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Title: ARPA-e Grid Optimization Competition, SCOPF Overview

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ARPA-e Grid Optimization Competition

SCOPF Overview

Carleton Coffrin

Advanced Network Science Initiative

https://lanl-ansi.github.io/

Managed by Triad National Security, LLC for the U.S. Department of Energy's NNSA

My Role in the Competition

- **Faux Competitor**
  - Early test driving of the platform and datasets
  - Develop and run various simple solutions approaches

- **Instance Analysis**
  - Searching for basic data errors and unexpected issues
  - Estimating problem difficulty (e.g. optimality gaps)

- **Preparing PowerModelsSCOPF.jl**
  - An open-source platform for R&D on SCOPF algorithms

Seeking Post-Doc / Post-Bac to help

What is the competition problem?

Why is it challenging?

Competition Problem Specification Document

- https://gocompetition.energy.gov/challenges/challenge-1/formulation
- **87 Pages (mathematical program 18-32)**
- **I will focus on key intuitions**
  - Skip lots of the details

GRID OPTIMIZATION (GO) COMPETITION

SCOPF Problem Formulation: Challenge 1

Grid Optimization Competition

Updated: April 9, 2019

1 Background

This document contains the official formulation that will be used for evaluation in Challenge 1 of the Grid Optimization (GO) Competition. Minor changes may occur within the formulation. Entrants will be notified when a new version is released. Changes are not expected to be of any significance, to cause a change in approach for the Entrants.

This formulation builds upon the formulation published in ARPA-E DE-FOA-0001952. **Entrants will be judged based on the current official formulation posted on the GO Competition website (this document, which is subject to change), not the formulation posted in DE-FOA-0001952.** Entrants are permitted and encouraged to use any alternative problem formulation and modeling convention within their own software (such as convex relaxation, decoupled power flow formulations, current-voltage formulations, etc.) in an attempt to produce an exact or approximate solution to this particular mathematical program. However, the judging of all submitted approaches must conform to the official formulation presented here.

The following mathematical programming problem is a type of a security-constrained (AC based) optimal power flow, or SCOPF. There are many ways to formulate the SCOPF problem; this document presents multiple equivalent options for specified constraints. Entrants are strongly encouraged to study this formulation precisely and to engage with the broader community if anything is not clear (please see the FAQs and forum on the GO Competition website, `https://gocompetition.energy.gov/`).

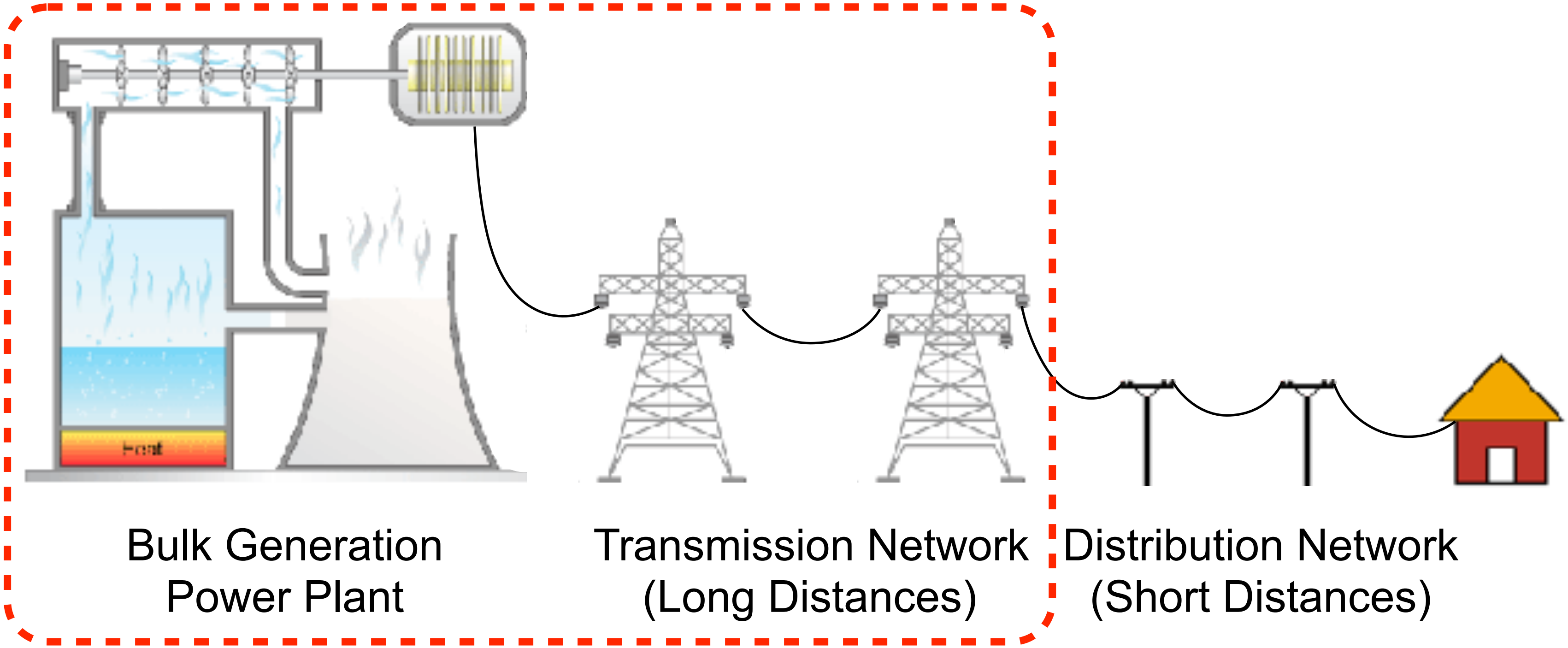
This SCOPF problem is defined to be an alternating current (AC) formulation, which is based on a bus-branch power system network model and considers security constraints. In general, Entrants are tasked with determining the optimal dispatch and control settings for power generation and grid control equipment in order to minimize the cost of operation, subject

...

