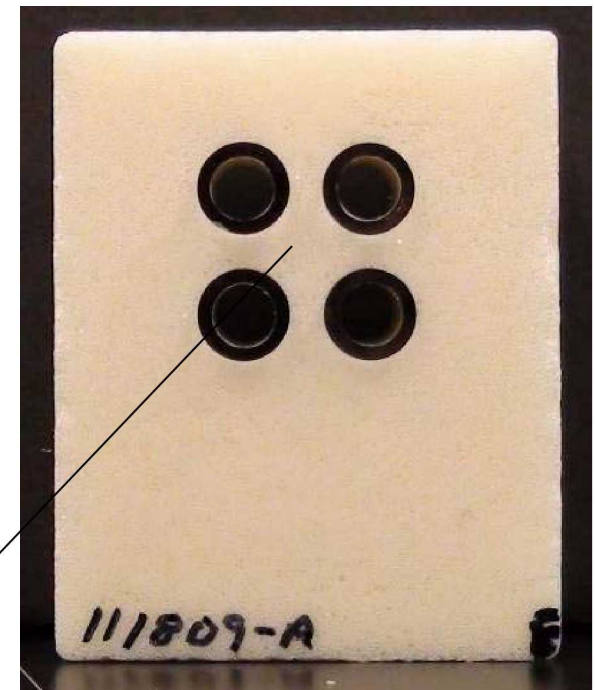
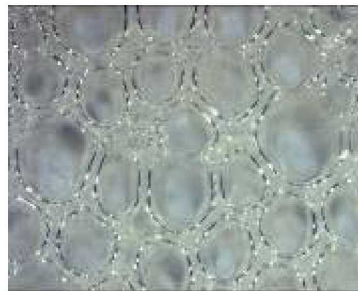


**Foam with improved processing stability:** Goals are controllable cure, broad processing window and stable foam features, good flow stability under strain

## Basic approach for a chemically blown, epoxy foam:

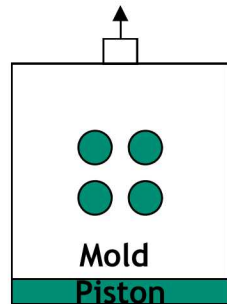
Base epoxy resin: Epon 154, 100 parts  
Anhydride curative ECA100, 75 phr → Main epoxy resin cure  
Reactive polyol AD310, 8.7 phr  
Isocyanate 9561, 12.2 phr } Initial cell stabilizing urethane condensation  
BOC foaming aid, 1 phr }  
Surfactant DC5986, 0.5% in total resin } Contribute to foaming  
Catalyst, imidazole, 1% in total resin }

Note: Traces of water in resins will react with isocyanate to produce some CO<sub>2</sub>

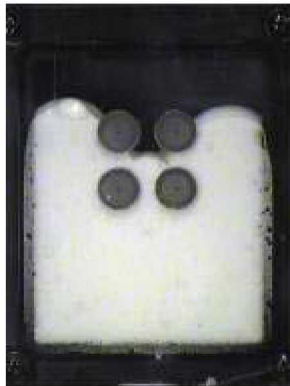


Pushed foam into mold  
(density~0.18g/cc)

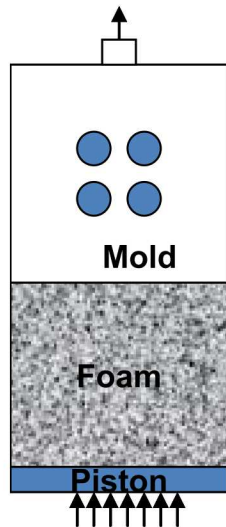
# Candidate Foams Tested in “Push” Process



Low viscosity



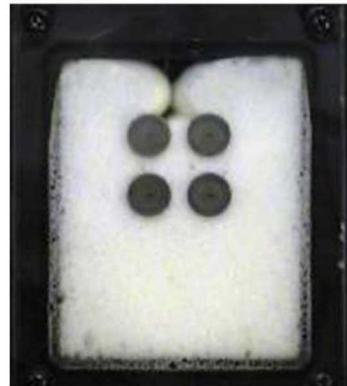
High viscosity



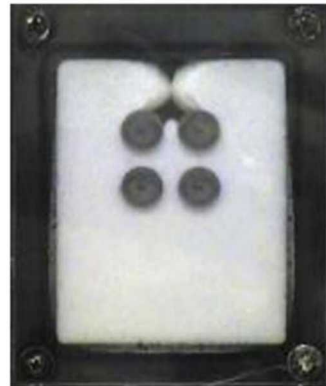
Free-rise

1032

Push



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- In general, lower viscosity resins fill narrow spaces better when self-rising (pictured here) or when pushed.
- Optimized candidate shows good filling in either process, self-rising or pushed after foamed.
- Pushing compresses material to become slightly more dense.
- Pushed here for 1 minute
- Free rise took about 5 minutes

