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Selection of a Nominal Device Using Functional Data Analysis

PRESENTED BY

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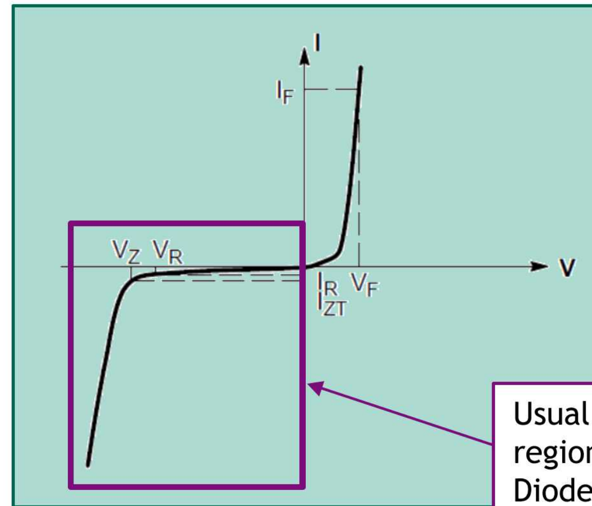
Motivation

Zener Diode

MMSZ5239BT1G
Zener Diode
9.1V 500 mW,
SOD123,
Cu wire bond

- An **ideal** diode is an electronic component that only allows current to flow when a critical voltage is reached.
- A **real** Zener diode, as dictated by the physical characteristics of a semiconductor P-N junction, has a current voltage-characteristic that approximates ideal diode behavior.

Current Voltage Characteristic of a Zener Diode



Usual operational region of a Zener Diode in an electrical circuit

Electrical parameters defining the diode electrical behavior

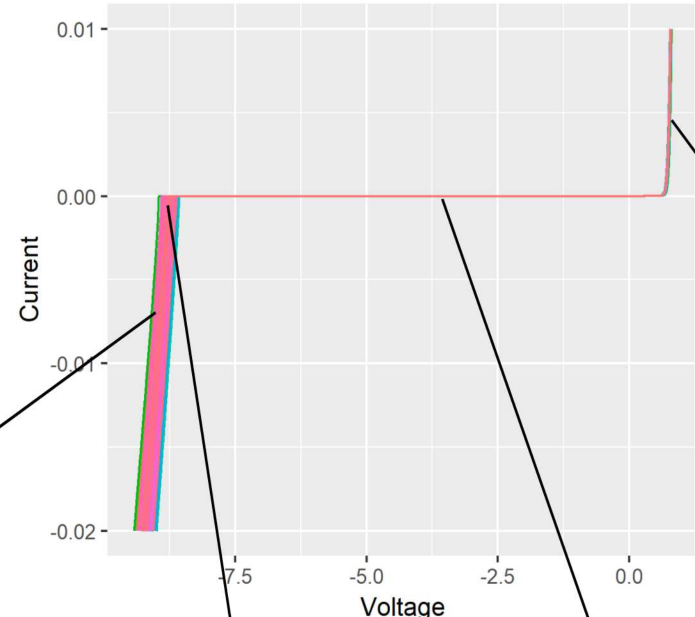
Symbol	Parameter
V_Z	Reverse Zener Voltage @ I_{ZT}
I_{ZT}	Reverse Current
Z_{ZT}	Maximum Zener Impedance @ I_{ZT}
I_{ZK}	Reverse Current
Z_{ZK}	Maximum Zener Impedance @ I_{ZK}
I_R	Reverse Leakage Current @ V_R
V_R	Reverse Voltage
I_F	Forward Current
V_F	Forward Voltage @ I_F