First Question:
What is Security Constrained Optimal Power Flow?

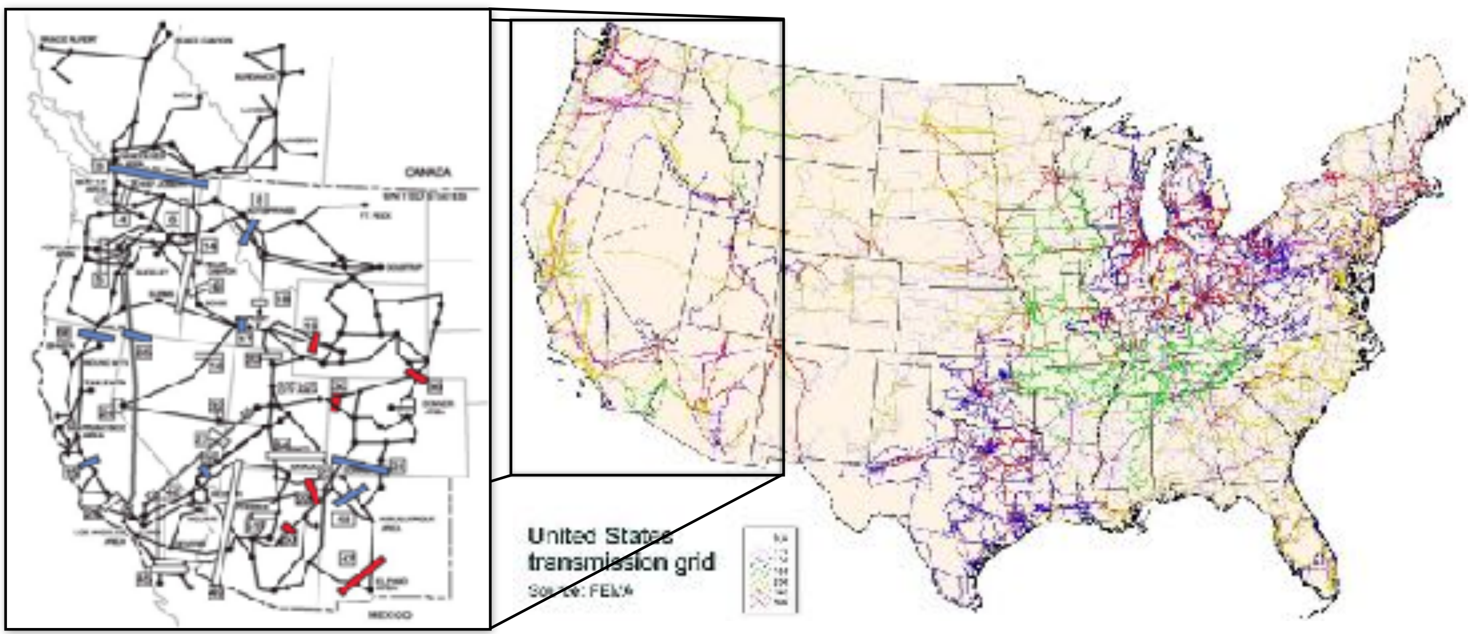
Start with Optimal Power Flow "building block"

Optimization of Bulk Electricity Transmission

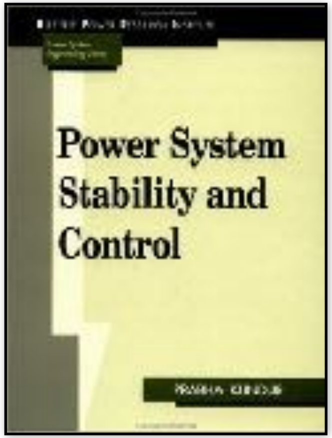


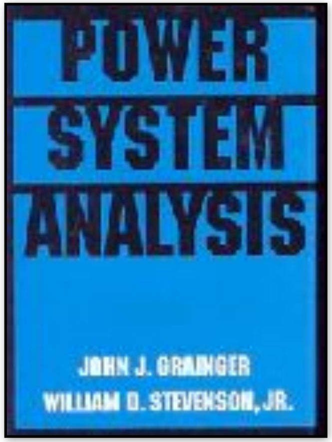
The United States Transmission System

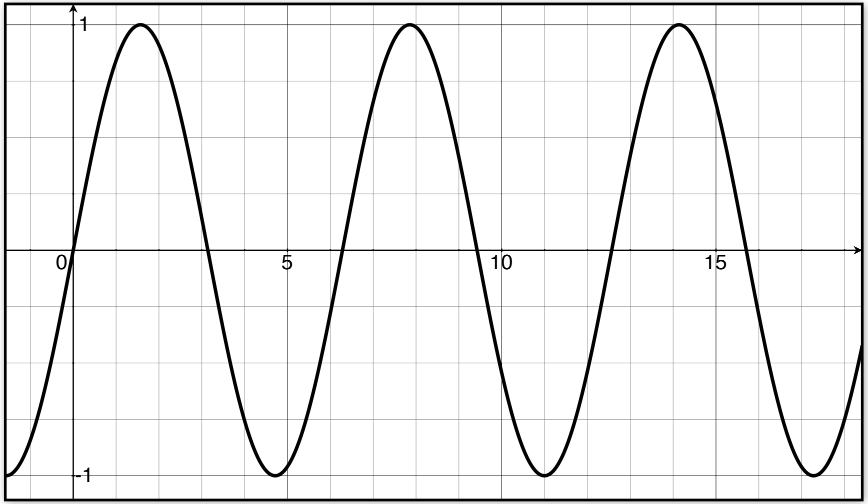
- **A graph with approximately 100,000 nodes and 130,000 edges**
  - Planar (mostly)
  - Operated on sub-graphs around 30,000-60,000 nodes



