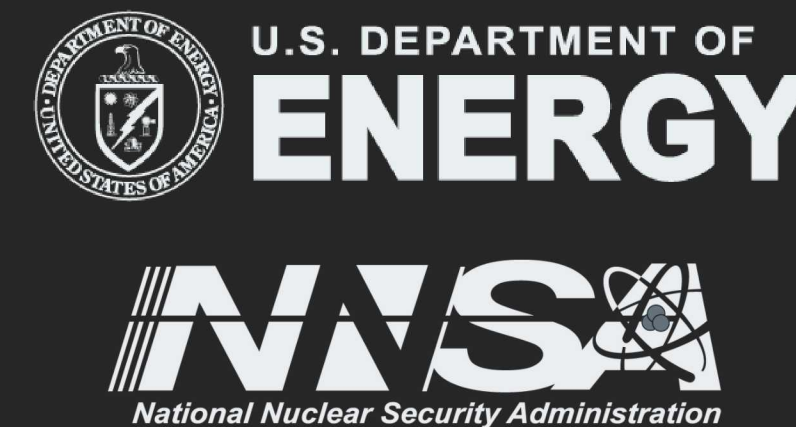


Compressed Optimization of Device Architectures (CODA) for Semiconductor Quantum Devices

Adam Frees

John King Gamble, Daniel R. Ward, Robin Blume-Kohout, M. A. Eriksson,
Mark Friesen, and S. N. Coppersmith
March 6, 2018

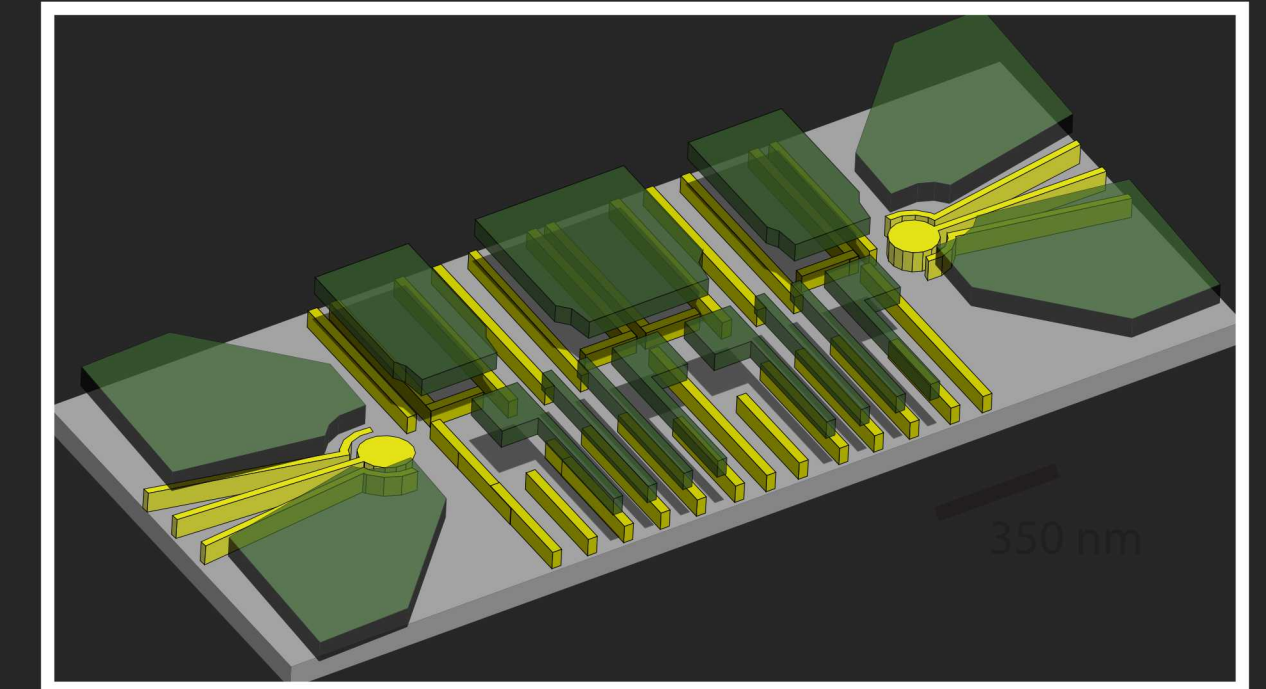
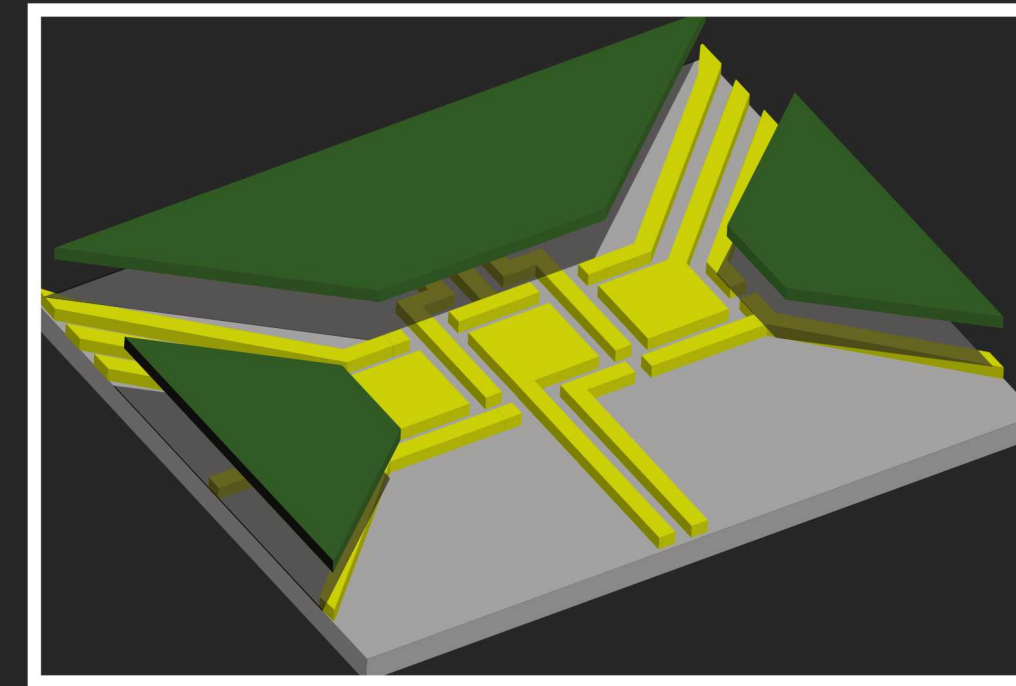


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The goal: to develop a robust tool and scalable tool for optimizing the tuning and design of semiconductor devices

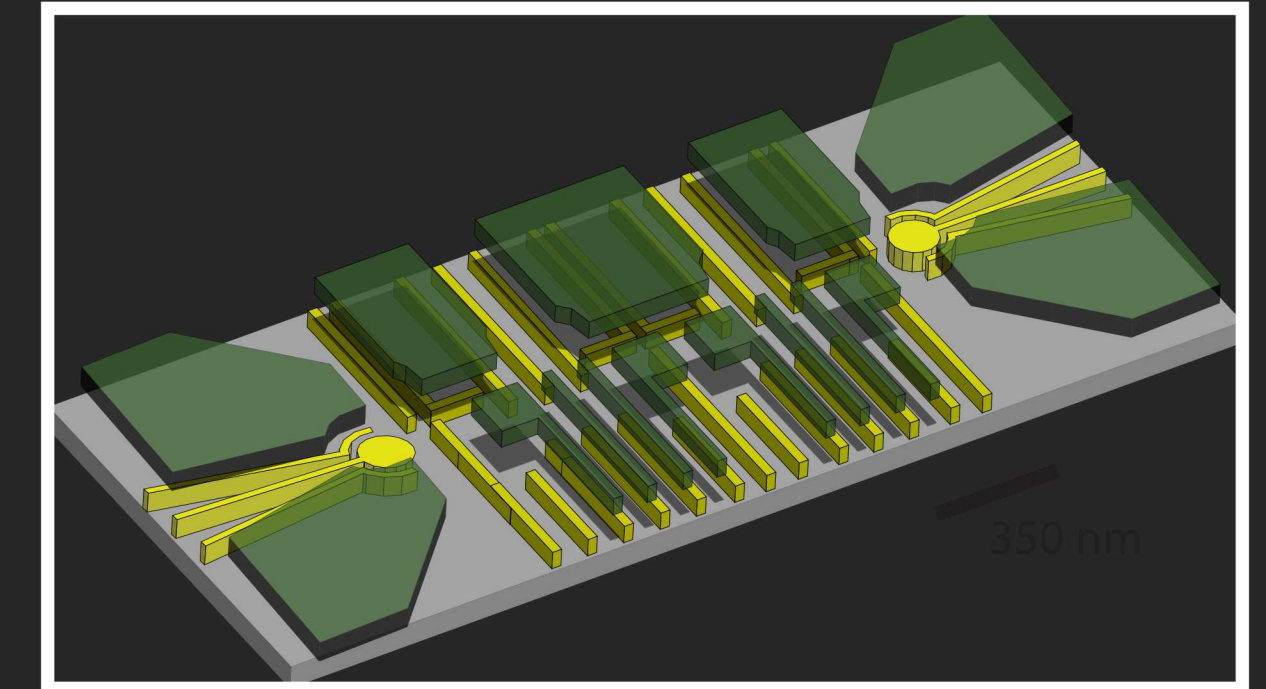
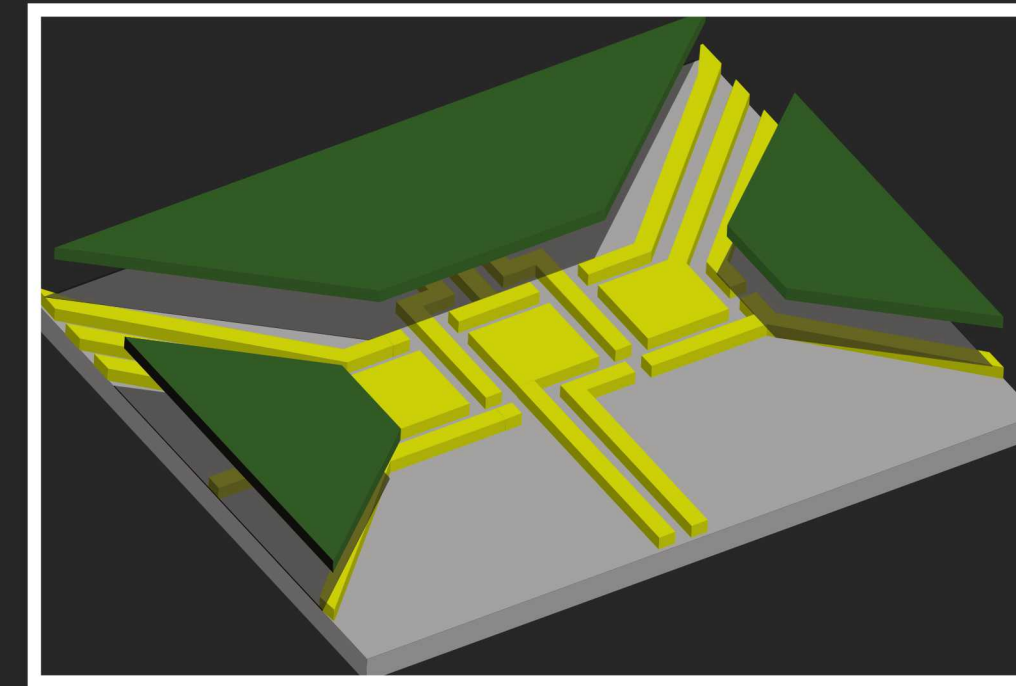
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- In particular, designing devices which are easily "tunable" can become challenging
 - "Tunable" - able to independently control a quantity of interest (e.g. tunnel coupling) by changing voltages on a small number of electrodes
- The CODA procedure can automatically and objectively compare the "tunability" of different device designs



A metric for comparing different ways to tune a device can be used to compare the "tunability" of different devices.