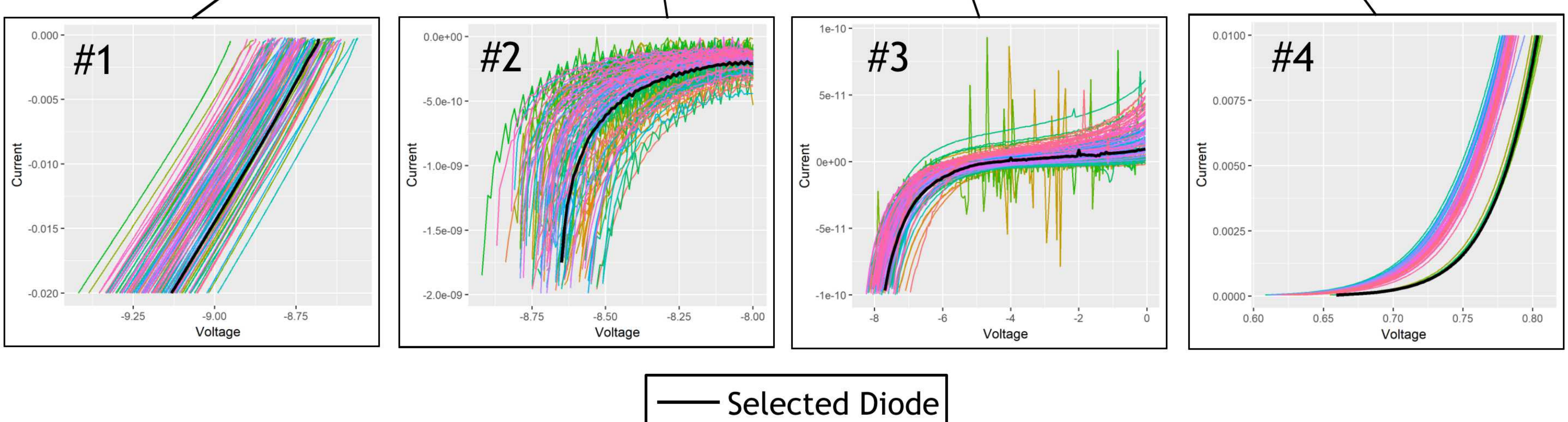
Voltage at which diode allows current to pass

Zener Diode Data

- Data taken in four different measurement sweeps
- Relevant measured behavior spans eight orders of magnitude
- Electrical behavior of a single diode shifts within a distribution

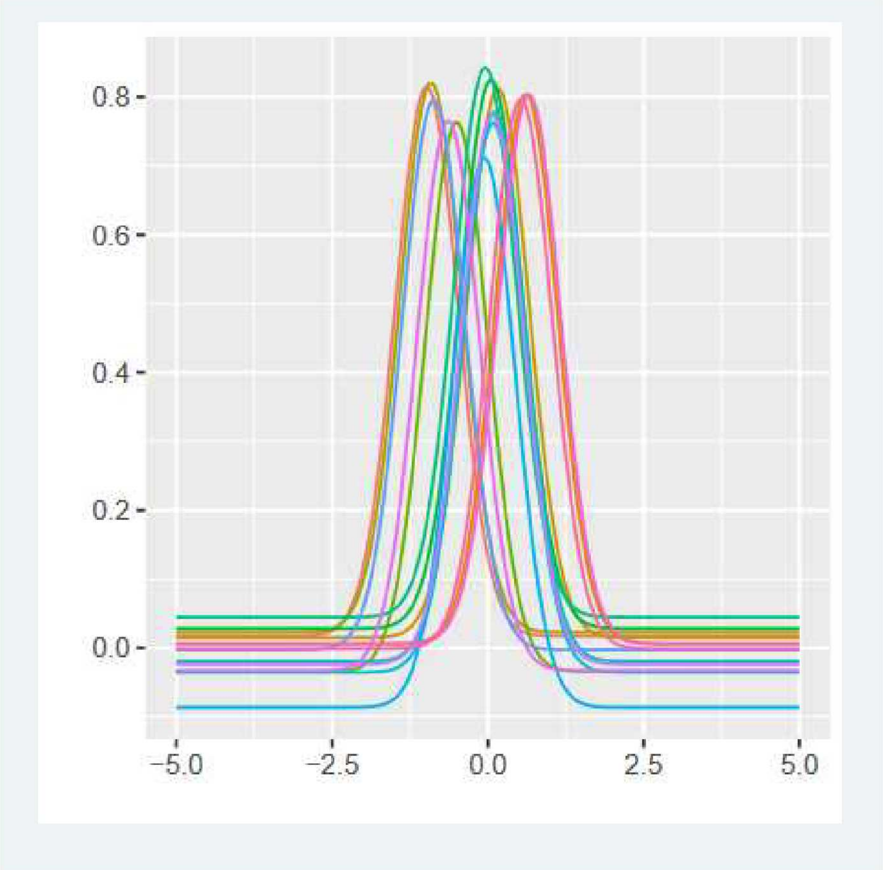
How does one define a representative diode from this dataset?