Alternating Current makes everything Complex







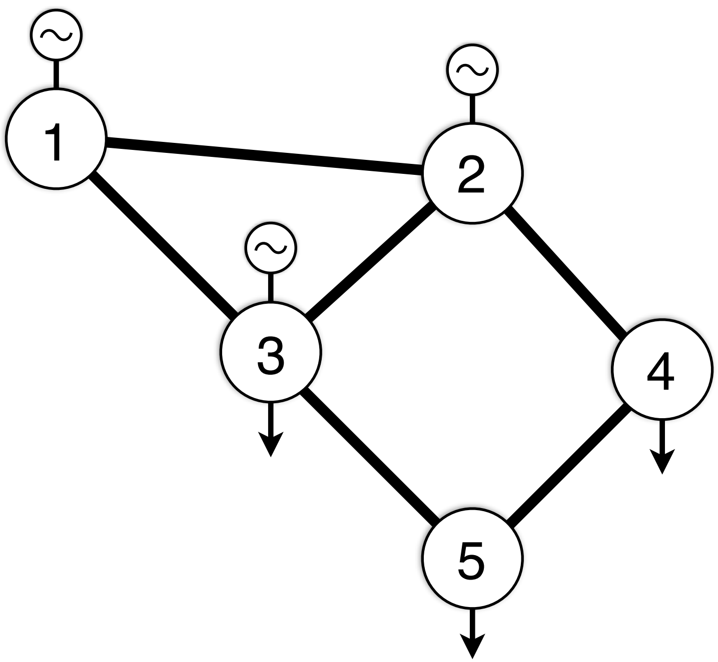
x Typical O.R.

$$X = x + \boldsymbol{i}y$$

Power Systems O.R.

What is Optimal Power Flow?

- **A natural and fundamental transmission network optimization problem**
  - analogous to minimum-cost network flow
- **Given**
  - A Power Transmission Network
  - Required Demands (i.e. customer loads)
  - Generator Cost Functions
- **Minimize**
  - The cost of generating power
- **Subject To**
  - Meeting the required demands
  - Network and generator operation constraints



Physics of power networks leads to non-convex nonlinear problem!

A Conical OPF Formulation

variables: — Power Generation, Node Voltage

$$S_i^g \ \ \forall i \in G$$

$$V_i \ \ \forall i \in N$$

minimize: — Minimize Generation Cost (convex)

$$\sum_{i \in G} f(S_i^g)$$

subject to:

Voltage Bounds:

$$(\boldsymbol{v}_i^{\boldsymbol{l}})^2 \leq V_i V_i^* \leq (\boldsymbol{v}_i^{\boldsymbol{u}})^2 \ \ \forall i \in N$$

Generation Bounds:

$$\boldsymbol{S}_i^{\boldsymbol{gl}} \leq S_i^g \leq \boldsymbol{S}_i^{\boldsymbol{gu}} \ \ \forall i \in G$$

Nodal Power Balance:

$$\sum_{k \in G_i} S_k^g - \boldsymbol{S}_i^d = \sum_{(i,j) \in E_i \cup E_i^R} S_{ij} \ \ \forall i \in N$$

Line Power Flow (Ohm's Law):

$$S_{ij} = \boldsymbol{Y}_{ij}^* V_i V_i^* - \boldsymbol{Y}_{ij}^* V_i V_j^* \ \ (i,j) \in E \cup E^R$$

Line Flow Limit:

$$|S_{ij}|^2 \leq (\boldsymbol{s}_{ij}^{\boldsymbol{u}})^2 \ \ \forall (i,j) \in E \cup E^R$$

The Competition OPF Variant

variables:

$$S_i^g \;\; \forall i \in G$$

$$V_i \;\; \forall i \in N$$

$$\Delta S_i \;\; \forall i \in N$$

$$\delta s_{ij}^u \;\; \forall (i,j) \in E \cup E^R$$

Slack Variables

minimize:

$$\sum_{i \in G} f(S_i^g) + \sum_{i \in N} \boldsymbol{\rho} |\Delta S_i| + \sum_{(i,j) \in E} \boldsymbol{\rho} \delta s_{ij}^u$$

subject to:

$$(\boldsymbol{v}_i^{\boldsymbol{l}})^2 \le V_i V_i^* \le (\boldsymbol{v}_i^{\boldsymbol{u}})^2 \;\; \forall i \in N$$

$$\boldsymbol{S}_i^{\boldsymbol{gl}} \le S_i^g \le \boldsymbol{S}_i^{\boldsymbol{gu}} \;\; \forall i \in G$$

Hard Constraints
(variable bounds)

$$\Delta S_i + \sum_{k \in G_i} S_k^g - \boldsymbol{S}_i^d = \sum_{(i,j) \in E_i \cup E_i^R} S_{ij} \;\; \forall i \in N$$

$$S_{ij} = \boldsymbol{Y}_{ij}^* V_i V_i^* - \boldsymbol{Y}_{ij}^* V_i V_j^* \;\; (i,j) \in E \cup E^R$$