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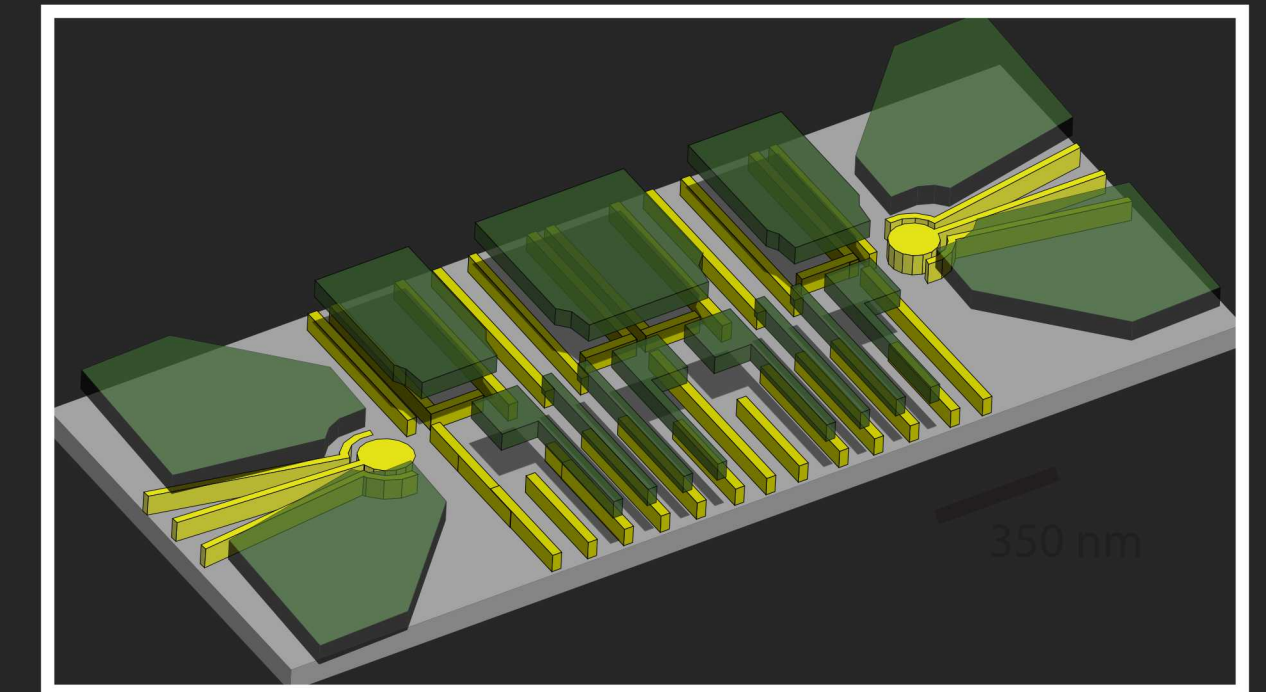
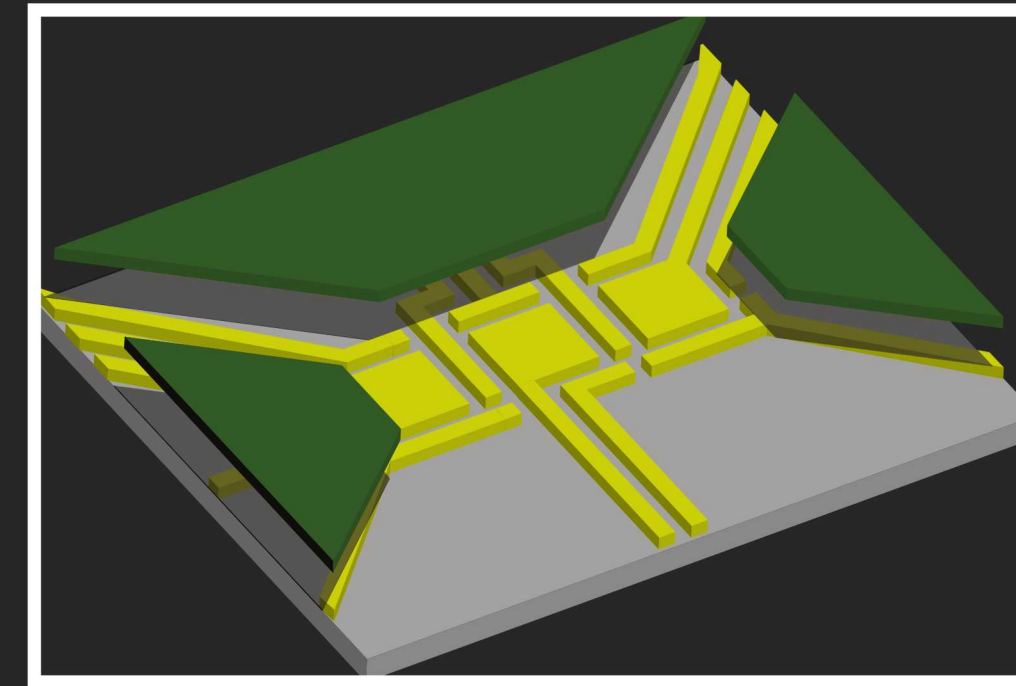
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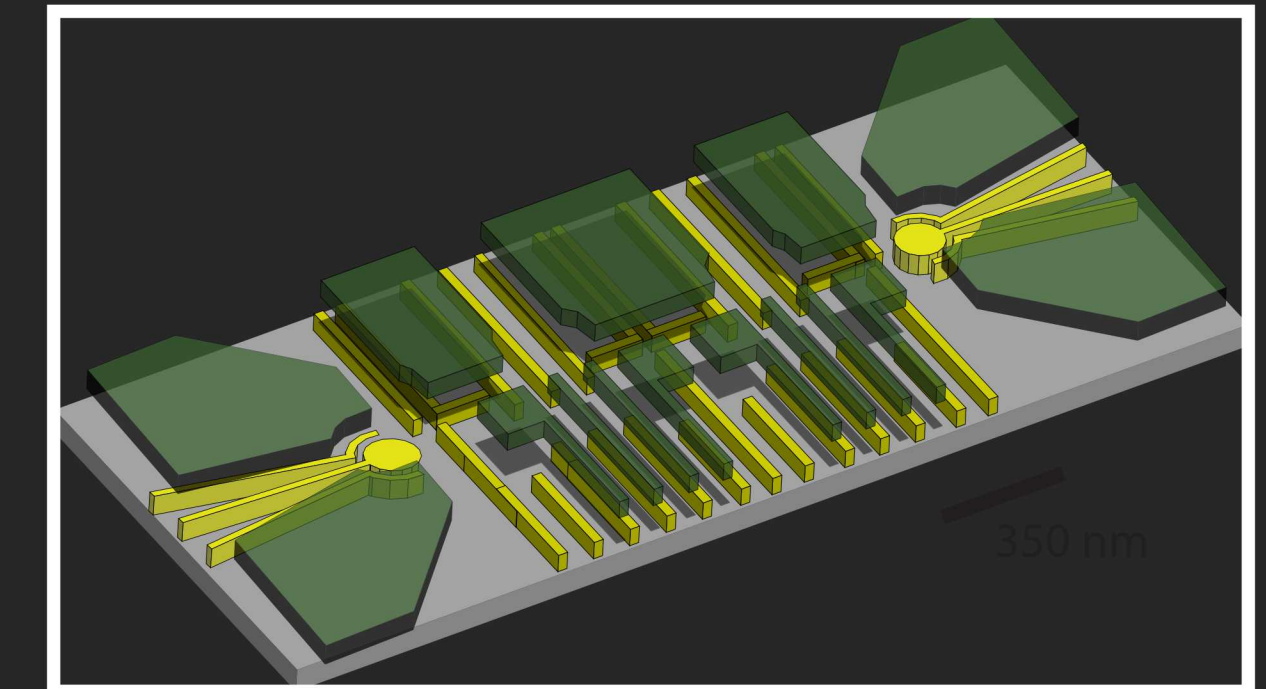
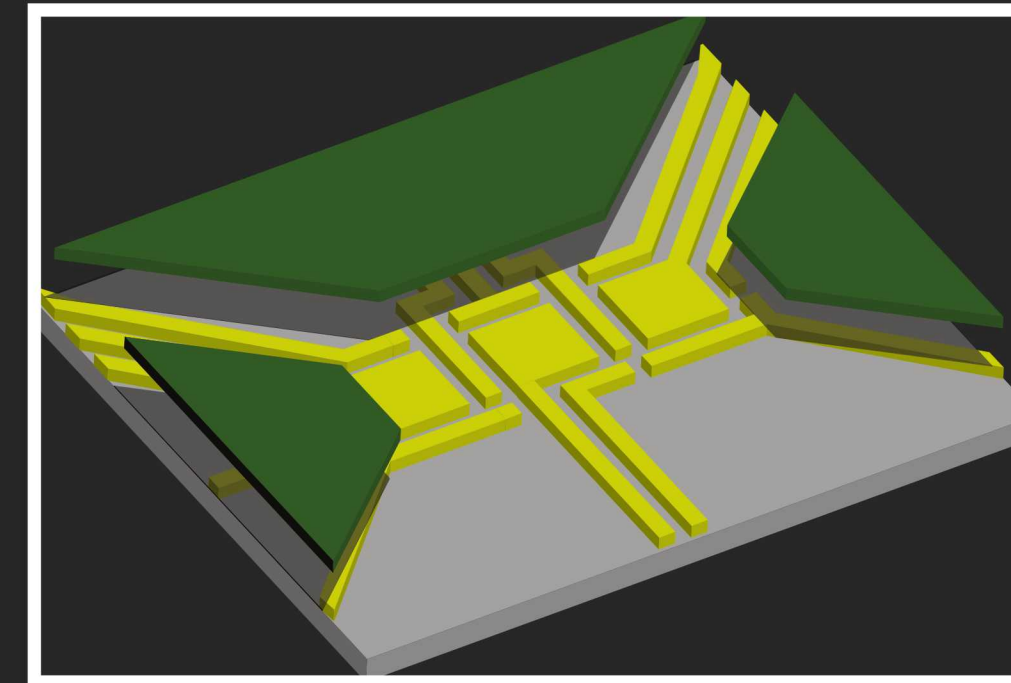
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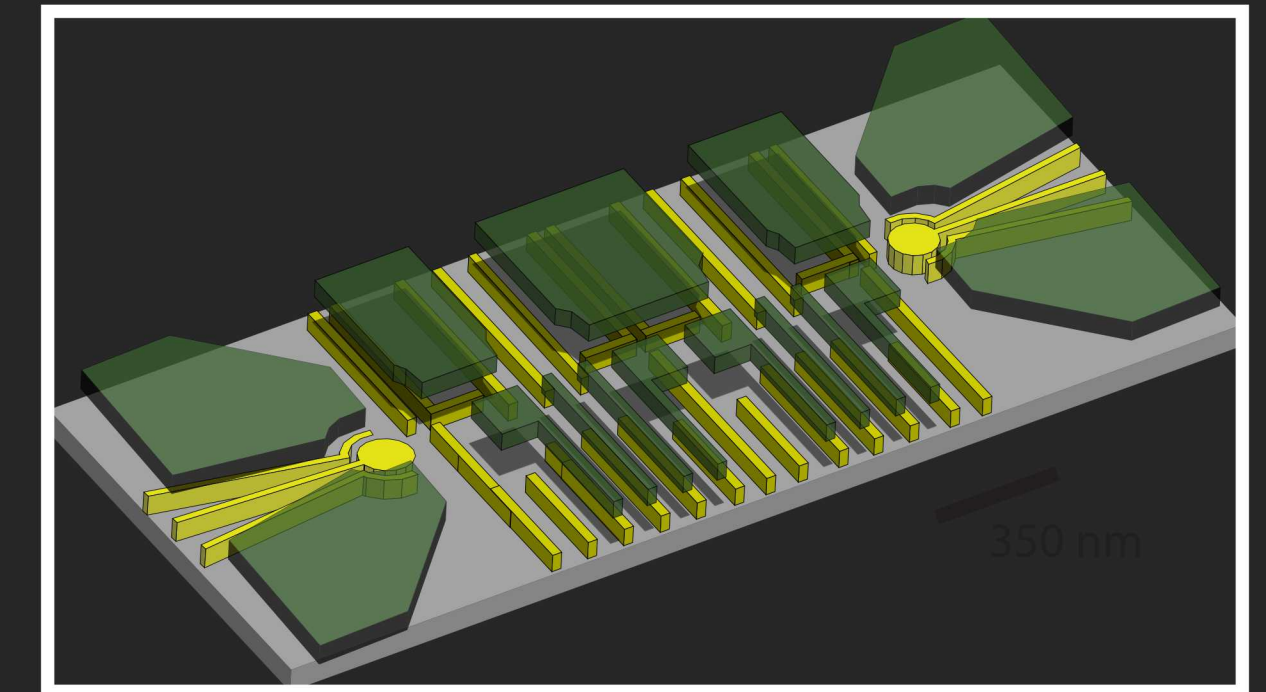
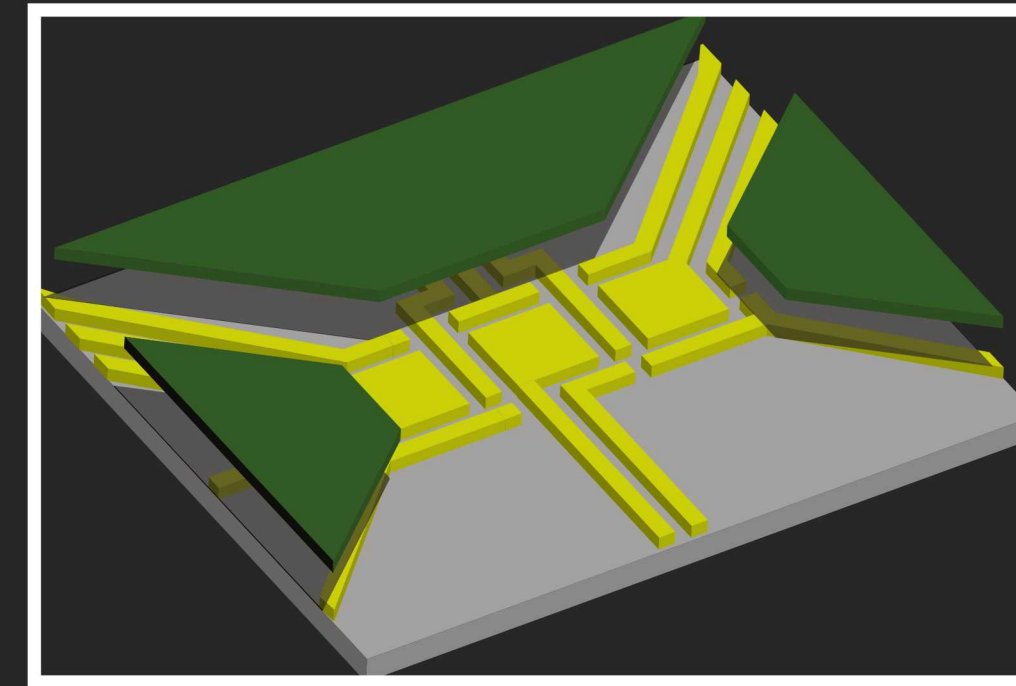
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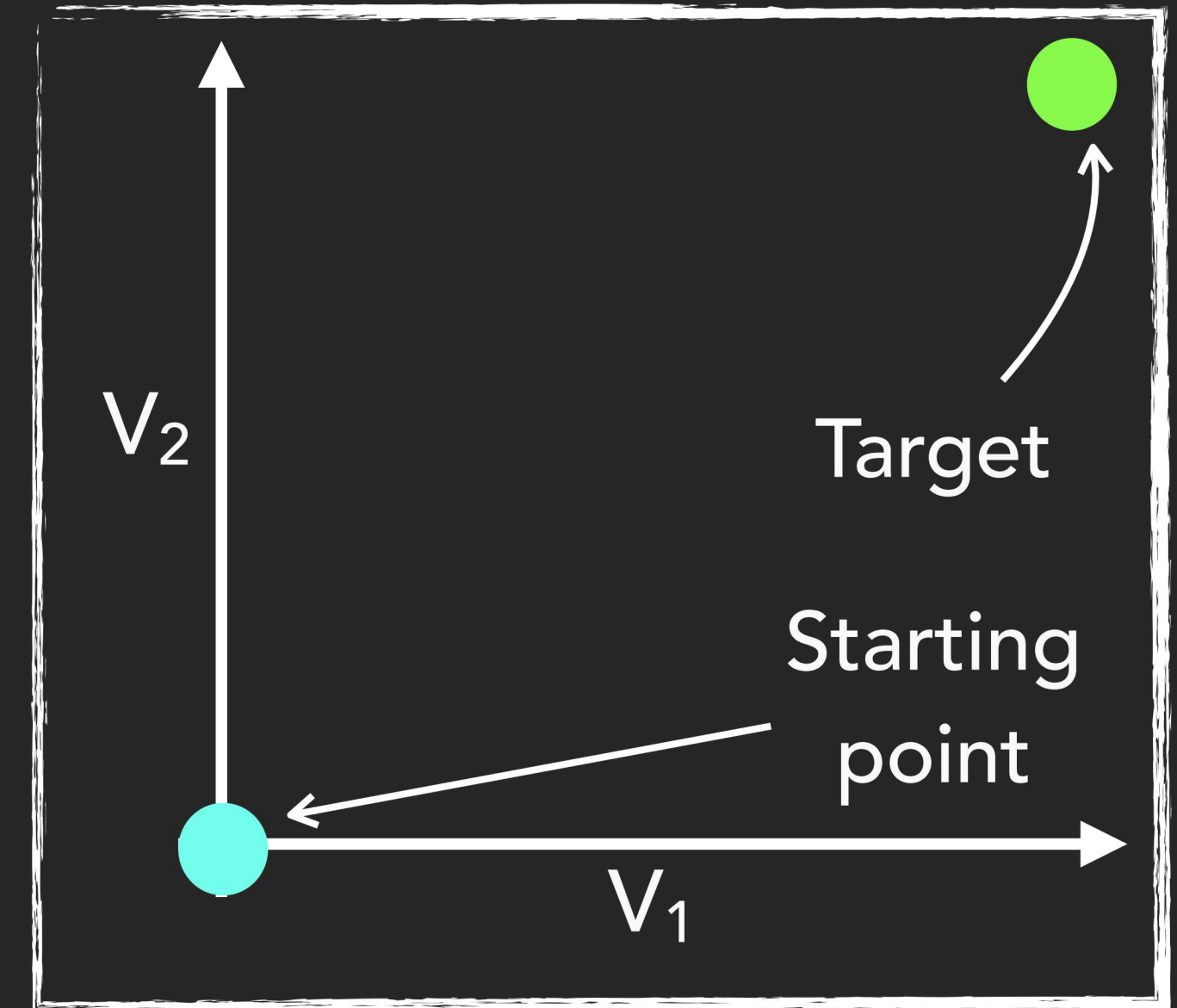
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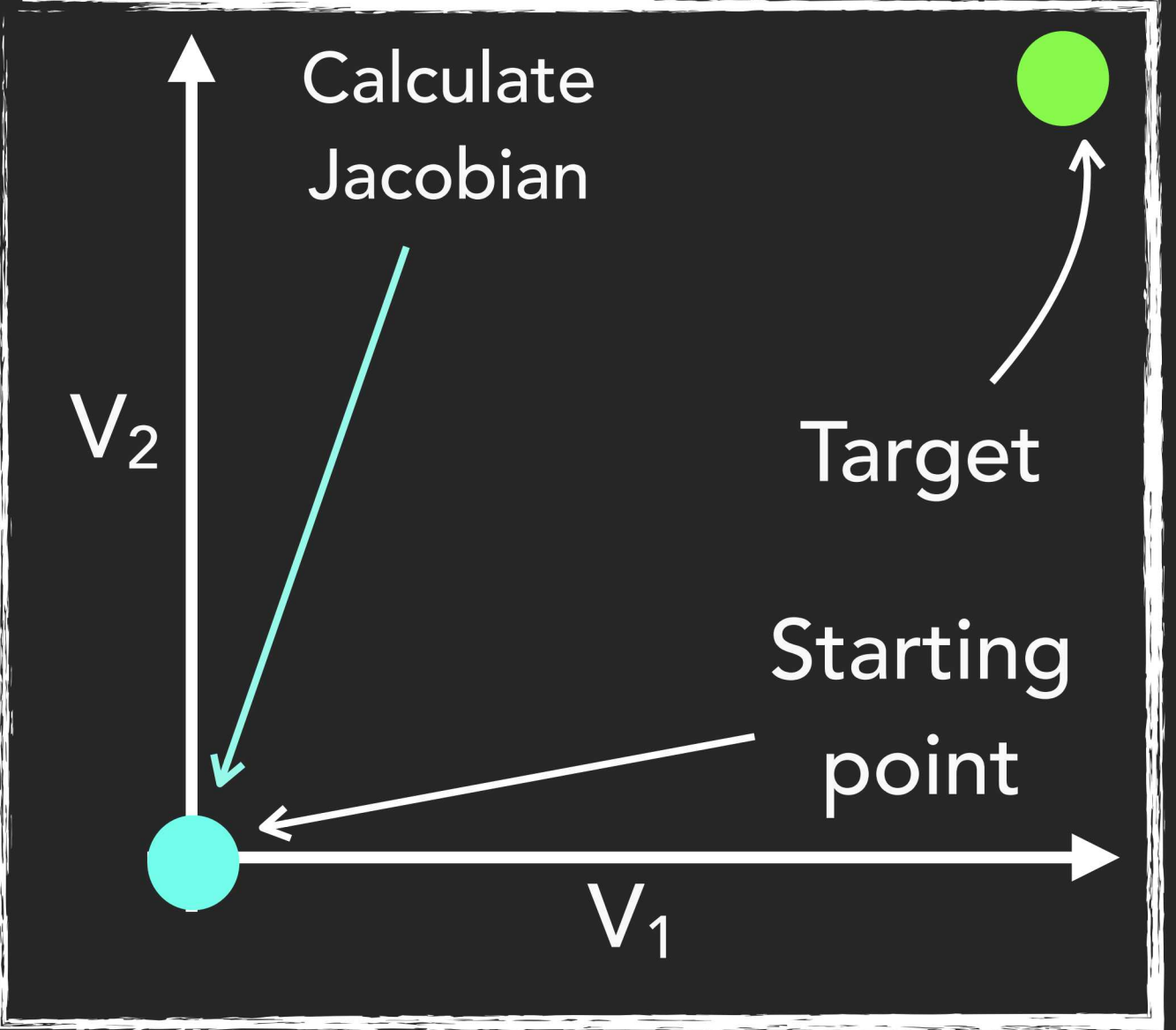
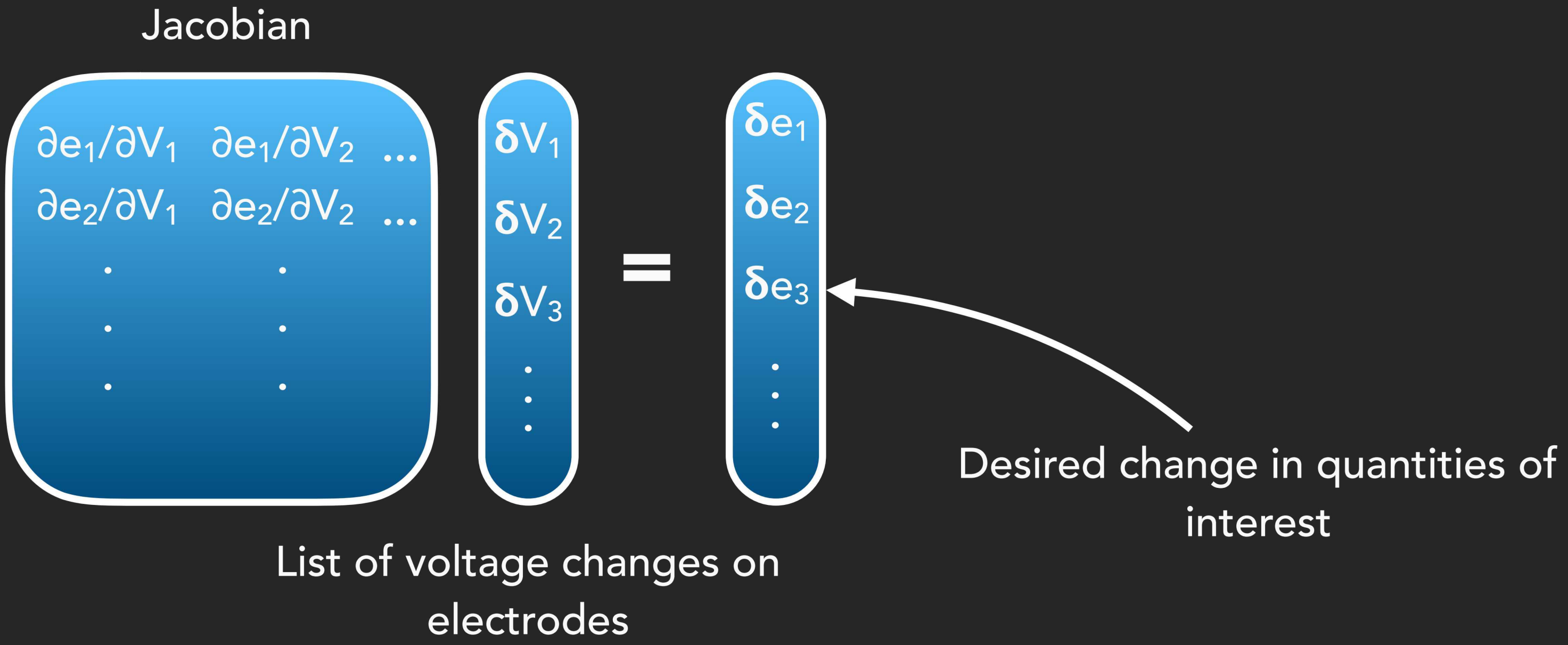
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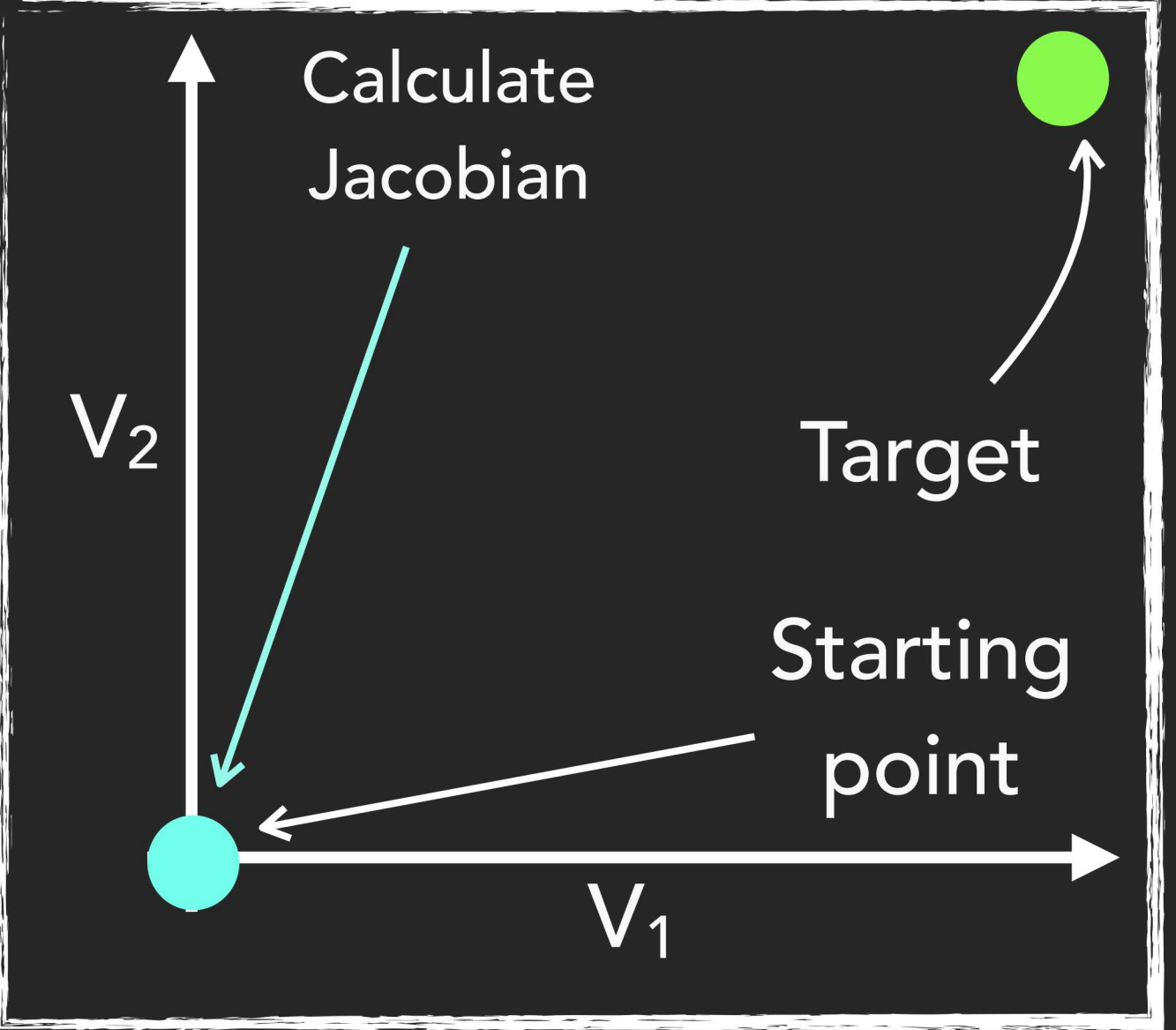
