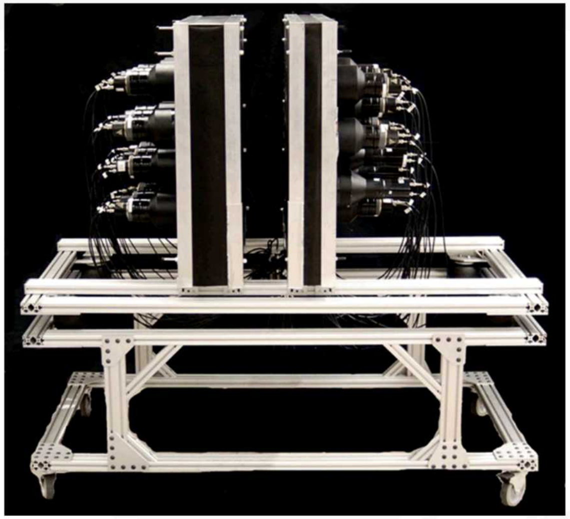
Fast Neutron Detection

SAND2018-12870PE



Peter Marleau
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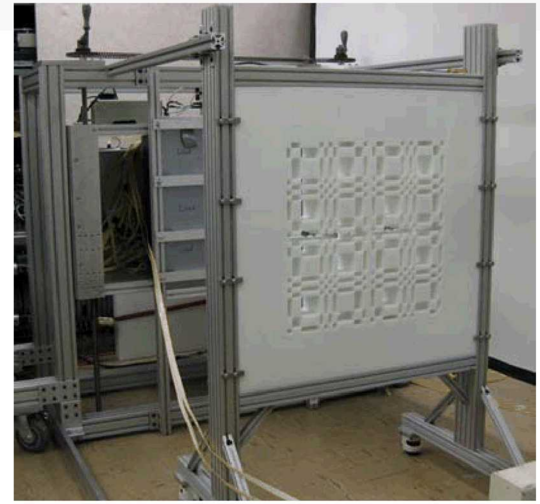

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Outline

- **Neutrons?**
  - Discovery
  - Fission
  - Chain reactions
- **Special Nuclear Material**
  - What’s so special?
  - Characterization methods
- **Multiplicity analysis**
  - He-3 detectors
- **Scintillator detectors**
  - dE/dx – quenching and pulse shape discrimination
  - PSD methods
- **Correlated timing (TOFFEE)**

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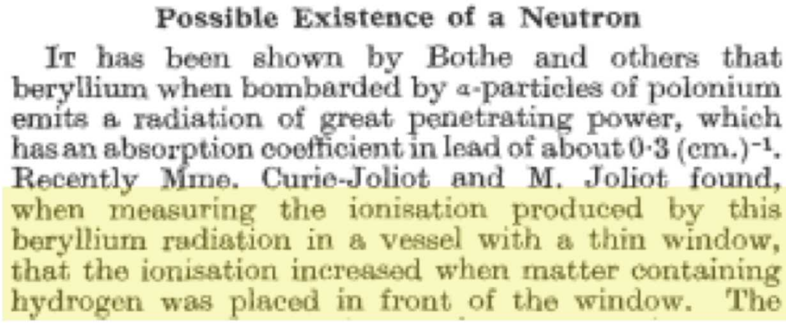
Neutrons?

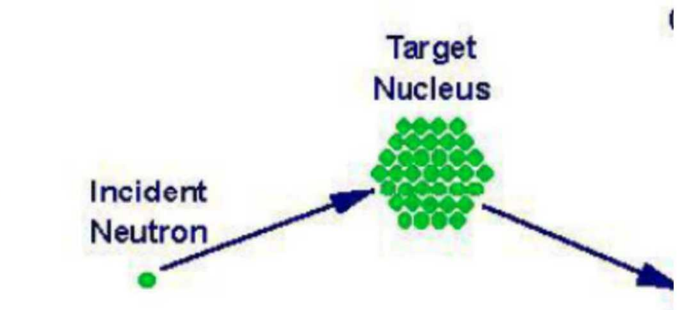
Sir James Chadwick - 1932



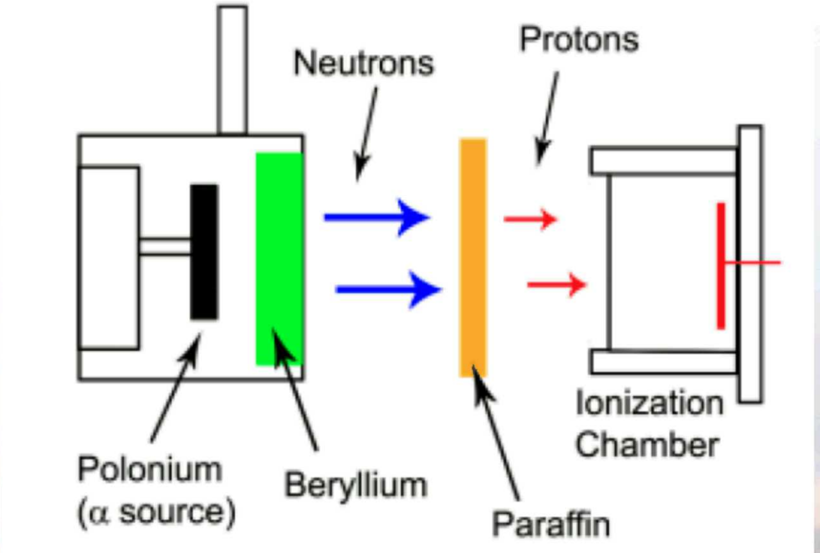
Possible Existence of a Neutron

It has been shown by Bothe and others that beryllium when bombarded by α-particles of polonium emits a radiation of great penetrating power, which has an absorption coefficient in lead of about 0·3 (cm.)$^{-1}$. Recently Mme. Curie-Joliot and M. Joliot found, when measuring the ionisation produced by this beryllium radiation in a vessel with a thin window, that the ionisation increased when matter containing hydrogen was placed in front of the window. The

Ref.: J. Chadwick, Nature 132 (1932) 3252

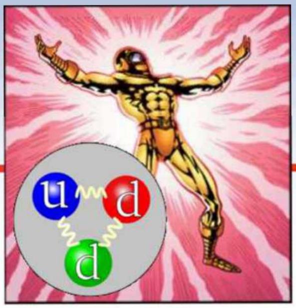


- Atoms were thought to be composed only of protons and electrons.
- Chadwick explained the Curie-Joliot experiment as caused by neutrons rather than photons.
- First measurement of the neutron mass.

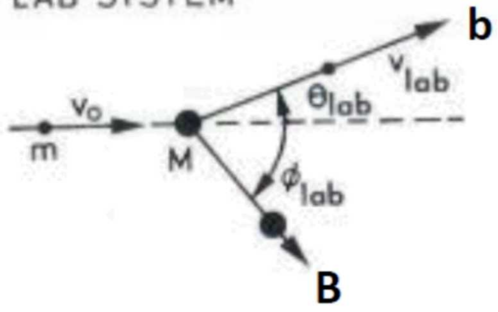


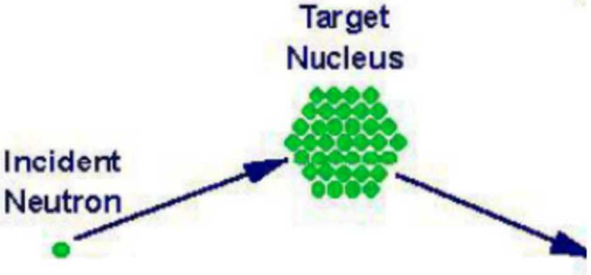
Credit: Ralf Nolte, “Detection of Neutrons: Part 1”

Neutron Interactions - Elastic

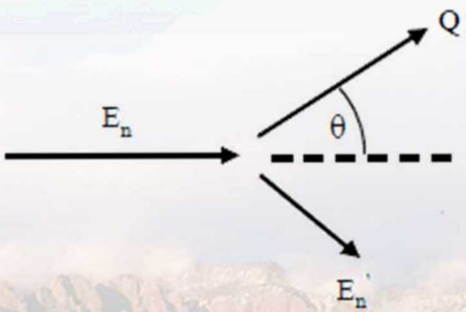


$$E_B = E_0 \frac{4A}{(A+a)^2} \cdot \cos^2 \phi_B^{lab}$$



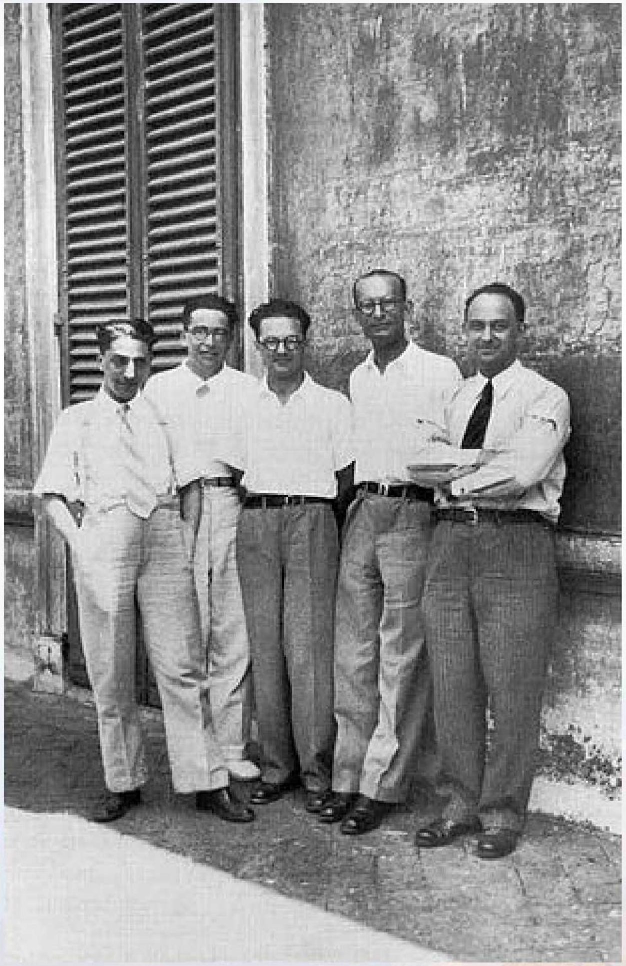


$$Q_{max} = \frac{4mME_n}{(M+m)^2}$$



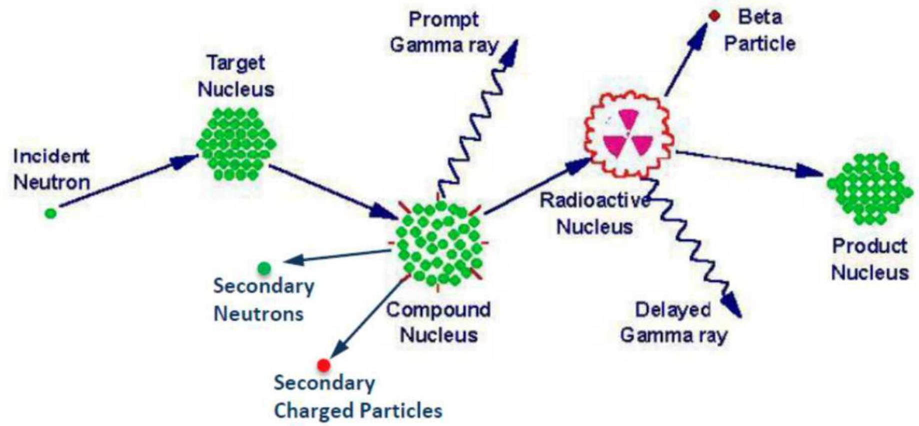
Nucleus	Q_{max}/E_n
$^{1}_{1}H$	1.000
$^{2}_{1}H$	0.889
$^{4}_{2}He$	0.640
$^{9}_{4}Be$	0.360
$^{12}_{6}C$	0.284
$^{16}_{8}O$	0.221
$^{56}_{26}Fe$	0.069
$^{118}_{50}Sn$	0.033
$^{238}_{92}U$	0.017

Neutron Induced Radioactivity



Enrico Fermi and students

Inelastic interactions



- By 1934 Enrico Fermi used neutrons to induce radioactivity in 22 different elements.
- He discovered that slowing (or moderating or thermalizing) the neutrons increased the rate of radioactivity.
- In 1938, he received a Nobel Prize in Physics for this work.

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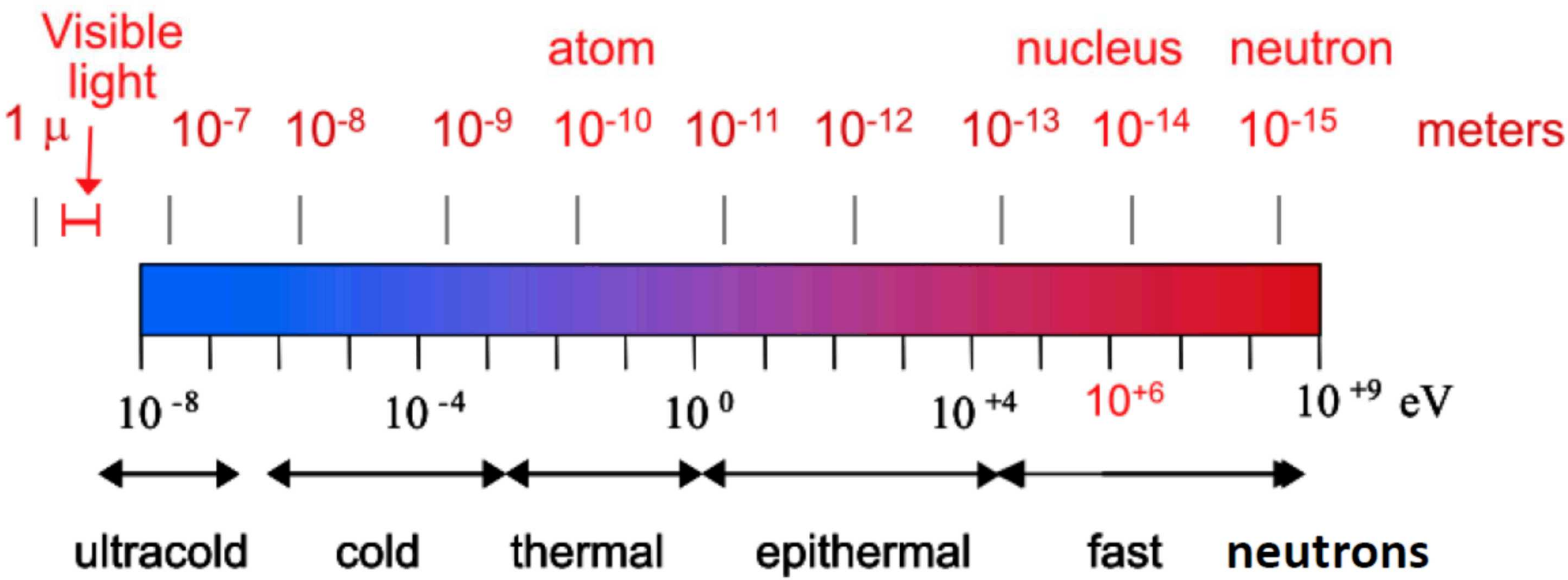
Neutron properties

de Broglie wavelength: $\lambda = h / p$



Credit: Ralf Nolte, "Detection of Neutrons: Part 1"

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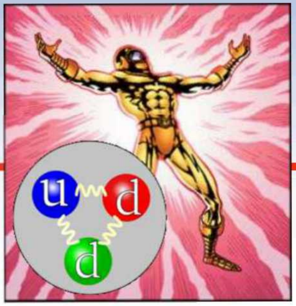
Slow Neutron Interactions/Detection

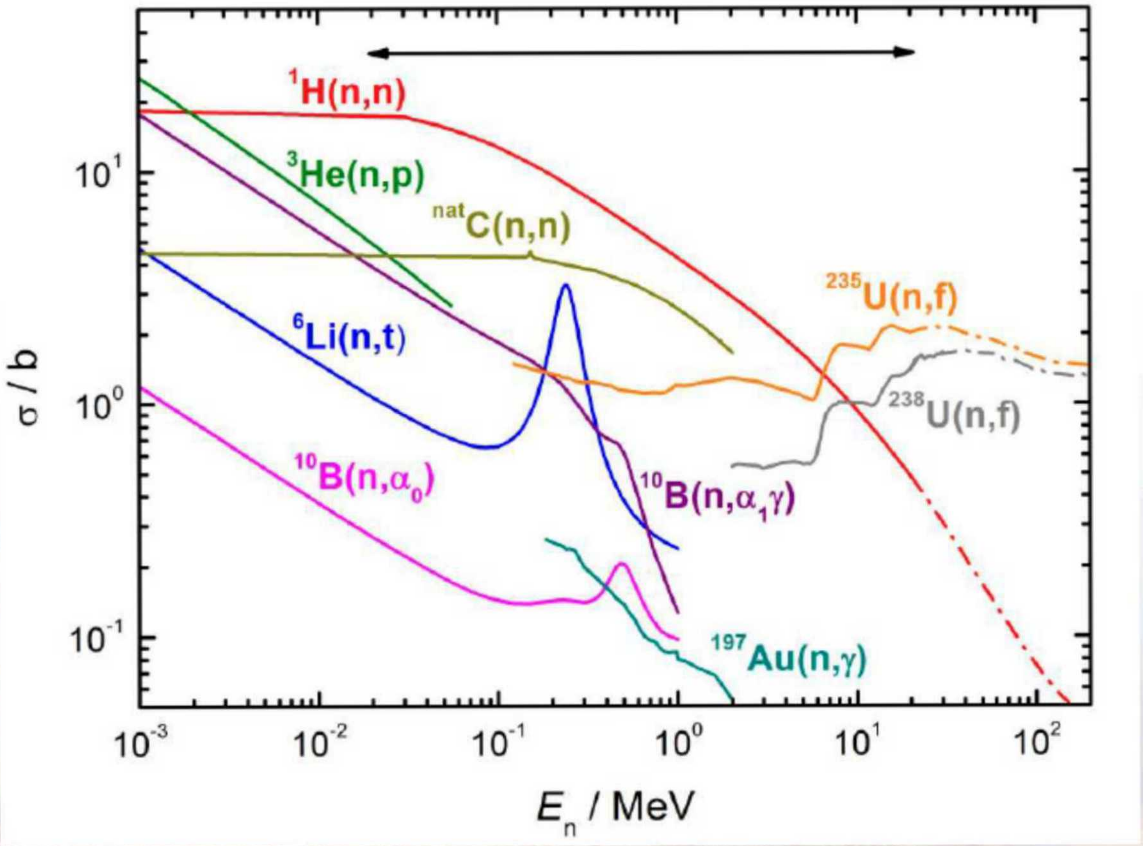


- High thermal cross section
- High Q-value

$n + {}^{3}He \rightarrow {}^{3}H + {}^{1}H + 0.764\ MeV$

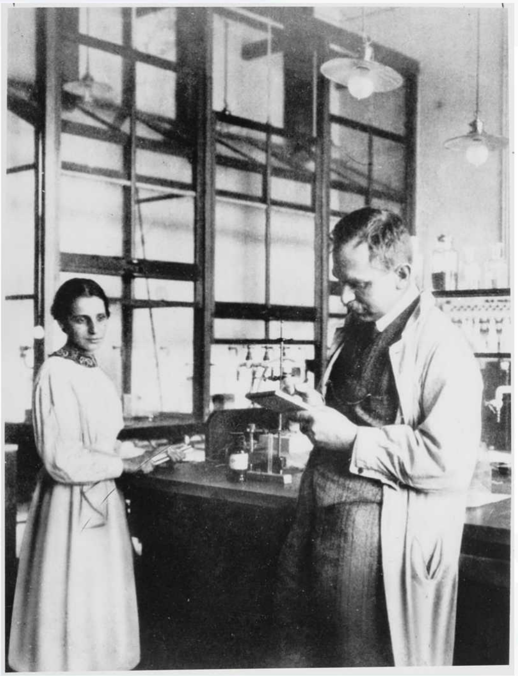
$n + {}^{10}B \rightarrow {}^{7}Li + {}^{4}He + 2.31\ MeV$