What is Functional Data?

- **Functional data** is data that various continuous across an independent variable.
- We want to analyze the data in its **entirety**.



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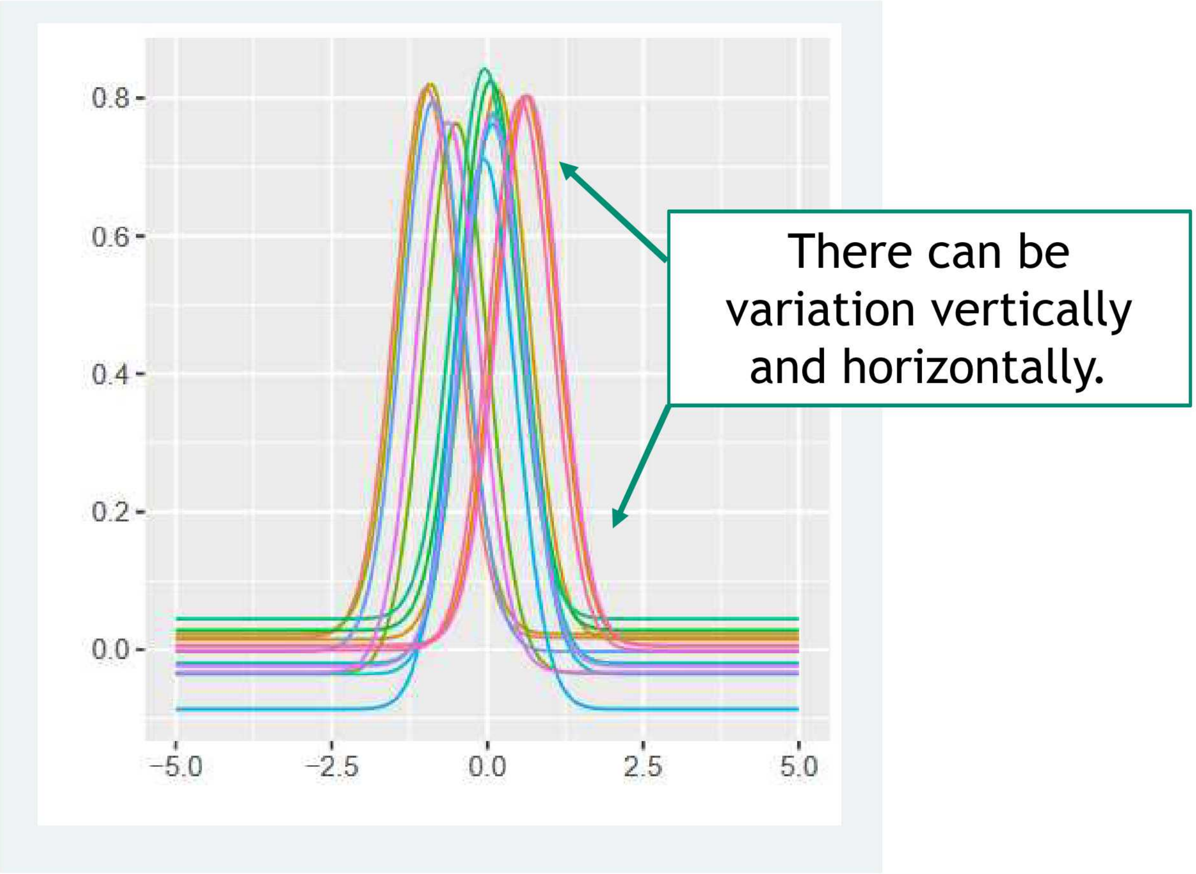