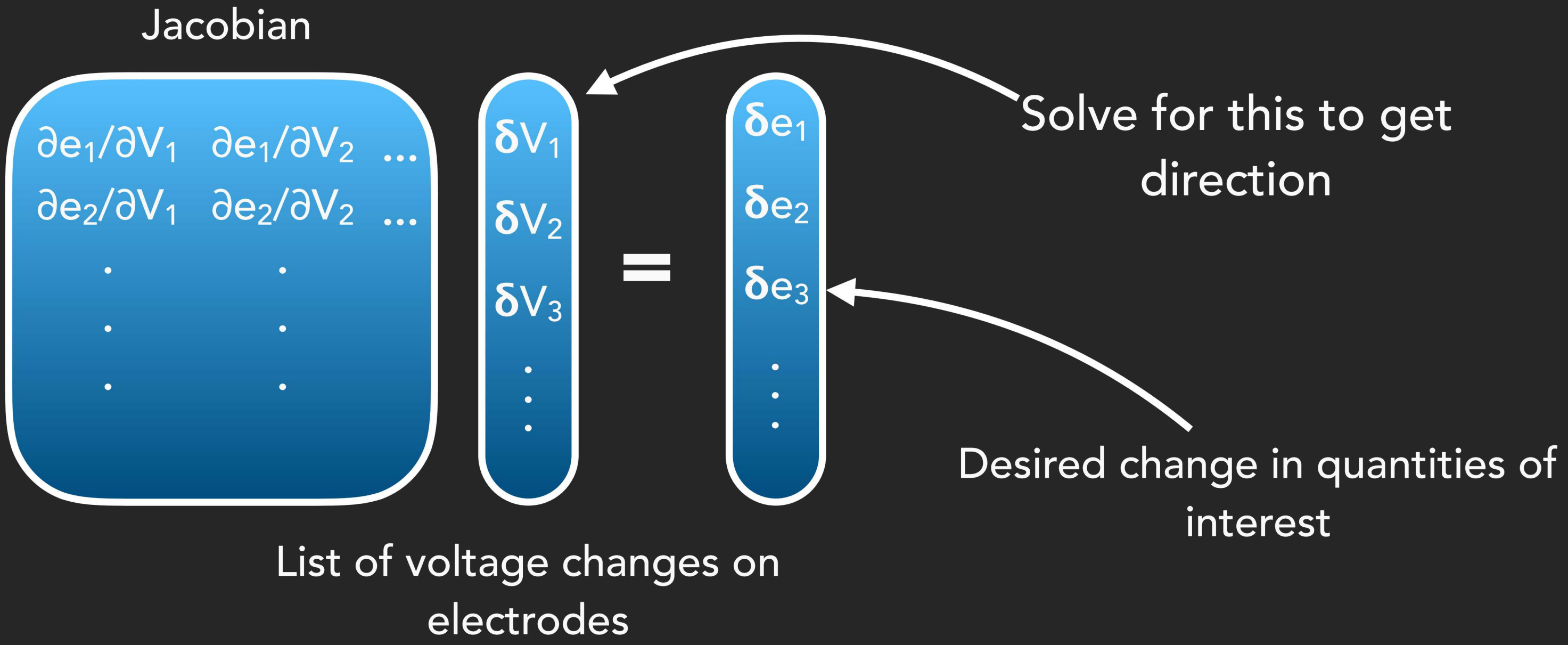
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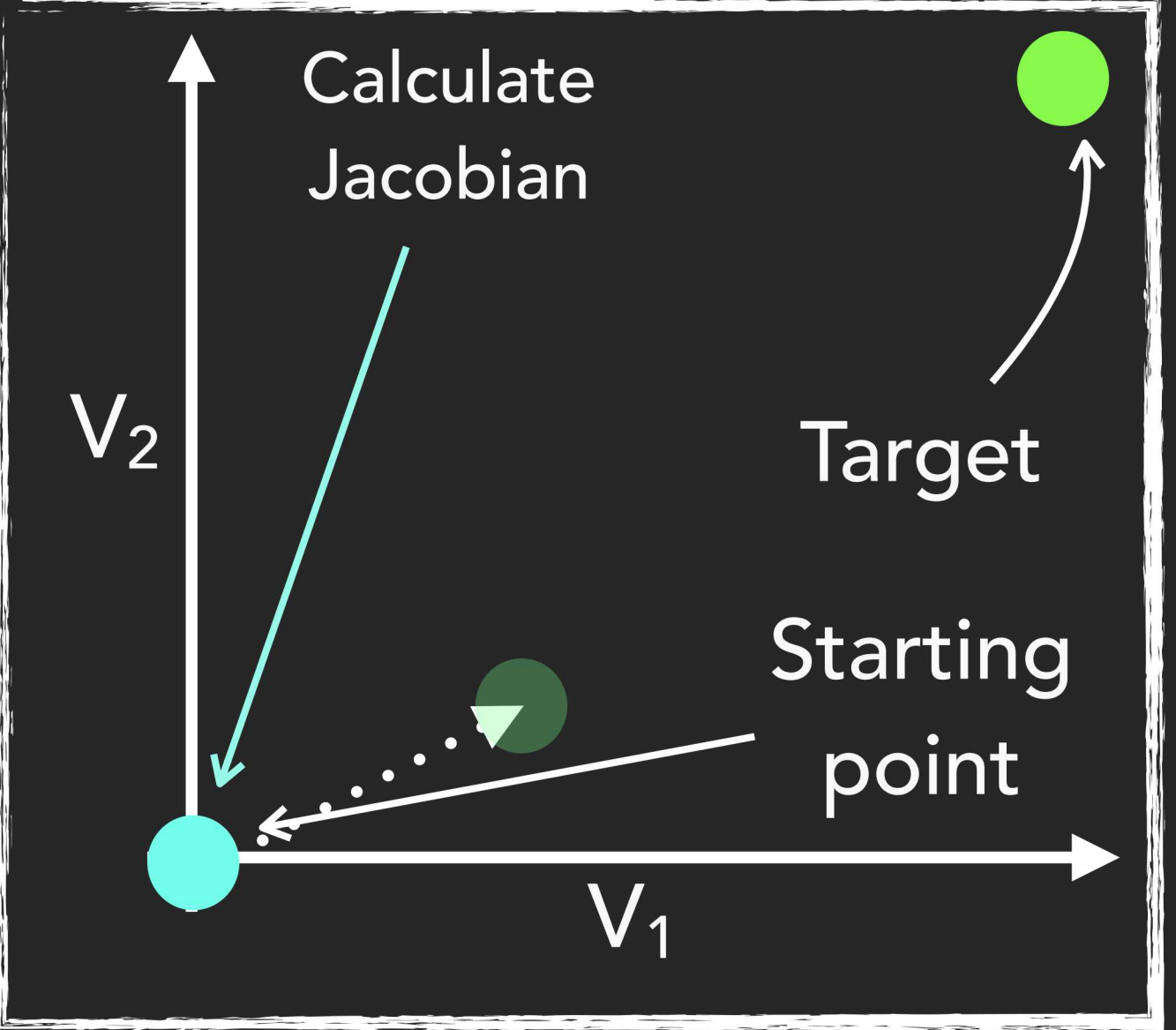
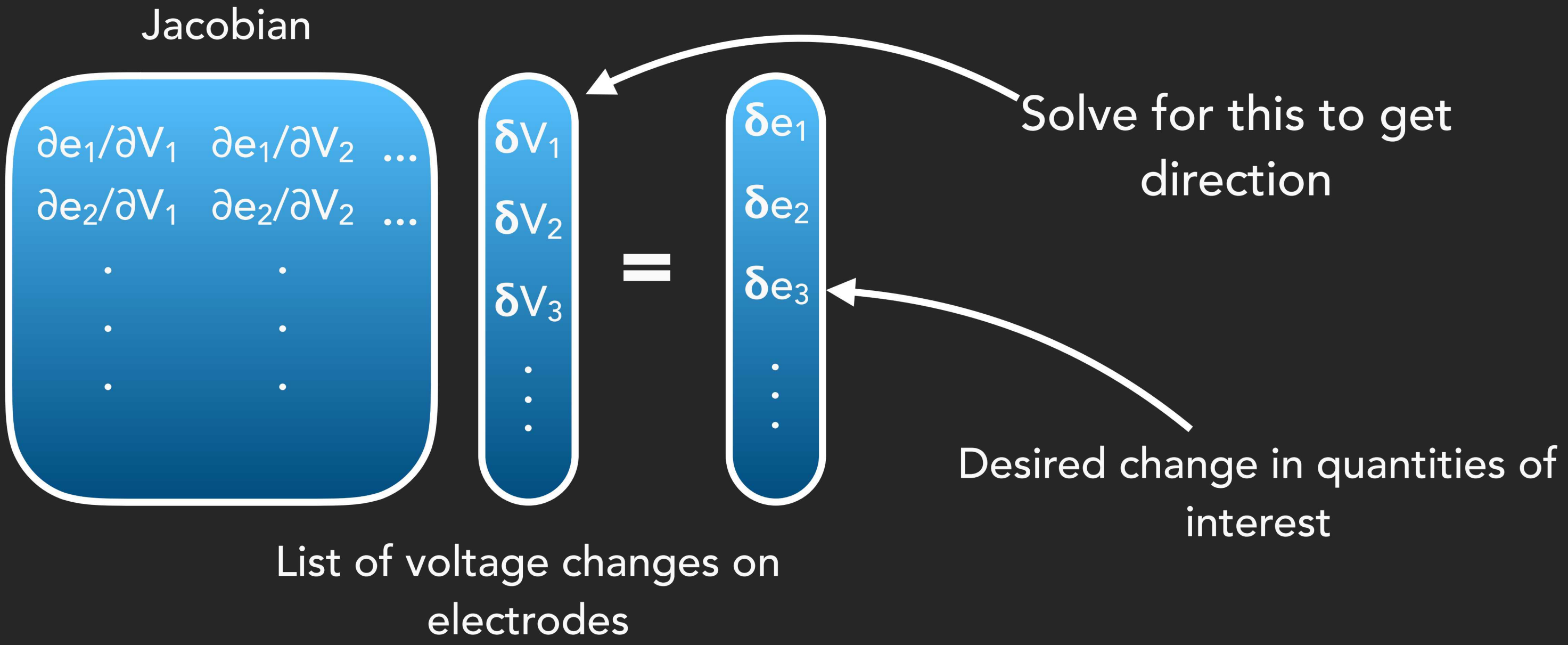
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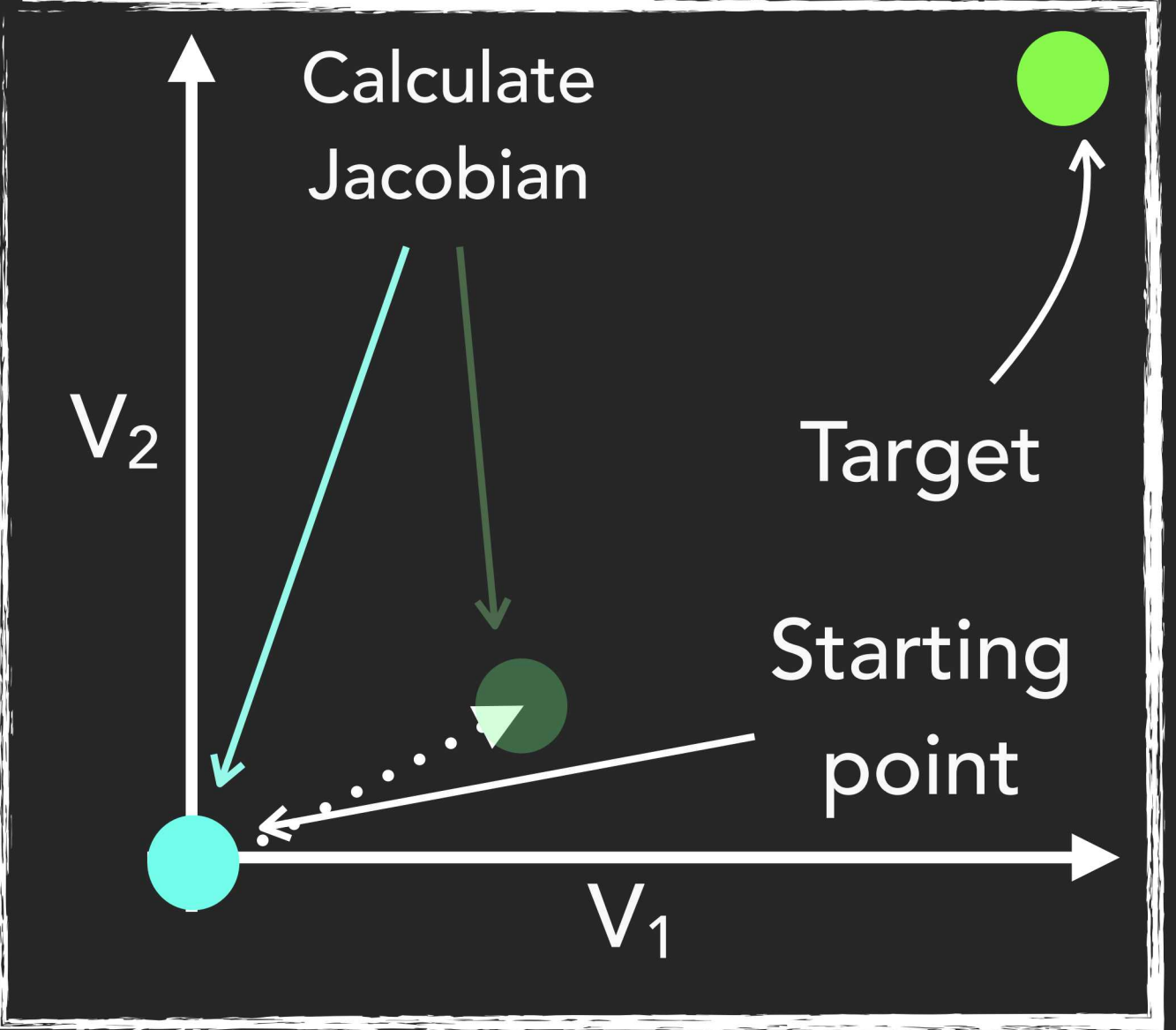
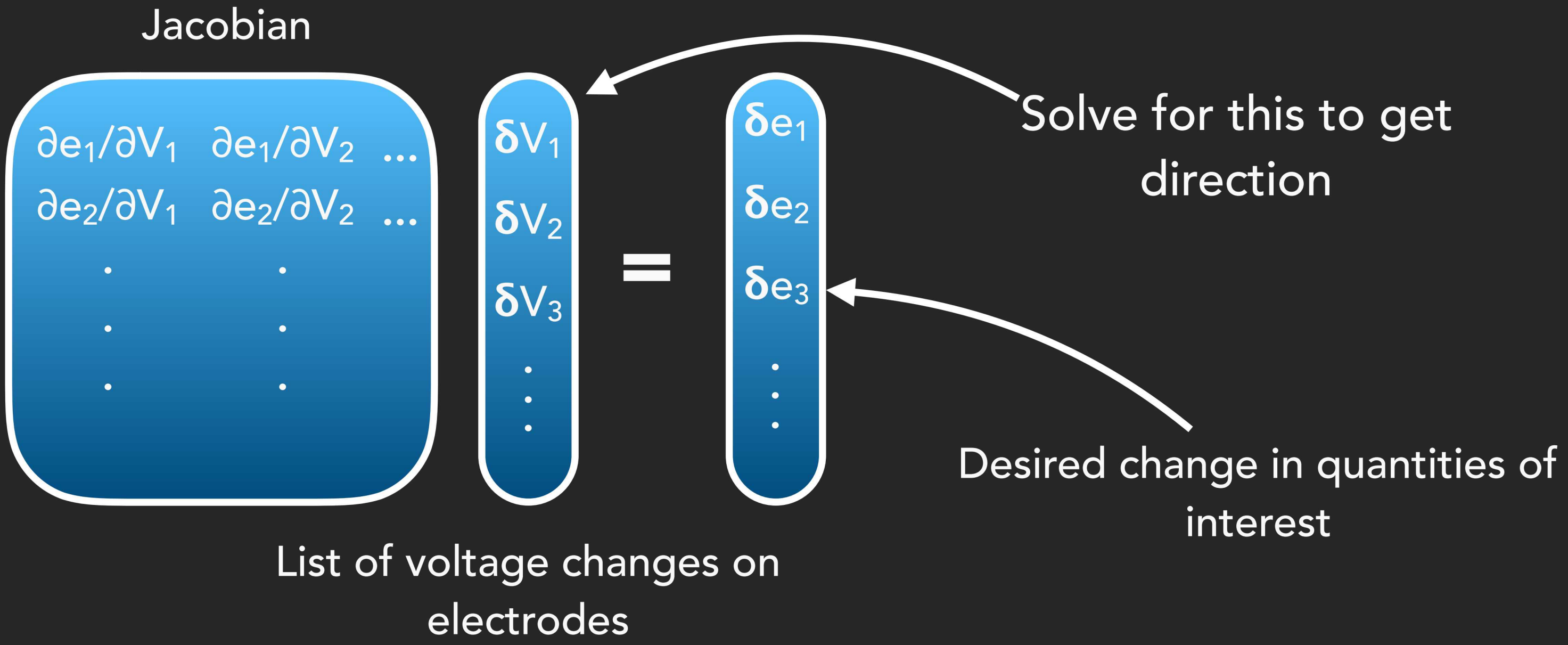
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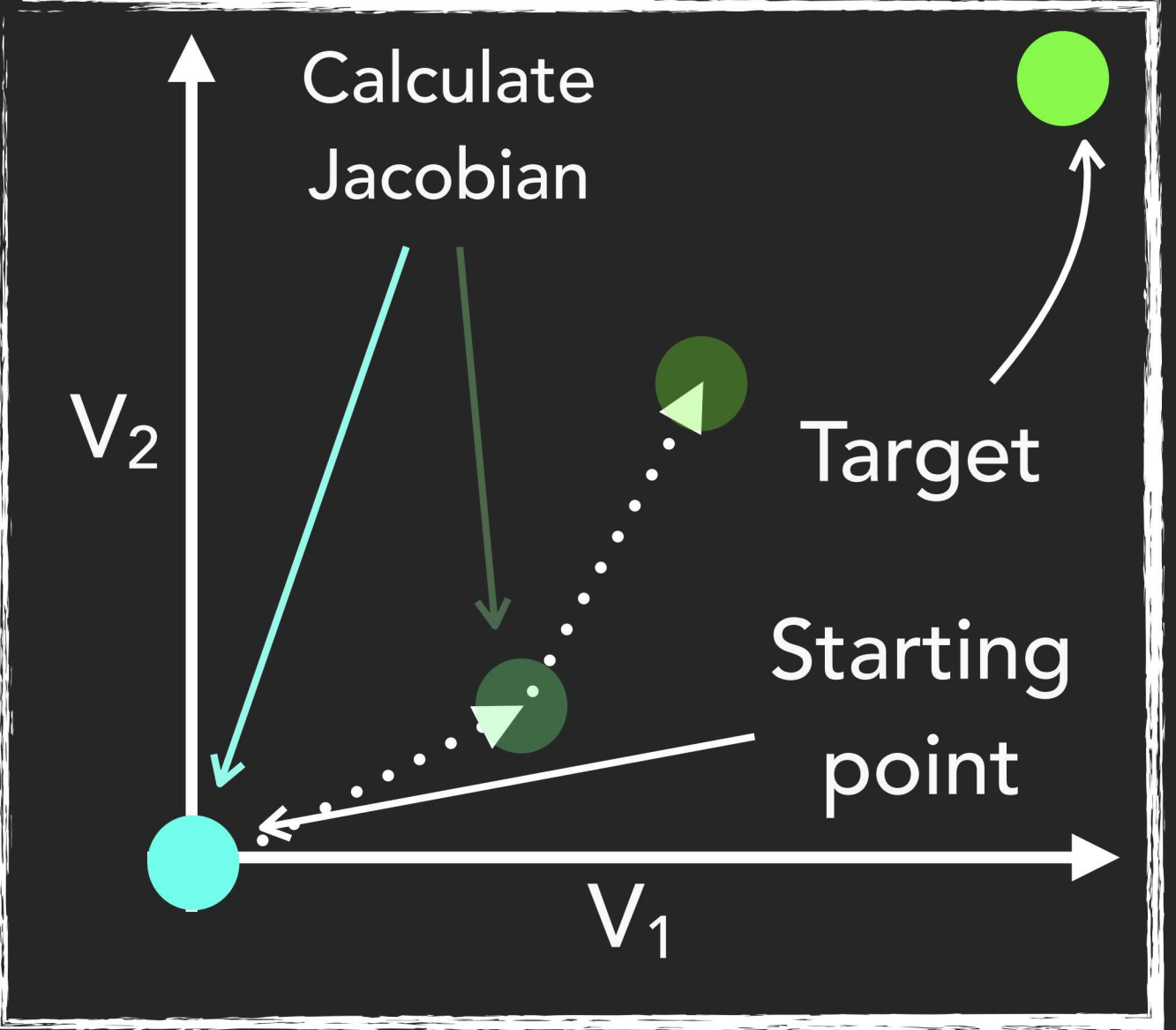
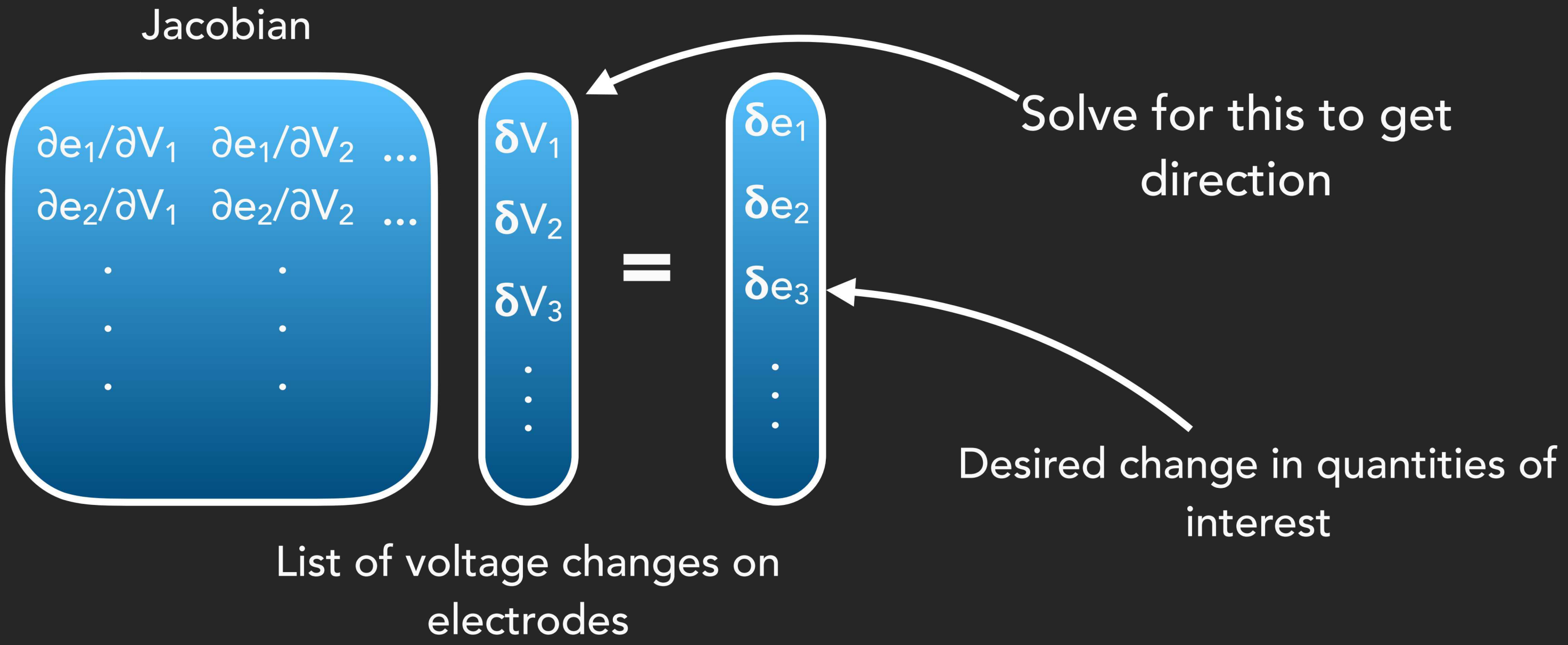
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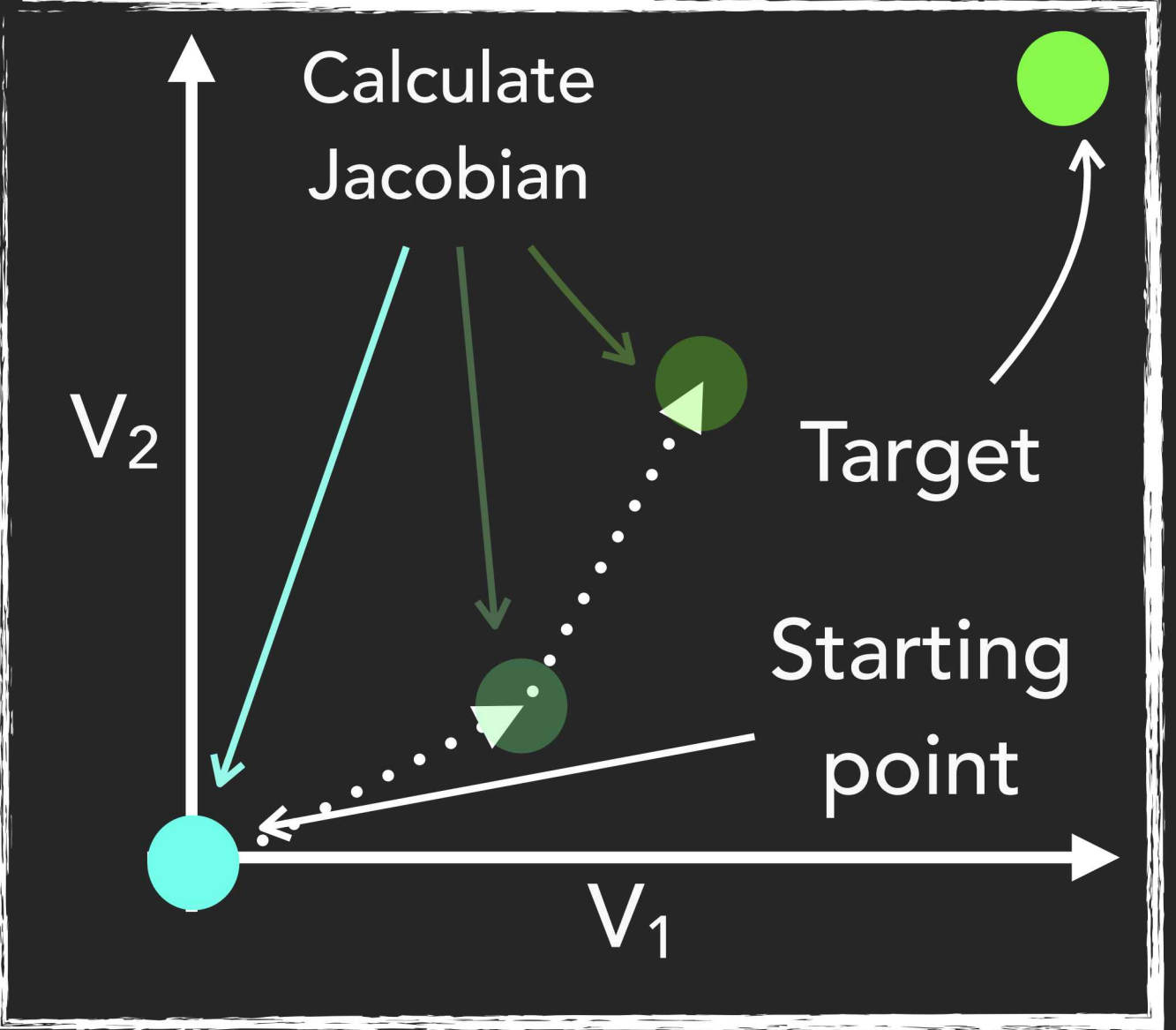
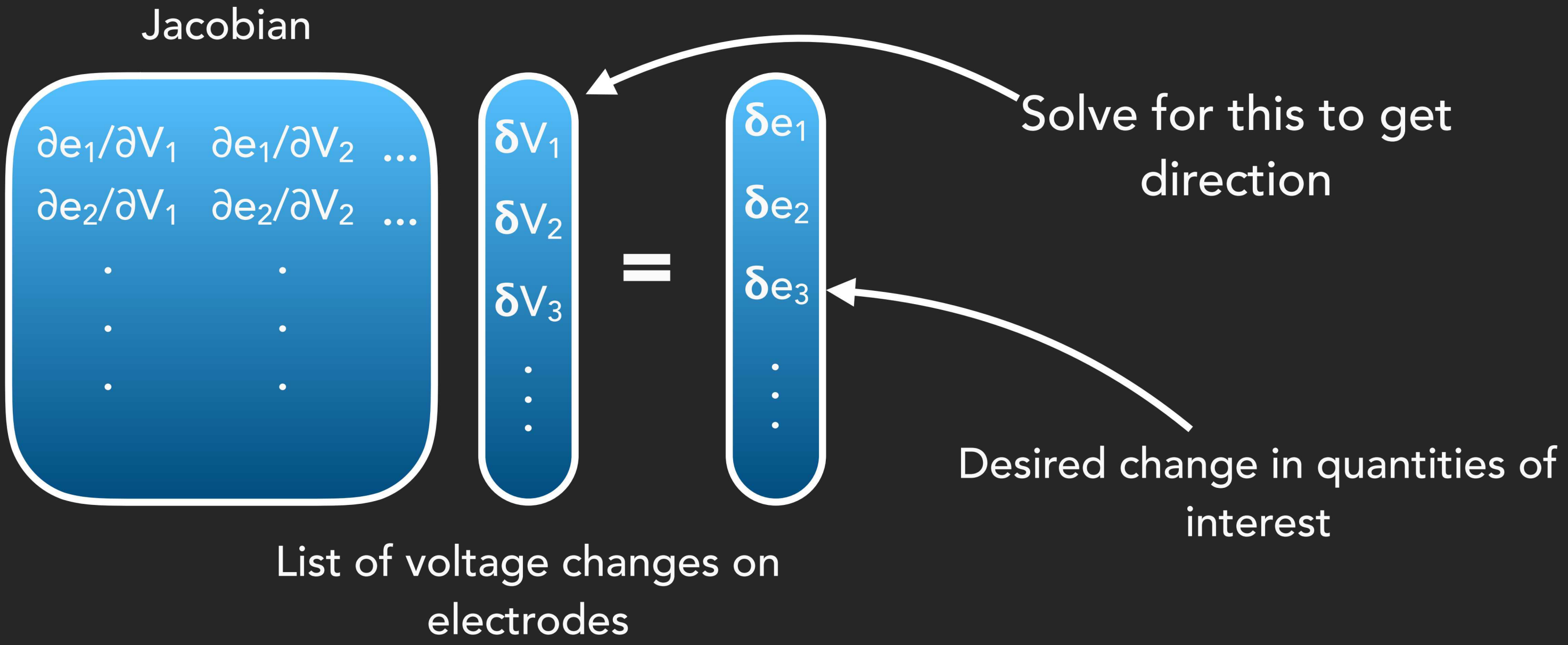
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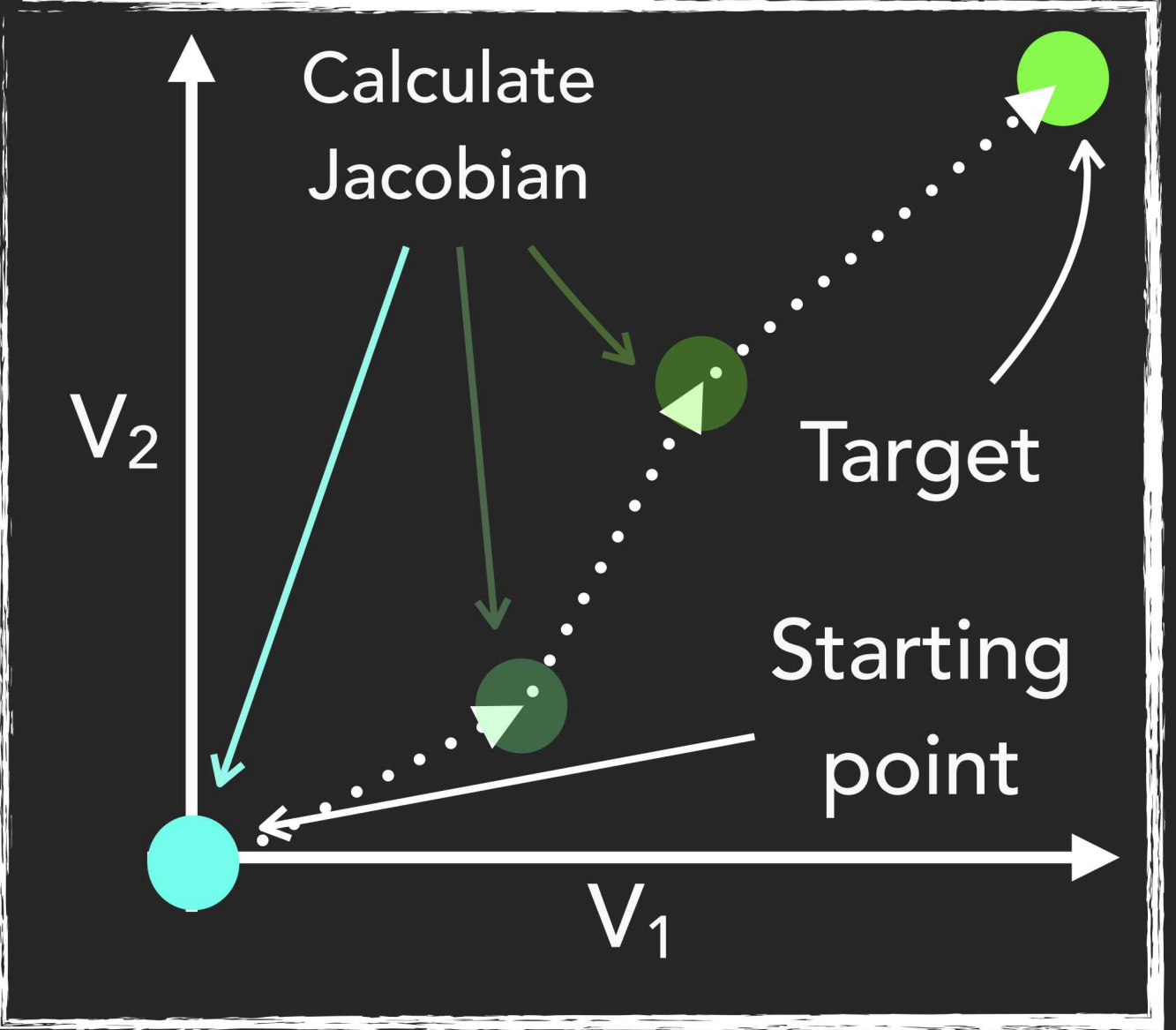
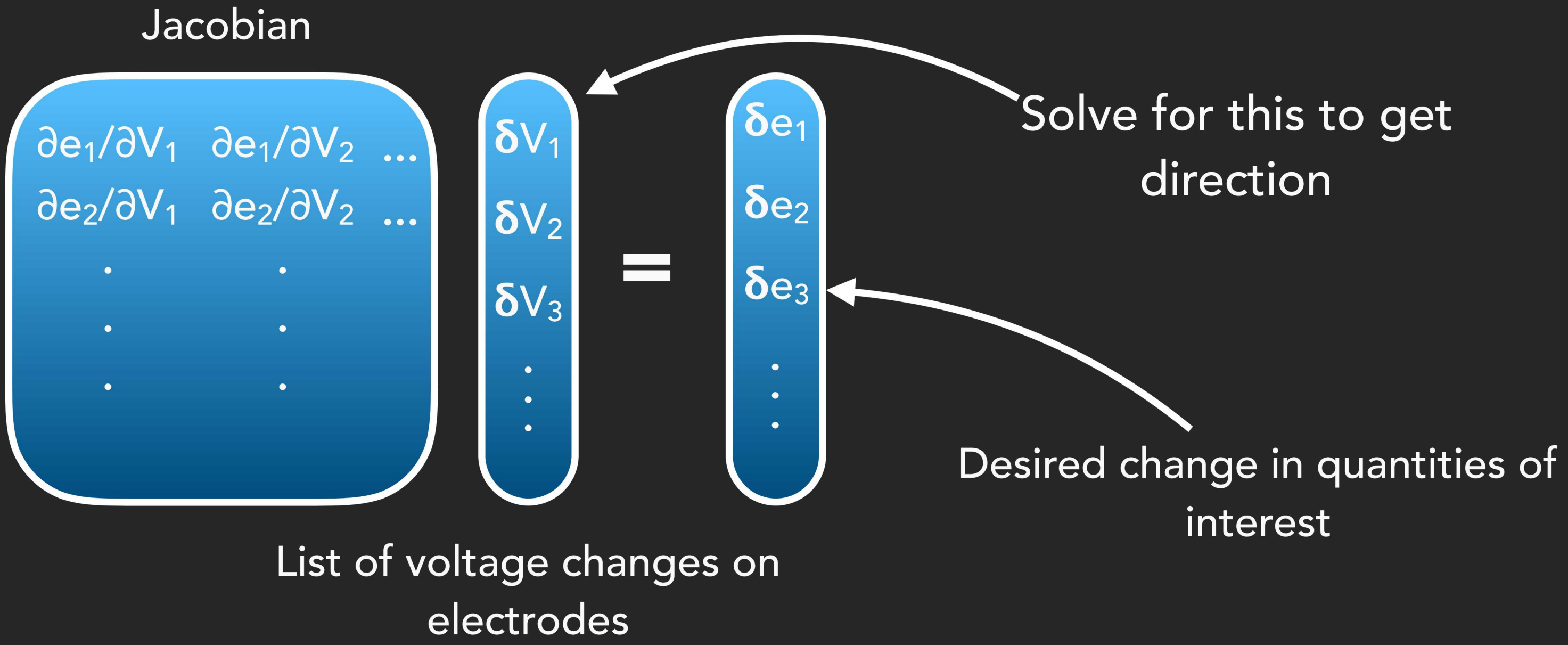