$n + {}^{6}Li \rightarrow {}^{3}H + \alpha + 4.78\ MeV$



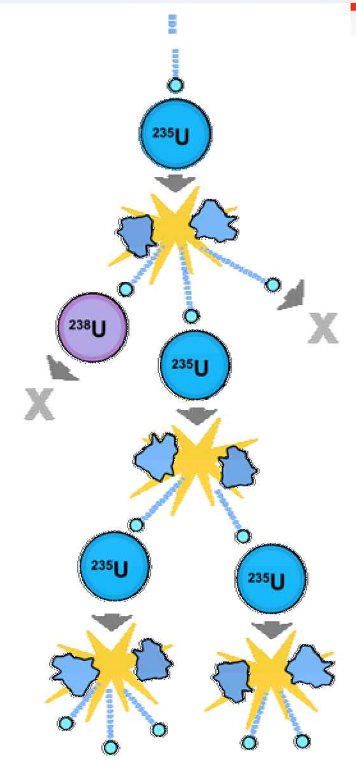
Taken from ENDF database

Neutrons and fission

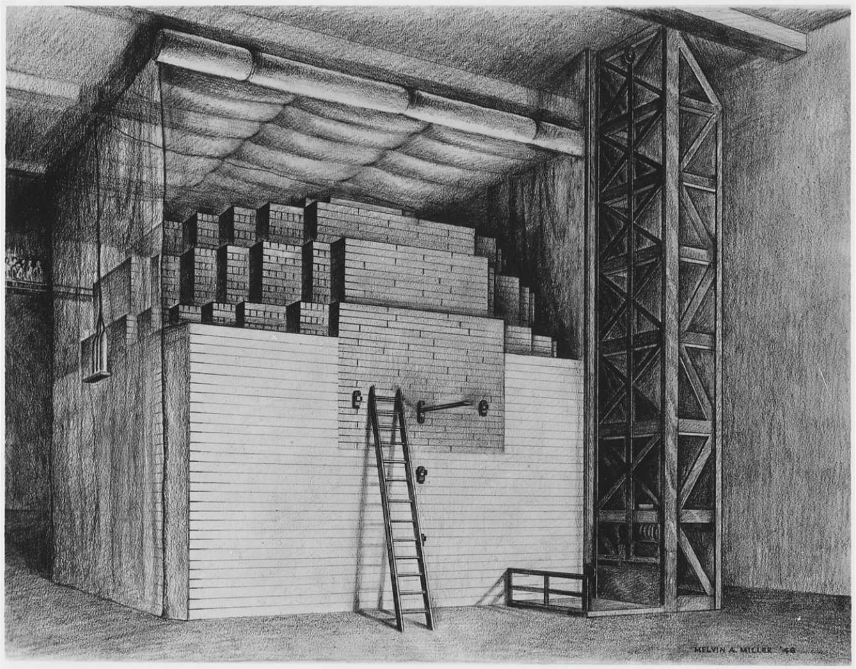


Lisa Meitner and Otto Hahn

- 1938 – Lisa Meitner, Otto Hahn, and Fritz Strassman bombarded uranium with neutrons in Berlin.
- 1939 - Nuclear fission (term coined by Otto Frisch), the neutron induced splitting of nuclei into lighter elements releasing 100s of MeV of energy, was discovered.
- Hahn and Strassman predicted the liberation of additional neutrons during fission. Joliot discovered the phenomenon of chain reactions.
- Hahn received the 1944 Nobel Prize in Chemistry for the discovery of the fission of heavy atomic nuclei.



Reactor, bombs, and the Manhattan Project



1939 – Leo Szilard proposed the possibility of sustainable and exponentially increasing neutron driven chain reactions.

1939 – Chadwick proposed an atomic bomb utilizing tons of natural uranium.

1939 – Oppenheimer proposed a cube of uranium detueride might "blow itself to hell"

1942 – the "Chicago Pile" reactor demonstrated sustained critical nuclear chain-reactions.

1942 – the Manhattan Project began.

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What's so special?

As defined by the IAEA:

Nuclear Material – metals uranium, plutonium, and thorium in any form.

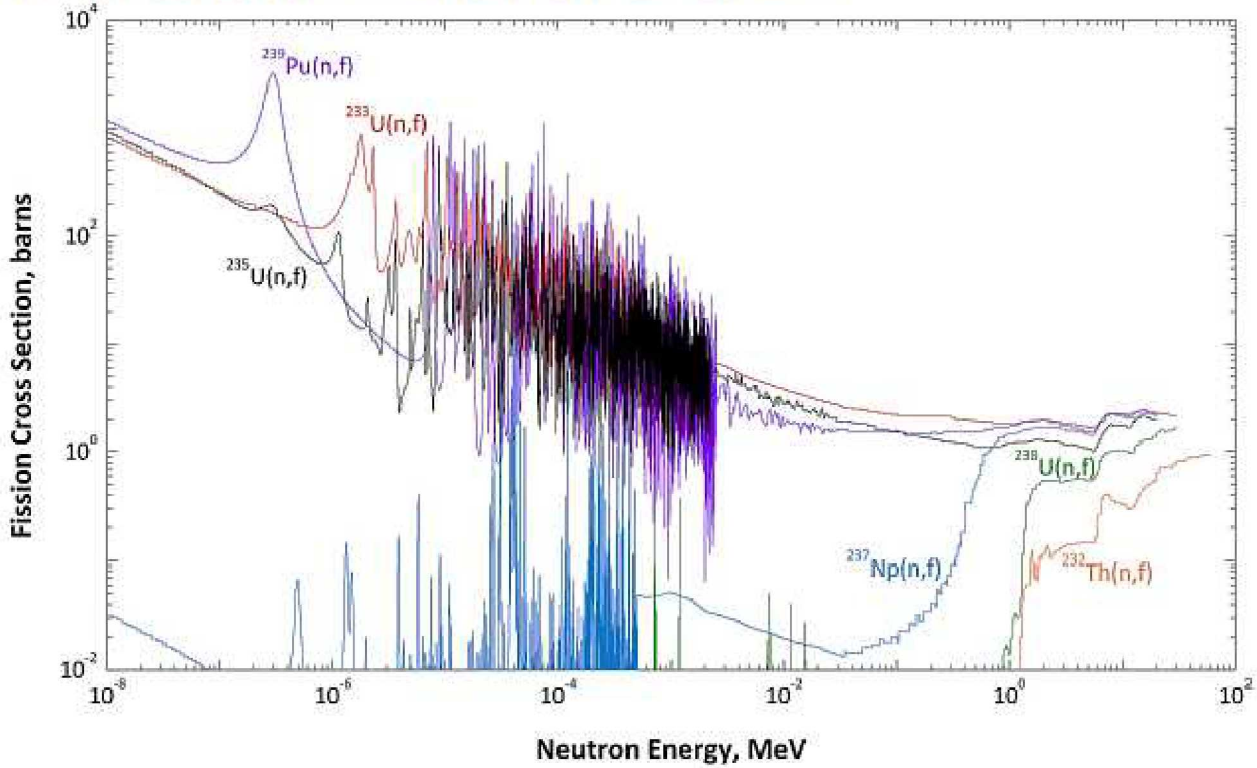
Special Nuclear Material – U-233, U-235, Pu-239

Source Material – everything that is not special

What's so special?

Isotope	Induced Thermal Fission Multiplicity ν
^{232}U	3.13
^{233}U	2.4
^{234}U	2.4
^{235}U	2.41
^{236}U	2.2
^{238}U	2.3
^{237}Np	2.70
^{238}Pu	2.9
^{239}Pu	2.88
^{240}Pu	2.8
^{241}Pu	2.8
^{242}Pu	2.81
^{244}Cm	3.46
^{252}Cf	4.06

The Neutron Fission Cross Sections



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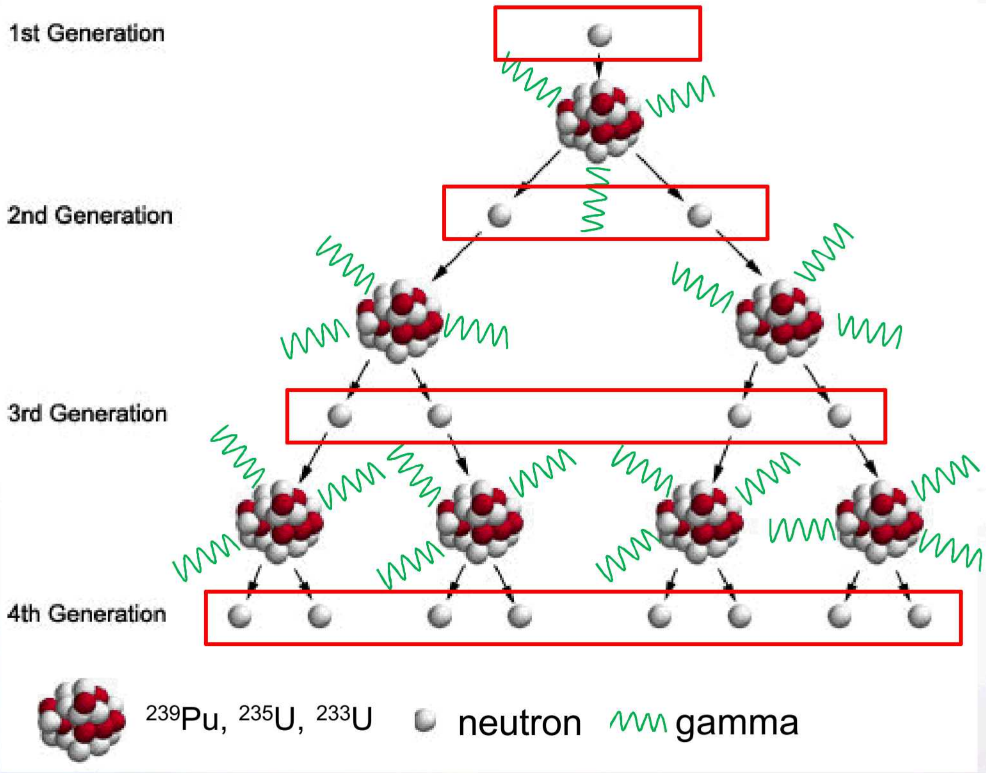
What is so special about SNM?
Basic vocabulary and definitions



http://www.atomicarchive.com/Fission/Fission2.shtml

Fission products:

- 2 - 4 neutrons per fission ($\bar{\nu}_n$)
- 7 - 9 gammas per fission

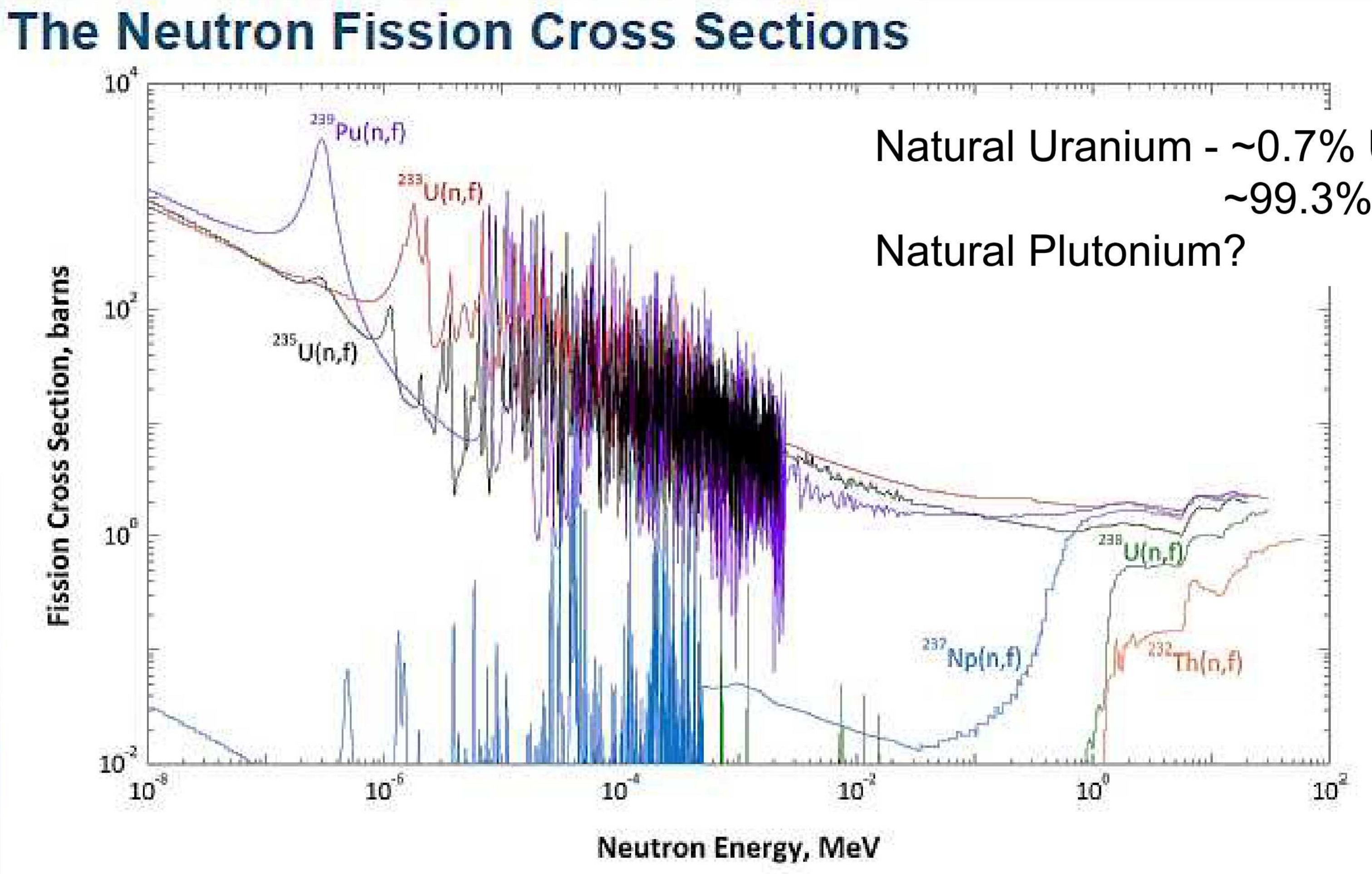
Quantities of Interest:

$$k = \frac{neutrons\ in\ one\ generation}{neutrons\ in\ preceeding\ generation}$$

Subcritical multiplication – *total* neutrons created per source neutron:

$$\boldsymbol{M = \frac{1}{1-k}}$$

Where can I get some?

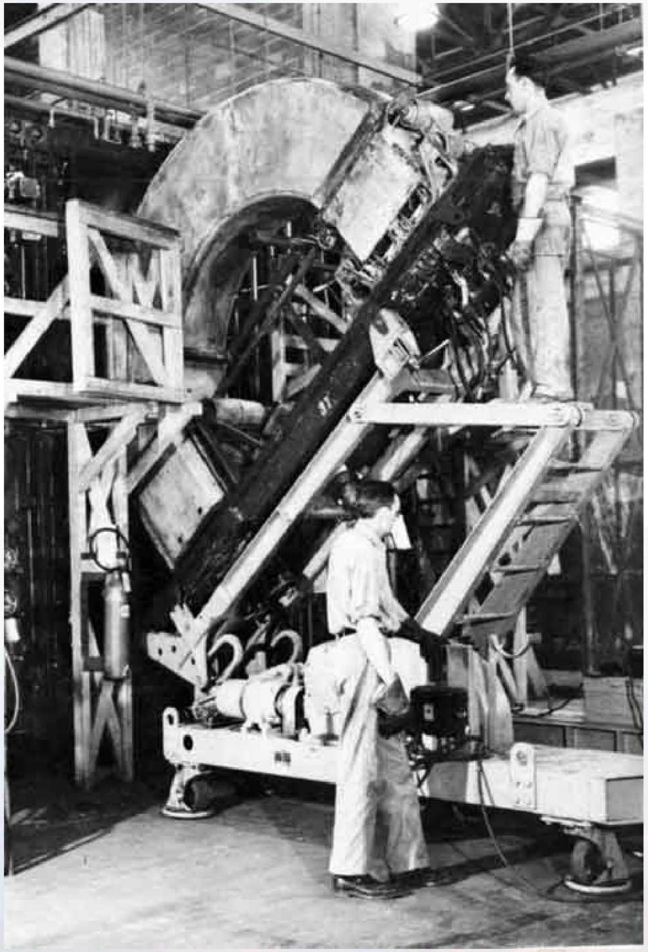


Natural Uranium - ~0.7% U-235
~99.3% U-238

Natural Plutonium?

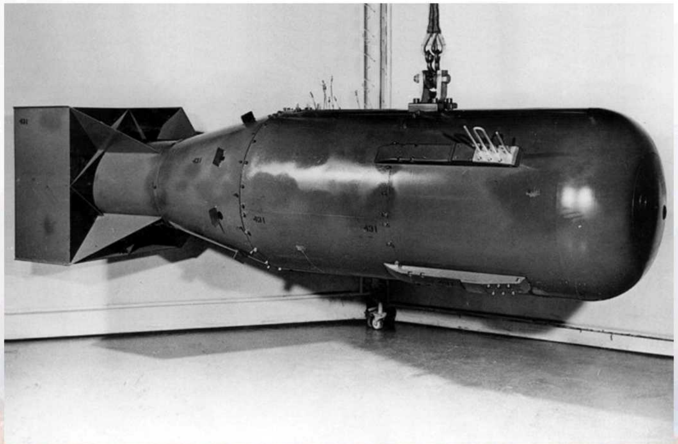
Figure courtesy of David Chichester, INL

Uranium enrichment and Oak Ridge



Recovery of U-235 from a Calutron

- Ernest Lawrence invented the Calutron, based on the Cyclotron, at UC Berkeley.
- 1943 - The Y-12 plant in Oak Ridge, TN was stood up to enrich natural uranium.
- 1945 – Little Boy was ready.

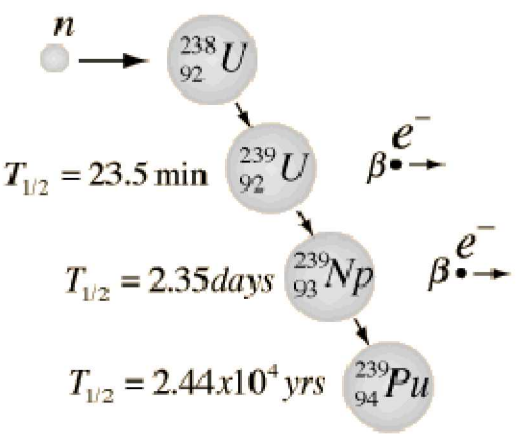


Can I make it?

