Effects of shear-rate dependent viscosity on the flow of a cement slurry

Chengcheng Tao[1], Eilis Rosenbaum[1], Barbara Kutchko[1], Mehrdad Massoudi[1]
[1]US Department of Energy, National Energy Technology Laboratory, Pittsburgh, PA, 15236

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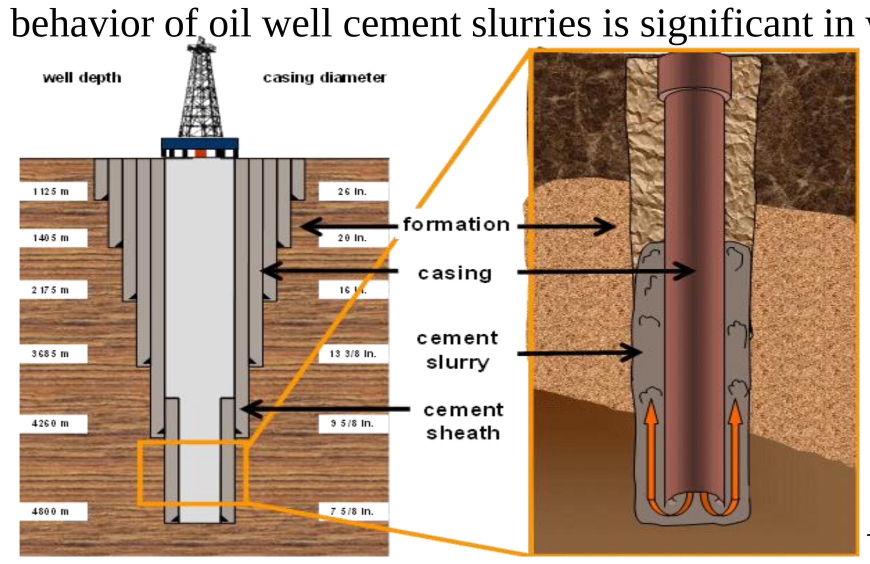
MOTIVATION

- Well cementing is the process of placing a cement slurry in the annulus space between the well casing and the surrounding for zonal isolation
- Goal is to eliminate fluids migration in the well
- Challenges in oil well cementing operations: high temperature, high pressure, weak or porous formations, corrosive fluids
- Cement slurry design for the oil well is a function of various parameters: well bore geometry, casing hardware, drilling mud characteristics, filtration and mixing conditions etc
- Rheological behavior of oil well cement slurries is significant in well cementing operation



Piot, B. (2009)

METHODS

- In this paper, we assume that the cement slurry behaves as a non-Newtonian fluid.
- We use a constitutive relation for the viscous stress tensor which is based on a modified form of the second grade (Rivlin-Ericksen) fluid.
- The motion is steady and fully developed.
- The flow is assumed to be one-dimensional.

Mathematical model

Conservation of mass

$$\frac{\partial \rho}{\partial t} + div(\rho \boldsymbol{v}) = 0$$

ρ: density of cement slurry, which is related to ρ_f (density of pure fluid), $\rho = (1-\phi)\,\rho_f$

Conservation of linear momentum

$$\rho \frac{d\boldsymbol{v}}{dt} = div\boldsymbol{T} + \rho\boldsymbol{b}$$

d/dt: total time derivative, given by $\frac{d(.)}{dt} = \frac{\partial(.)}{\partial t} + [grad(.)]\boldsymbol{v}$

$\boldsymbol{b}$: body force vector

Conservation of angular momentum

$$\boldsymbol{T} = \boldsymbol{T}^T$$

Viscous stress tensor of fluid

$$\boldsymbol{T}_v = -p\boldsymbol{I} + \mu_{eff}(\phi, \boldsymbol{A_1})\boldsymbol{A_1} + \alpha_1\boldsymbol{A_2} + \alpha_2\boldsymbol{A_1}^2$$

p: pressure

ϕ: volume fraction

$\boldsymbol{A_n}$: n-th order Rivlin-Ericksen tensors

where $\boldsymbol{A_1} = \nabla\boldsymbol{v} + \nabla\boldsymbol{v}^T$ $\quad \boldsymbol{A_2} = \frac{d\boldsymbol{A_1}}{dt} + \boldsymbol{A_1}\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^T\boldsymbol{A_1}$