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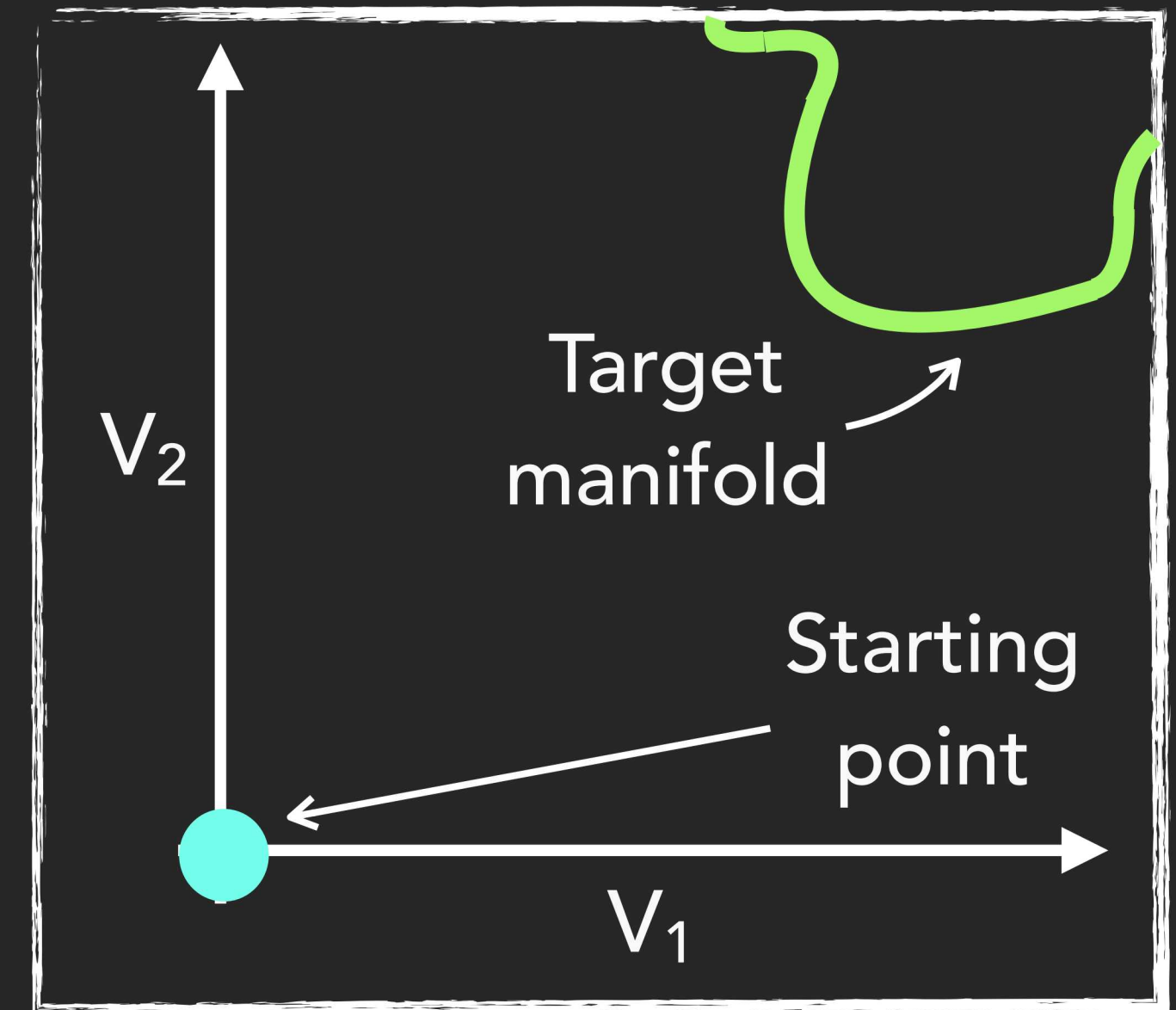
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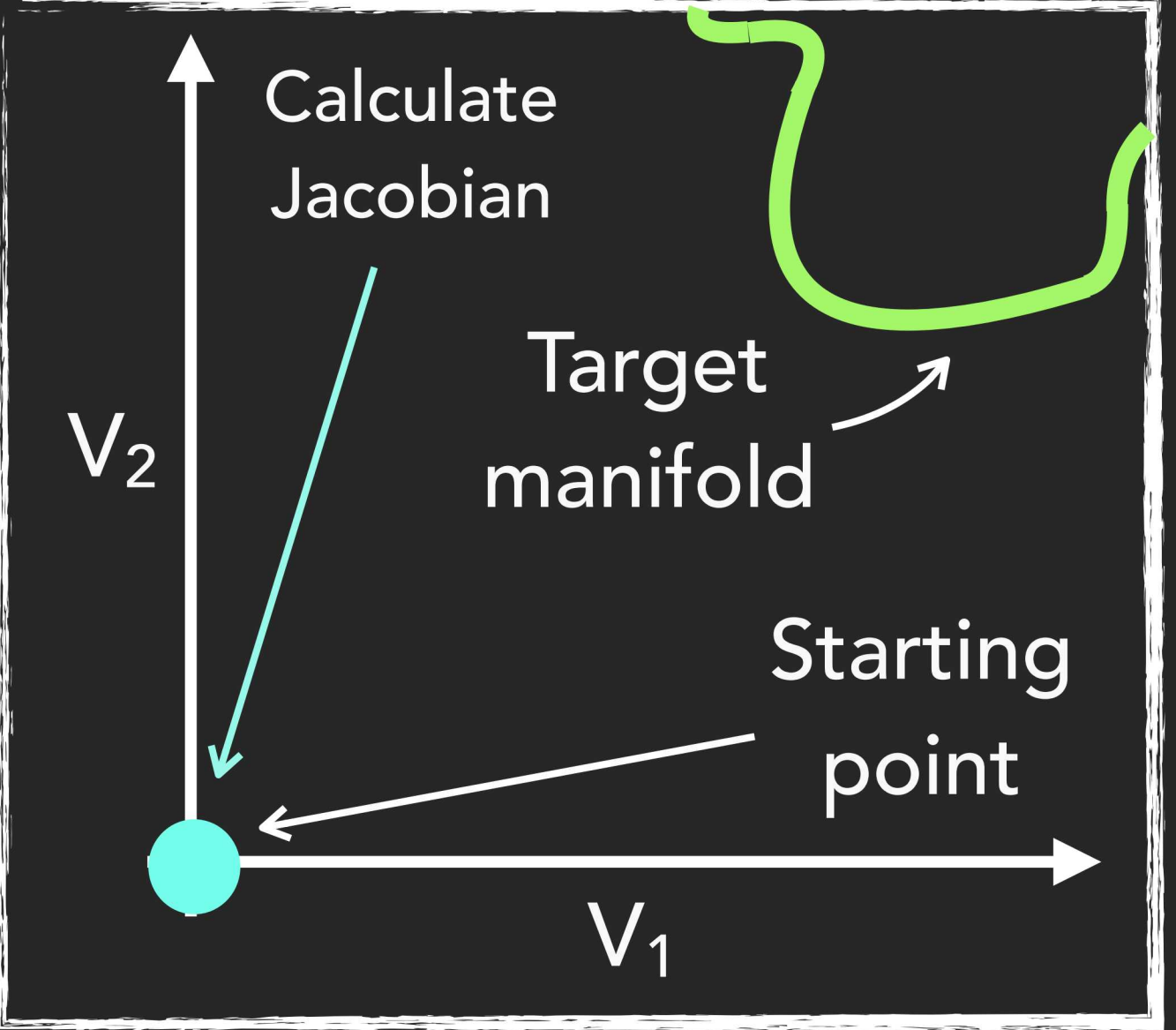
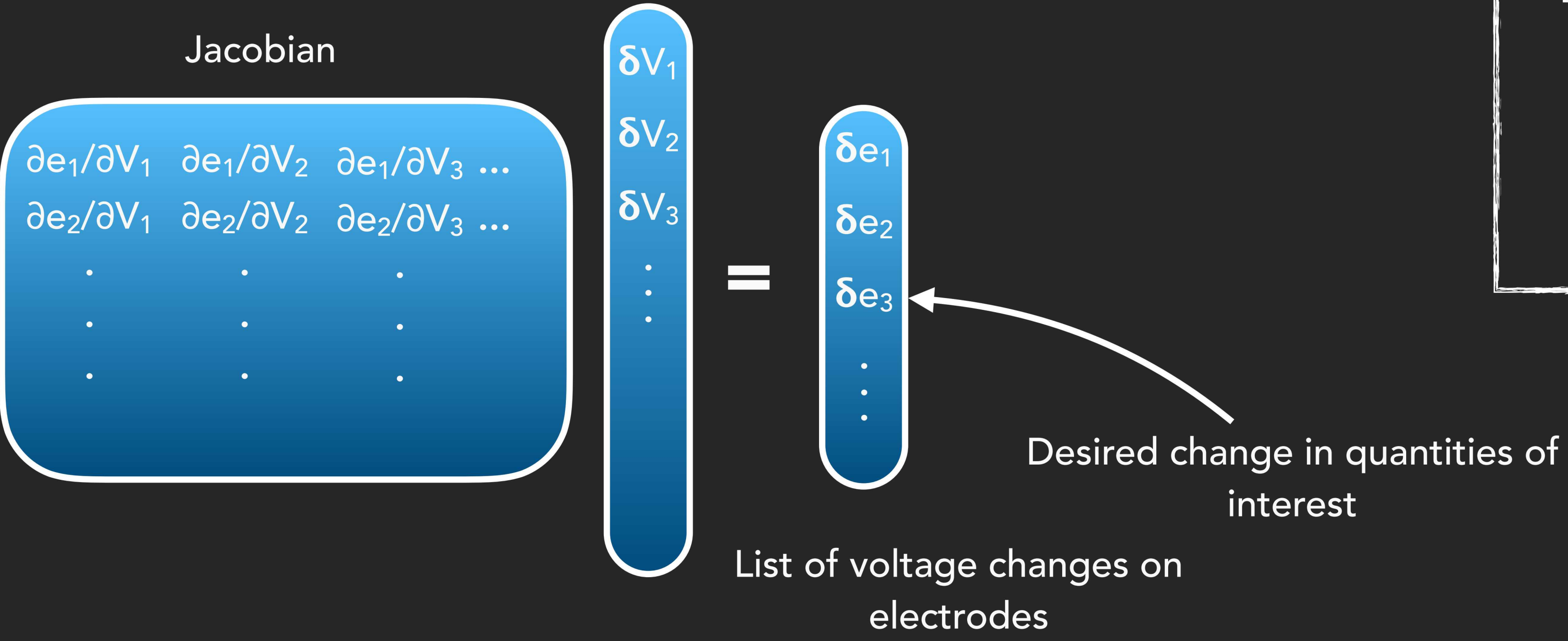
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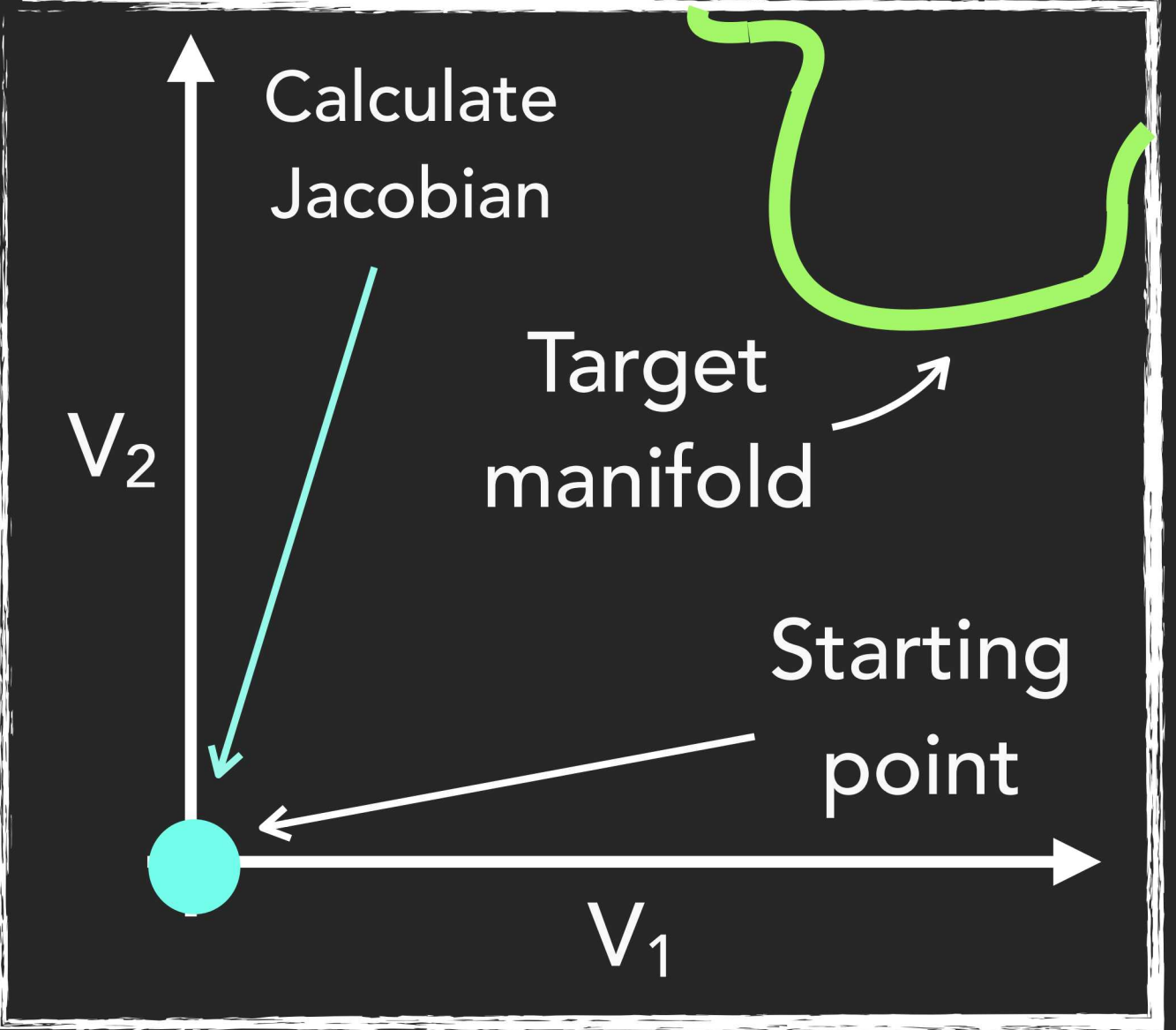
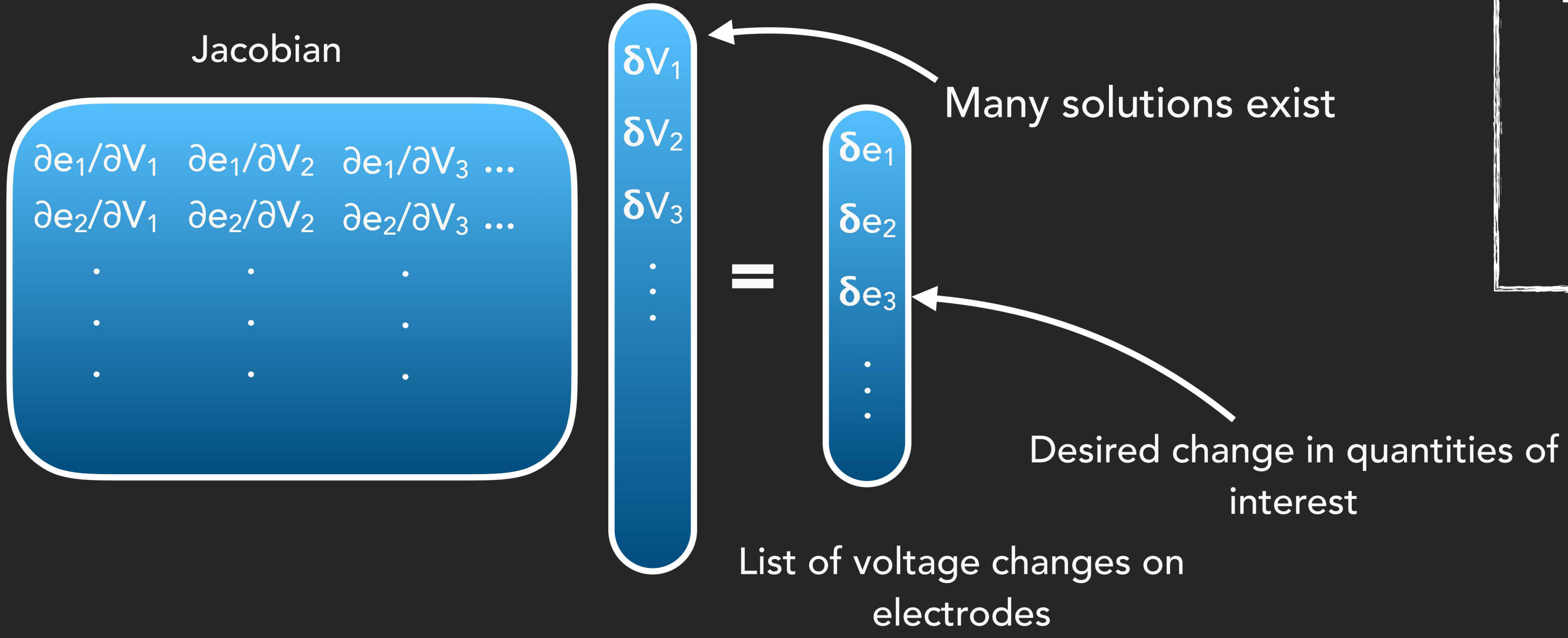
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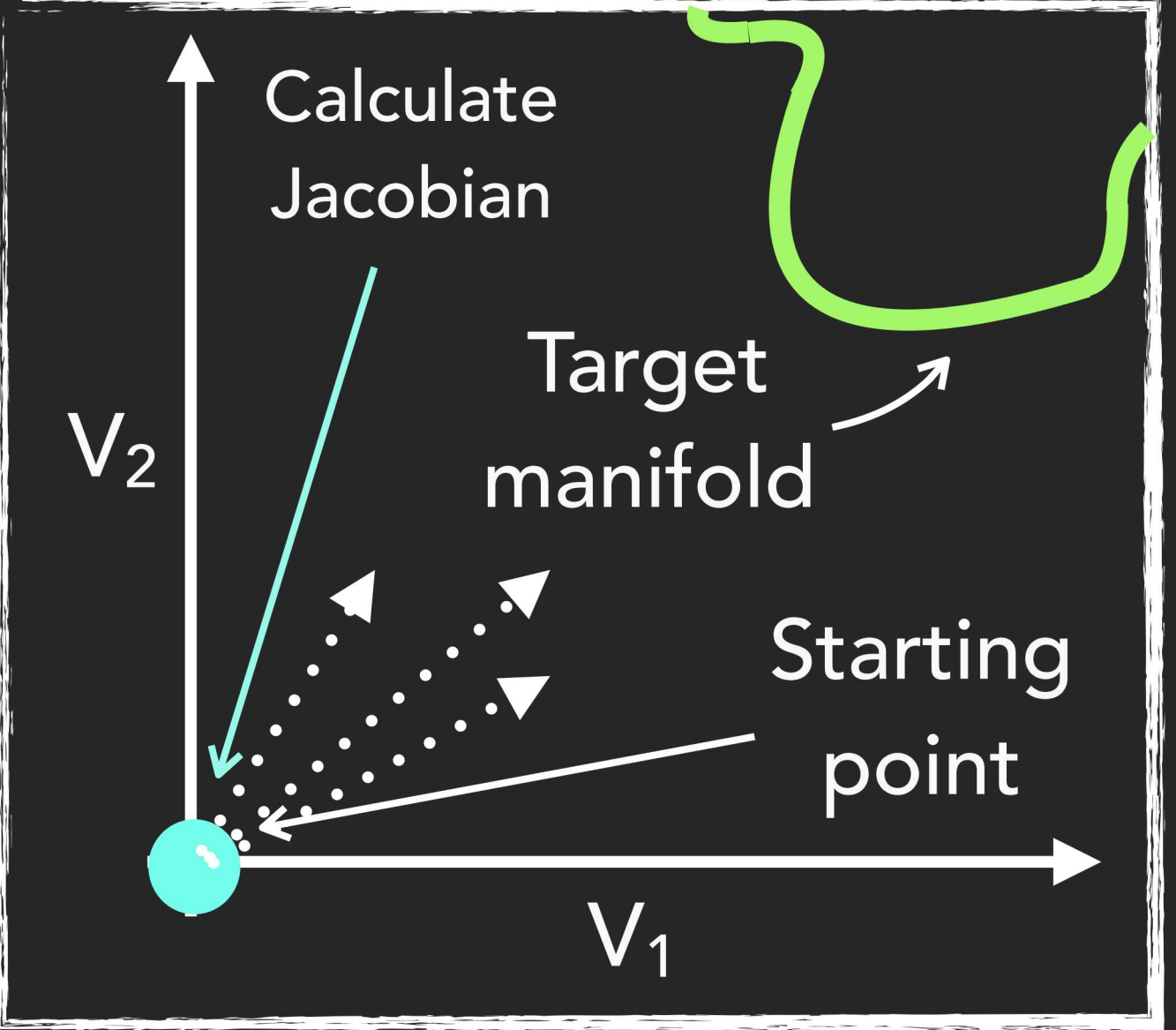
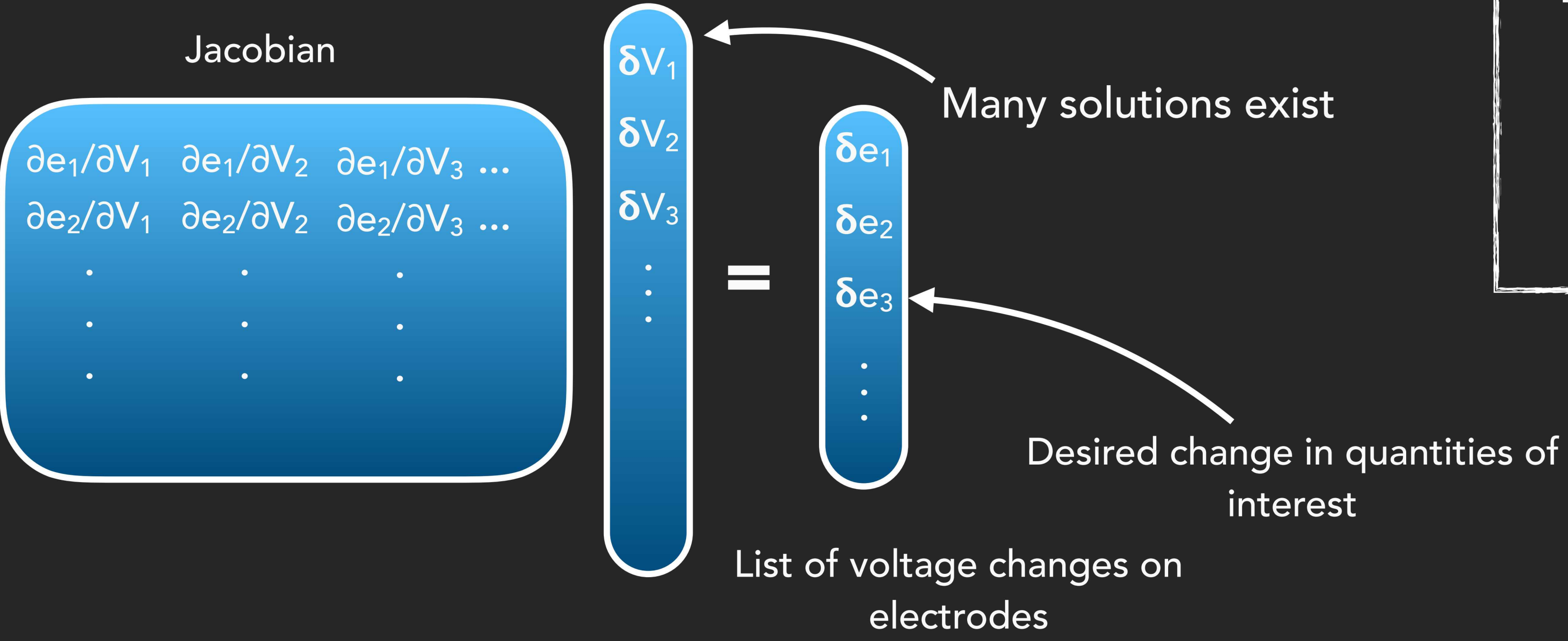
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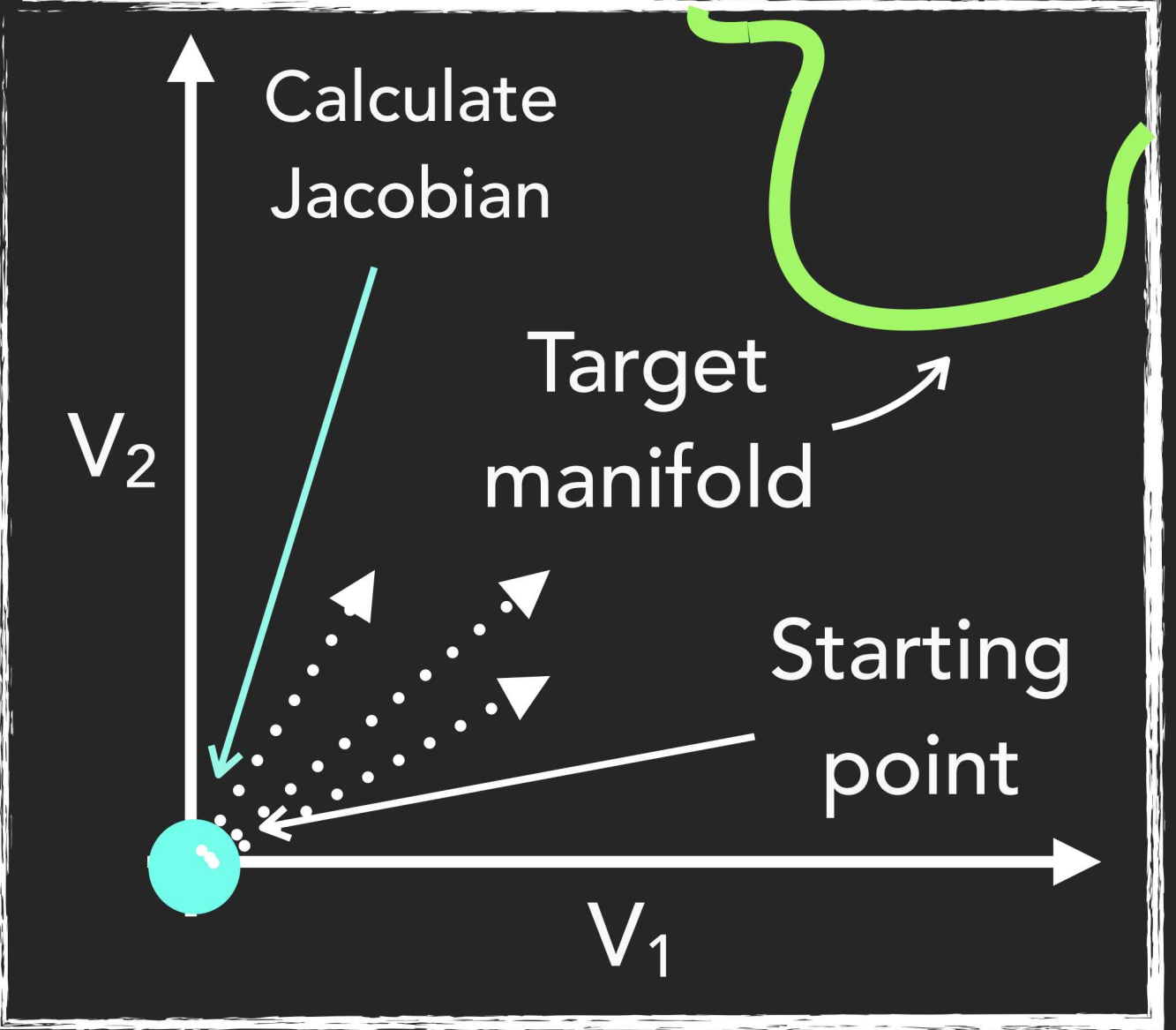
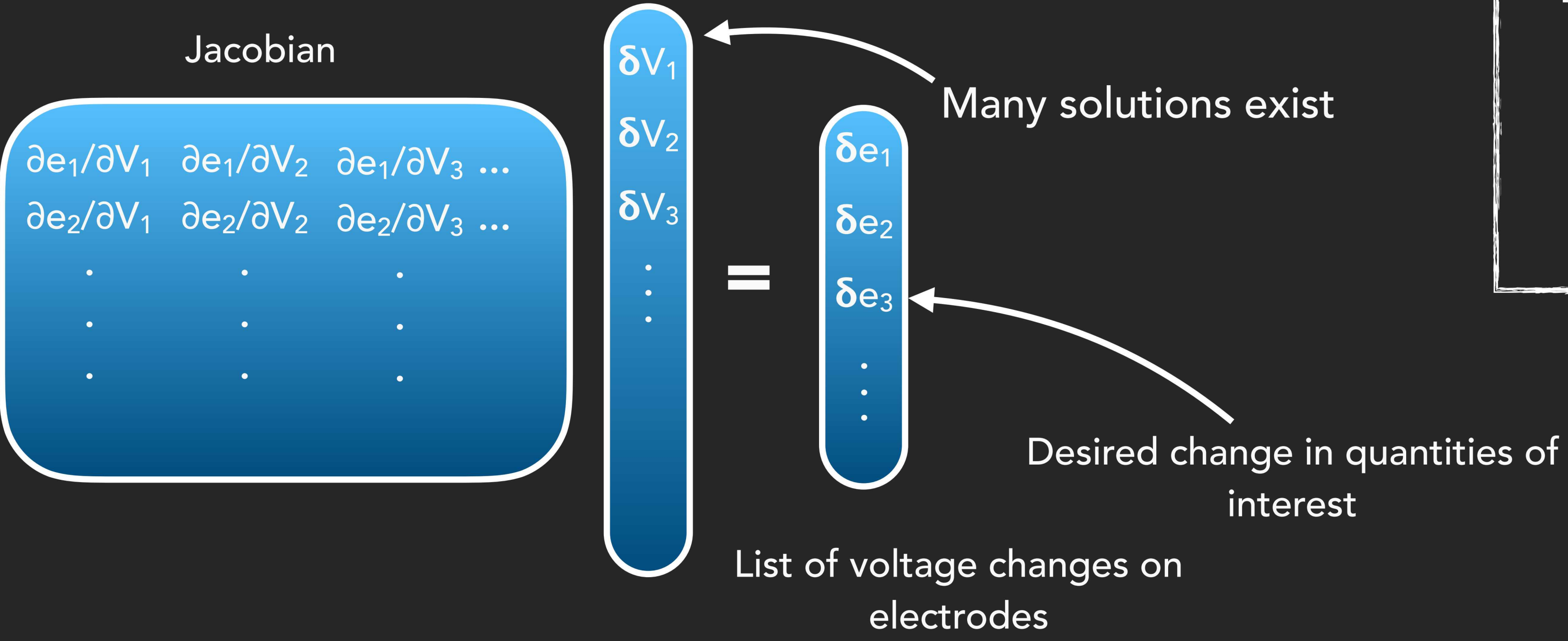
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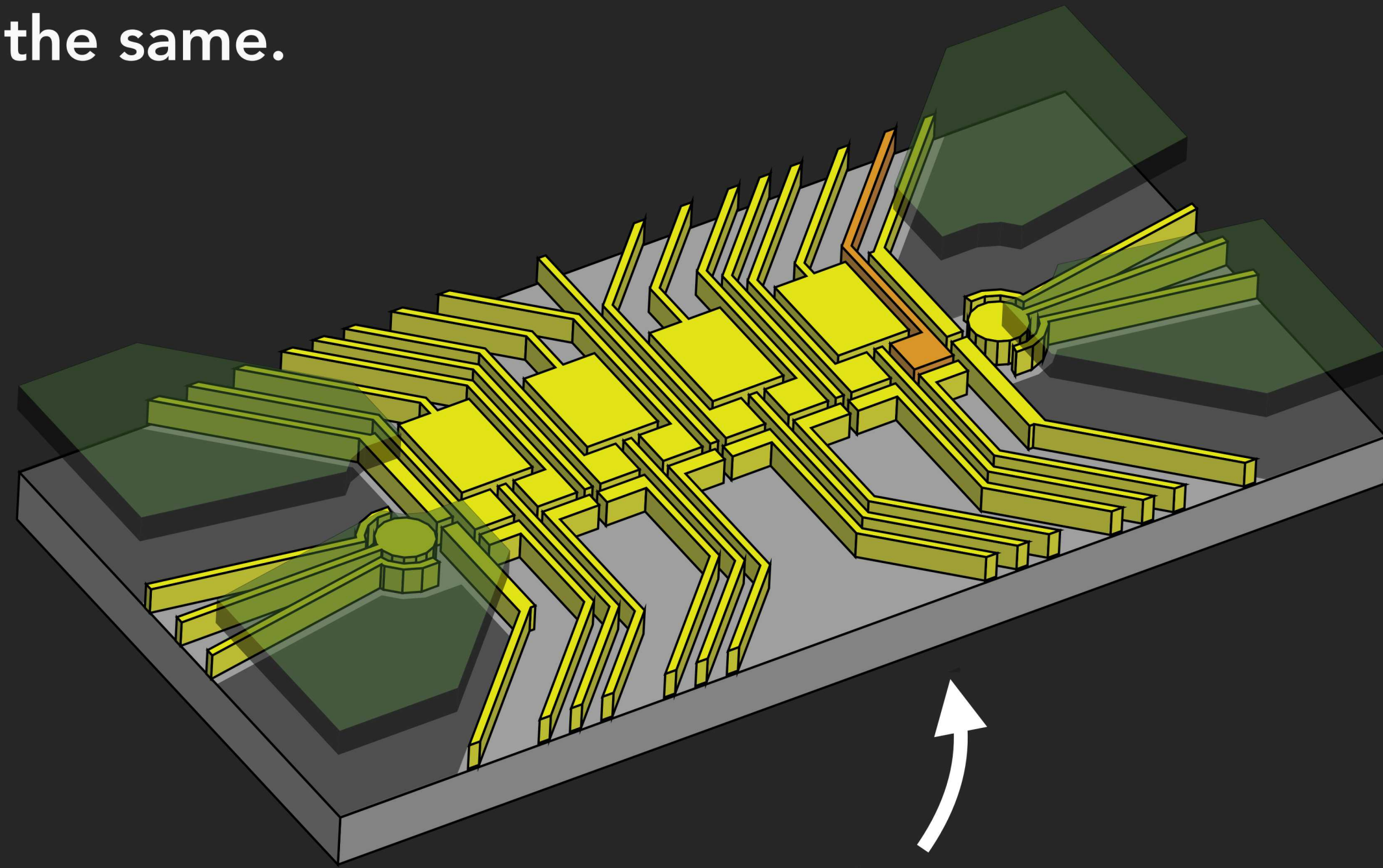