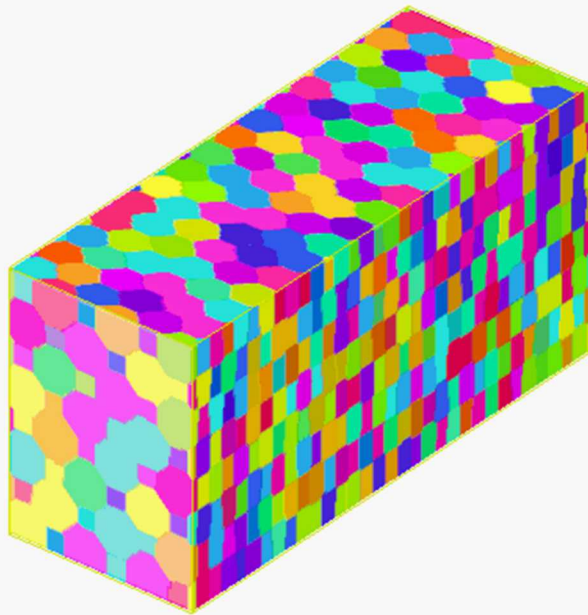


# Conclusions and Future Work

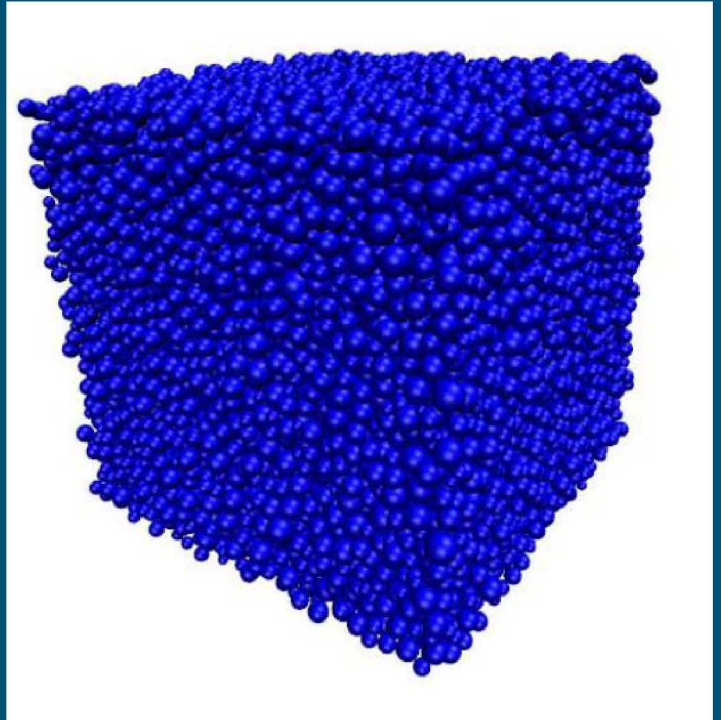


- **Current model is adequate for production calculation**
  - Determining metering, initial placement, voids, gate, and vent location, manufacturing stresses and initial foam shape
  - Current model is “first order.” We are working to make the model more predictive
- **Next generation model needs to include**
  - Equation of state for density approach for gas phase
  - Two-phase CO<sub>2</sub> generation model: solubilized CO<sub>2</sub> in the polymer and CO<sub>2</sub> gas in the bubbles
  - Foam depressurization and its linkage to shape change
- **Include local bubble size and bubble-scale interactions**
  - Predict bubble size with Rayleigh-Plesset equation using an influence volume approach
  - From the bubble size and number density, predict foam density
  - Bubble-scale modeling to include gelation and gas pressure in density model to make it more predictive for both foaming and aging

# Questions?



Pott's model of foam  
bubbles in shear flow  
(Veena Tikare, SNL)



Polydisperse bubble  
microstructure generated with  
LAMMPS and Aria/CDFEM  
(Dan Bolintineanu , SNL)