The Passive Neutron Signatures

Isotope	Half Life	Spontaneous Fission Yield (n/s-kg)	Spontaneous Fission Multiplicity ν	Induced Thermal Fission Multiplicity ν
^{232}U	71.7 yr	1,300	1.71	3.13
^{233}U	1.59×10^5 yr	0.86		
^{234}U	2.45×10^5 yr	5.02		
^{235}U	7.04×10^8 yr	0.299		
^{236}U	2.34×10^6 yr	5.49		
^{238}U	4.47×10^9 yr	13.6		
^{237}Np	2.14×10^6 yr	0.114		
^{238}Pu	87.7 yr	2.59×10^6		
^{239}Pu	2.41×10^4 yr	21.8		
^{240}Pu	6.56×10^3 yr	1.02×10^6		
^{241}Pu	14.35 yr	50 ±		
^{242}Pu	3.76×10^5 yr	1.72×10^6		
^{244}Cm	18.1 yr	1.08×10^{10}		
^{252}Cf	2.65 yr	2.34×10^{15}		

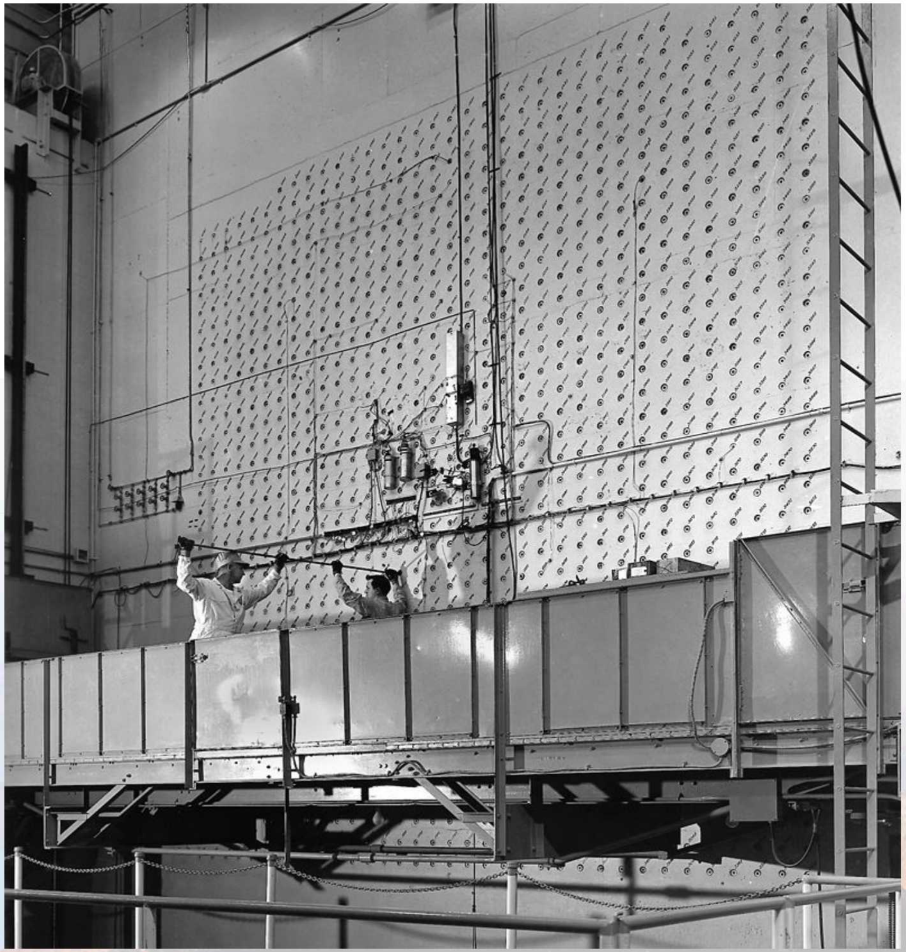
There isn't natural Plutonium to be found, but …



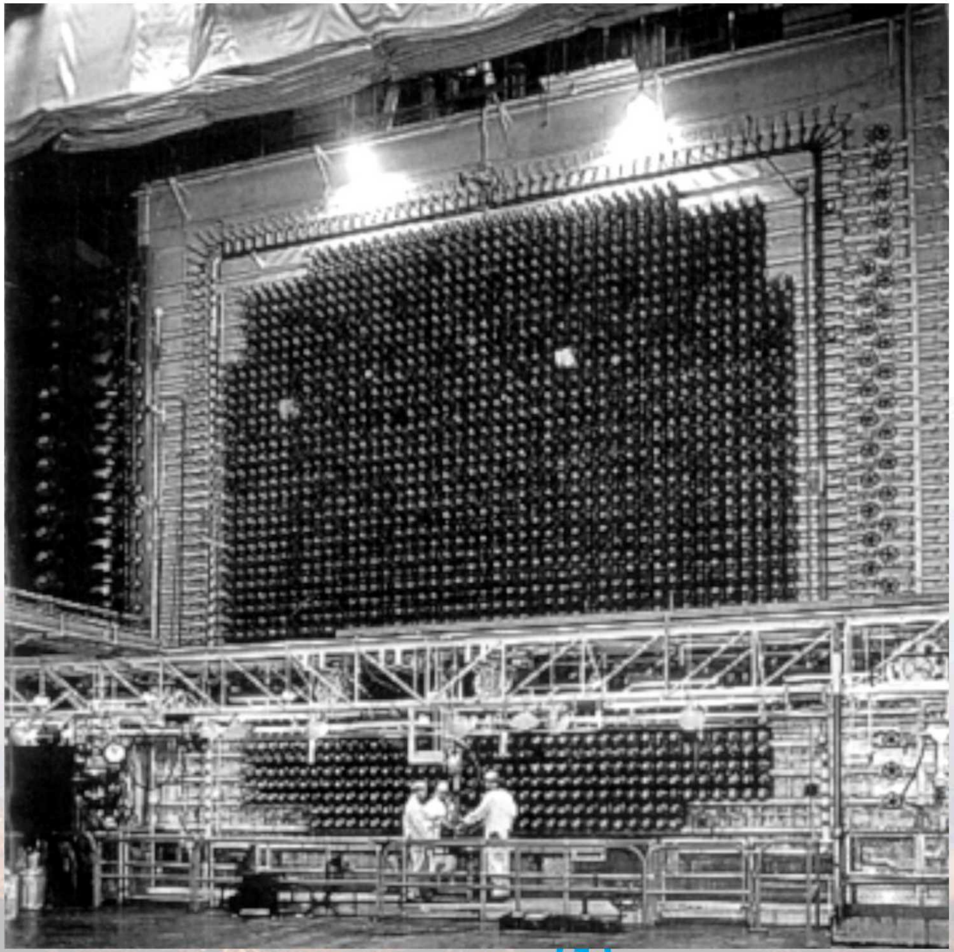
Ref: "Panda Book", values with ± have significant uncertainty

First Nuclear Reactors

1943 - ORNL X-10 Graphite Reactor
graphite moderated, air cooled

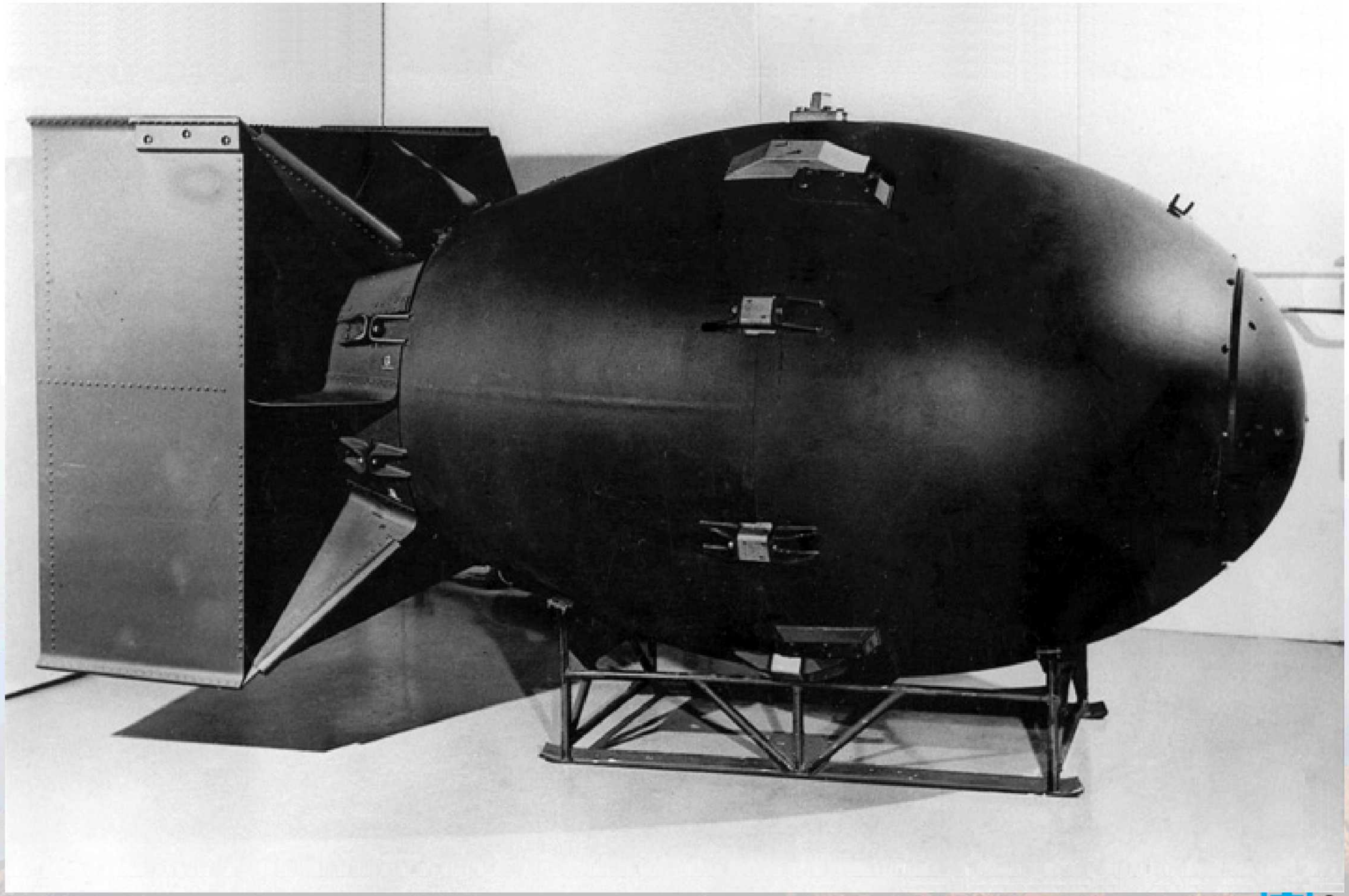


1944 - Hanford B Reactor
graphite moderated, water cooled



Fat Man - 1945

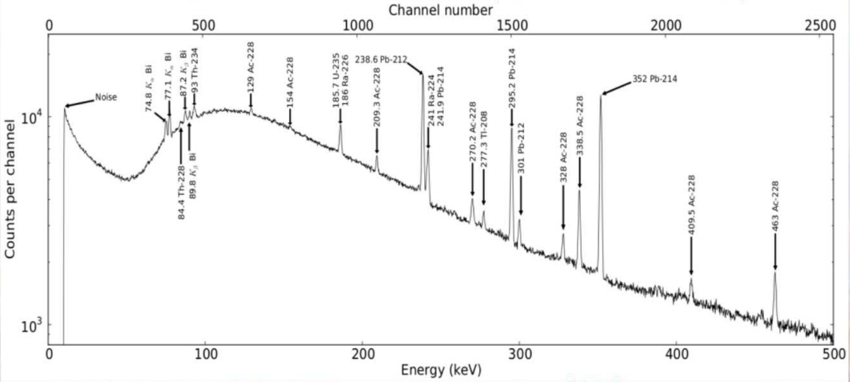
Weapons Grade Plutonium (WGPu): >93% Pu-239



Measurable Unique Signatures of Fissile Material

1. **Radiography**
   - Geometry
2. **Gamma spectrum**
   - Isotopic content + surface area/shielding
3. **Total neutron rate**
   - Assay of the contents of specific materials
4. **Correlated counts**
   - Multiplicity analysis (singles, doubles, triples): fission rate, multiplication, (α, n) component
   - Relative multiplication:
     - Rossi-alpha distribution
     - Feynman variance technique

Plutonium Scrap Multiplicity Counter, used for accurate assays of plutonium metal, oxide, mixed oxide, or scrap *(LANL PANDA Manual).*



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19

Current Methods
Drawbacks and Limitations

1. Geometry/Radiography
   - Insensitive to element/isotope type
   - Poor penetration
2. Gamma spectrum
   - Attenuation and self-shielding
3. Total neutron rate
   - Spontaneous & induced fission and (α,n) sources are indistinguishable
4. Correlated counts
   - Requires high efficiency, necessitates large detection system
   - Efficiency has to be well known
   - Detector lifetime of 10-30 μs is long compared to fission chain processes
   - Neutron energy information is lost due to moderation

→ He-3 based technologies

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20

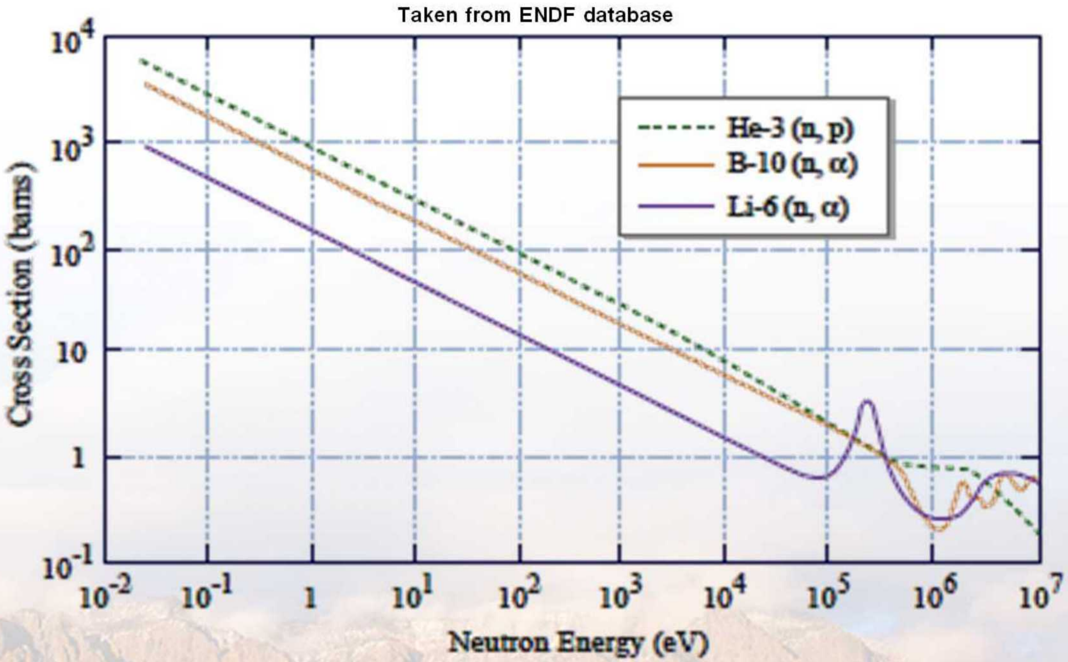
Thermal Neutron Detection

- **High thermal cross section** **(efficiency)**
- **High Q-value** **(discrimination)**

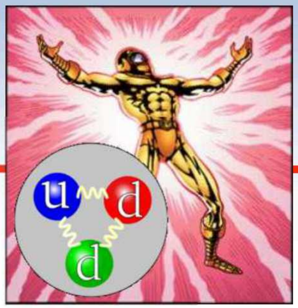
$n + {}^{3}He \rightarrow {}^{3}H + {}^{1}H + 0.764\ MeV$

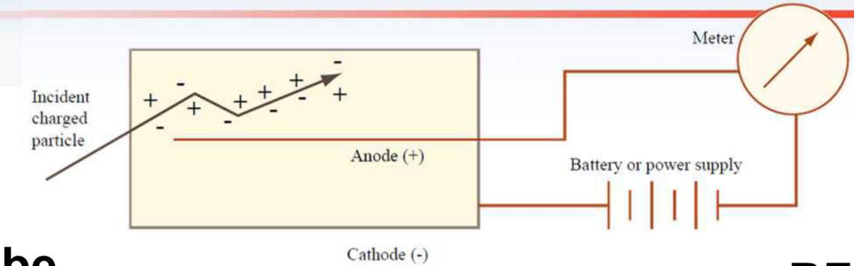
$n + {}^{10}B \rightarrow {}^{7}Li + \alpha + 2.31\ MeV$