We want to find a tuning which limits the number of electrodes used.



Minimizing L_2 norm of voltages finds solutions which use many electrodes

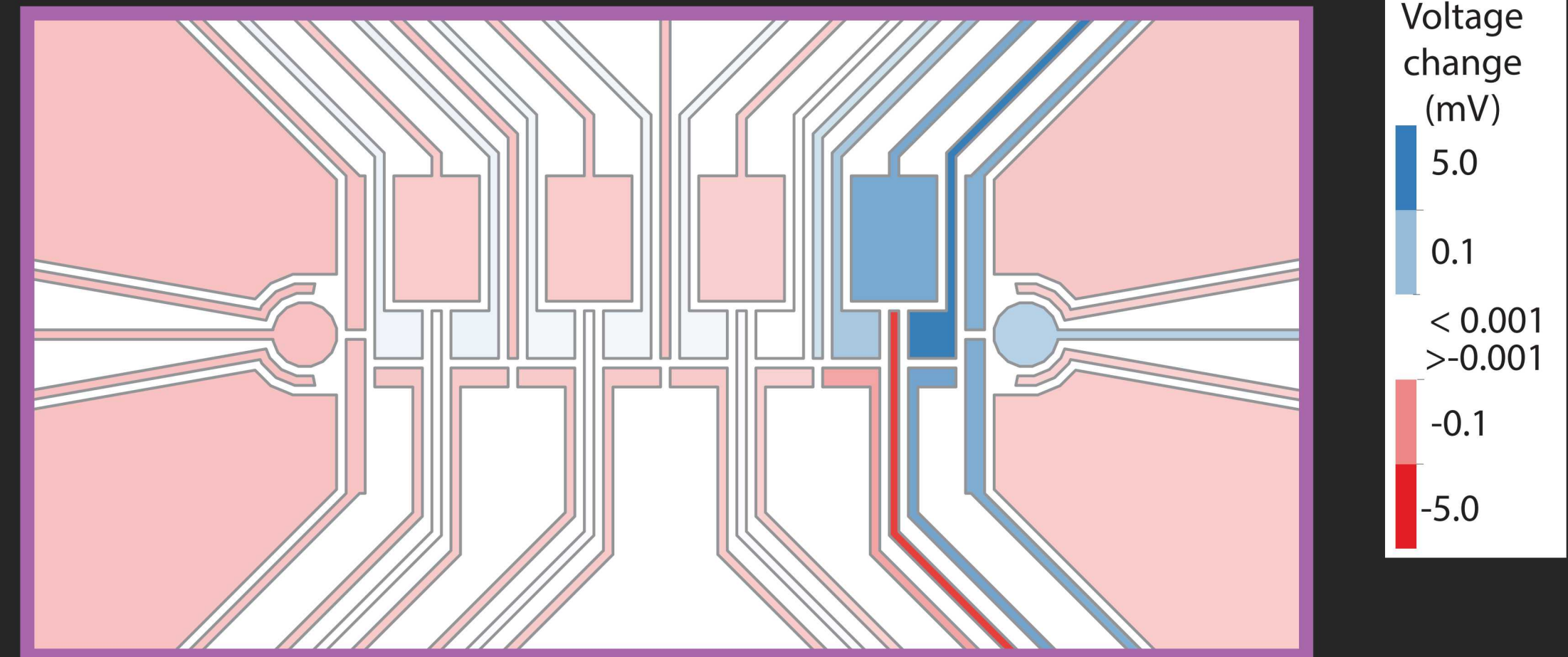
Objective: add one electron to the **right-most dot**, keeping all **tunnel couplings / other dot occupations** the same.