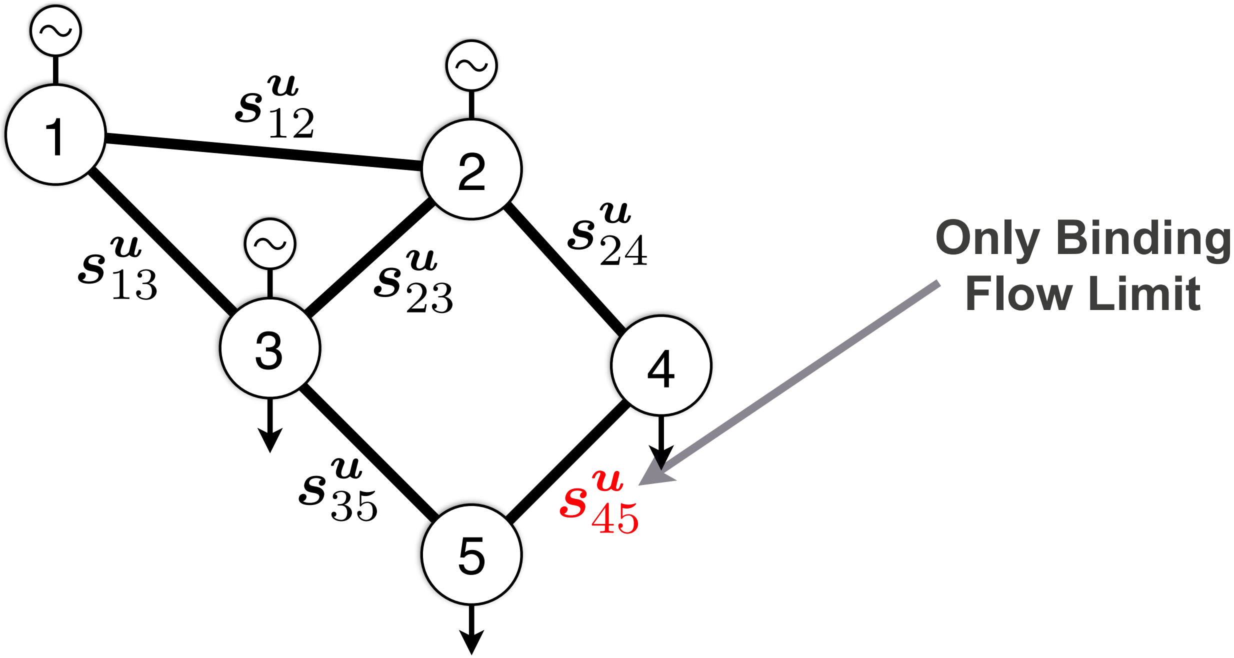
$$|S_{ij}|^2 \le (\boldsymbol{s}_{ij}^{\boldsymbol{u}} + \delta s_{ij}^u)^2 \;\; \forall (i,j) \in E \cup E^R$$

Soft Constraints

Key OPF Insight

- **Only a very small set of line flow constraints bind in the optimal solution**

The physics of power networks leads to very few binding flow constraints



Solution Approaches and Scalability

- **Interior Point Algorithms (e.g. Ipopt, KNITRO)**
  - Second-Order Gradient Decent like approaches
  - Only provide locally optimality, but seems to be very near globally optimal in practice
- **Sequential Linear/Quadratic Programming (e.g. gurobi, cplex)**
  - Linearized around an operating point; Optimize; Repeat
  - Only provide locally optimality, but seems to be sufficient in practice
- **Problem Scales**
  - 10,000 node network in 60-90 seconds
  - Cases curated here, https://github.com/power-grid-lib/pglib-opf

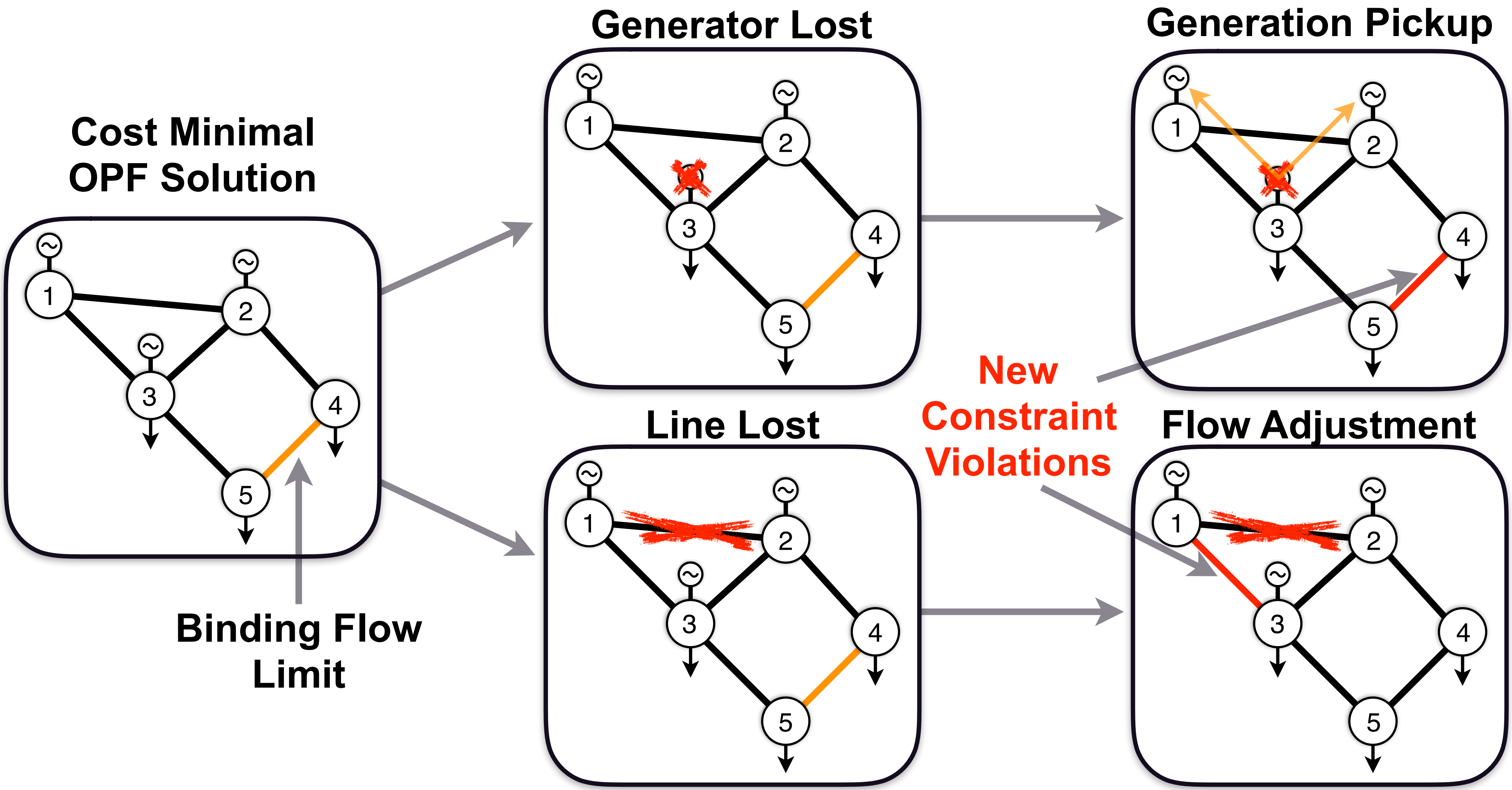
First Question Redux:
What is Security Constrained Optimal Power Flow?