$n + {}^{6}Li \rightarrow {}^{3}H + \alpha + 4.78\ MeV$



Gas Filled Neutron Counters



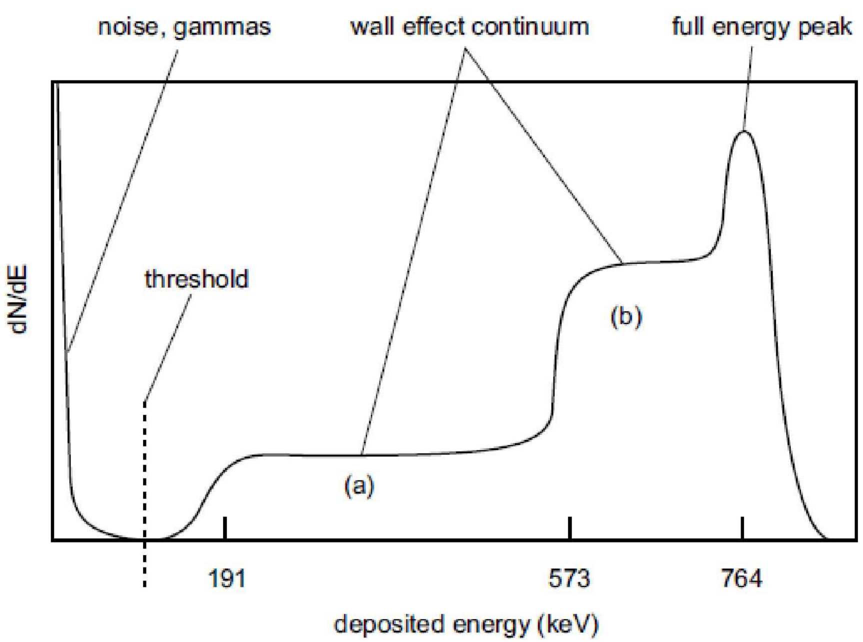


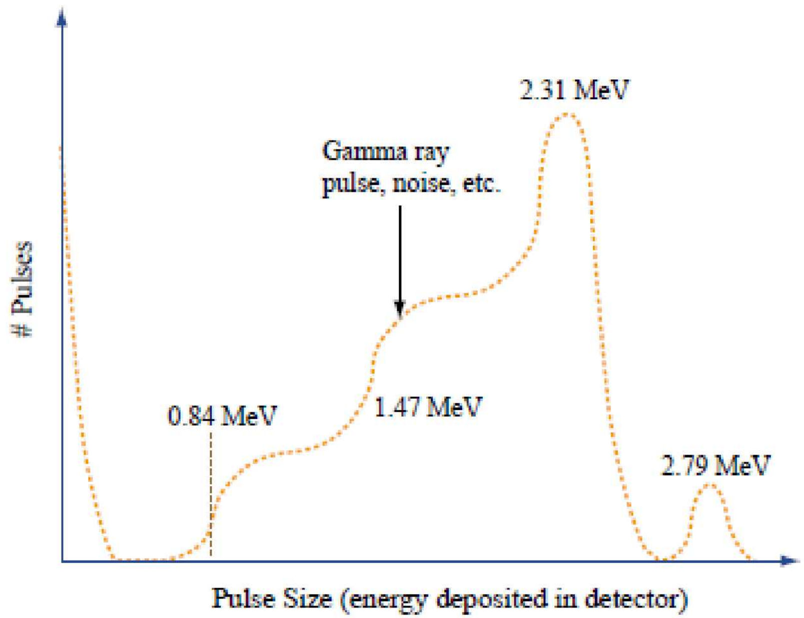
He3 Tube

BF3 Tube

$n + {}^{3}He \rightarrow {}^{3}H + {}^{1}H + 0.764\ MeV$

$n + {}^{10}B \rightarrow {}^{7}Li + {}^{4}He + 2.31\ MeV$





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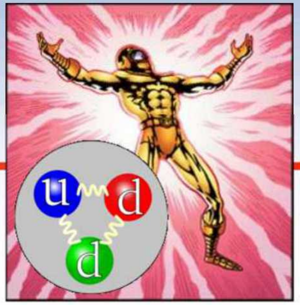
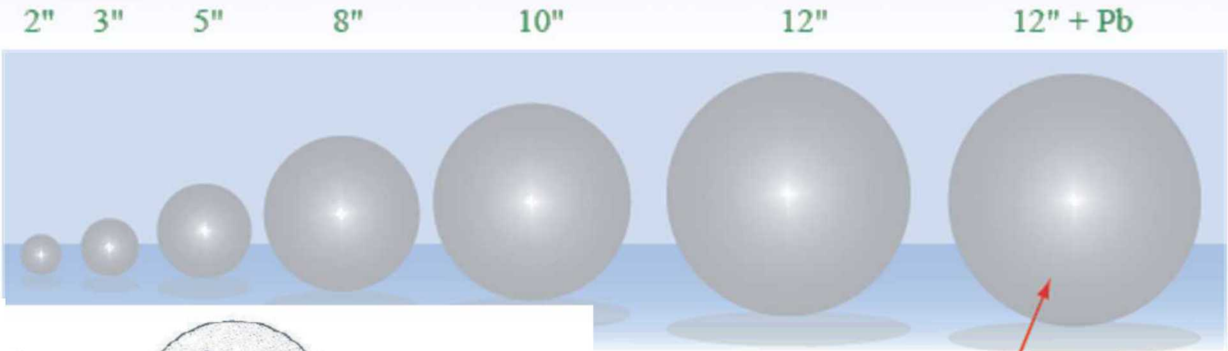
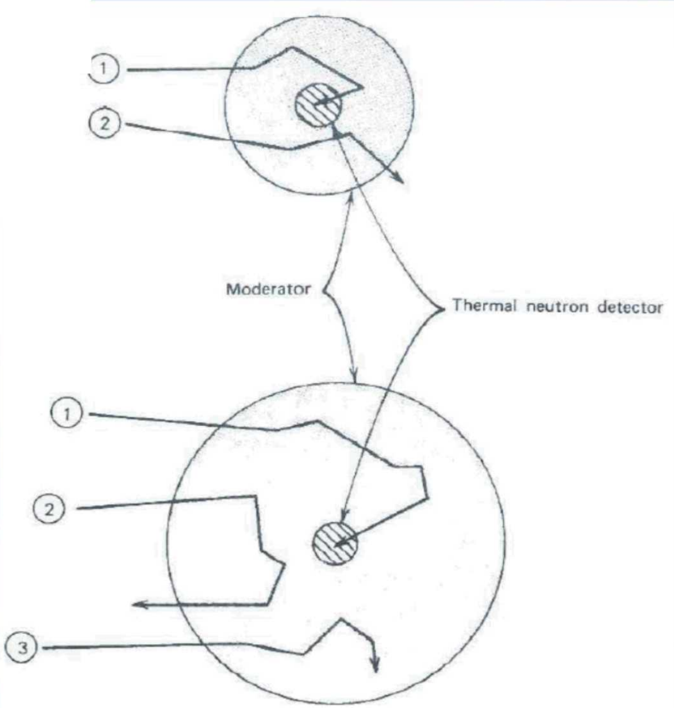
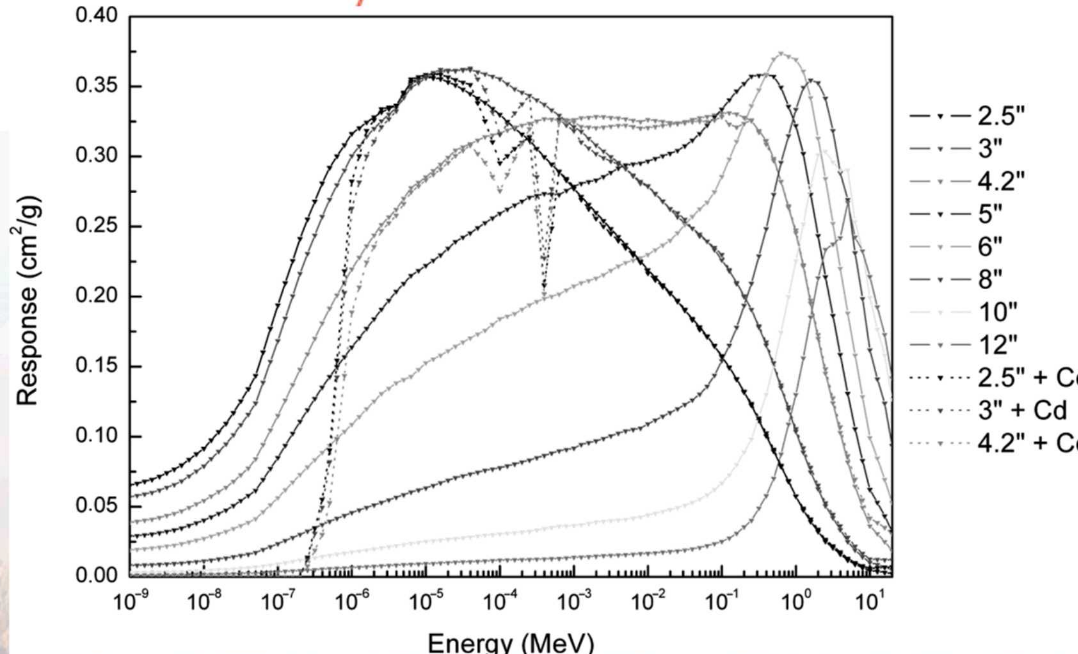
Fast Thermal Neutron Counters





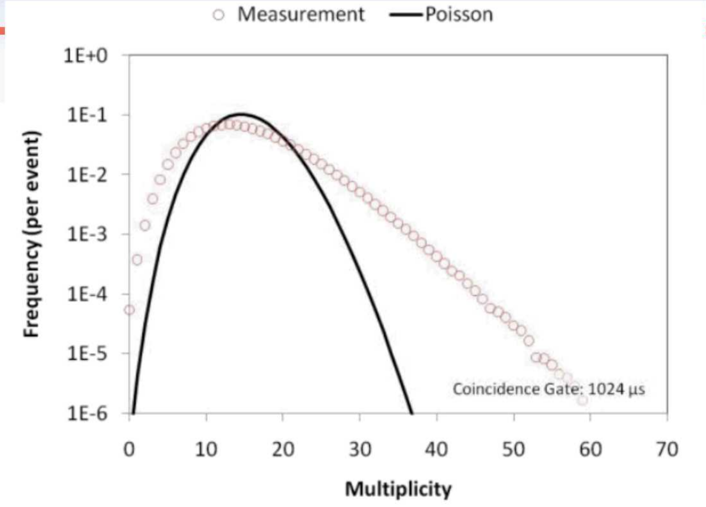


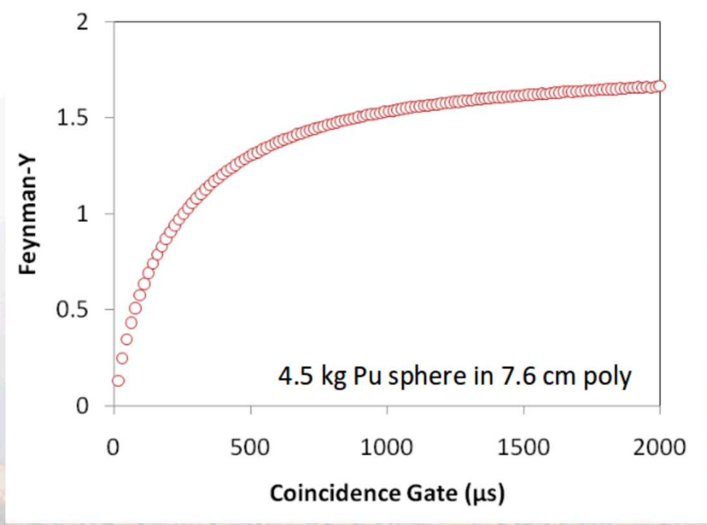


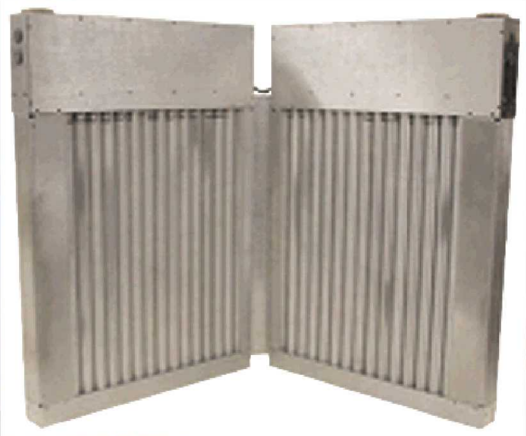
Ref.: G. Knoll, Radiation detection and measurement, 3rd ed., p. 539

Feynman-Y Approach

- Measures correlated counts in a fixed gate
- Fission chains create variance in excess of Poisson distribution
- $\frac{\sigma^2}{\mu} = 1 + Y$
  - σ^2: variance
  - μ: mean







Figures courtesy of John Mattingly: http://web.ornl.gov/sci/nsed/outreach/presentation/2011/Mattingly.pdf

Neutron Coincidence Counting Equations

1. $S = F\epsilon M v_{s1}(1+\alpha)$
2. $D = \frac{F\epsilon^2 f_d M^2}{2}\left[v_{s2} + \left(\frac{M-1}{v_{i1}}\right) v_{s1}(1+\alpha)v_{i2}\right]$
3. $T = \frac{F\epsilon^3 f_t M^3}{6}\left[v_{s2} + \left(\frac{M-1}{v_{i1}-1}\right)[2v_{s2}v_{i2} + v_{s1}(1+\alpha)v_{i3}] + 3\left(\frac{M-1}{v_{i1}}\right)^2 v_{21}(1+\alpha)v_{i2}^2\right]$

- **F = spontaneous fission rate**
- ϵ **= neutron detection efficiency**
- **M = neutron leakage multiplication,**
- $\alpha = (\alpha, n)$ **to spontaneous fission neutron ratio**
- f_d**= doubles gate fraction**
- f_t**= triples gate fraction**
- v_{s1}, v_{s2}, v_{s3}**= factorial moments of the spontaneous fission neutron distribution**
- v_{i1}, v_{i2}, v_{i3}**= factorial moments of the induced fission neutron distribution**

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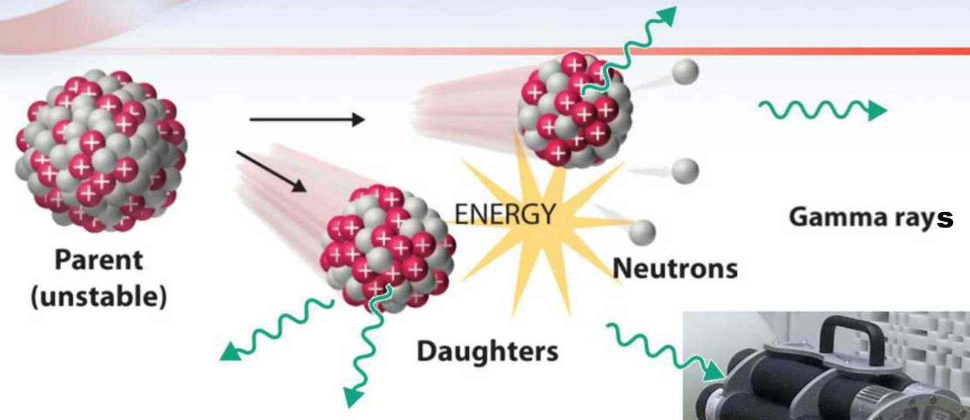
Targeting Multiplicity 2 correlations only

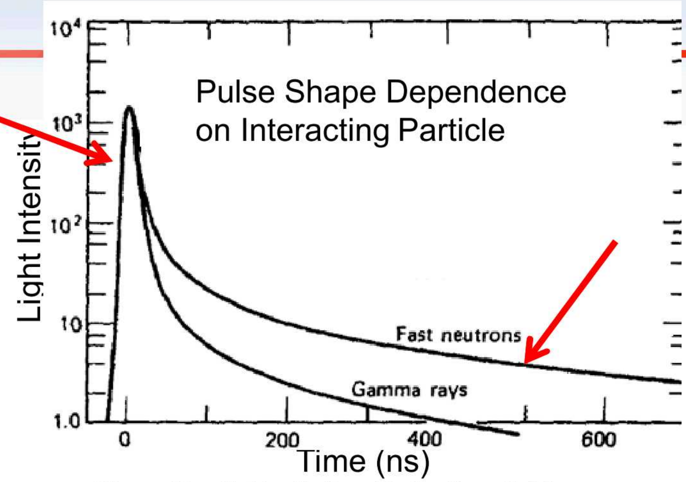


- Neutrons and gamma-rays are emitted nearly simultaneously during the fission process.
- Minimum multiplicity equals two:
  1. Gamma-gamma: great for timing, but a lot of detector cross talk and uncorrelated background.
  2. Neutron-neutron: without event by event energy, expected correlated timing is spread on the order of the spread in fission chain dynamics.
  3. Gamma-neutron: gamma starts precise clock, neutron creates certainty that fission has taken place (also more penetrating).

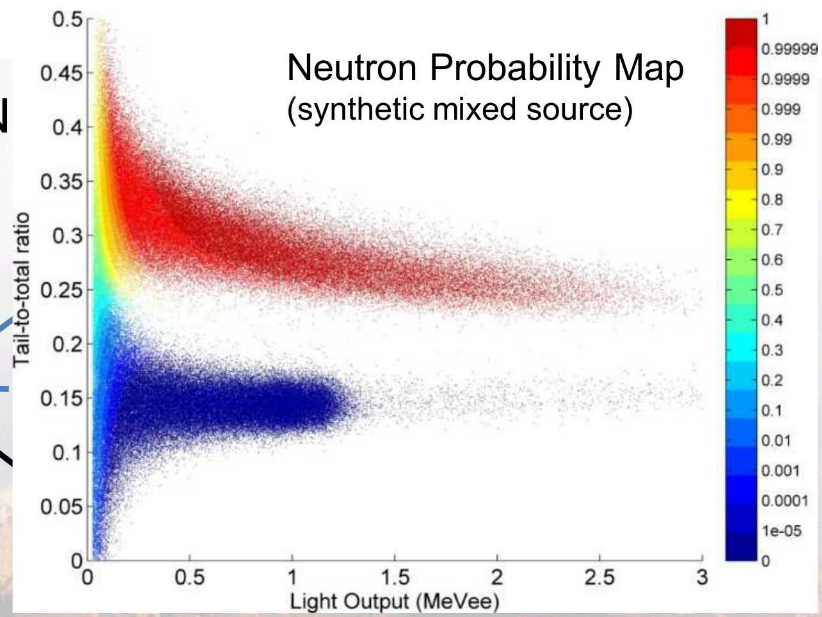
26

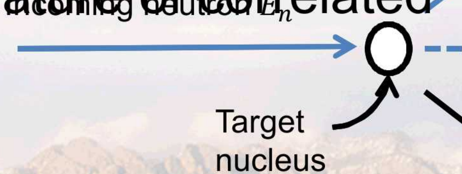
Fast Organic Scintillators
Three main new advantages

1. Fast rise time
   - Sub-nanosecond timing resolution
2. Incident neutron energy
   - Light output proportional to proton recoil
3. PSD Capable
   - Discriminate neutrons and gamma rays
   - Gamma rays provide clean timing signature of correlated event



Glenn Knoll Radiation Detection & Measurement 3rd Edition





Detection Efficiency

Table 13-1. Neutron and gamma-ray interaction probabilities in typical gas proportional counters and scintillators

Thermal Detectors	Interaction Probability: Thermal Neutron	Interaction Probability: 1-MeV Gamma Ray
^3He (2.5 cm diam, 4 atm)	0.77	0.0001
Ar (2.5 cm diam, 2 atm)	0.0	0.0005
BF_3 (5.0 cm diam, 0.66 atm)	0.29	0.0006
Al tube wall (0.8 mm)	0.0	0.014

Fast Detectors	Interaction Probability: 1-MeV Neutron	Interaction Probability: 1-MeV Gamma Ray
^4He (5.0 cm diam, 18 atm)	0.01	0.001
Al tube wall (0.8 mm)	0.0	0.014
Scintillator (5.0 cm thick)	0.78	0.26

From T.W. Crane and M.P. Baker, Chapter 13, Neutron Detectors

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Energy Deposition

Table 13-2. Neutron and gamma-ray energy deposition in typical gas proportional counters and scintillators

Thermal Detectors	Alpha or Proton Range (cm)	dE/dx for 400-keV Electron (keV/cm)	Average Neutron Reaction Energy Deposited (keV)	Electron Energy Deposited (keV)[a]	Ratio of Neutron to Electron Energy Deposition
^3He (2.5 cm diam, 4 atm)	2.1	1.1	~500	4.0	125
^3He (2.5 cm diam, 4 atm) + Ar (2 atm)	0.5	6.7	~750	24.0	30
BF_3 (5.0 cm diam, 0.66 atm)	0.7	3.6	~2300	25.7	90
Fast Detectors					
^4He (5.0 cm diam, 18 atm)	0.1	6.7	1000	48	20
Scintillator (5.0 cm thick)	0.001	2000	1000	400	2.5

[a]This calculation assumes a path length of $\sqrt{2} \times$ tube diameter.