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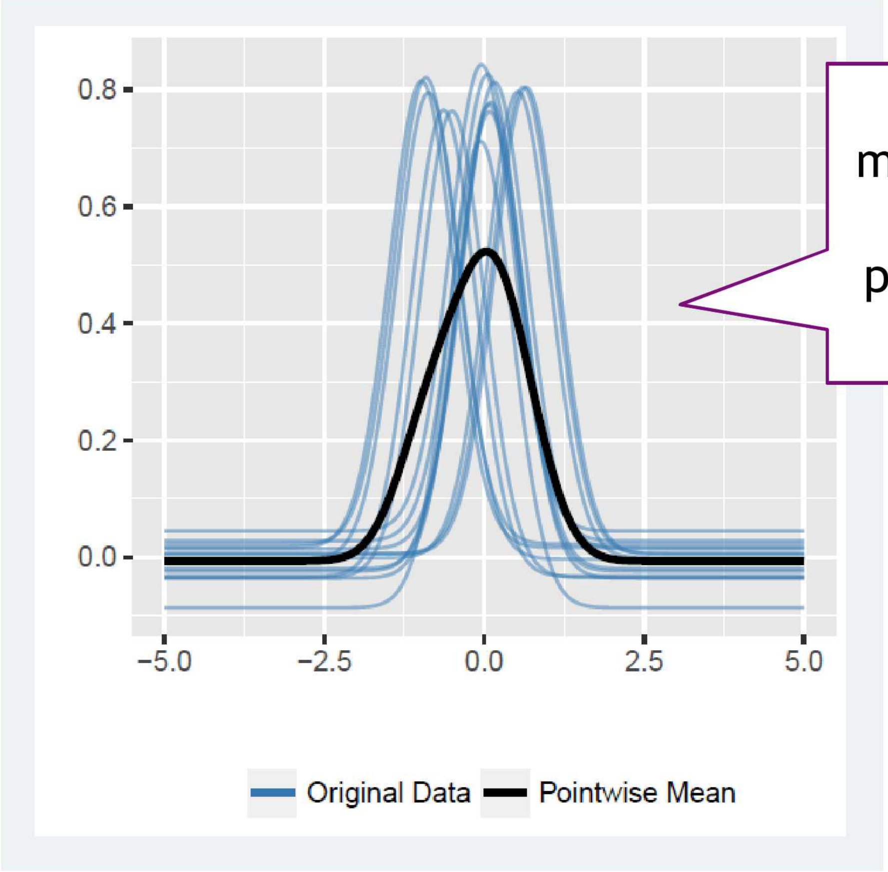
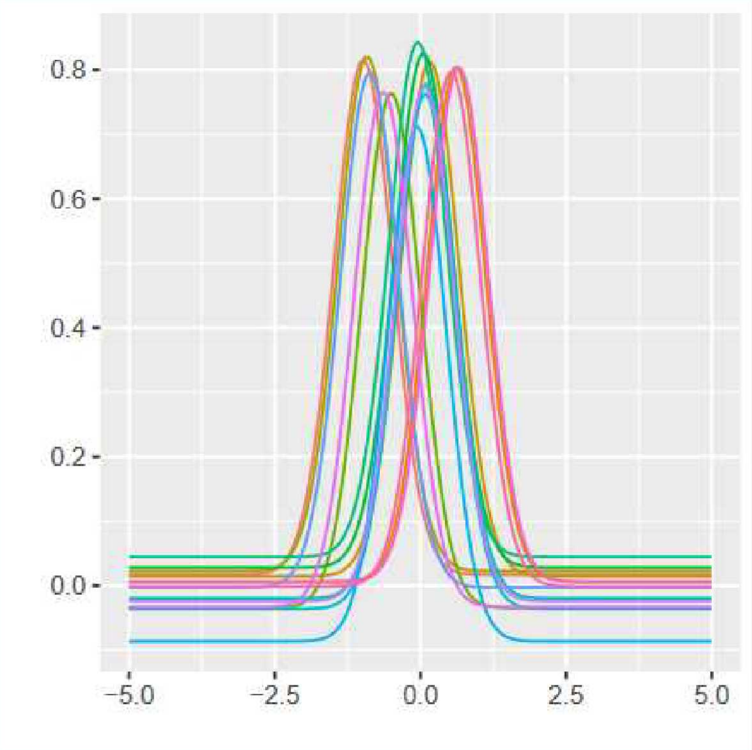