Balance Cost and Resilience

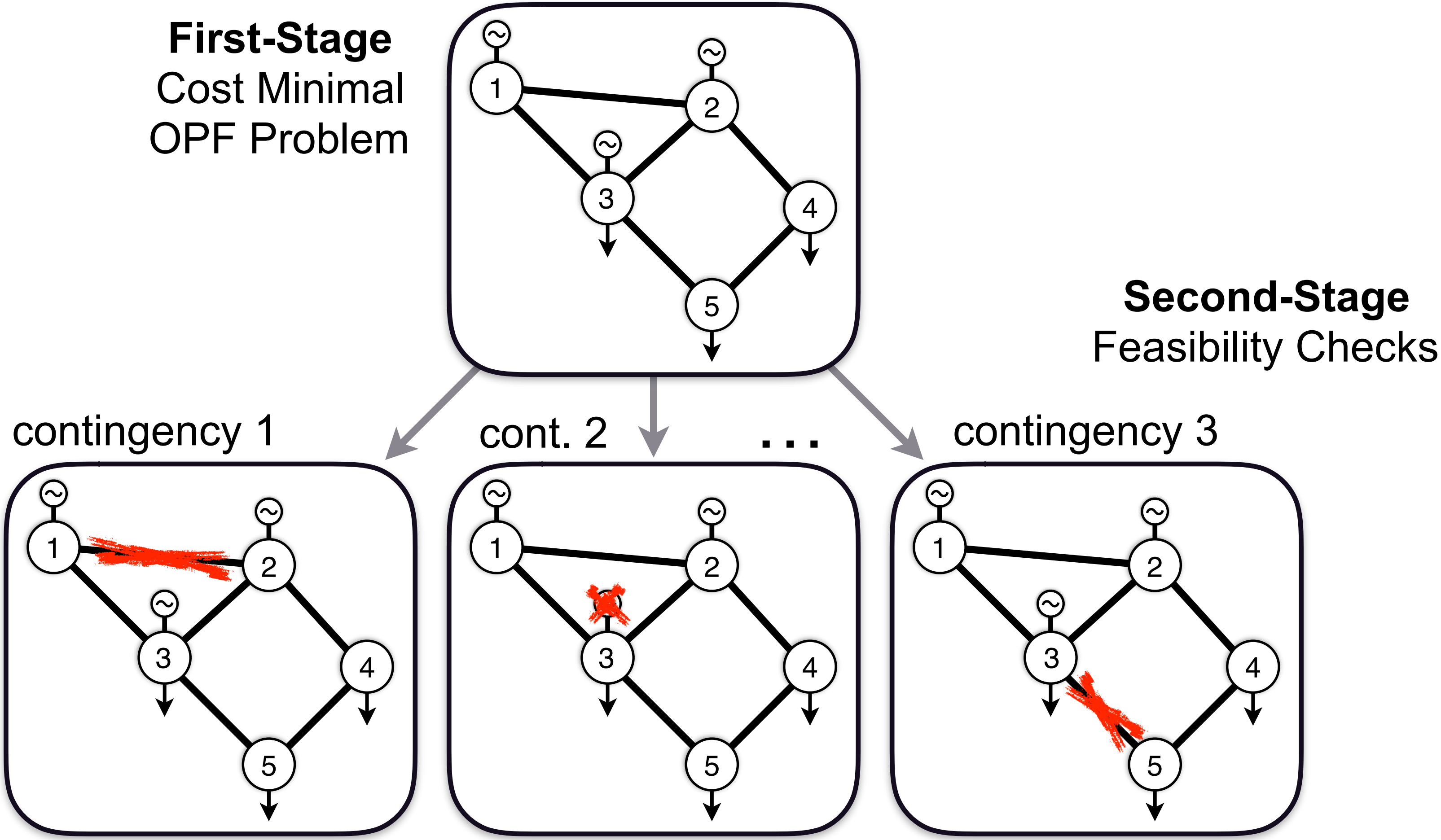
- **Power Networks are regulated to be "N-1 Secure"**
  - Intention, the network can withstand the spontaneous failure of any single component during daily operations

- **Lots of details in practice**
  - Focus on a specific subset of component failures
  - Many constraints can be exceeded for a short amount of time
  - What time scale? (e.g. seconds vs minutes)
  - Available recourse actions?