From T.W. Crane and M.P. Baker, Chapter 13, Neutron Detectors

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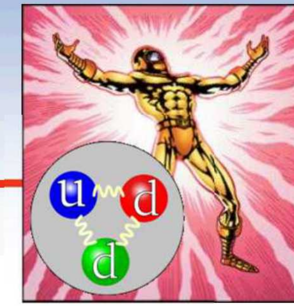
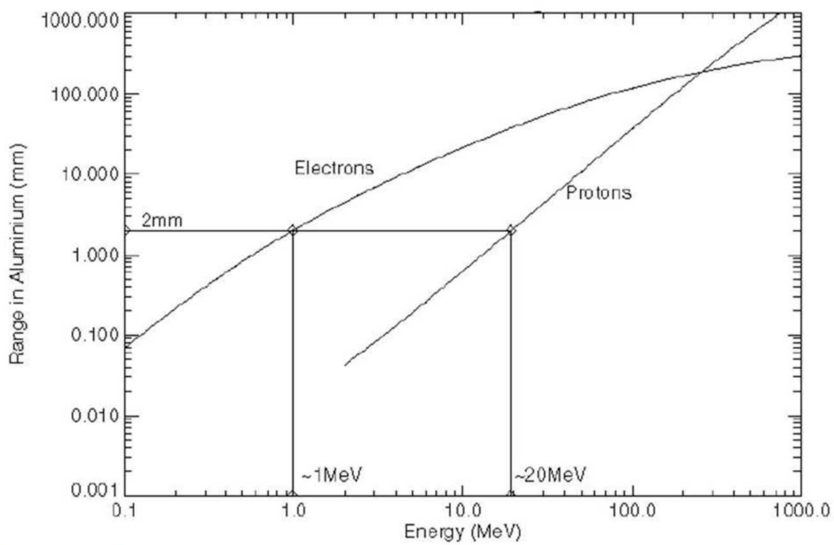
Discrimination – It's all about dE/dx



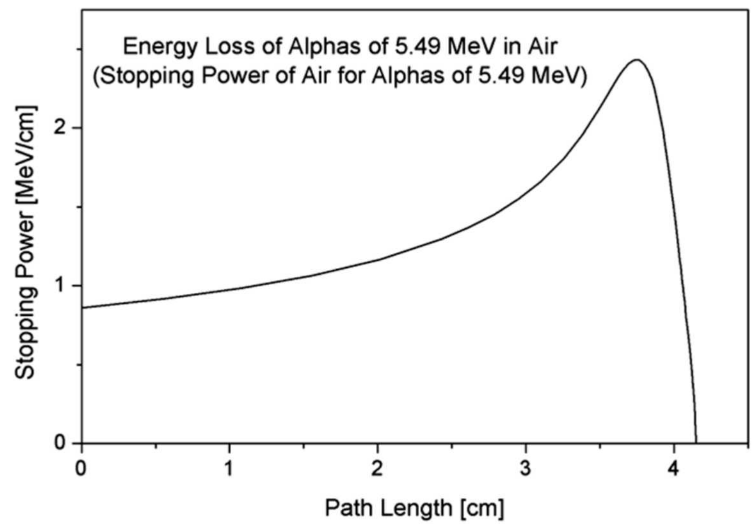
$-dE/dx \sim M Z^2/E$

Me = 511 keV

Mp = 938 MeV



Mean ranges of protons and electrons in aluminum [Source: E.J. Daly, A.Hilgers, G. Drolshagen, and H.D.R. Evans, "Space Environment Analysis: Experience and Trends," ESA 1996 *Symposium on Environment Modelling for Space-based Applications*, Sept. 18-20, 1996, ESTEC, Noordwijk, The Netherlands.]



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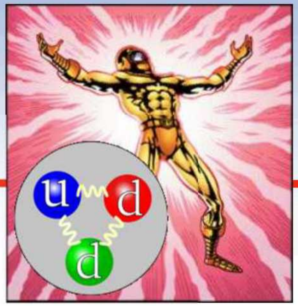
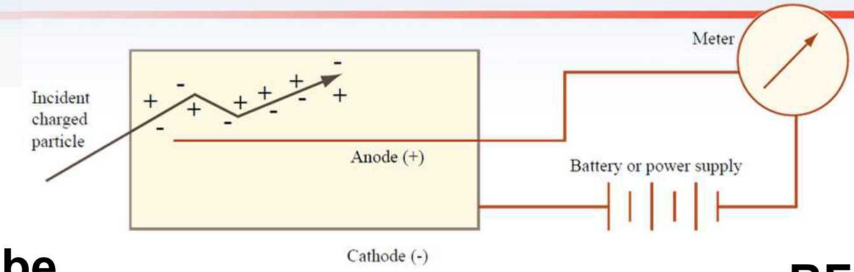
Discrimination – It's all about dE/dx



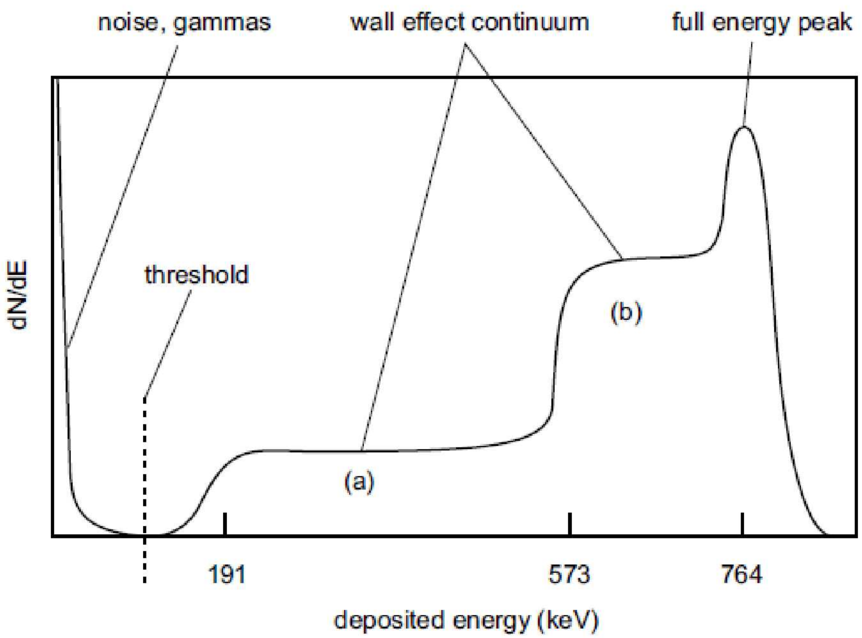


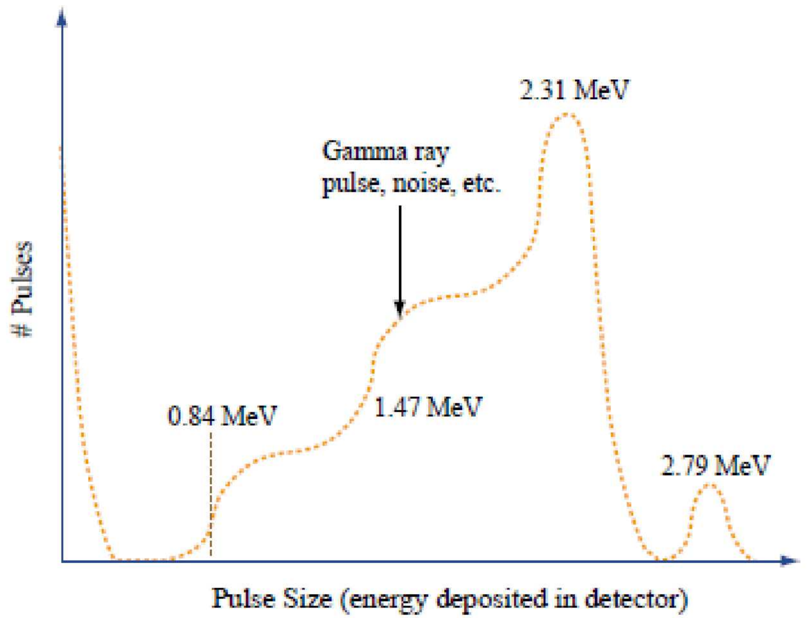
He3 Tube

BF3 Tube

n + ^{3}He → ^{3}H + ^{1}H + 0.764 MeV

n + ^{10}B → ^{7}Li + ^{4}He + 2.31 MeV





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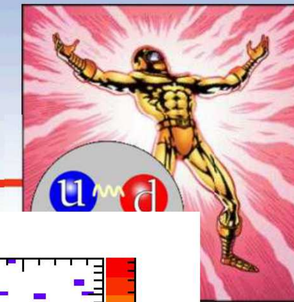
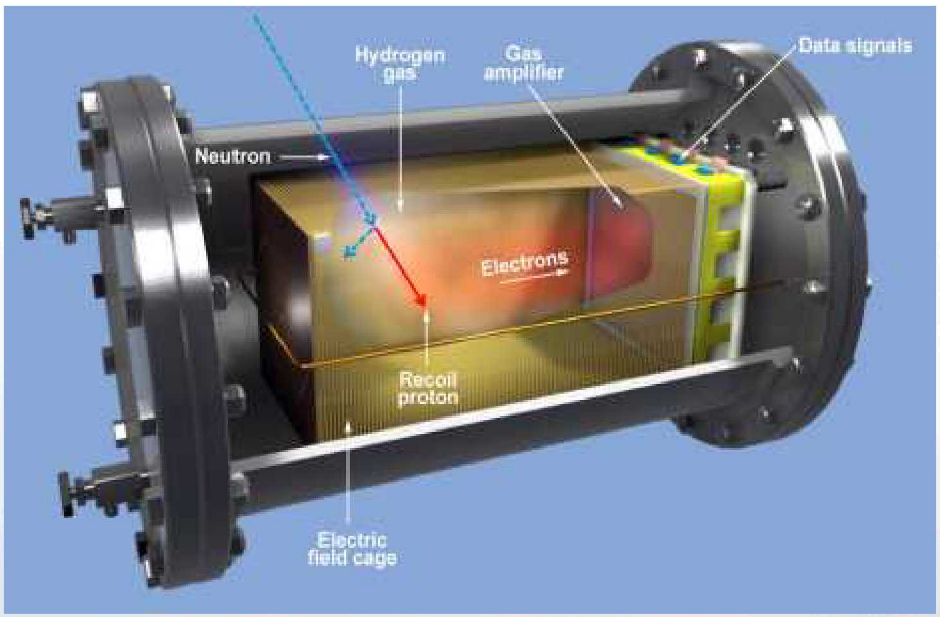
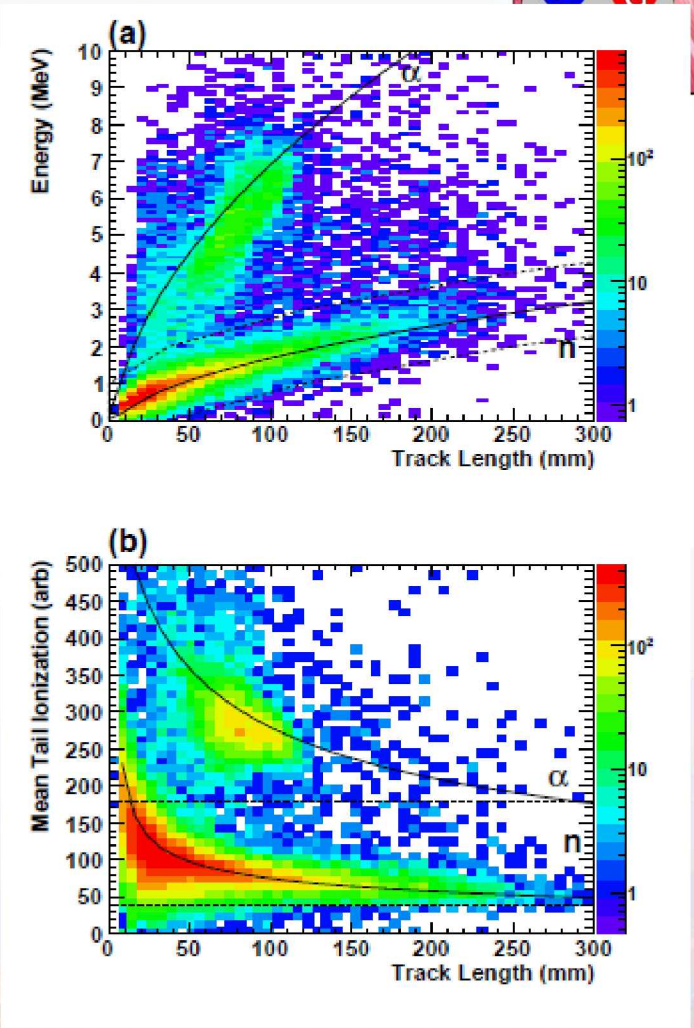
Discrimination – It's all about dE/dx



$-dE/dx \sim M\,Z^2/E$





Bowden, Heffner, et al, "Directional Fast Neutron Detection Using a TPC", NIMA, 2010

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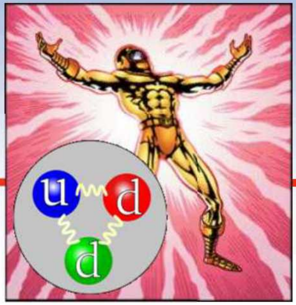
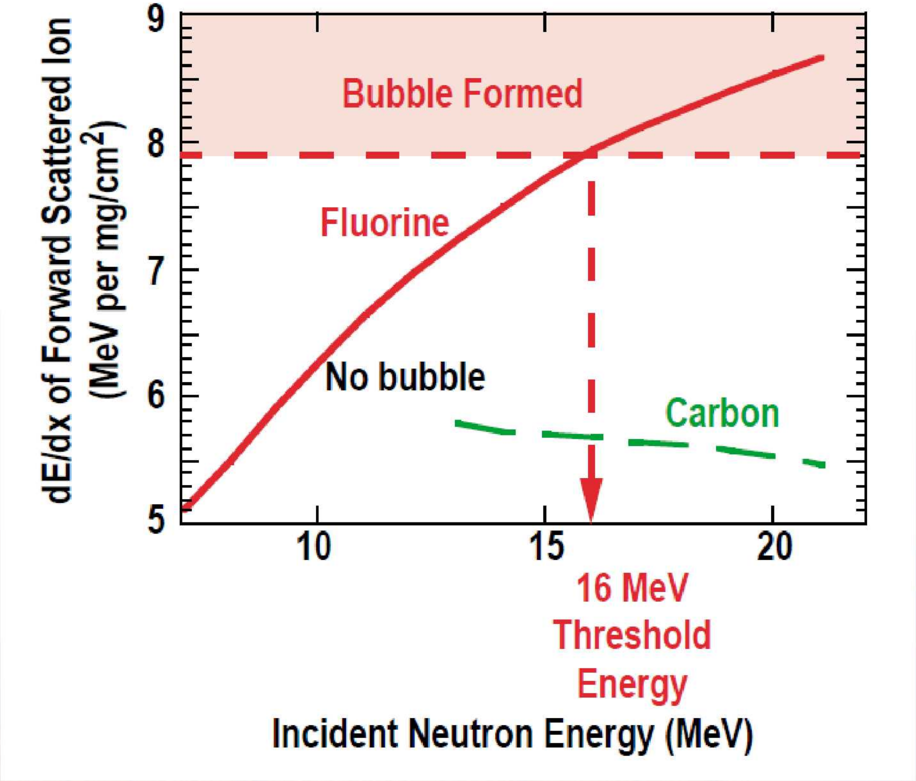
Discrimination – It's all about dE/dx



$-dE/dx \sim M Z^2/E$



http://www.bubbletech.ca/radiation_detectors_files/Bubble%20Detectors.html

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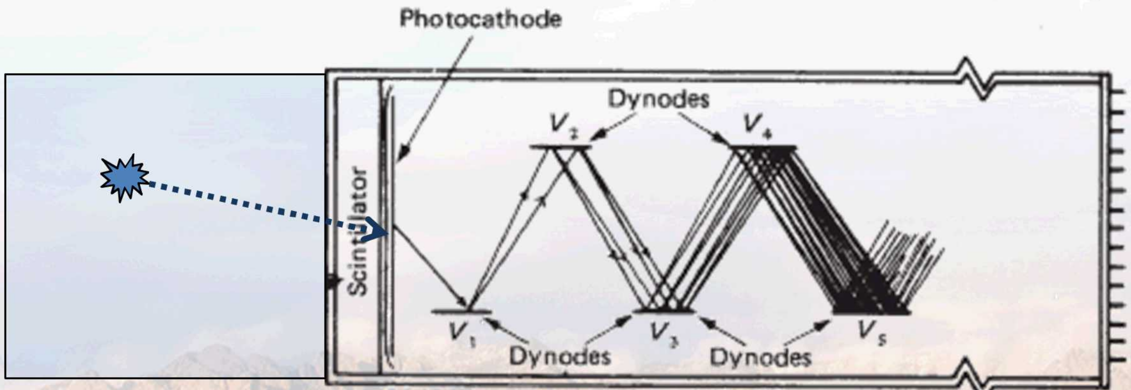
Neutron Detection – Scintillator

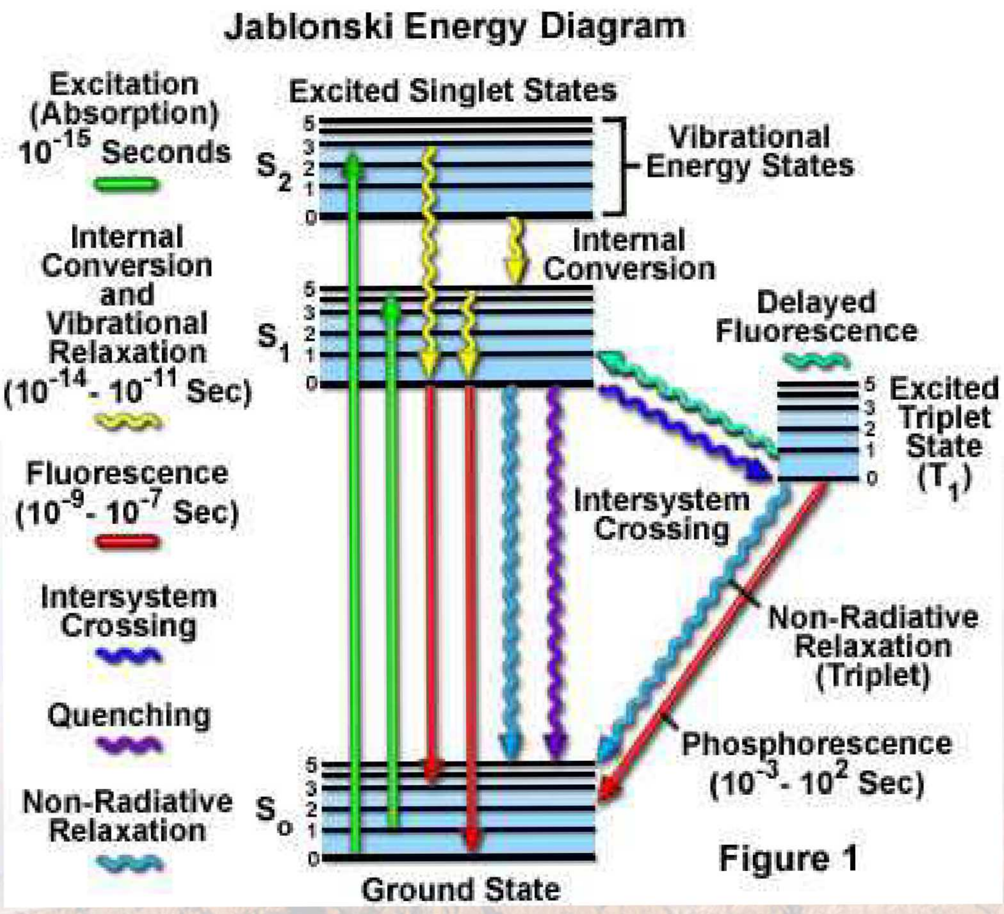
- Luminescence - When a material is excited and it subsequently gives off light.
- How it is excited determines the type of luminescence.
- Scintillation – luminescence produced by ionizing radiation excitation.
- Fluorescence – photoluminescence or scintillation that has a fast decay time (ns to μs).
- Phosphorescence – same as fluorescence, but with much slower decay time (ms to seconds)