COMSOL simulation.

Each dot contains one electron (calculated via Thomas-Fermi¹ approximation), each pair of dots have appreciable tunnel coupling (calculated using the WKB² approximation)

Minimal L_2 norm



[1] Stopa, M. *Phys. Rev. B* 54, 13767–13783 (1996).

[2] R. Shankar, *Principles of Quantum Mechanics*, 2nd ed (Springer, 1994)



L_1 regularization of underconstrained systems leads to sparse solutions

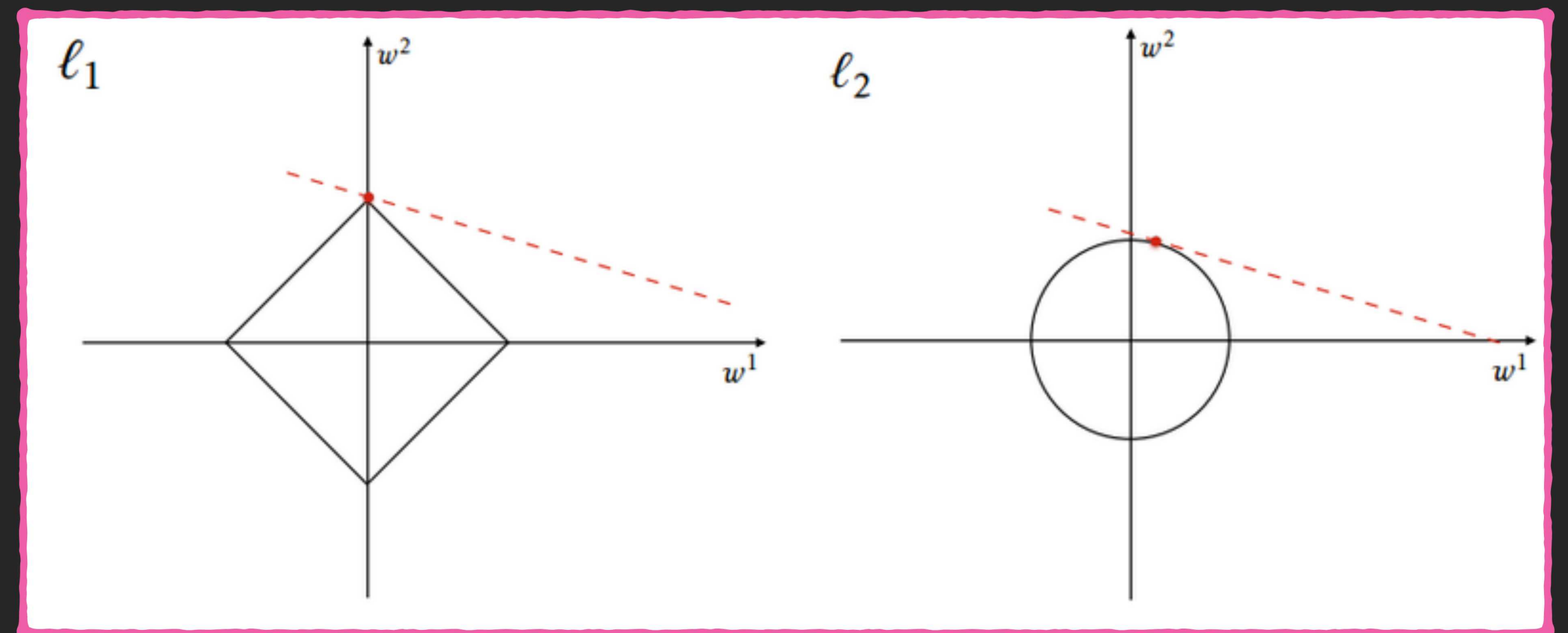
Regularization: finding the solution to an underconstrained linear system which minimizes a particular norm.

L_2 norm: $(x_1^2 + x_2^2 + x_3^2 + \dots)^{1/2}$

L_1 norm: $(|x_1| + |x_2| + |x_3| + \dots)$

L_0 norm: # of nonzero elements

L_1 regularization is a good proxy for L_0 regularization



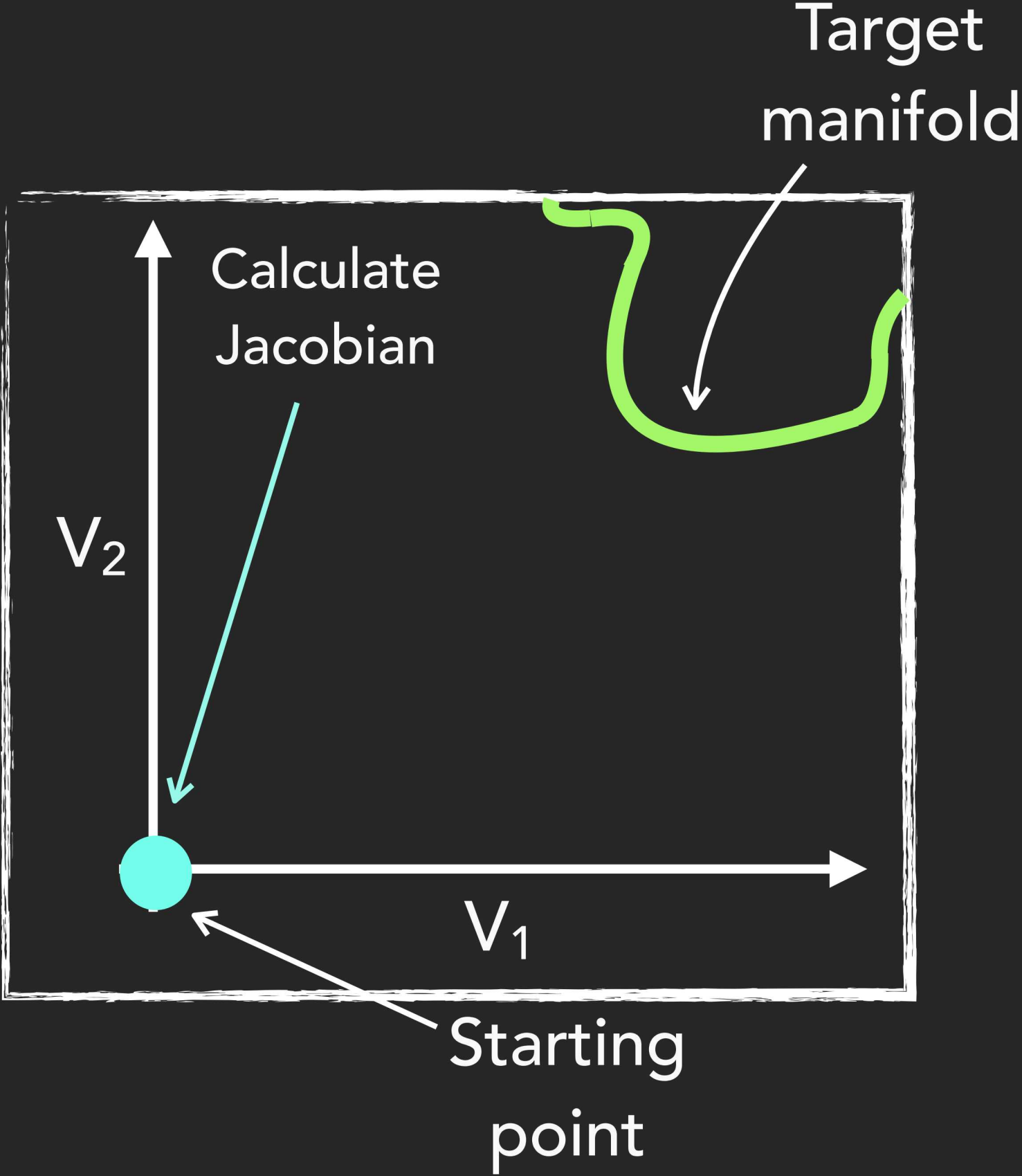
Source: Wikipedia - "Regularization (mathematics)"



Compressed sensing techniques can be used to find voltage tuning with minimal L_1 norm

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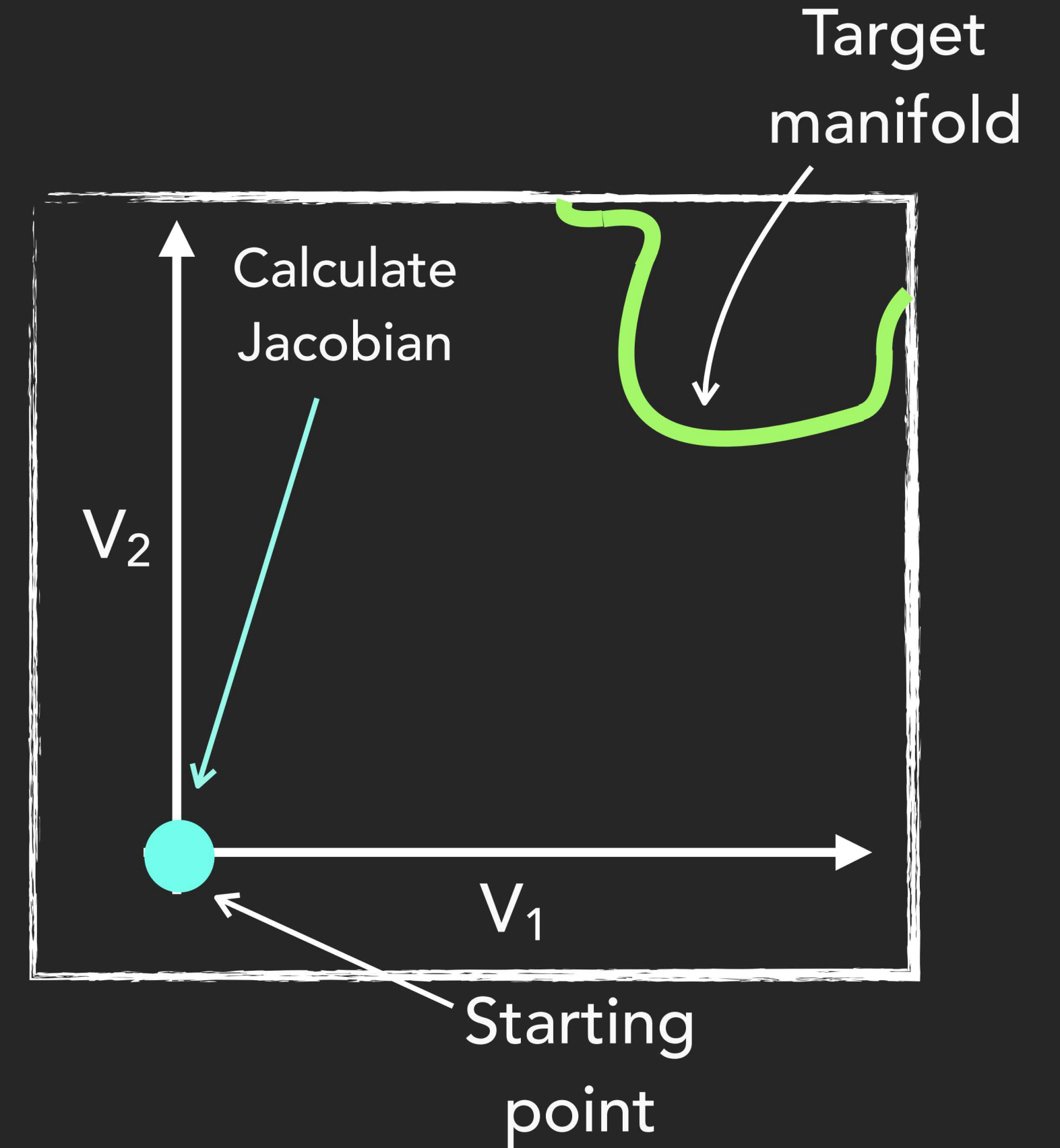


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Using methods from compressed sensing¹⁻³, we can find the solution which minimizes the L_1 norm



- [1] E. J. Candès, et al., Communications on Pure and Applied Mathematics, 59, 1207 (2006)
- [2] D. Donoho, IEEE Transactions on Information Theory, 52, 1289 (2006)
- [3] Diamond, S. & Boyd, S. CVXpy Journal of Machine Learning Research 17, 1–5 (2016)

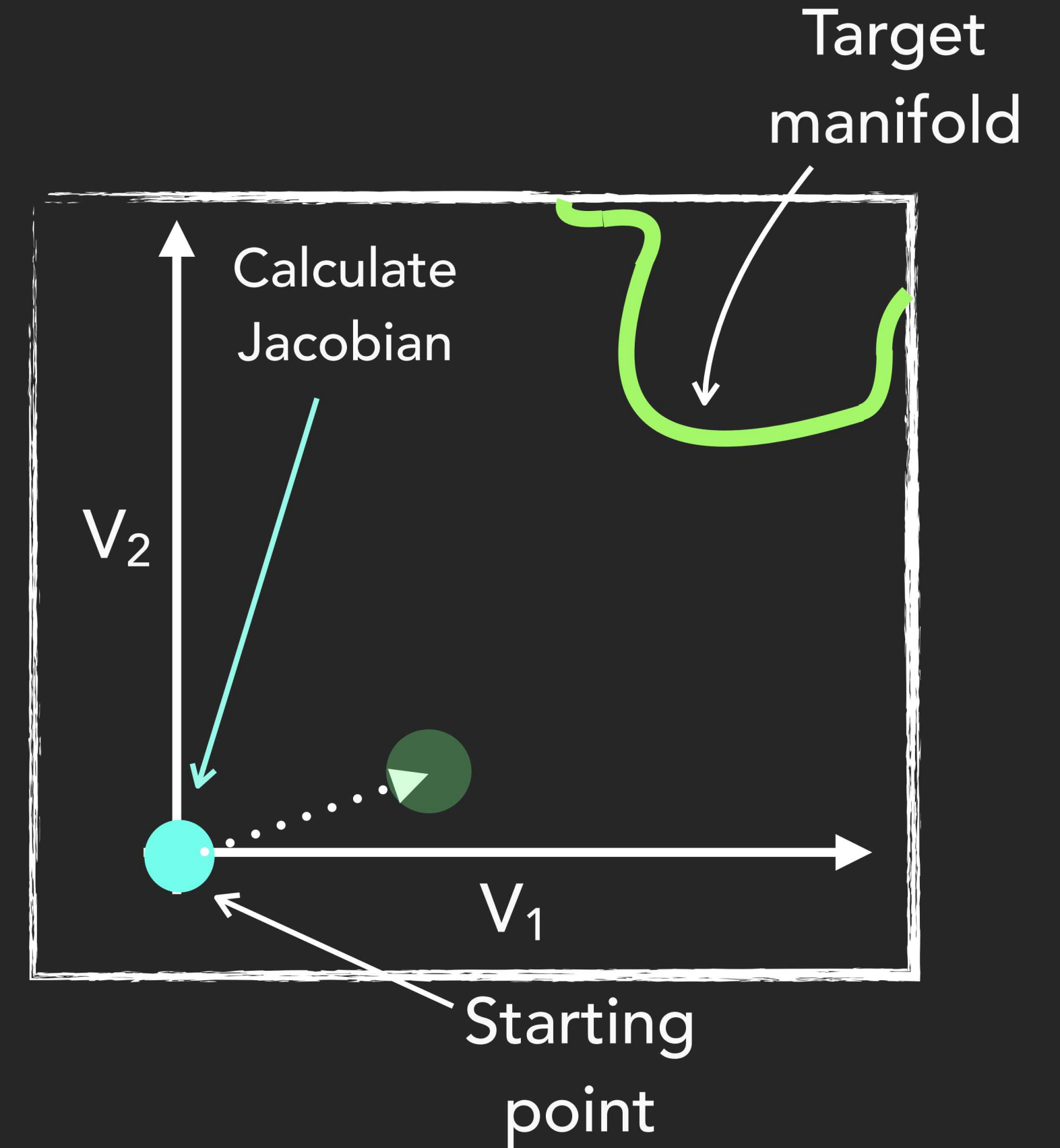


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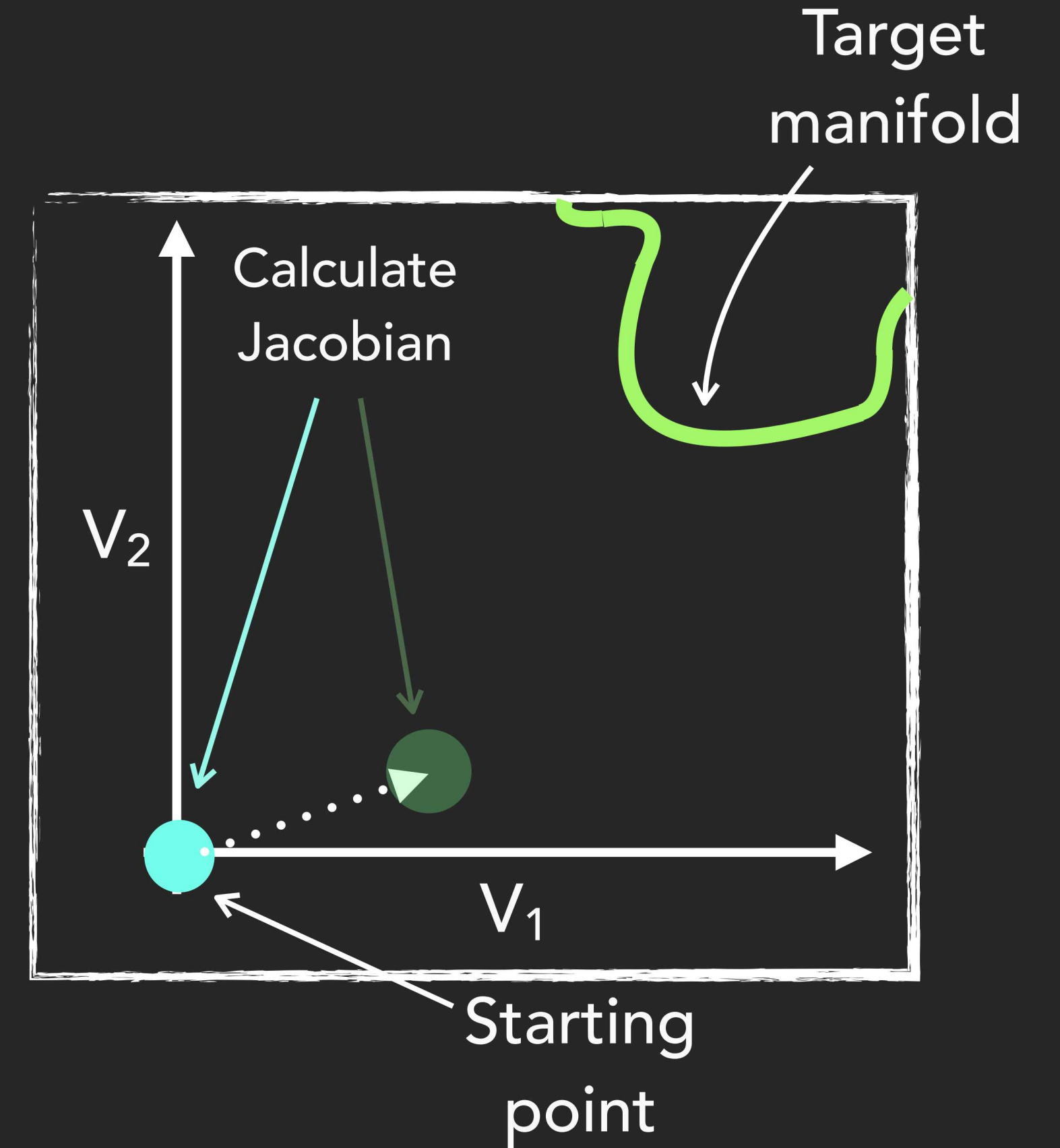


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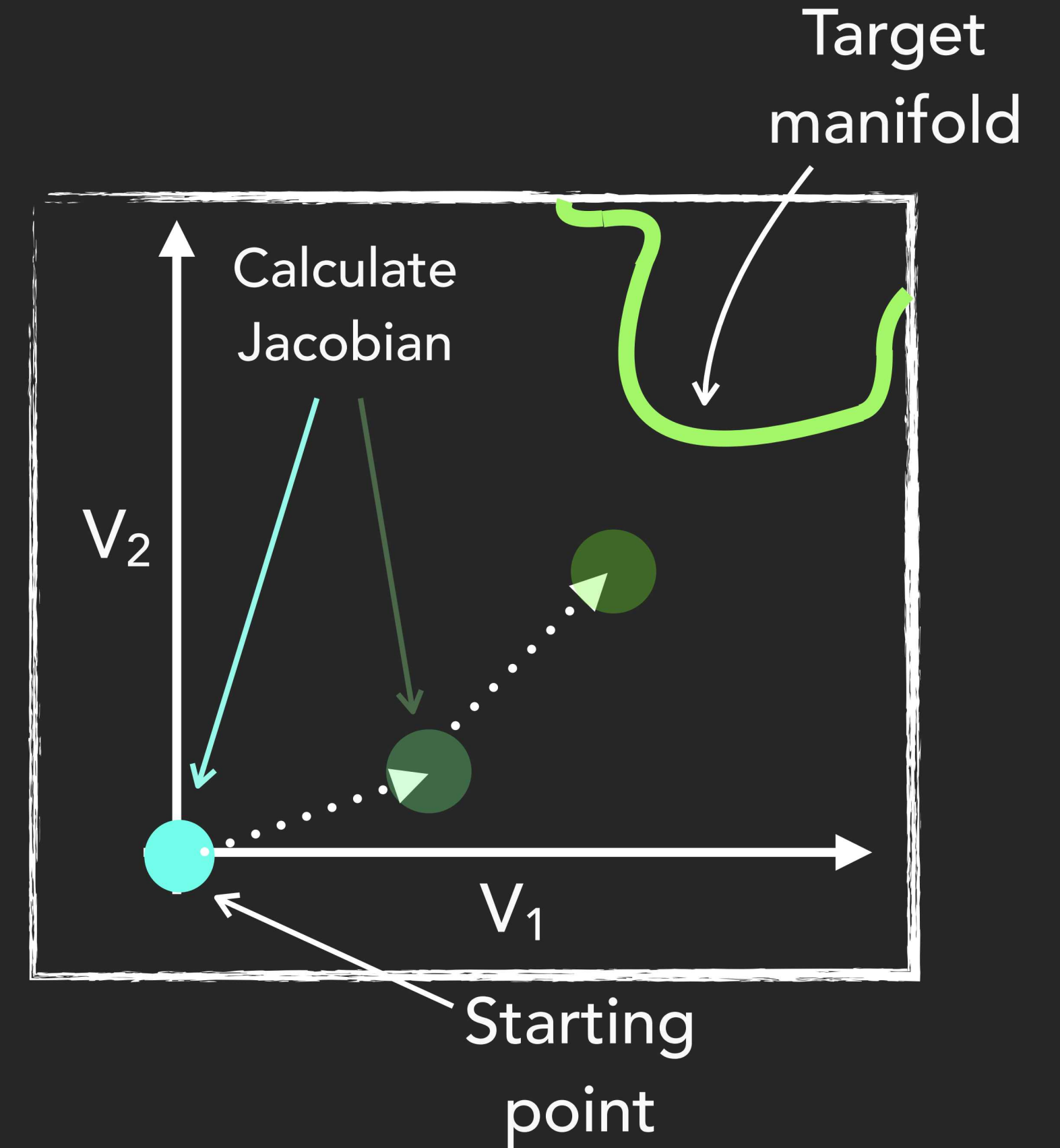