The Core Challenge of Security Constraints



A Two-Stage Mathematical Program



The Second Stage Model

$\forall c \in Contingencies$

variables:

$$S^{g}_{ci} \;\; \forall i \in G_c$$
$$V_{ci} \;\; \forall i \in N$$
$$\Delta S_{ci} \;\; \forall i \in N$$
$$\delta s^{u}_{cij} \;\; \forall (i,j) \in E_c \cup E^R_c$$

subject to:

$$generator\_response\_active(\Re(S^g_i), \Re(S^g_{ci})) \;\; \forall i \in G_c$$
$$generator\_response\_reactive(V_i, V_{ci}, \Im(S^g_{ci})) \;\; \forall i \in G_c$$
$$(\boldsymbol{v}^{\boldsymbol{l}}_i)^2 \le V_{ci}V^*_{ci} \le (\boldsymbol{v}^{\boldsymbol{u}}_i)^2 \;\; \forall i \in N$$
$$\boldsymbol{S}^{\boldsymbol{gl}}_i \le S^g_{ci} \le \boldsymbol{S}^{\boldsymbol{gu}}_i \;\; \forall i \in N$$
$$\Delta S_{ci} + \sum_{k \in G_{ci}} S^g_{ck} - \boldsymbol{S}^d_i = \sum_{(i,j) \in E_c \cup E^R_c} S_{cij} \;\; \forall i \in N$$
$$S_{cij} = \boldsymbol{Y}^*_{ij} V_{ci} V^*_{ci} - \boldsymbol{Y}^*_{ij} V_{ci} V^*_{cj} \;\; (i,j) \in E_c \cup E^R_c$$
$$|S_{cij}|^2 \le (\boldsymbol{s}^{\boldsymbol{u}}_{ij} + \delta s^u_{cij})^2 \;\; \forall (i,j) \in E_c \cup E^R_c$$

- **Similar to first-stage feasibility set**
- **Response constraints “link” to first stage decisions**

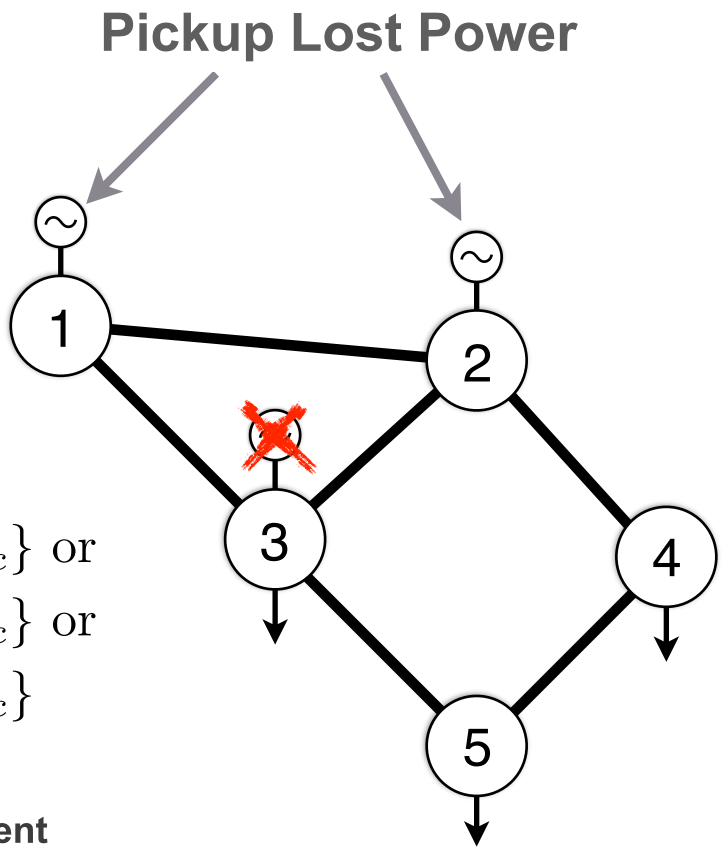
Second Stage Generator Active Power Response

- **Pre-defined linear response function**
  - Non-convex due to generation saturation

$$S^g_{ci} = p^g_{ci} + \boldsymbol{i} q^g_{ci}$$

$$\boldsymbol{p}^{\boldsymbol{gl}}_i \leq p^g_{ci} \leq \boldsymbol{p}^{\boldsymbol{gu}}_i$$

$$p^g_{ci} = \begin{cases} \{\boldsymbol{p}^{\boldsymbol{gl}}_i \leq p^g_{ci} \leq \boldsymbol{p}^{\boldsymbol{gu}}_i & \text{and } p^g_{ci} = p^g_i + \boldsymbol{\alpha}_i \Delta_c\} \text{ or} \\ \{p^g_{ci} = \boldsymbol{p}^{\boldsymbol{gu}}_i & \text{and } p^g_{ci} \leq p^g_i + \boldsymbol{\alpha}_i \Delta_c\} \text{ or} \\ \{p^g_{ci} = \boldsymbol{p}^{\boldsymbol{gl}}_i & \text{and } p^g_{ci} \geq p^g_i + \boldsymbol{\alpha}_i \Delta_c\} \end{cases}$$