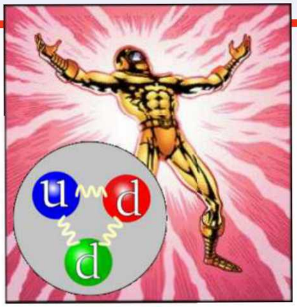
Neutron Detection – Scintillator

- Desirable Properties:
  - High conversion efficiency
  - Short decay times
  - Transparent to its own emission
  - Wavelength matched to photodetector
  - Inorganic – NaI(TI), CsI(TI), LaBr, BGO
  - Organic – Anthracene, Stilbene, Plastic, Liquid



Neutron Detection – Organic Scintillator

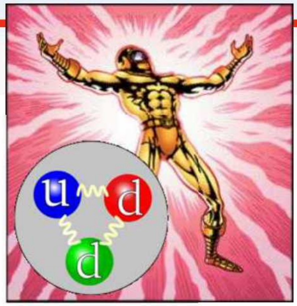




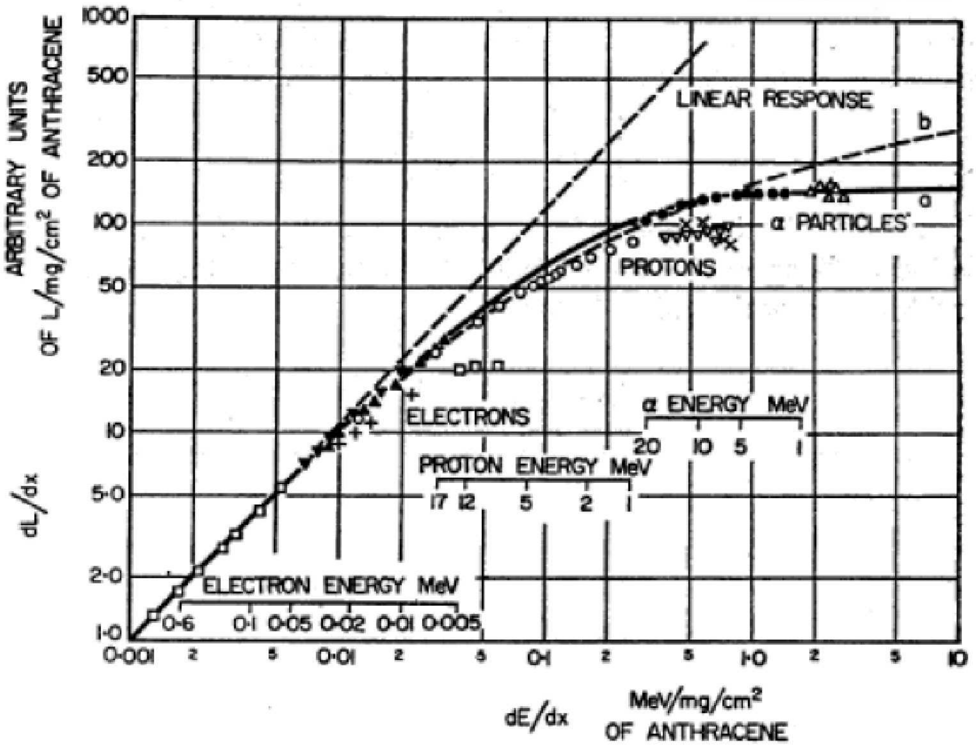
$T1 + T1 = S0 + S1$

http://micro.magnet.fsu.edu/primer/techniques/fluorescence/fluorescenceintro.html

Neutron Detection – Quenching, Birk's Law



- start with linearity at low ionization density $\frac{dL}{dx} = A\frac{dE}{dx}$
- let the density of excited molecules be proportional to the ionization density $B\frac{dE}{dx}$
- let k be the fraction that is quenched
$$\frac{dL}{dx} = \frac{A\frac{dE}{dx}}{1 + kB\frac{dE}{dx}}$$
- for small dE/dx, approaches linearity
- for large dE/dx, approaches saturation $\frac{dL}{dx} = \frac{A}{kB}$



from Birks

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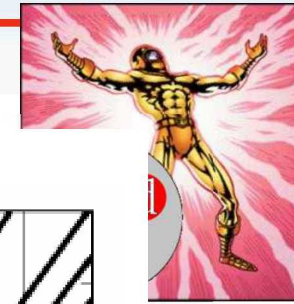
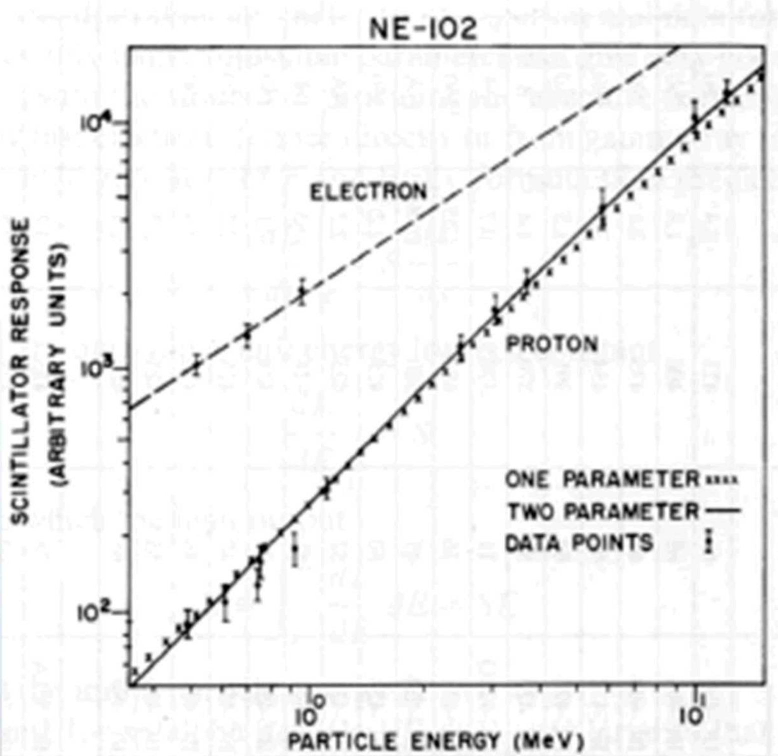
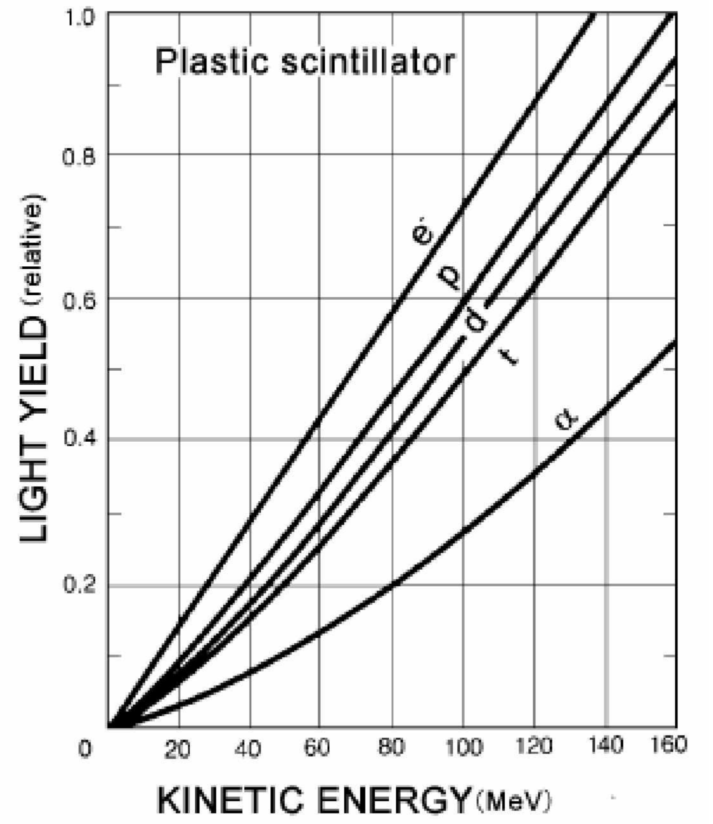
Neutron Detection – Quenching, dE/dx

$$L(E) = \int_0^E \frac{S \cdot (\mathrm{d}E/\mathrm{d}x)}{1 + kB \cdot (\mathrm{d}E/dx)} \left(\frac{dE}{dx}\right)^{-1} \mathrm{d}E$$

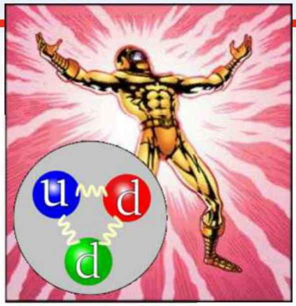


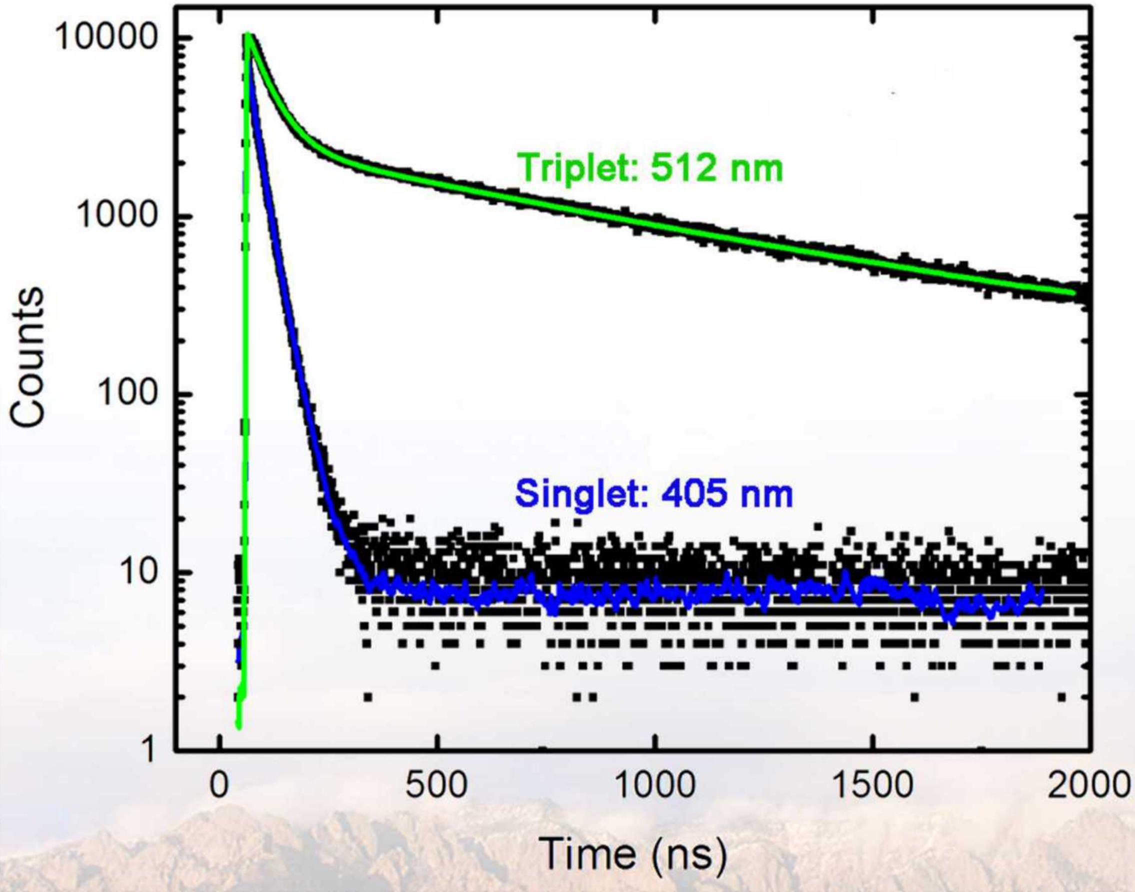


from Gooding and Pugh

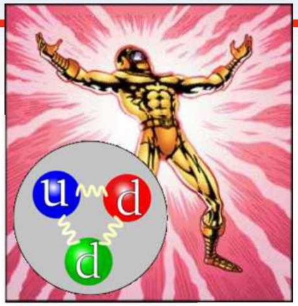


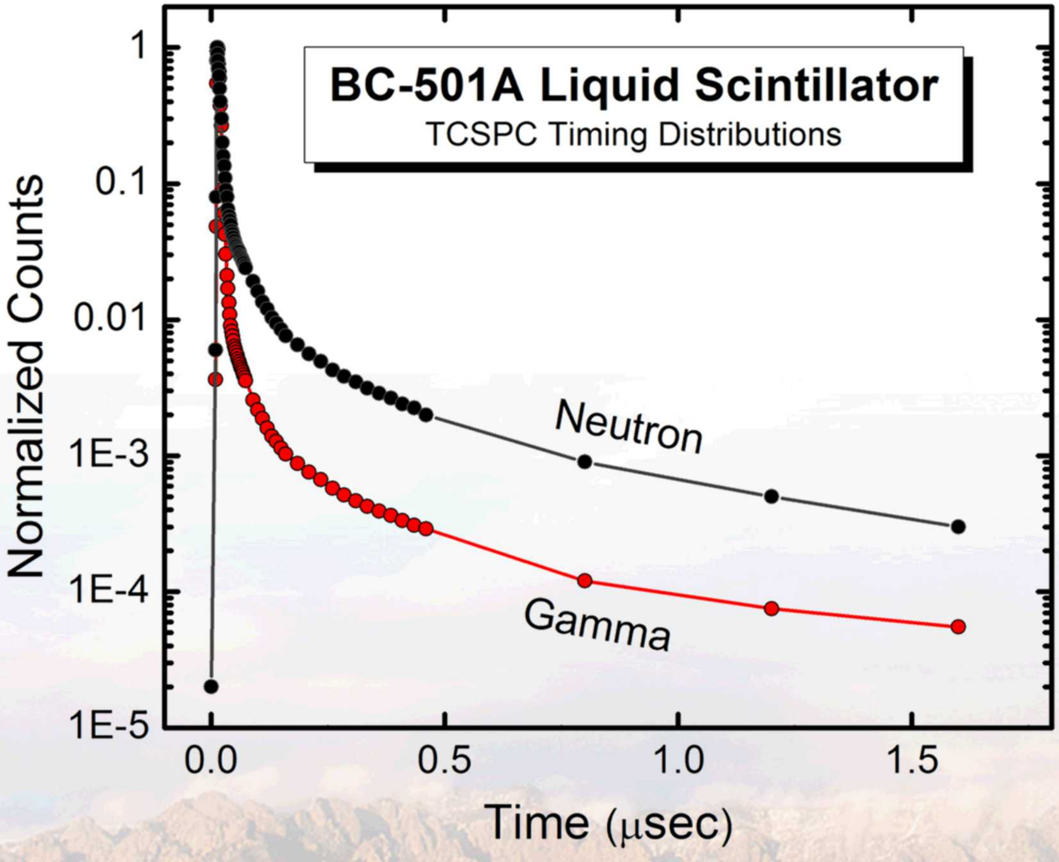
Neutron Detection – Slow Component





Pulse Shape Discrimination





*Szczesniak, T. et al *IEEE Trans. Nucl. Sci.* **2010**, 57, 3846.

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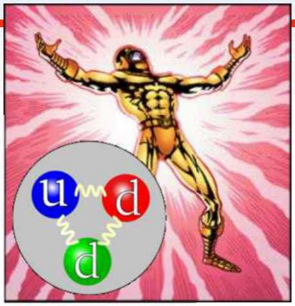
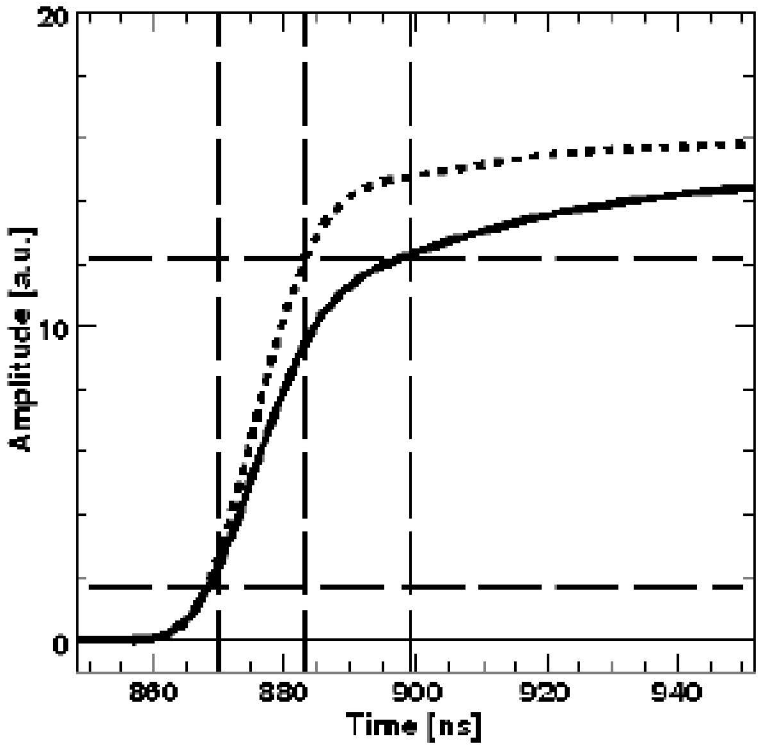
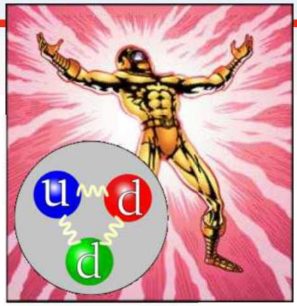
PSD - Integrated Rise Time



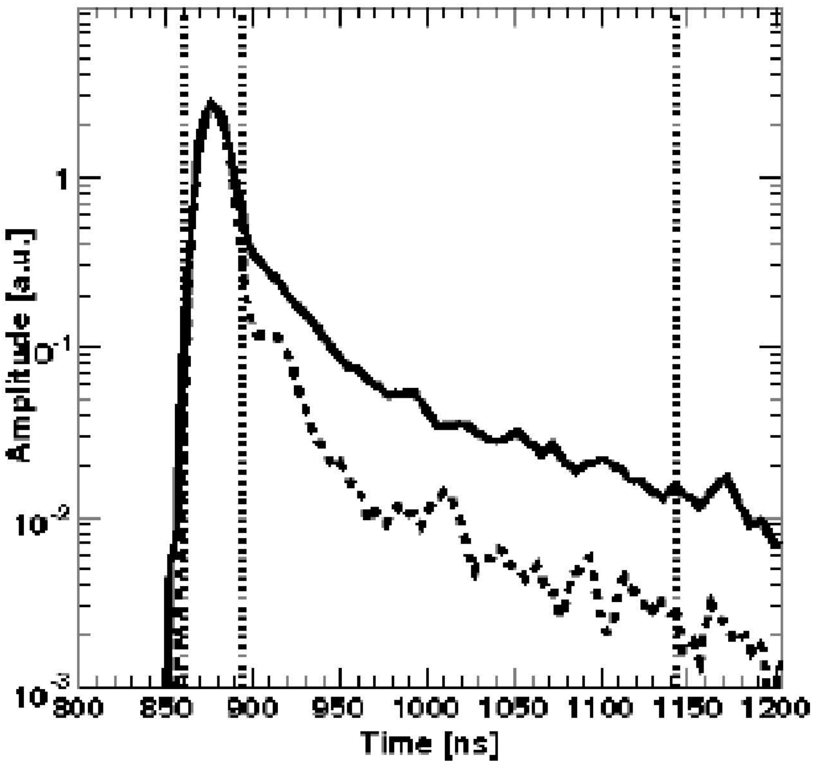
- Digitally integrate the pulse and take the rise time.
- PSD is taken as time between 10% and some fraction of total integral.