α_1, α_2: normal stress coefficients

μ_{eff}: effective viscosity, where $\mu_{eff} = \mu(\phi)$

$\mu(\phi)$: viscosity correlation for cement slurry, which is given by Krieger and Dougherty viscosity model $\mu(\phi) = \mu_0(1-\frac{\phi}{\phi_m})^{-\beta\phi_m}$

μ_0: viscosity of the cement slurry without particles

β: intrinsic viscosity of cement slurry

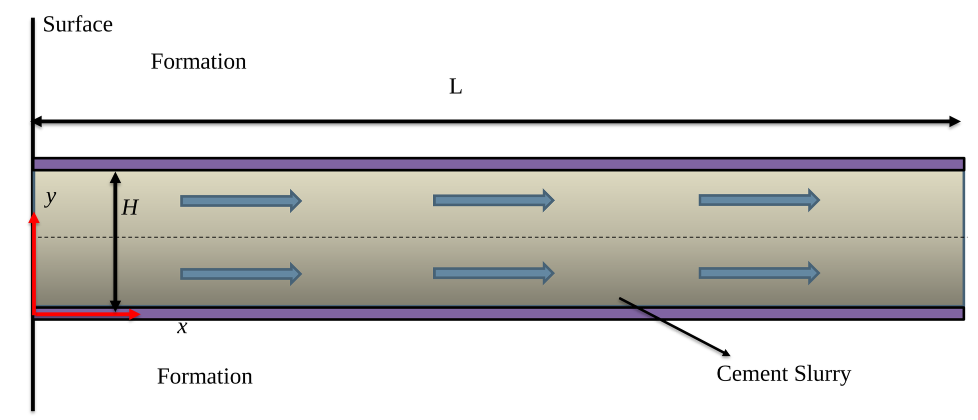
ϕ_m: maximum volume concentration of solids

Geometry of the flow

The velocity and the volume fraction forms

$$\begin{cases} \phi = \phi(y) \\ \boldsymbol{v} = v(y)\boldsymbol{e_x} \end{cases}$$

Schematic diagram



Dimensionless forms of the equations

$$\bar{y} = \frac{y}{H} \qquad \bar{v} = \frac{v}{V}$$

$$\begin{cases} \frac{\partial}{\partial \bar{y}}\left[\left(1-\frac{\phi}{\phi_m}\right)^{-\beta}\frac{\partial \bar{v}}{\partial \bar{y}}\right] = N_1 \\ N_2\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} = \frac{\partial \phi}{\partial \bar{y}} + N_3(1-\phi) \end{cases}$$