Formulations on pg. 28 in the specification document

Second Stage Generator Reactive Power Response

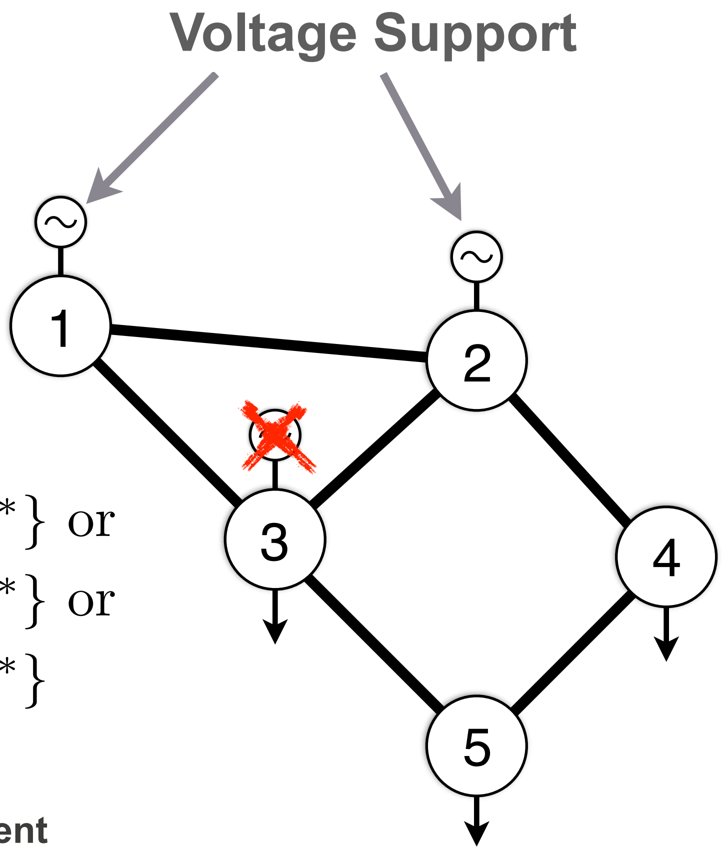
- **Maintain the first-stage voltage, if possible**

$$S^g_{ci} = p^g_{ci} + \boldsymbol{i} q^g_{ci}$$

$$\boldsymbol{q}^{\boldsymbol{gl}}_i \leq q^g_{ci} \leq \boldsymbol{q}^{\boldsymbol{gu}}_i$$

$$(\boldsymbol{v}^{\boldsymbol{l}}_i)^2 \leq V_{ci} V^*_{ci} \leq (\boldsymbol{v}^{\boldsymbol{u}}_i)^2$$

$$q^g_{ci} = \begin{cases} \{\boldsymbol{q}^{\boldsymbol{gl}}_i \leq q^g_{ci} \leq \boldsymbol{q}^{\boldsymbol{gu}}_i & \text{and } V_{ci} V^*_{ci} = V_i V^*_i\} \text{ or} \\ \{q^g_{ci} = \boldsymbol{q}^{\boldsymbol{gu}}_i & \text{and } V_{ci} V^*_{ci} \leq V_i V^*_i\} \text{ or} \\ \{q^g_{ci} = \boldsymbol{q}^{\boldsymbol{gl}}_i & \text{and } V_{ci} V^*_{ci} \geq V_i V^*_i\} \end{cases}$$



Formulations on pg. 30 in the specification document

Competition Objective Function (simplified)

- **Recall the Soft-OPF Objective**

$$\sum_{i \in G} f(S_i^g) + \sum_{i \in N} \boldsymbol{\rho} |\Delta S_i| + \sum_{(i,j) \in E} \boldsymbol{\rho} \delta s_{ij}^u$$

- **Soft-SCOPF Objective**

$$\sum_{i \in G} f(S_i^g) + \sum_{i \in N} \boldsymbol{\rho} |\Delta S_i| + \sum_{(i,j) \in E} \boldsymbol{\rho} \delta s_{ij}^u + 1/|C| \sum_{c \in C} \left(\sum_{i \in N} \boldsymbol{\rho} |\Delta S_{ci}| + \sum_{(i,j) \in E} \boldsymbol{\rho} \delta s_{cij}^u \right)$$

So What?

Is a two-stage (MI)NLP that hard?

Practical SCOPF

- **Real-world power networks**
  - 30,000 - 60,000 nodes
  - 36,000 - 72,000 edges
  - 10,000 - 20,000 contingencies

AND solve in < 45 minutes

- **Writing down the SCOPF mathematical program as discribed**
  - 1,000,000,000 - 5,000,000,000 decision variables
  - comparable number of constraints