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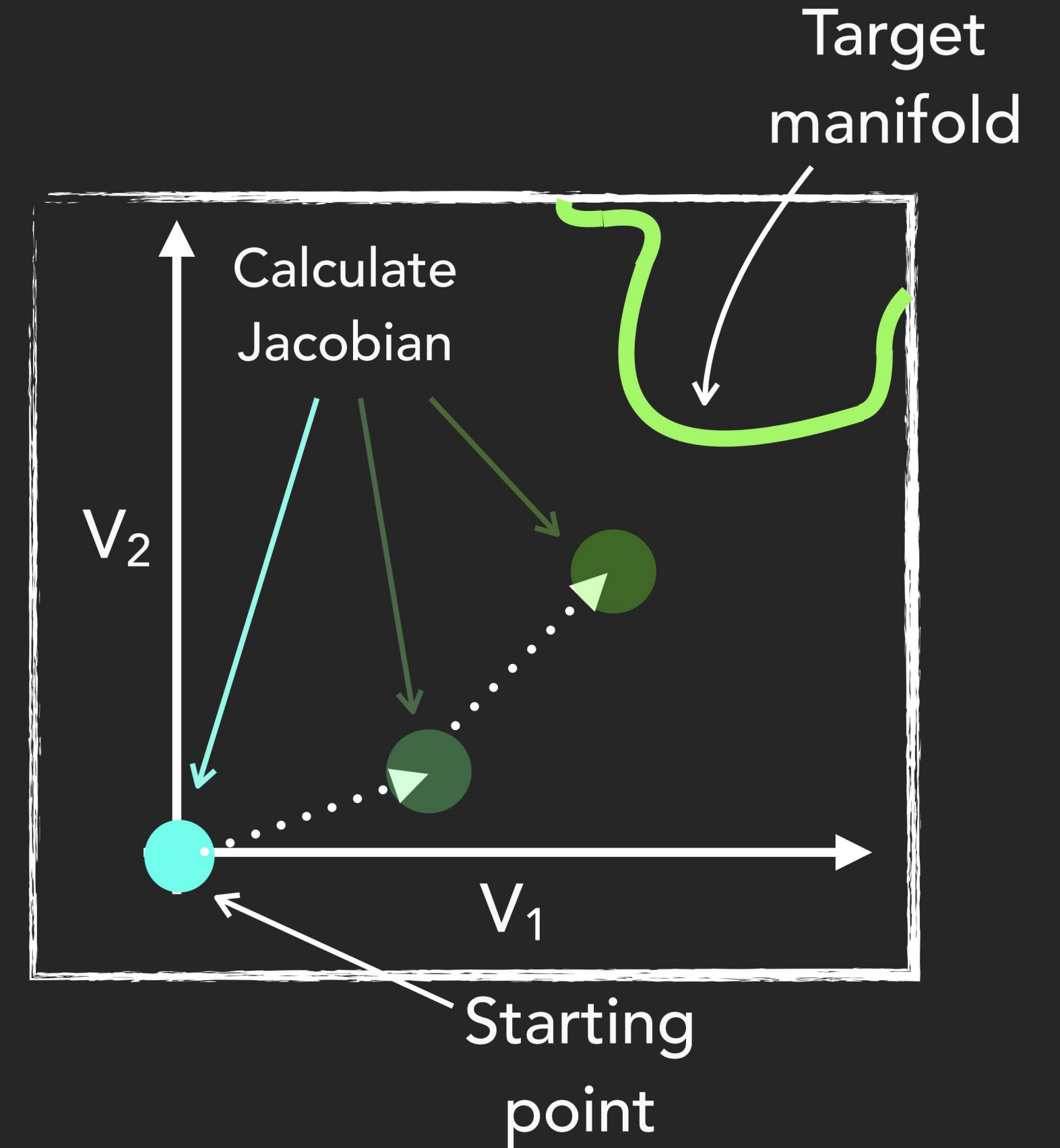


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- [2] D. Donoho, IEEE Transactions on Information Theory, 52, 1289 (2006)
- [3] Diamond, S. & Boyd, S. CVXpy Journal of Machine Learning Research 17, 1–5 (2016)

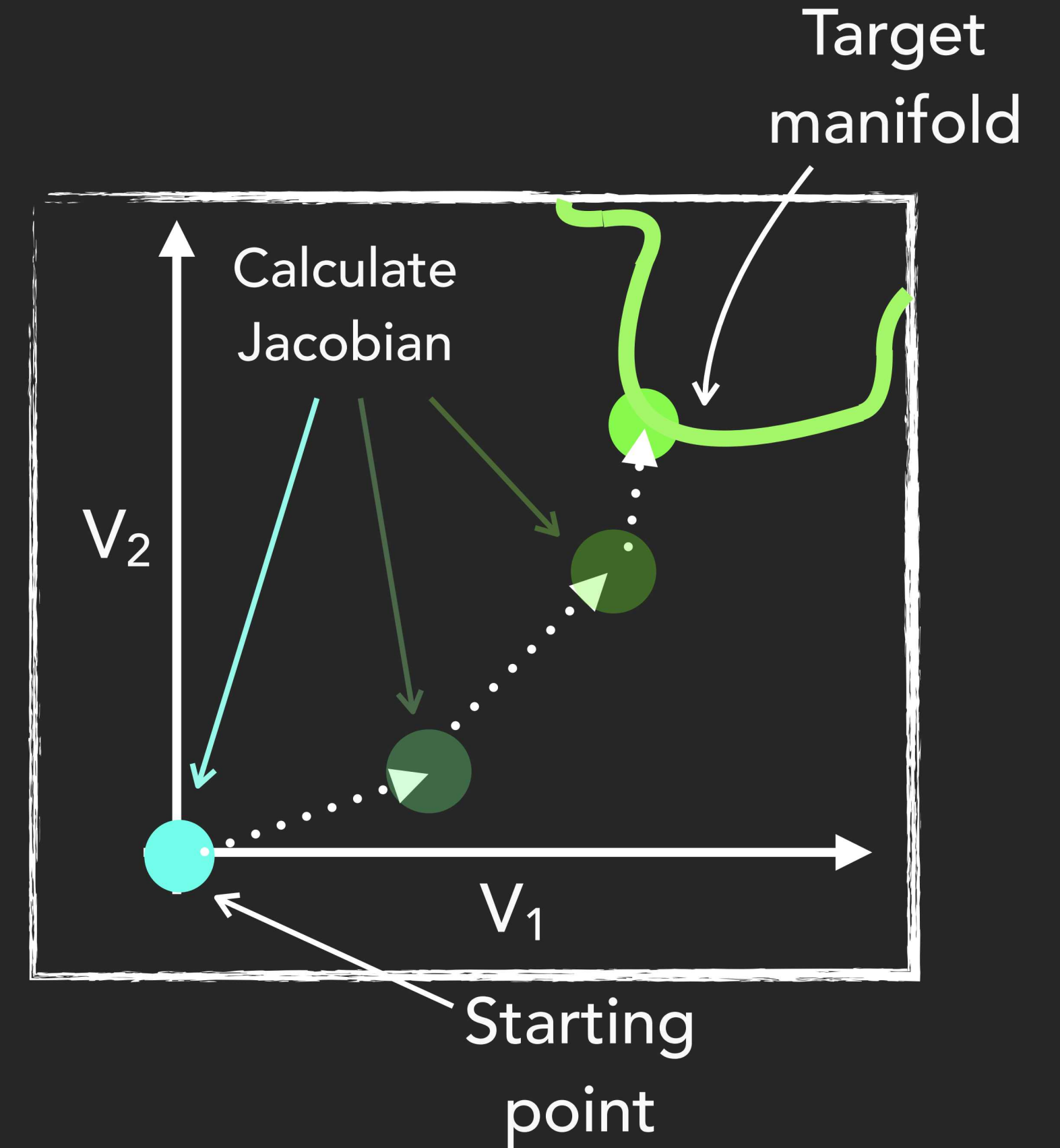


Compressed sensing techniques can be used to find voltage tuning with minimal L_1 norm

Given a nonlinear function that relates the voltages on $m > n$ electrodes to n quantities of interest (i.e. dot occupations, dot energies, tunnel couplings), find voltages which yield operational target (desired dot occupations, dot energies, tunnel couplings).

$$\begin{bmatrix} \frac{\partial e_1}{\partial V_1} & \frac{\partial e_1}{\partial V_2} & \frac{\partial e_1}{\partial V_3} & \dots \\ \frac{\partial e_2}{\partial V_1} & \frac{\partial e_2}{\partial V_2} & \frac{\partial e_2}{\partial V_3} & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \delta V_1 \\ \delta V_2 \\ \delta V_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} \delta e_1 \\ \delta e_2 \\ \delta e_3 \\ \vdots \end{bmatrix}$$

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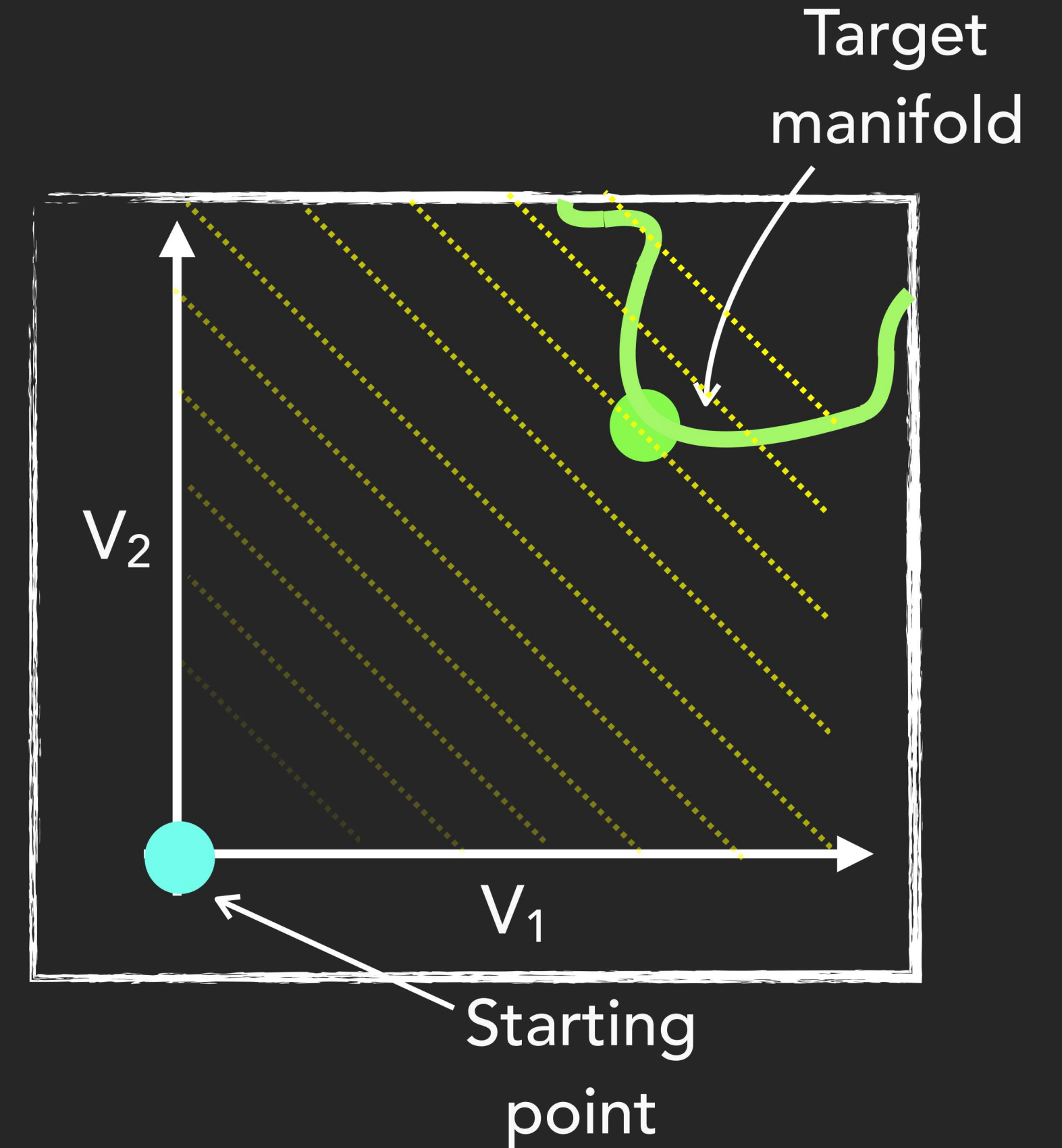


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Using methods from compressed sensing¹⁻³, we can find the solution which minimizes the L_1 norm



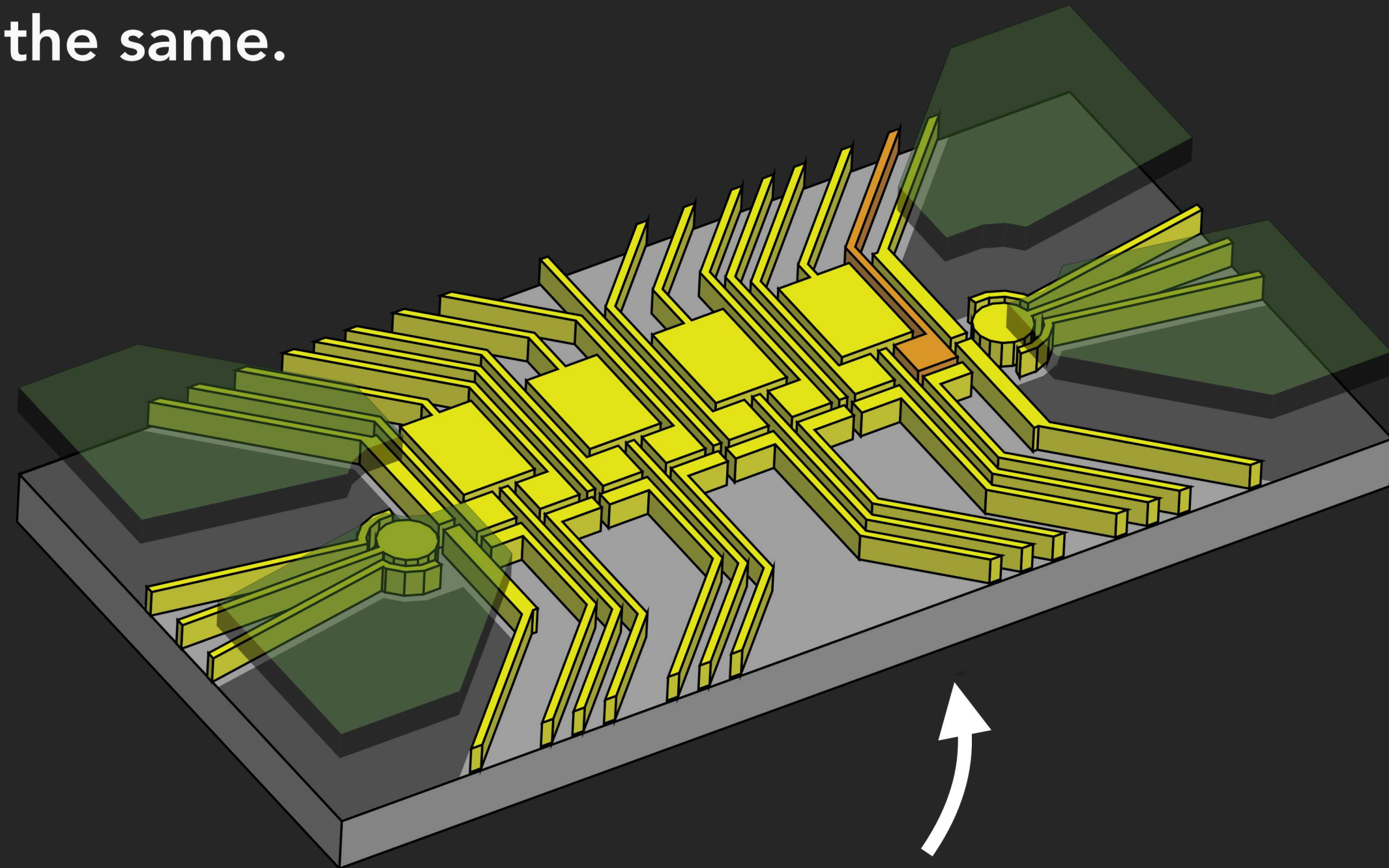
The solution chosen is guaranteed to be at a local minimum of the L_1 norm

[1] E. J. Candès, et al., Communications on Pure and Applied Mathematics, 59, 1207 (2006)
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Minimizing L_1 norm of voltages ensures small voltages on limited number of gates

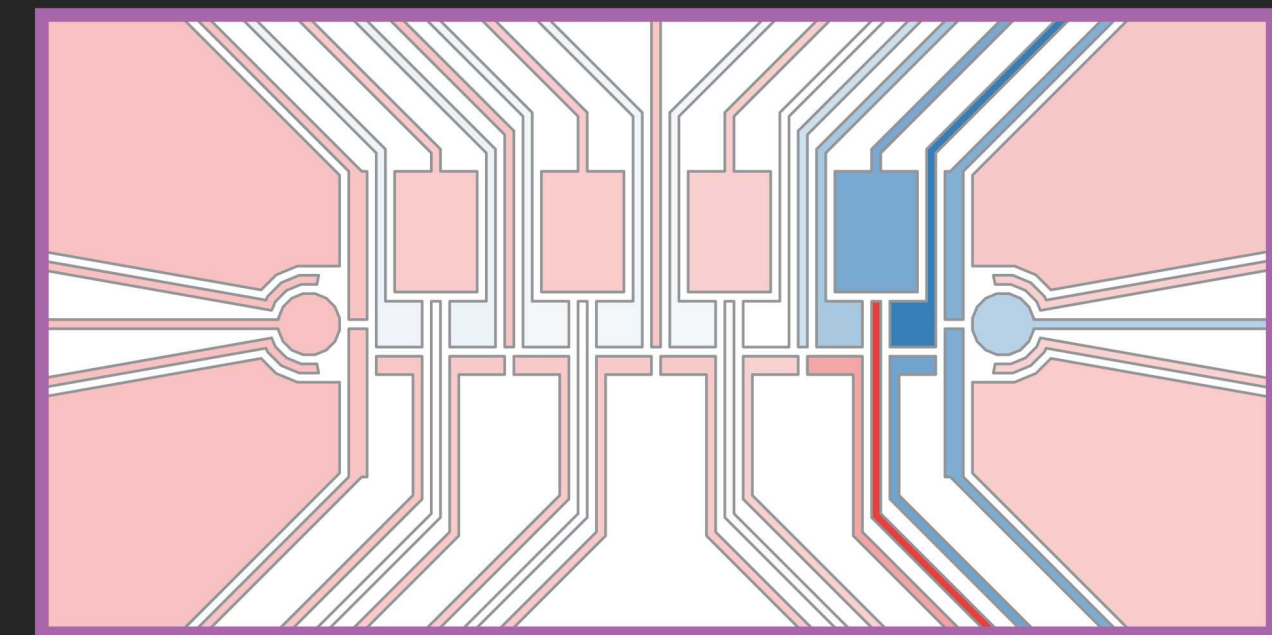
Objective: add one electron to the **right-most dot**, keeping all **tunnel couplings / other dot occupations** the same.



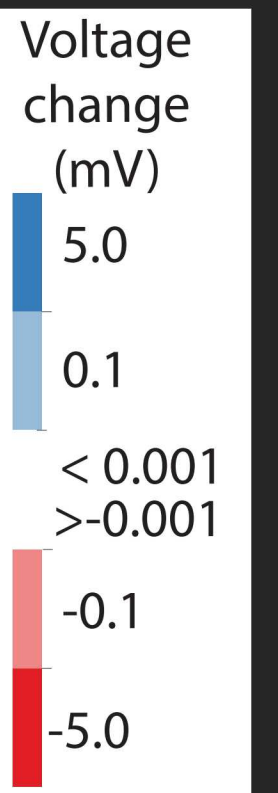
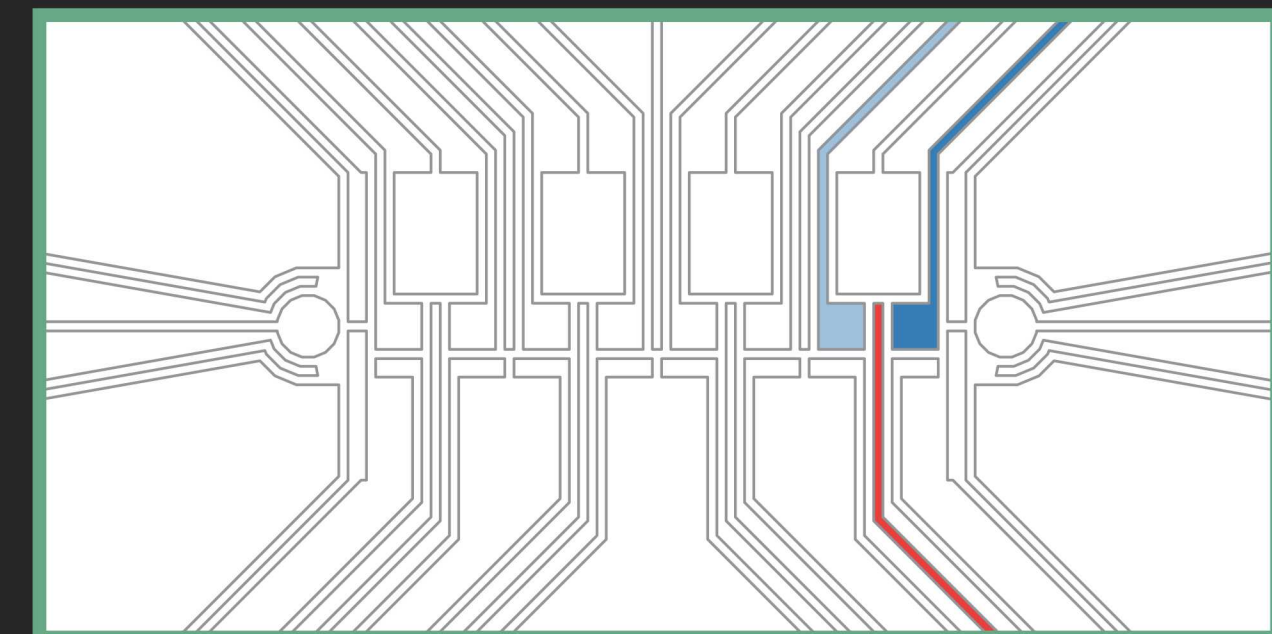
COMSOL simulation.

Each dot contains one electron (calculated via Thomas-Fermi¹ approximation), each pair of dots have appreciable tunnel coupling (calculated using the WKB² approximation)

Minimal L_2 norm



Minimal L_1 norm



The solution with the minimal L_1 norm will have **small voltages** on a **limited number of gates**

[1] Stopa, M. *Phys. Rev. B* 54, 13767–13783 (1996).