Where the dimensionless paremeters are

$$N_1 = f_1\frac{H^2}{\mu_0 V}$$

$$N_2 = \frac{2(2\alpha_1+\alpha_2)V^2}{f_2 H^2}$$

$$N_3 = \frac{\rho_f g H}{f_2}$$

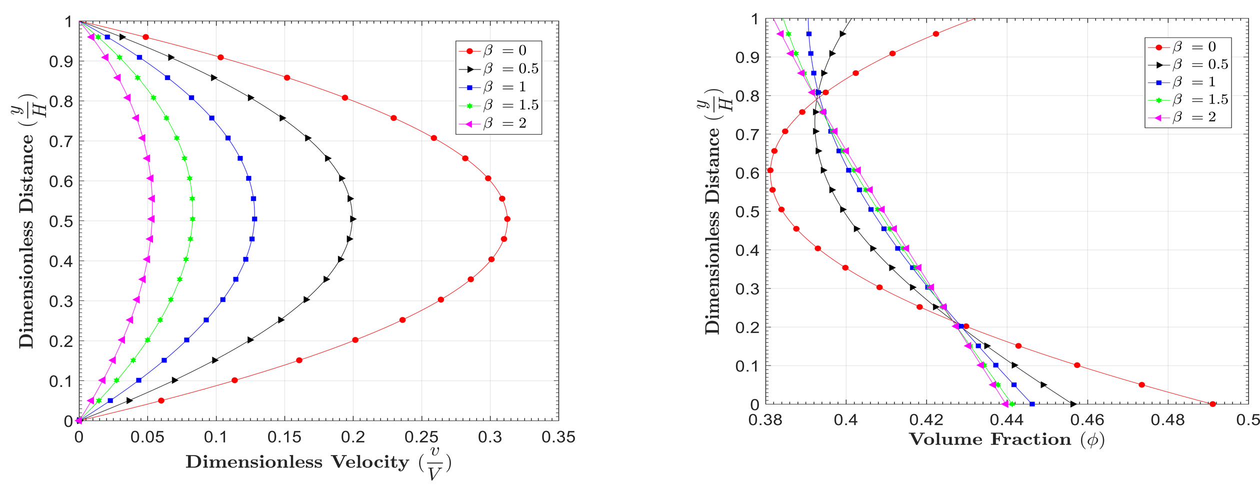
Boundary conditions

$$\begin{cases} \bar{v}(\bar{y}=0) = 0 \\ \bar{v}(\bar{y}=1) = 0 \\ \int_0^1 \bar{\phi}\, d\bar{y} = \bar{\phi}_{avg} \end{cases}$$

$\bar{\phi}_{avg}$: average value of $\bar{\phi}$ integrated over the cross section