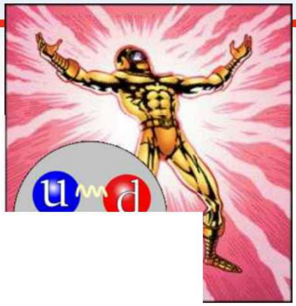
PSD - Slow component



- Locate the start of the slow decay component and compare integrals in prompt and slow.
- In this case:
  Fast = 0-33 ns
  Slow = 33-283 ns



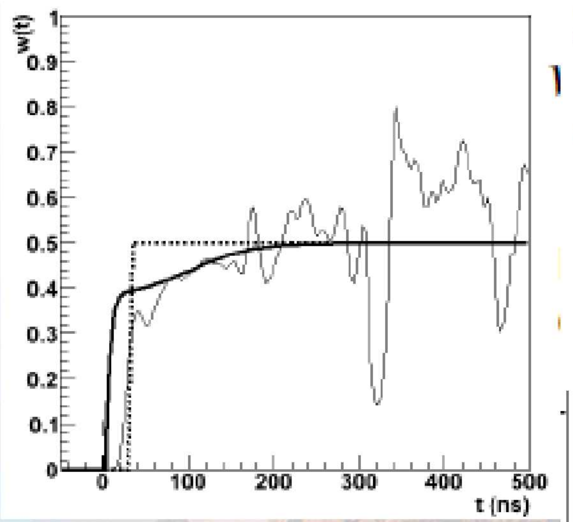
PSD – Gatti and De Martini integral



Starts by leading edge

Integrate the entire pulse with a suitable weight

$$S = \int_0^T p(t)\,w(t)\,dt$$



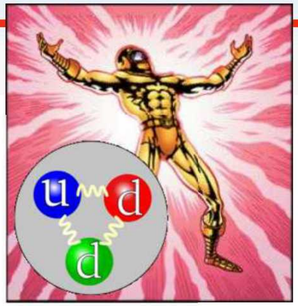
The optimal weight can be shown to be

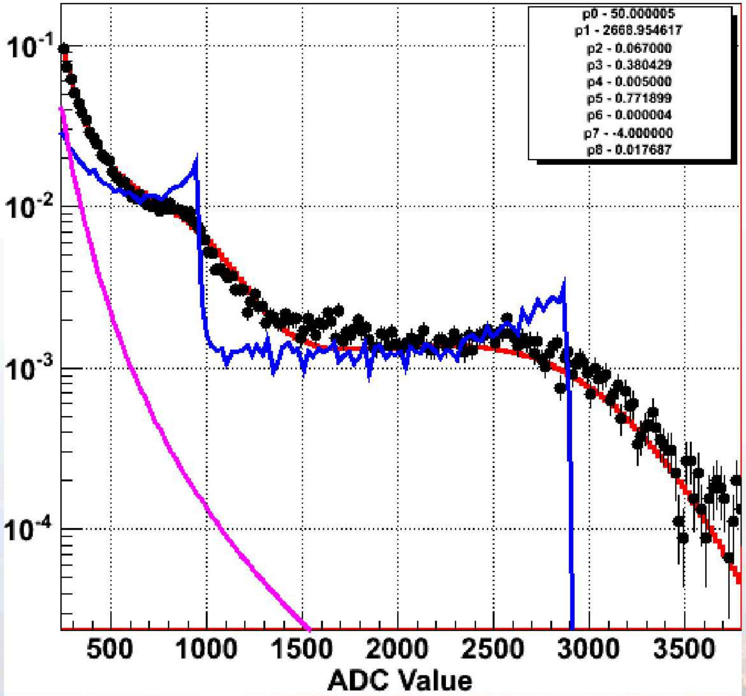
$$W(t) = \frac{\bar{n}(t) - \bar{\gamma}(t)}{\sigma_n^2(t) + \sigma_\gamma^2(t)}$$

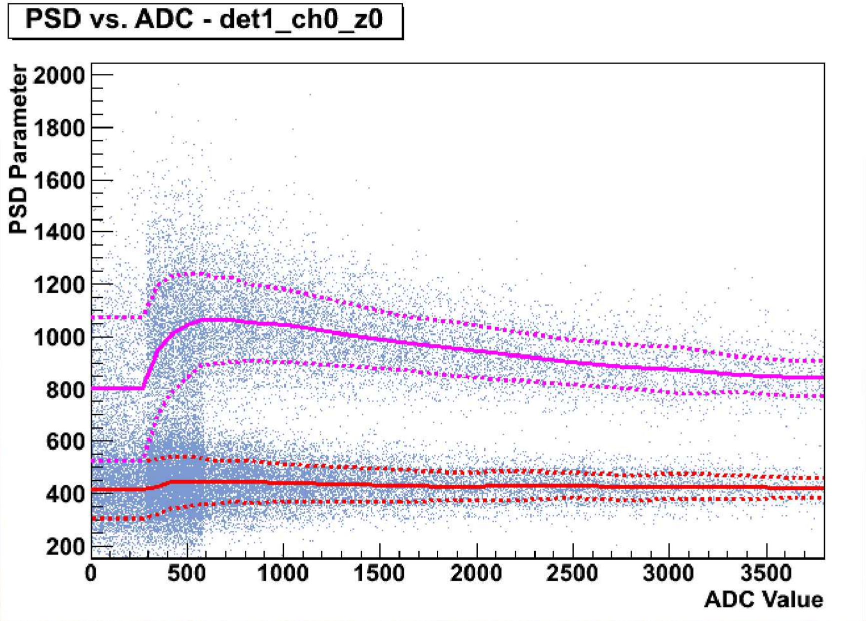
E. Gatti, F. de Martini, in: Nuclear Electronics, Proceedings of International Conference at Belgrade, Vol. II, IAEA, Vienna, 1962, p. 265.

PSD – Calibration

$$FOM = \frac{\left|X_\gamma - X_n\right|}{\left(W_\gamma + W_n\right)}$$







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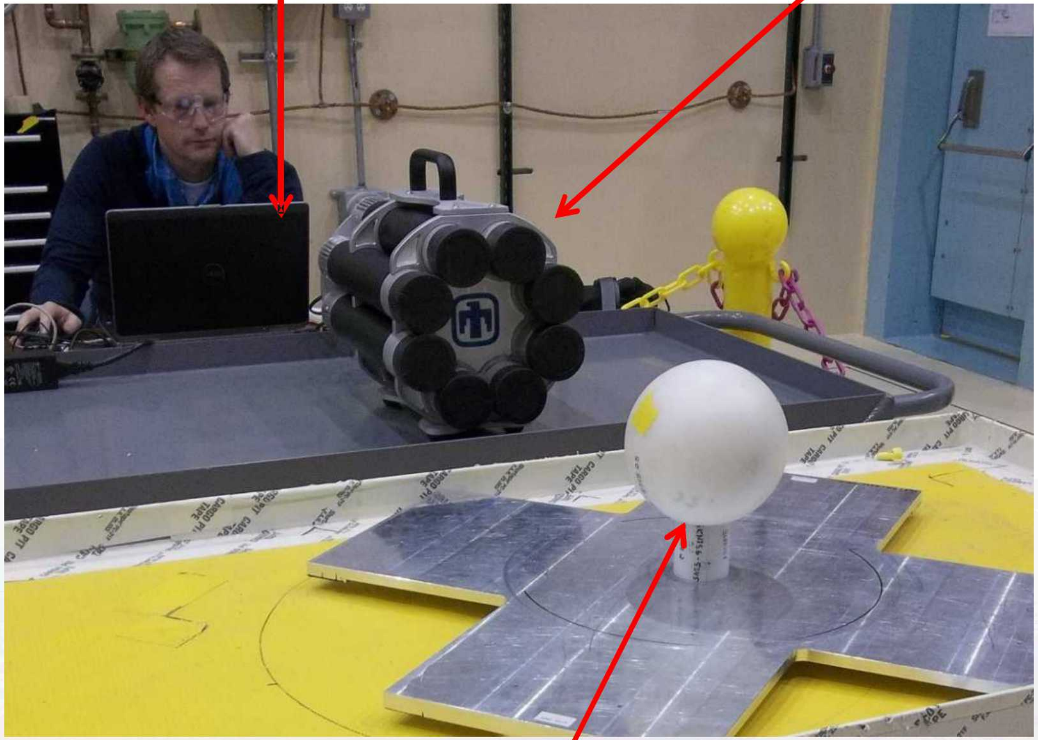
Correlated Radiation Signatures - Stilbene Array

Computer/digitizer

Stilbene array



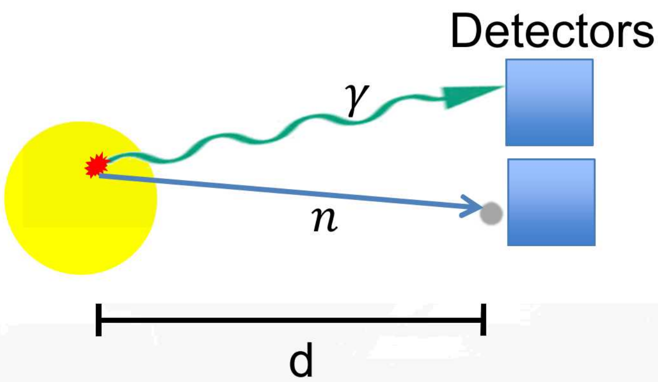
Beryllium Reflected Plutonium (BeRP) ball, 4.5 kg WGPu in a 1” shell of High Density Polyethylene

Same Fission Gamma-Neutron Correlations

Time Of Flight Fixed by Estimated Energy (TOFFEE)

E_n: neutron energy
R_γ: distance traveled by gamma ray
R_n: distance traveled by neutron

$$\Delta T_{n-\gamma} - \left(R_n \sqrt{\frac{m}{2E_n}} - \frac{R_\gamma}{c} \right) = 0$$

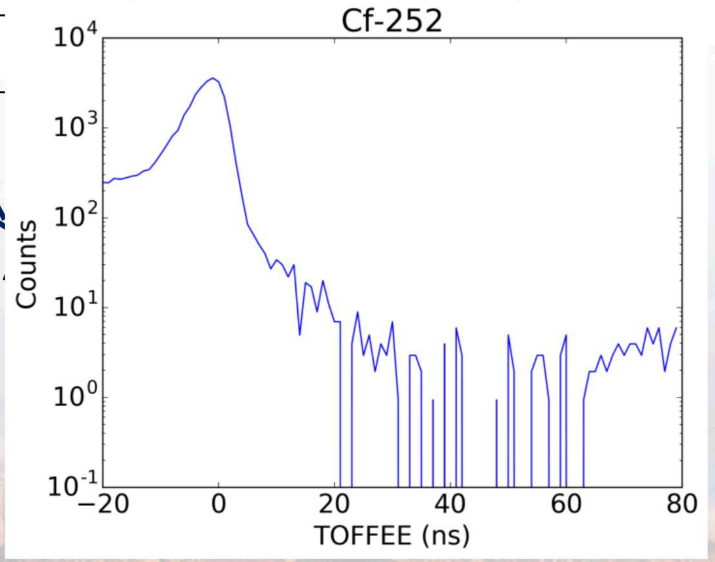


Time of arrival difference between γ and neutron	Time of flight between γ and neutron
Measured	

$$\Delta t_{n,\gamma} - \Delta \mathrm{t_p} \leq 0$$

Measured Quantities:

- Time between correlated γ and neutron: $\Delta t_{n,\gamma}$
- Proton recoil energy: $E_p \leq E_n$
- Source-to-detector distance: $d \approx R_n = R_\gamma$



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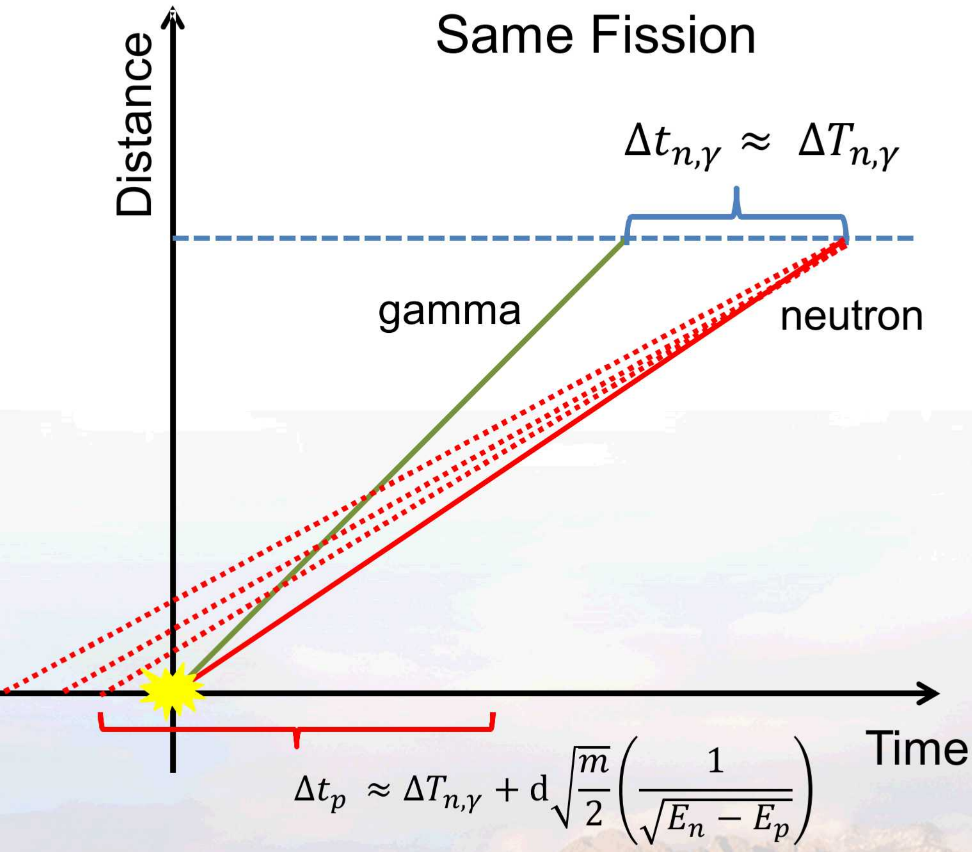
Same Fission Gamma-Neutron Detection Space-Time



1. Neutron and gamma are born from the same fission event
2. They travel the same distance the detectors
3. The measured and actual difference in time-of-flight is equivalent
4. When subtracting estimated time of flight of the neutron, the result is less than zero

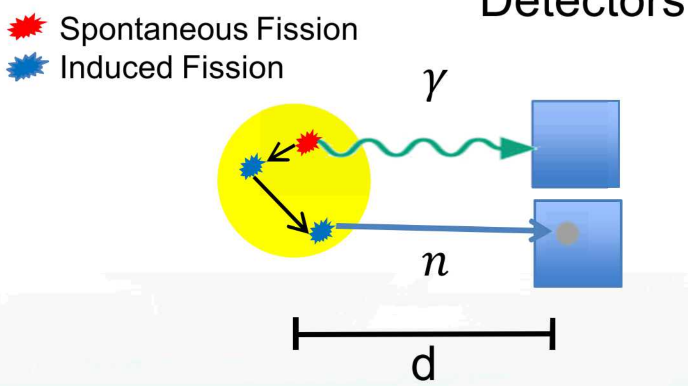
$$\Delta t_{n,\gamma} - \Delta \mathrm{t_p} \leq 0$$

Neutron-Gamma Correlation – Fission Chain

Time Of Flight Fixed by Estimated Energy (TOFFEE)

Time difference between fission events in a chain

Detectors

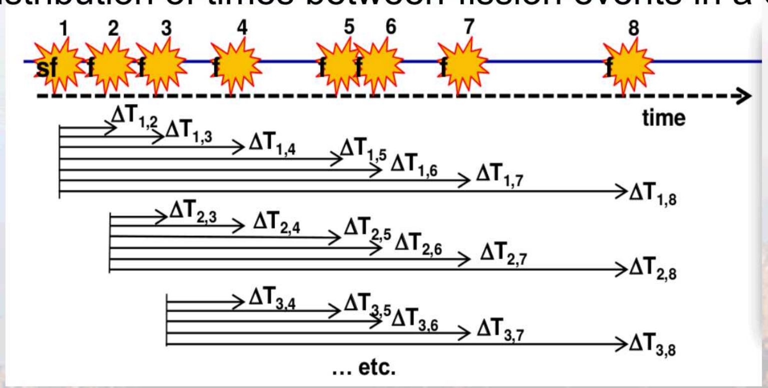


$$\Delta t_{n-\gamma} - \Delta t_p \leq \Delta T_g$$

Measured time difference between neutron and nearest γ

Estimated travel time difference between neutron and gamma

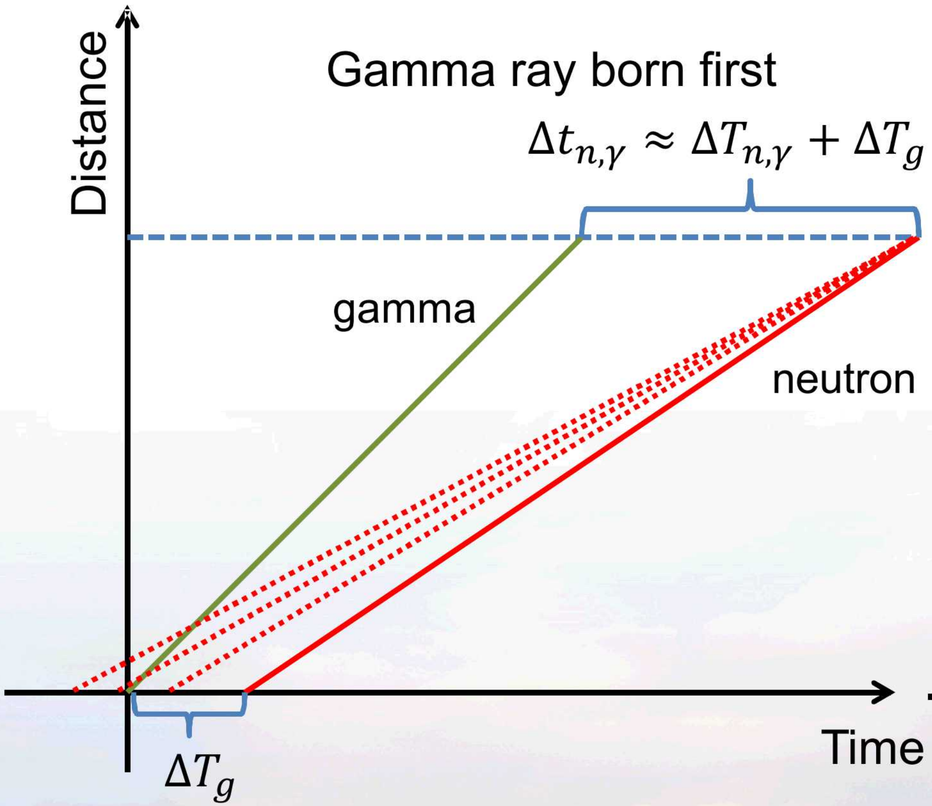
Distribution of times between fission events in a chain