In these cases, methods that require discretization can produce sub-optimal results.

Functional Data Analysis

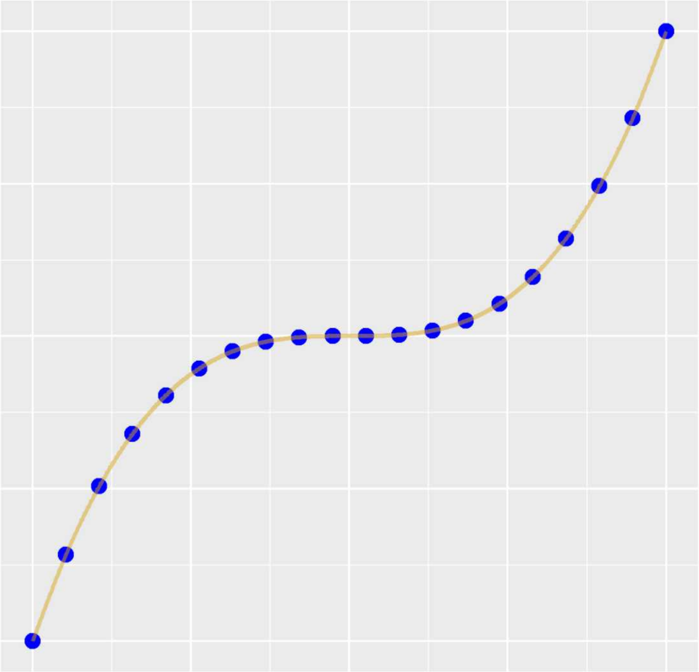


4 Steps

1. Data Smoothing & Interpolation
2. Data Warping
3. Mean Estimation
4. Characterization of Variability

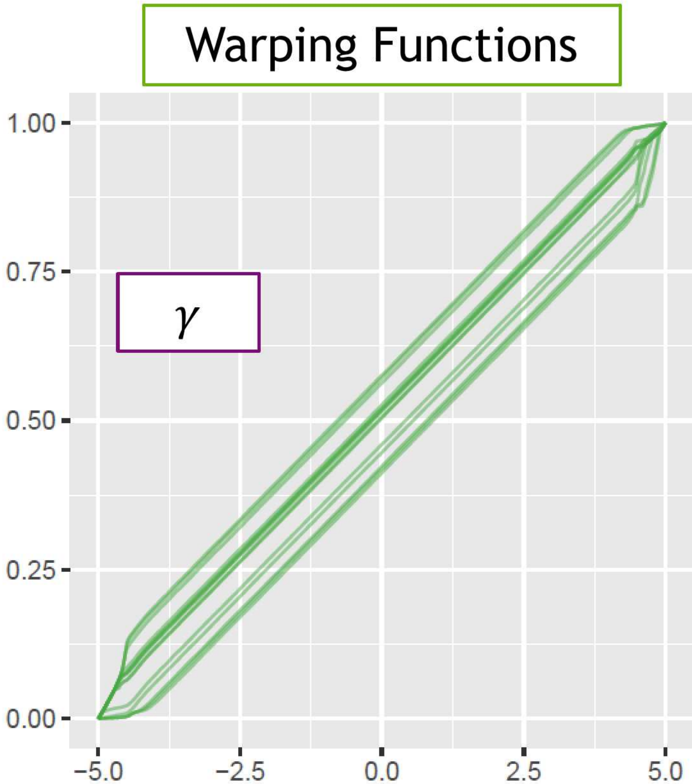
1. Smoothing & Interpolation

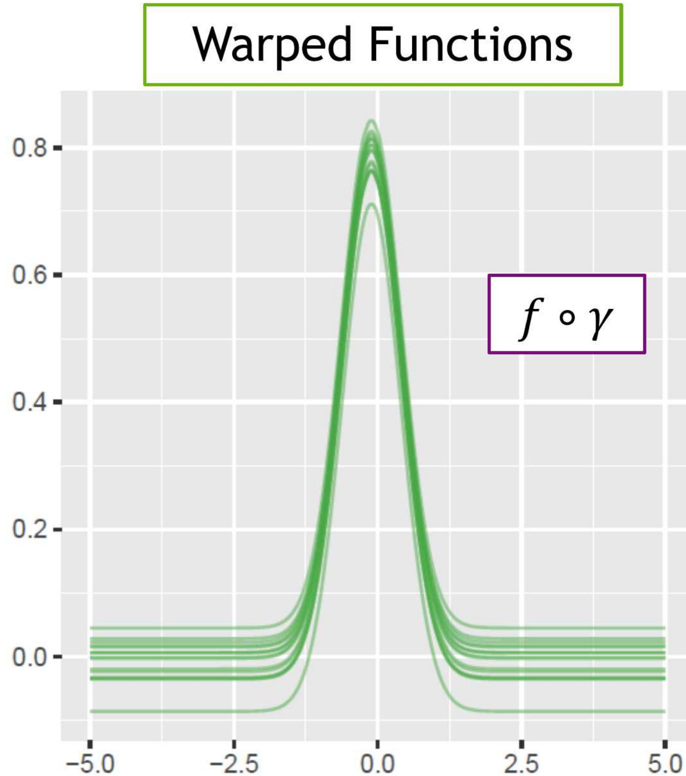
Since functional data is typically obtained from discrete measurements, the first step is to smooth the data (if needed) and then interpolate between points to get equally spaced points along the independent variable.



This can be done through a simple linear interpolation, or through more flexible methods such as splines.