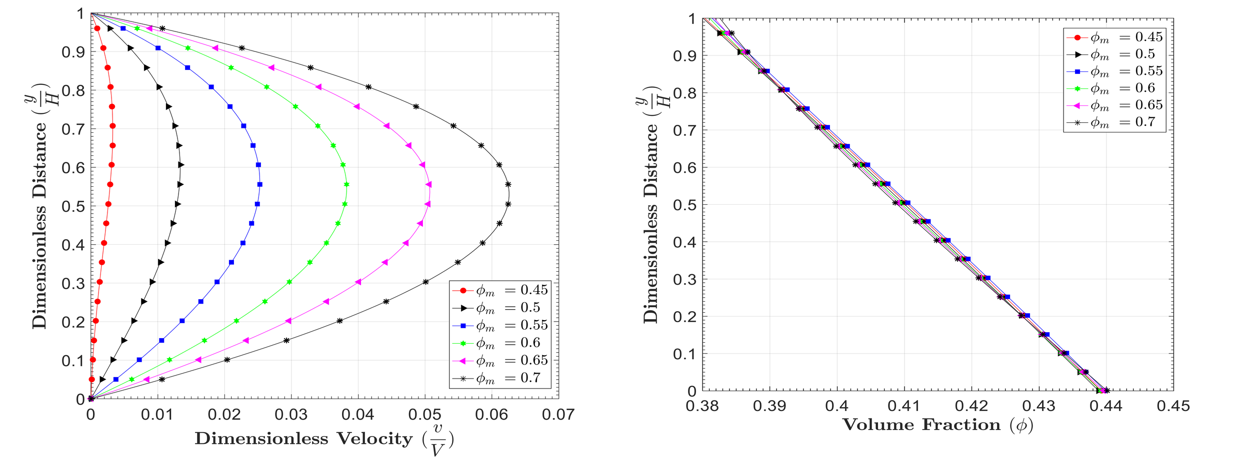
RESULTS

Effect of shear dependency viscosity parameter β



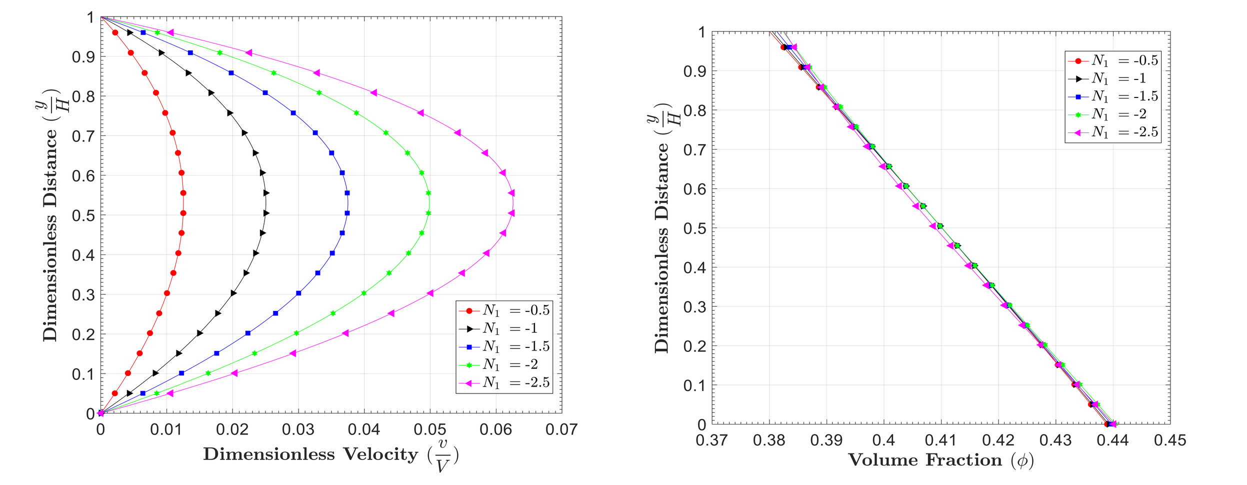
$\phi_m = 0.7, N_1 = -2.5, N_2 = 0.1, N_3 = 0.1, \phi_{avg} = 0.4$

Effect of maximum particle packing parameter ϕ_m



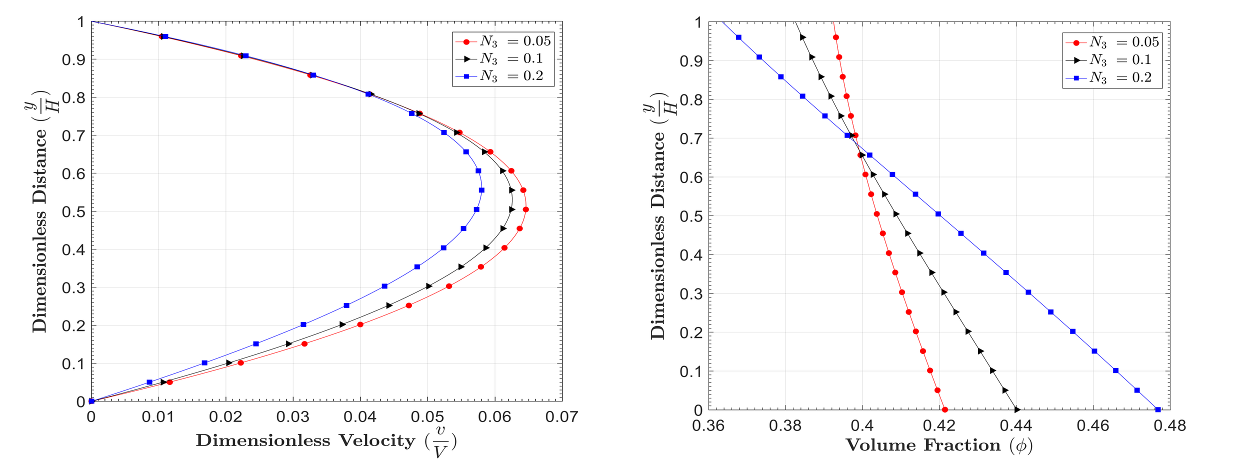
$\beta = 1.82, N_1 = -2.5, N_2 = 0.1, N_3 = 0.1, \phi_{avg} = 0.4$

Effect of x-axis pressure dependency parameter N_1



$\beta = 1.82, \phi_m = 0.7, N_2 = 0.1, N_3 = 0.1, \phi_{avg} = 0.4$

Effect of flux parameter N_3



$\beta = 1.82, \phi_m = 0.7, N_1 = -2.5, N_2 = 0.1, \phi_{avg} = 0.4$

CONCLUSIONS

- In this paper, we have modeled a cement slurry as a non-Newtonian suspension and have used a constitutive relation for the viscous stress tensor which is based on a modified form of the second grade (Rivlin-Ericksen) fluid.
- The governing equations for the fully developed flow of such a fluid inside a horizontal pipe were non-dimensionalized and numerically solved.
- The parametric study indicates that the flow field is affected by pressure and shear dependency parameter of the fluid.
- This study is simply a preliminary case and further studies will be performed where the effects of diffusion, heat transfer, such as viscous dissipation and yield stresses will be considered.

References

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