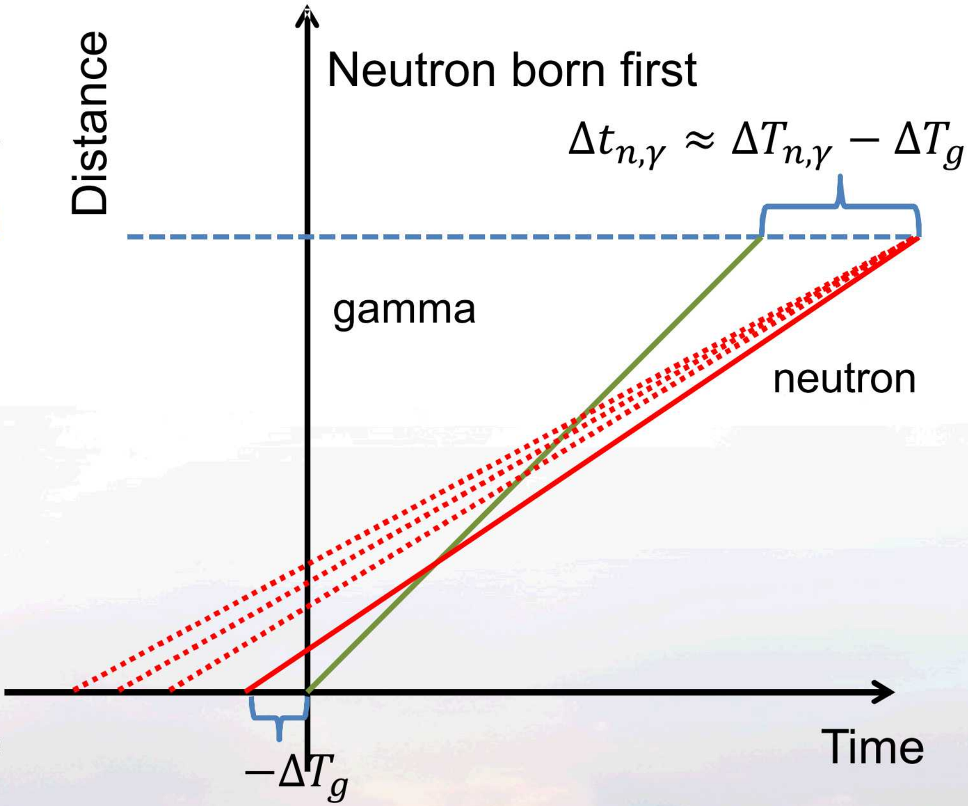
$$\Delta T_g > 0 \text{ if } f_1 = \gamma \,\&\, f_2 = n$$

$$\Delta T_g < 0 \text{ if } f_1 = n \,\&\, f_2 = \gamma$$

Fission Chains Gamma-Neutron Space-Time



$$\Delta t_{n,\gamma} - \Delta \mathrm{t_p} \le \Delta T_g$$



$$\Delta t_{n,\gamma} - \Delta \mathrm{t_p} \le -\Delta T_g$$

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TOFFEE Sensitivity to Fissile Material and Configuration

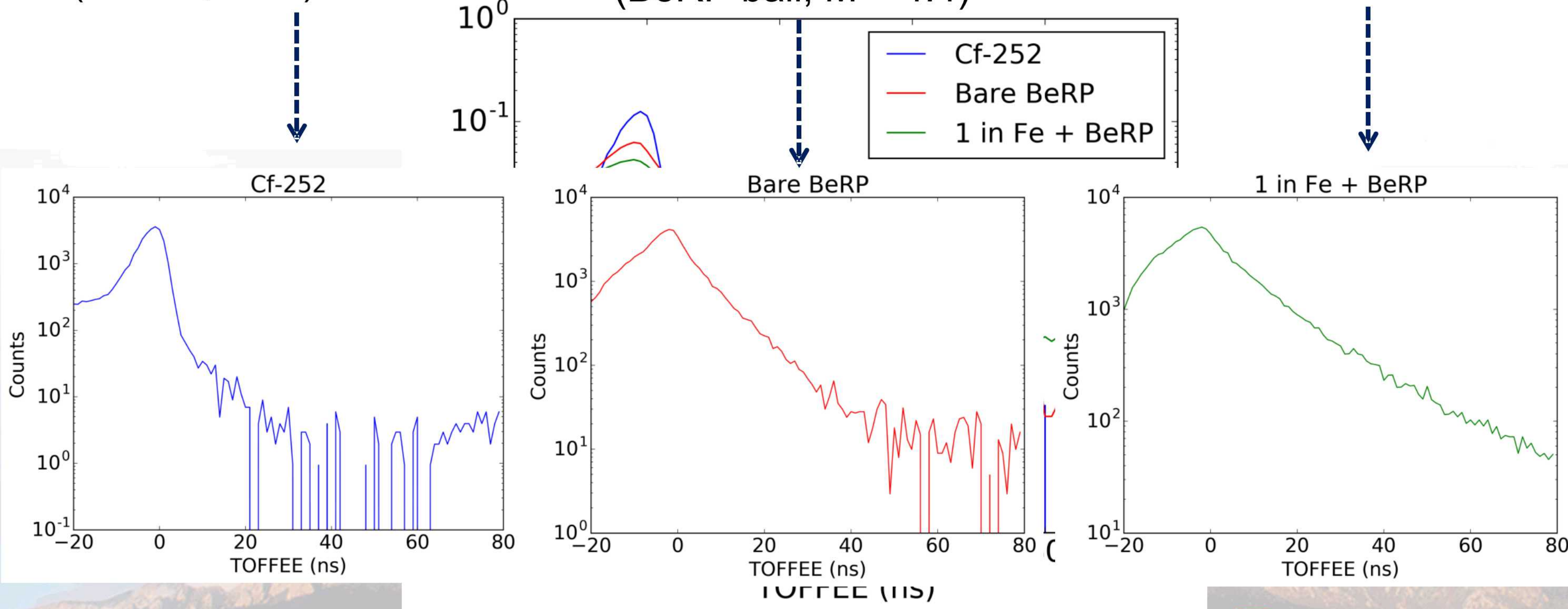
TOFFEE is sensitive to:

1. Fissile vs. fissionable
2. Fissile materials with different neutron multiplication
3. Bare vs. reflected fissile cores

Non-fissile fission source (Cf-252, M=0)

Fissile material (BeRP ball, M = 4.4)

Reflected Fissile material (BeRP ball, M = 6.48)



Extracting Physical Parameters from TOFFEE
Parametric Experiments and Simulations

- TOFFEE distribution was shown to discriminate between fissile assemblies
- A physical model is required to extract physical parameters by fitting the TOFFEE distribution
- Use point-kinetics model (energy and spatially independent), which describes the neutron population change, which tracks with the density of fissions in a fissile material

Measurements of BeRP ball
- Bare
- 0.5 in Steel
- 1.0 in Steel
- 1.5 in Steel
- 1.0 in Nickel

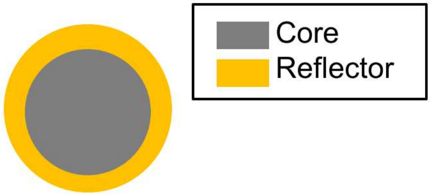
Simulations
- 0.5 – 6 in Steel
- 0.5 – 6 in Nickel
- 0.5 – 6 in Tungsten
- 0.5 – 6 in Aluminum

Two-region point kinetics

Core:

Neutron multiplication; Return from reflector; Loss in the core (abs & leakage)

$$\frac{dN_c}{dt} = \frac{k_c}{l_c} N_c + f_{rc} \frac{N_r}{l_r} - \frac{N_c}{l_c}$$



Influx from the core; Loss in the reflector

Reflector:

$$\frac{dN_r}{dt} = f_{cr} \frac{N_c}{l_c} - \frac{N_r}{l_r}$$

N_c & N_r: the number of neutrons in the fissile core and reflector regions
l_c & l_r: neutron lifetimes in the fissile core and reflector regions
k_c : multiplication factor in the fissile core region
$f = f_{cr} * f_{rc}$: fraction of neutrons that leak to reflector AND return to the core

Solution is a double exponential

$$N_c(t) = N_o[(1-R)e^{tr_1} + Re^{tr_2}]$$

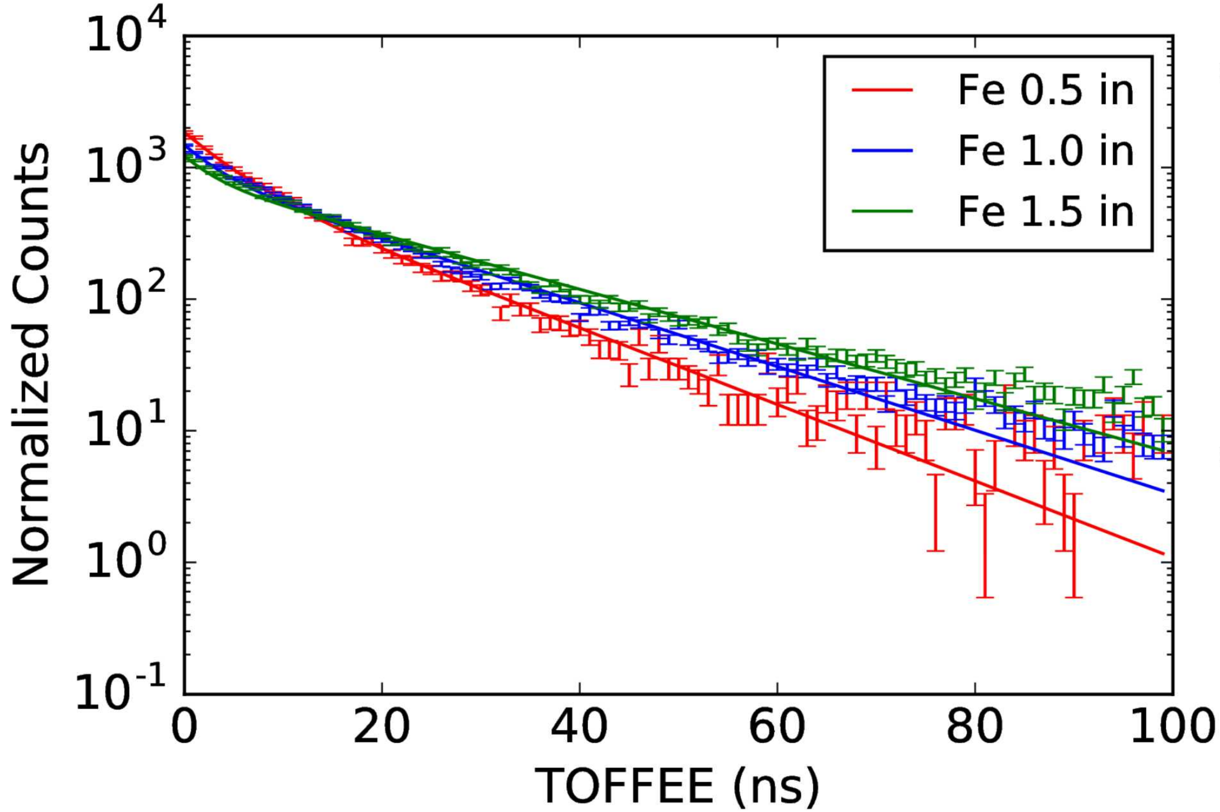
$$r_{1,2} = \frac{\pm\sqrt{4l_c l_r(f + k_c - 1) + \left(l_c - l_r(k_c - 1)\right)^2} - l_c + l_r(k_c - 1)}{2l_c l_r}$$

Scale Parameter: $R = \dfrac{r_1 - \alpha}{r_1 - r_2}$

Rate parameter (in a bare configuration): $\alpha = \dfrac{k_c - 1}{l_c}$

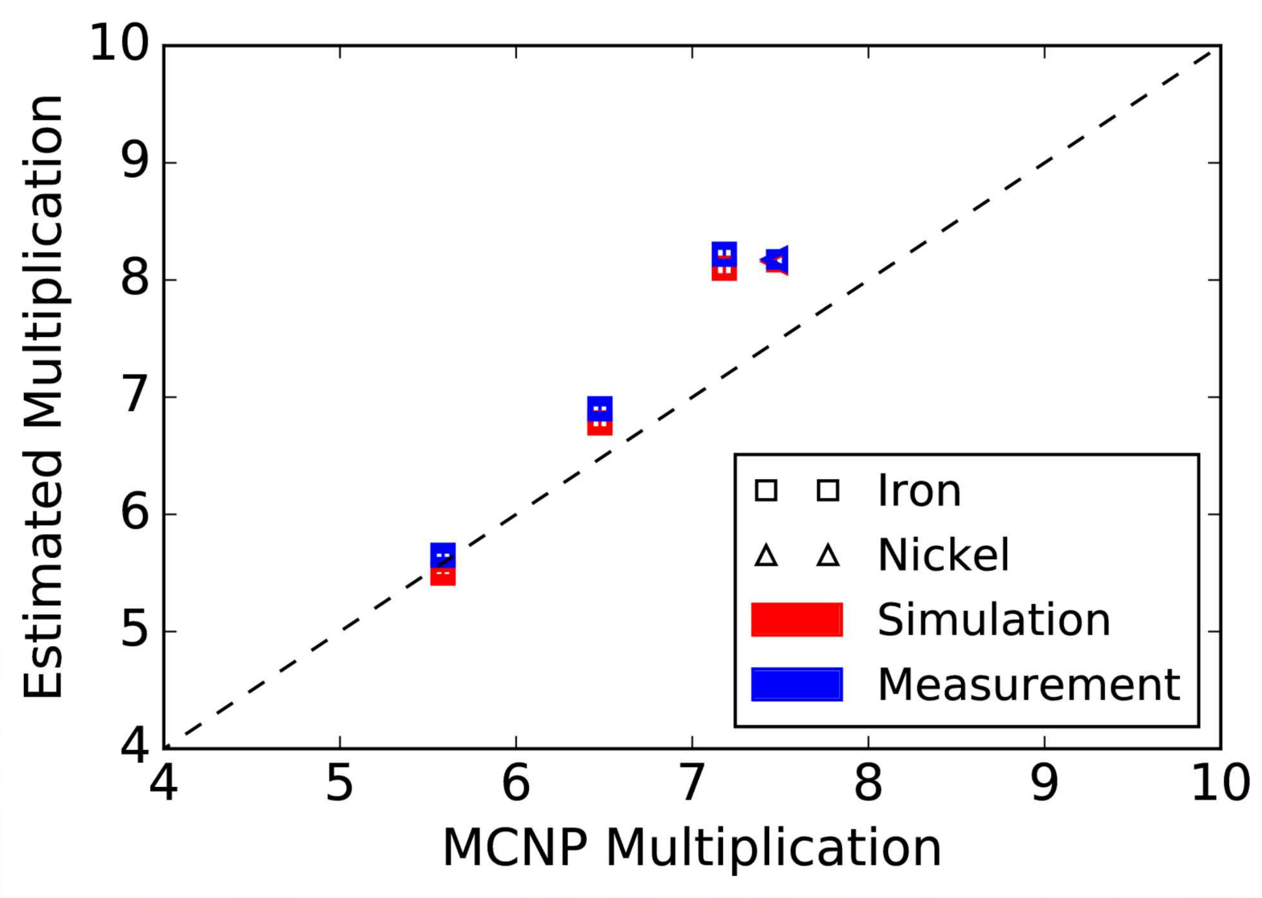
Fitting Strategy for Reflected Configurations

$$N_c(t) = N_o[(1-R)e^{tr_1} + Re^{tr_2}]$$



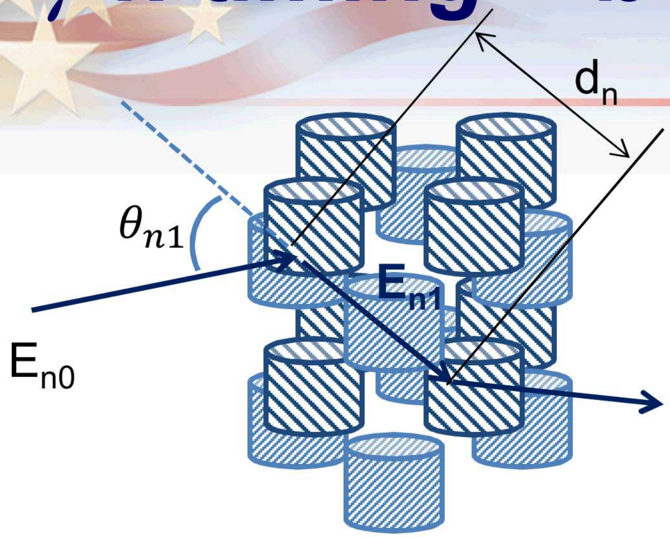
- Allowing reflection fraction (f) and neutron lifetime (l_r, l_c) produces muddled results
- Solution: fix k_c and l_c to what the core parameters are from the fit of the bare case

Estimating Multiplication of Reflected Assemblies
Measurement vs Simulation



$$k = \frac{k_c}{1 - f}$$

γ-n timing + better energy estimate



$$\Delta t_{n-\gamma} = \left(R_n \sqrt{\frac{m}{2E_{n0}}} - \frac{R_\gamma}{c} \right) + \Delta T_{f_1,f_2}$$

Time of arrival difference between γ and neutron	Time-of-flight difference between γ and neutron	Time difference between fissions in a chain

$$E_{n1} = \frac{m_n}{2} \times \frac{{d_n}^2}{TOF^2}$$

$$E_{n0} = E_p + E_{n1}$$

$$cos^2\theta_{n1} = \frac{E_{n1}}{E_{n0}}$$

MINER: the Mobile Imager of Neutrons for Emergency Response

- 16 independent 3”x3” EJ-309 liquid scintillator cells

$(\Delta t_{n-\gamma} - \Delta t_{En0})$ Distributions