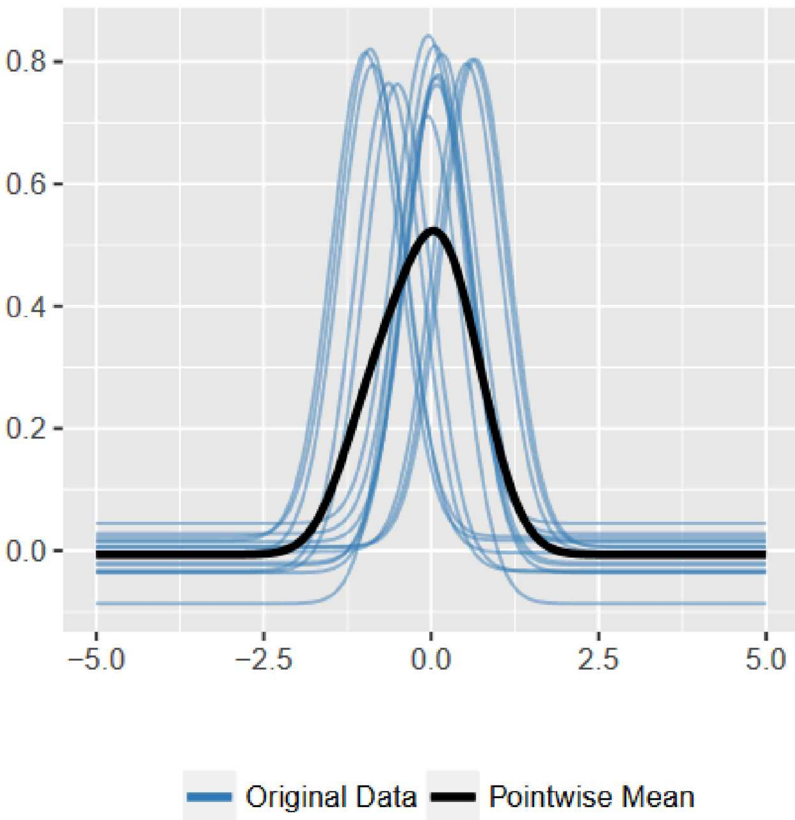
2. Data Warping

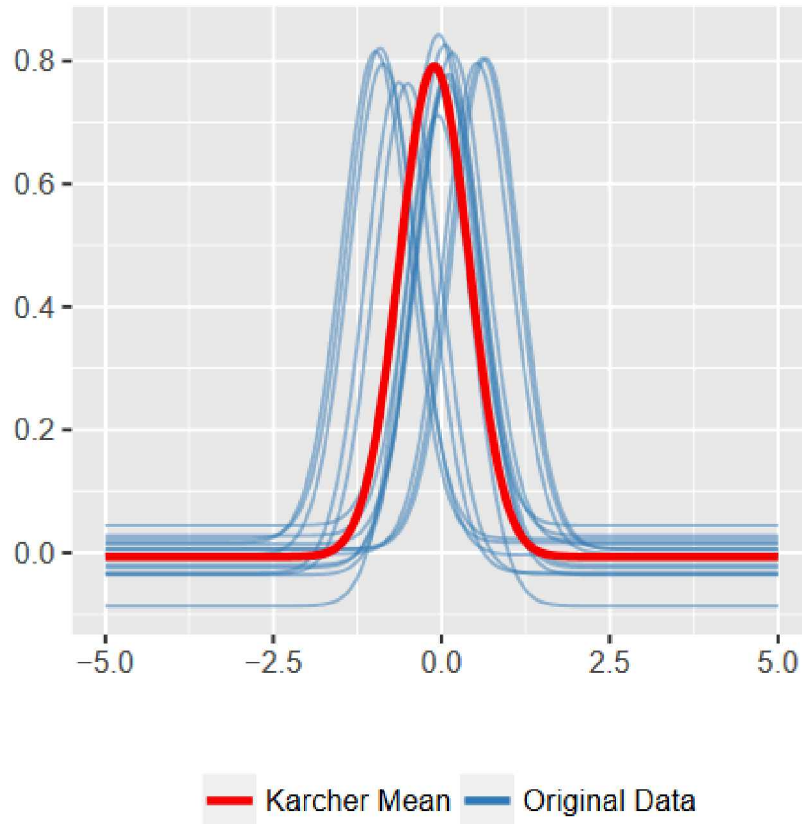




F: Set of functions on $[a, b]$
Γ: Set of diffeomorphisms on $[a, b]$
A diffeomorphism is a function γ that maps $[a, b]$ to $[a, b]$
and has constraints $\gamma(a) = 0$ and $\gamma(b) = 1$

3. Mean Estimation Using Karcher Mean





$$\mu_q = argmin_{f \in F} \sum_{i=1}^{n} d_a(f, f_i)^2$$

d_a is a distance metric

Variability Decomposed into 2 Parts

Amplitude Variability

$$d_a(f_1, f_2) = \inf_{\gamma \in \Gamma} ||q_1 - (q_2 \circ \gamma)\sqrt{\dot{\gamma}}\,||$$

$$q(t) = sign(\,\dot{f}(t))\sqrt{|\dot{f}(t)|}$$

Phase Variability

$$d_p(\gamma_1, \gamma_2) = \cos^{-1}\left(\int_0^1 \psi_1(t)\psi_2(t)dt\right)$$

$$\psi = \sqrt{\dot{\gamma}}$$

d_a and d_p are calculated between each function and the Karcher mean

Elastic Distance

A total distance – known as the **elastic distance** – can then be calculated for each function using a weighted sum of the amplitude and phase distances.