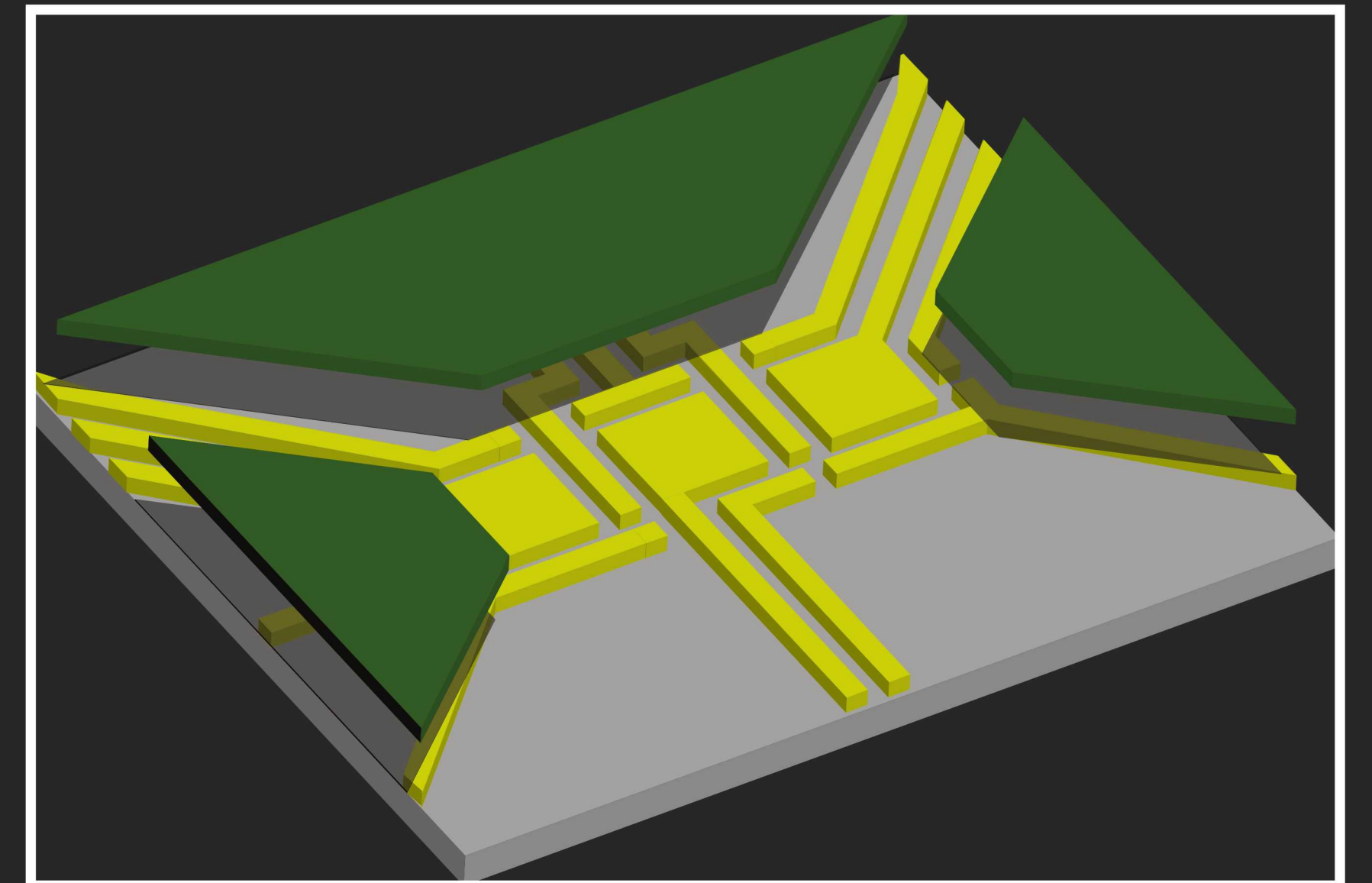
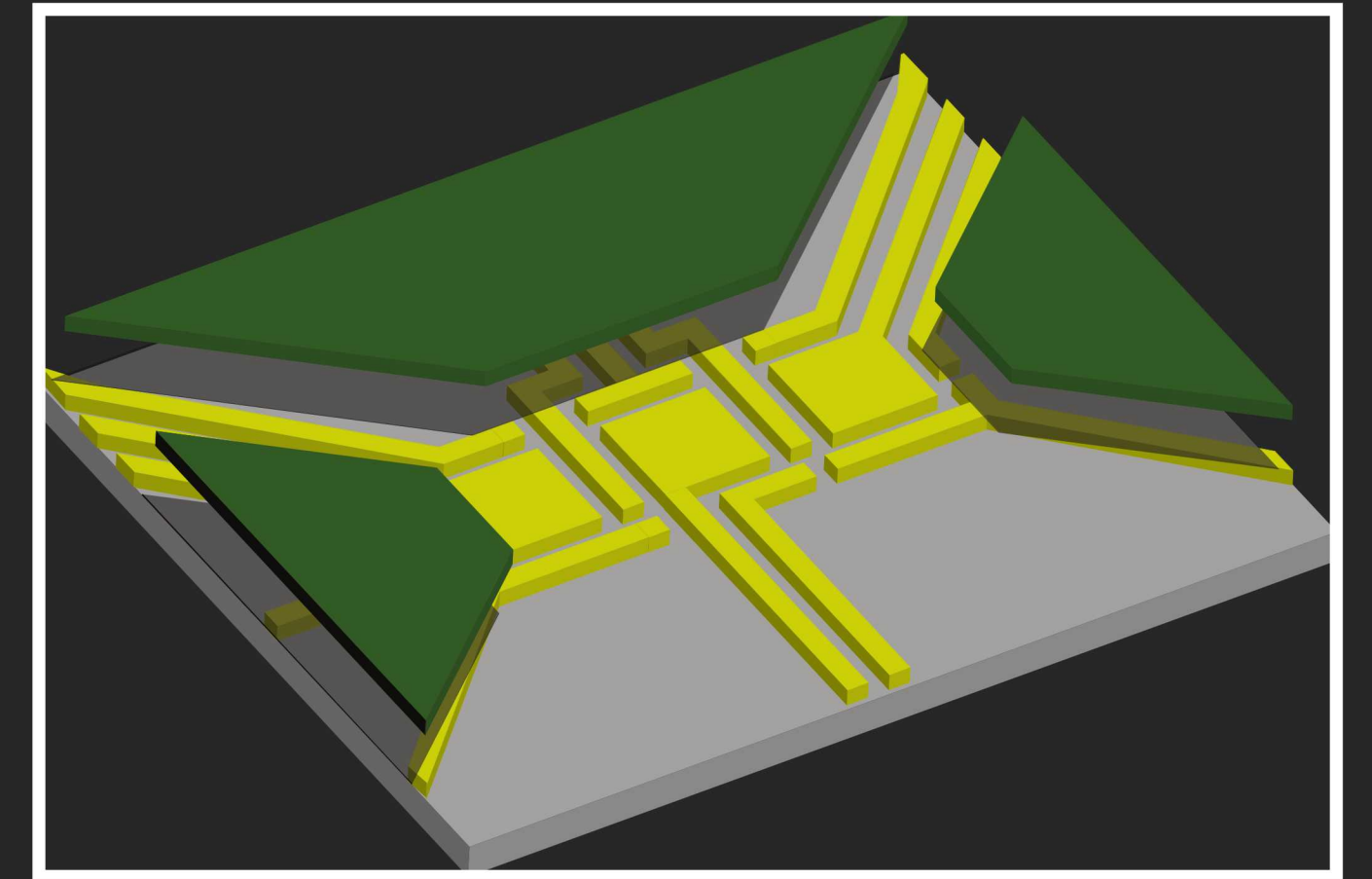
[2] R. Shankar, *Principles of Quantum Mechanics*, 2nd ed (Springer, 1994)



CODA can be applied to finding the optimal size of a device with a given gate design

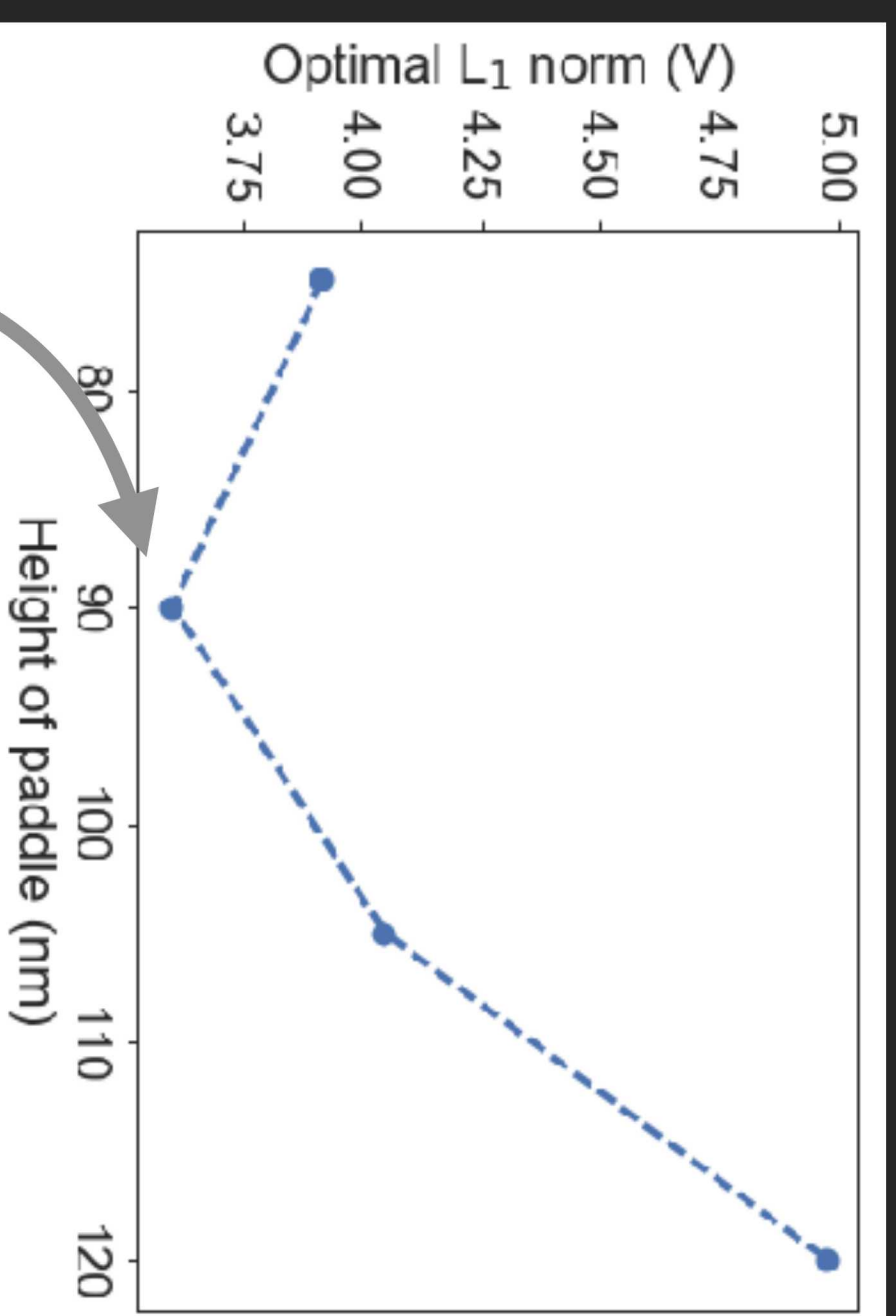
Test multiple device simulations with the same gate design

Operational target:

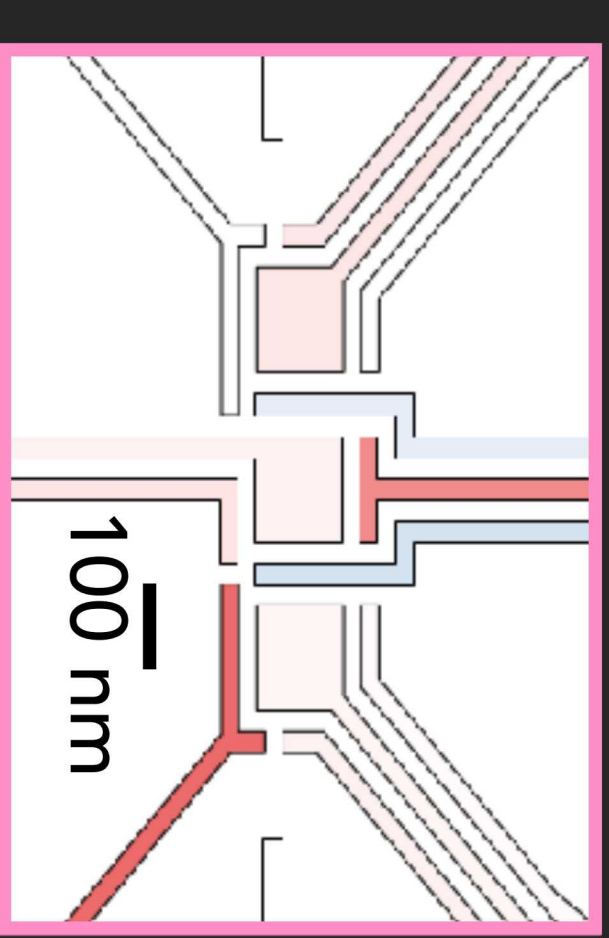
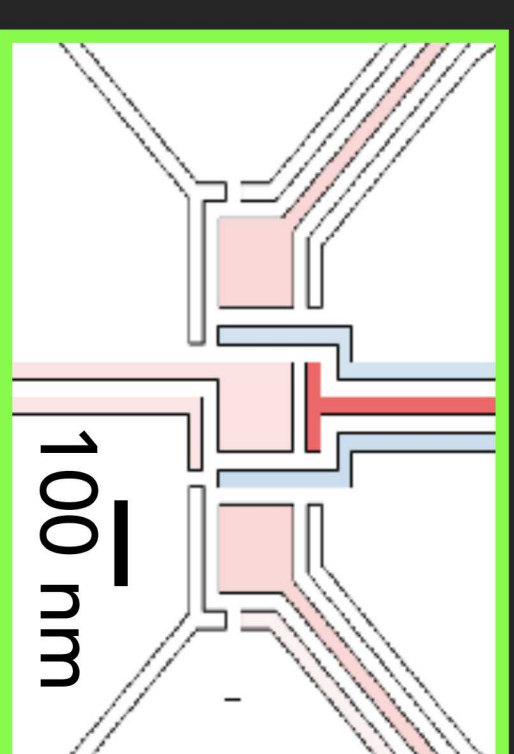
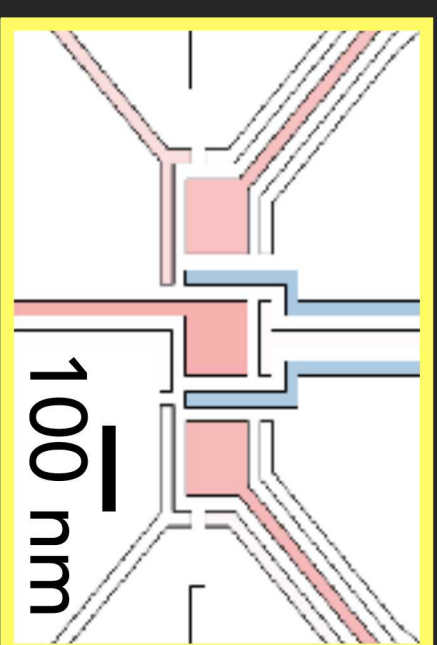


	How we calculate	What we want
Dot occupations	Thomas-Fermi approximation	Reduce from thirty electrons in all three dots to one electron.
Dot energies	Single electron Schrödinger solve	Increase / keep dot energies at 1 meV or above
Tunnel couplings	WKB approximation	Keep tunnel couplings between left / middle dots and middle / right dots constant.

Optimal achievable L_1 norm depends on device size, and has local minimum



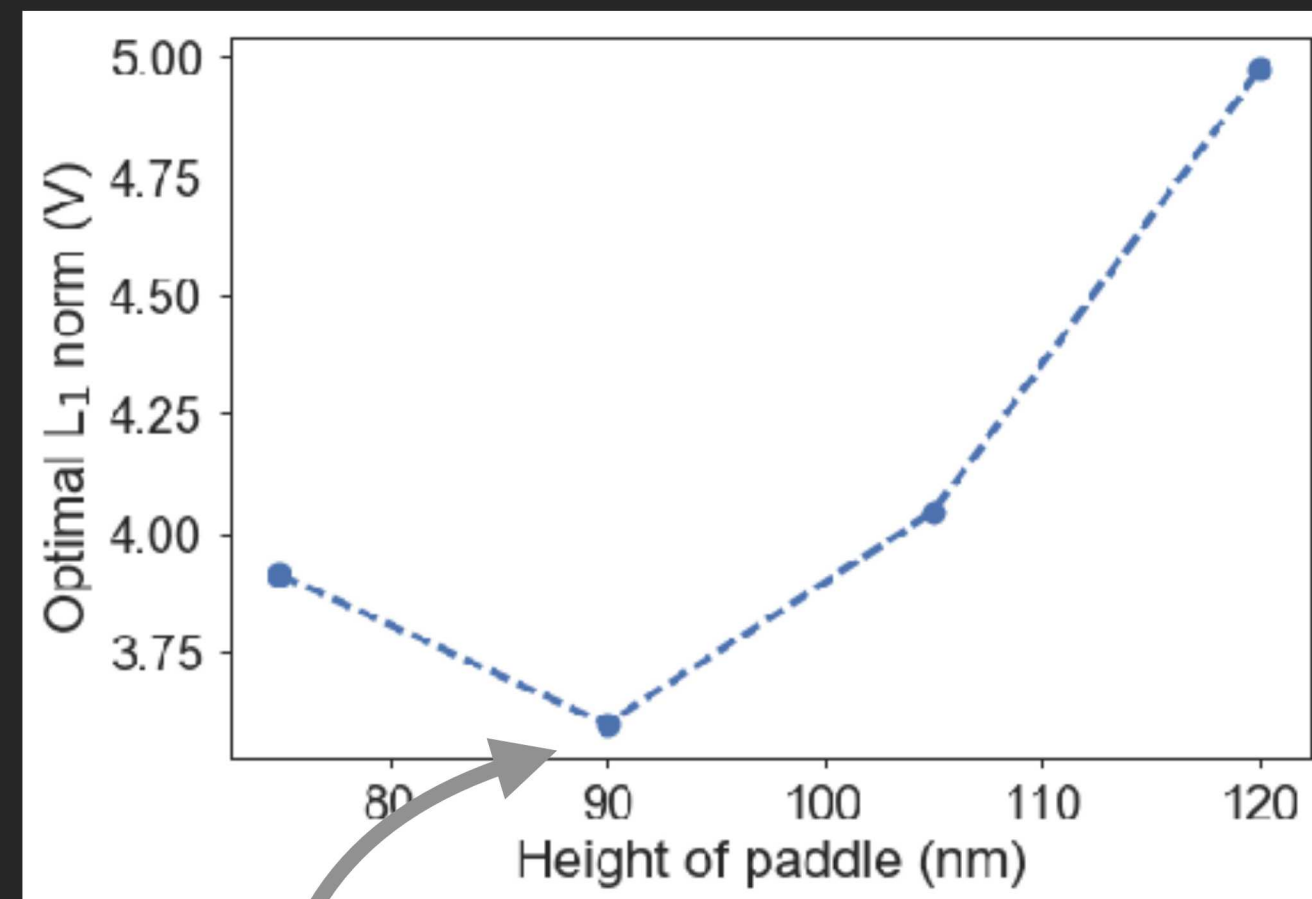
Local minimum in
device voltage L_1
norm



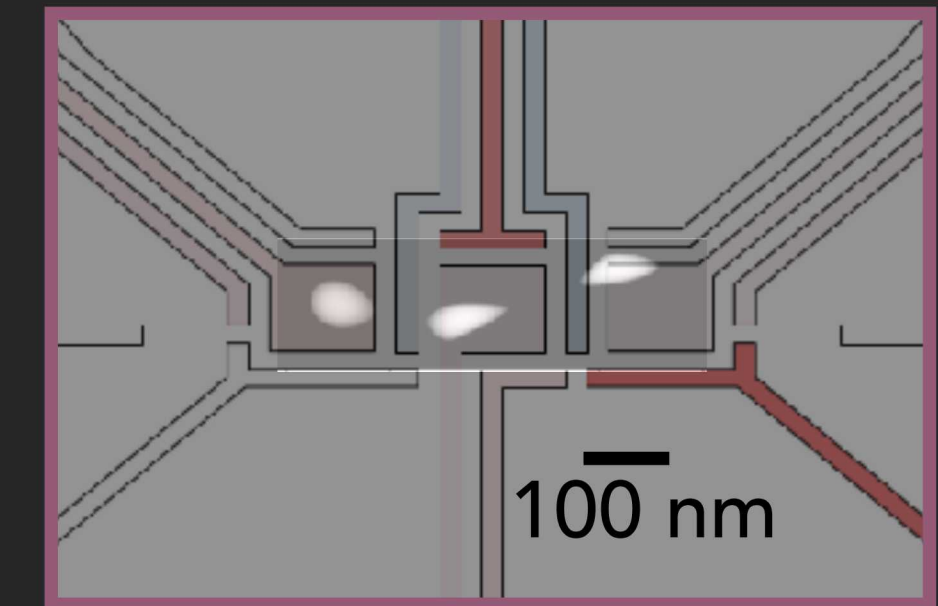
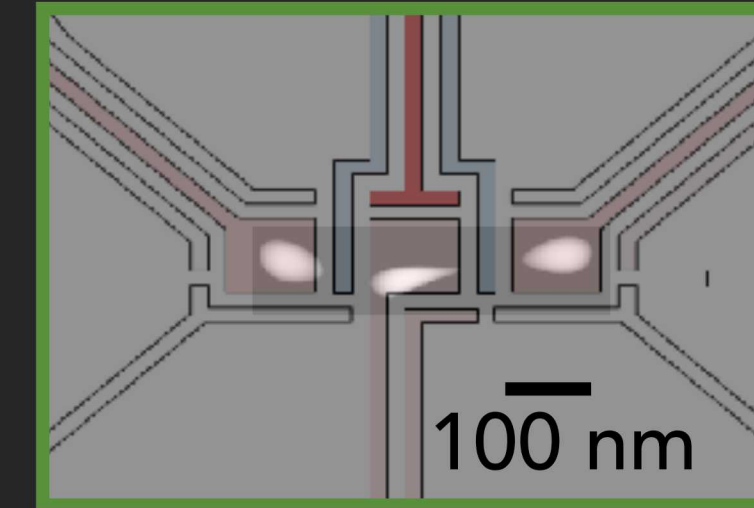
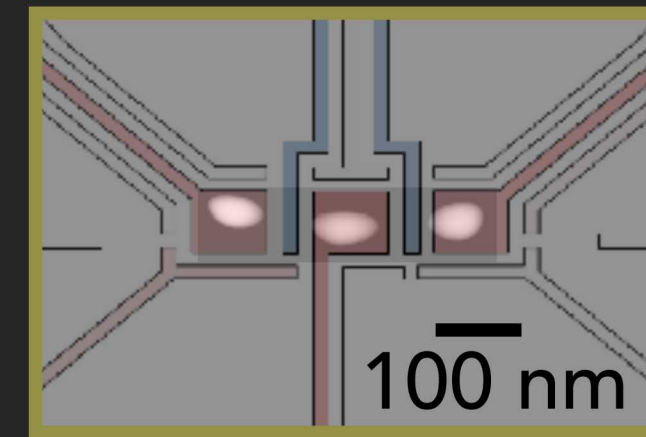
Smaller devices
need larger voltage
changes on paddles
and direct barrier
electrodes

Larger devices need
to use more
electrodes to control
the quantum dots

Optimal achievable L_1 norm depends on device size, and has local minimum



Local minimum in device voltage L_1 norm



←
Smaller devices need larger voltage changes on paddles and direct barrier electrodes

→
Larger devices need to use more electrodes to control the quantum dots

Conclusions

- The CODA procedure can automatically and objectively compare the "tunability" of different device designs
- It does this by finding the smallest L_1 norm of voltages required to perform a given action on a device
- Using this method we can find the optimal size of a device



Thank you!



Additional Slides