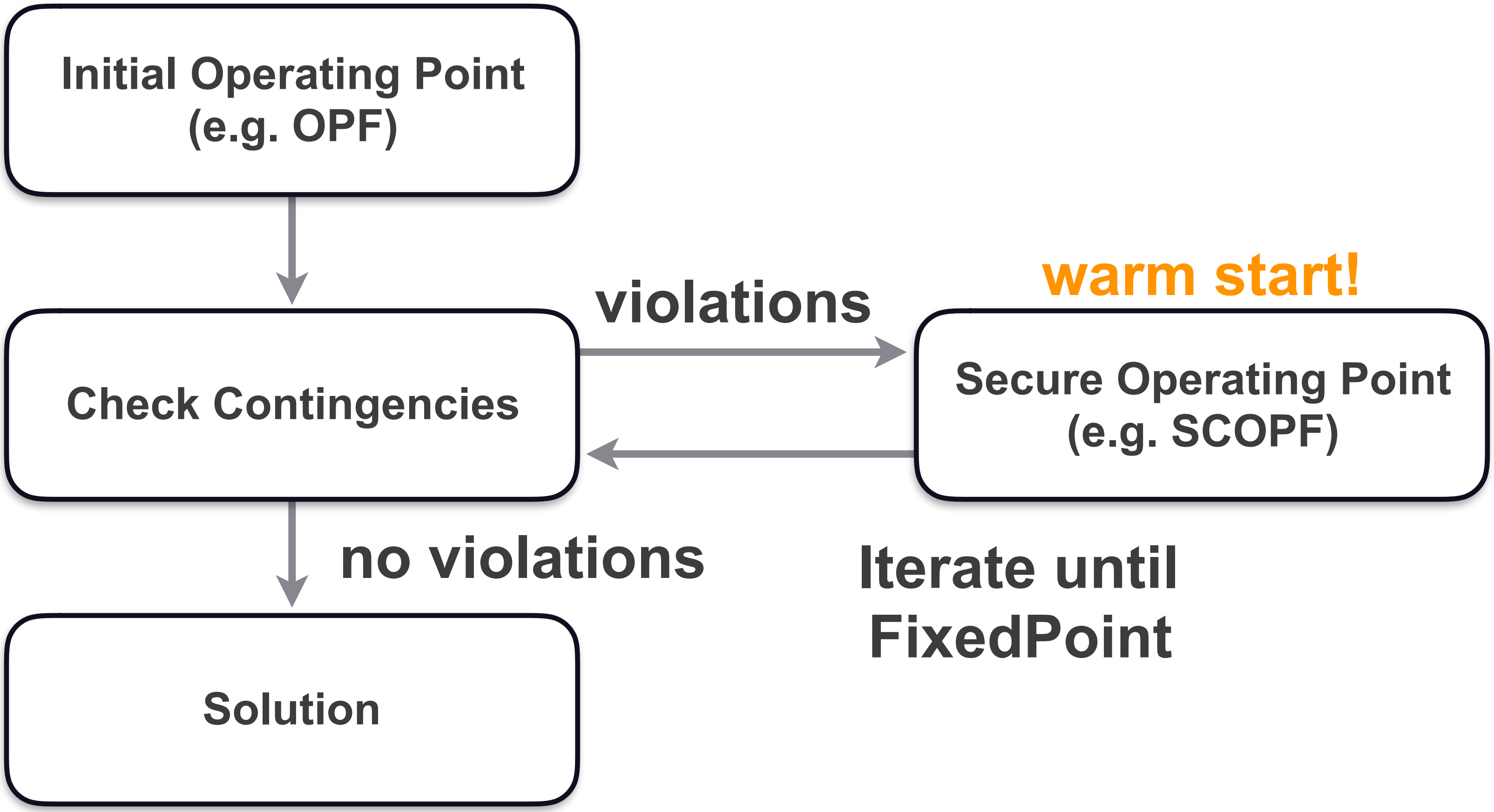
- **One example from the open competition datasets**
  - 30,000 buses; 10,810 contingencies
  - approximately 1,500,000,000 decision variables

O.R. Super Heroes

SCOPF Three Core Insights

- **A very small subset of contingencies include binding constraints**
  - suggests a constraint generation approach
  - contingency screening / filtering
- **In binding contingencies, only a very few flow constraints are binding**
  - similar to the OPF intuition that was mentioned
- **In very large networks, only a localized area is impacted by N-1 contingencies**
  - area-specific network reductions are possible in contingencies

Prototypical SCOPF Algorithm



Practical Example

- **One competition problem, Trial2_Offline-10x14/Network_70O-422, scenario_11**
  - Nodes 2312, Edges 3013, 995 contingencies
  - Approx. 10.5 Million decision variables
  - 3 Million line flow constraints

- **Contingency Screening (approx. 120 seconds per round)**
- **4 iterations to convergence**

- **3 Active generation outage contingencies**
- **5 Active line outage contingencies**
- **Only 8 active line flow constrains in contingencies! (0.00026%)**

Practical Example

- **Another competition problem, Trial2_Offline-10x14/ Network_09O-064, scenario_1**
  - Nodes 4918, Edges 6727, 5060 contingencies
  - Approx. 117.8 Million decision variables
  - 34 Million line flow constraints

- **Contingency Screening (approx. 1200 seconds per round)**
- **7 iterations to convergence**

- **10 Active generation outage contingencies**
- **58 Active line outage Contingencies**
- **Only 69 active line flow constrains in contingencies! (0.00018%)**

SCOPF In Practice

- **Linear approximations greatly improve scalability**
  - Commercial LP/QP solvers
  - Linearity results in lots of modeling “tricks”

- **DC Power Flow Model**

$$S_{ij} = \boldsymbol{Y}_{ij}^* V_i V_i^* - \boldsymbol{Y}_{ij}^* V_i V_j^*$$

$$p_{ij} = \boldsymbol{g}_{ij} v_i^2 - \boldsymbol{g}_{ij} v_i v_j \cos(\theta_i - \theta_j) - \boldsymbol{b}_{ij} v_i v_j \sin(\theta_i - \theta_j)$$

$$p_{ij} \approx -\boldsymbol{b}_{ij}(\theta_i - \theta_j)$$

Question: Is this approximation necessary and/or appropriate?

A Note on Solution Validation

- **The SCOPF competition has two phases**
- **Phase 1, solve the SCOPF problem**
  - 10-45 minutes depending on the division
  - can skip lots of unimportant contingencies
- **Phase 2, demonstrate that each contingency is secure**
  - Cannot skip any contingencies
  - 2 wall-clock seconds per contingency (but have 6 computers with 24 cores each; 288 seconds per contingency in theory)
- **Both tasks are computationally challenging given time limits**

GRID OPTIMIZATION (GO) COMPETITION

Shameless Plug

- **PowerModels.jl**
  - Transmission network modeling in Julia / JuMP

- **PowerModelsSCOPF.jl (forthcoming)**
  - Study SCOPF instances with billions of variables!
  - Compares AC and DC formulations
  - A variety of contingency filters
  - A variety of recourse models
  - Parallel contingency filters
  - Solution validation tools

Seeking Post-Doc
Post-Bac

Thanks!