$$d_\tau\left(f_i, \mu_q\right) = (1 - \tau)d_a\left(f_i, \mu_q\right) + (\tau)d_p(f_i, \mu_q)$$

$$\tau \in [0,1]$$

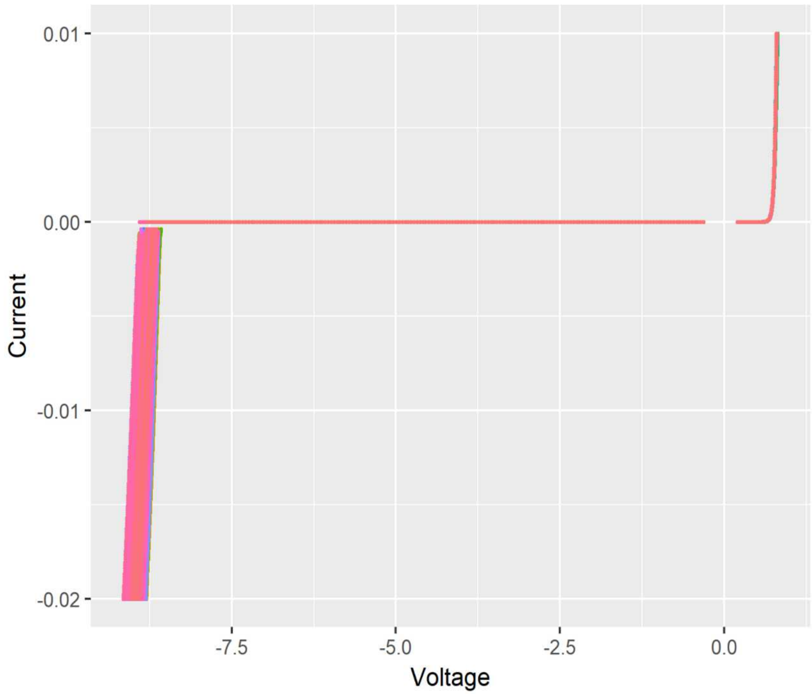
The **nominal device** can be chosen as the device with the smallest elastic distance from the Karcher mean.

Applying FDA to Zener Diode Data

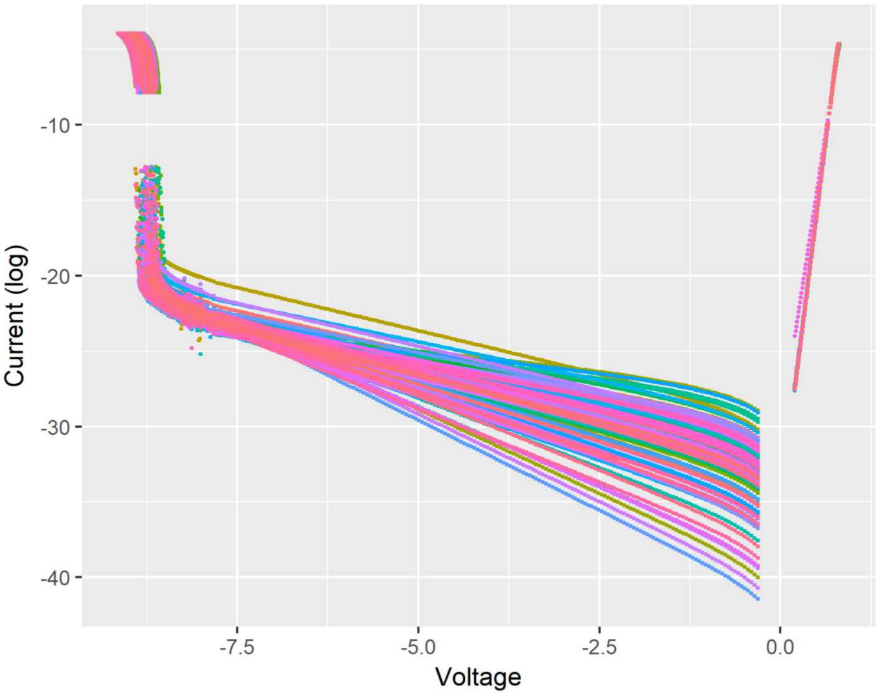
Experimental Data

120 Zener Diodes

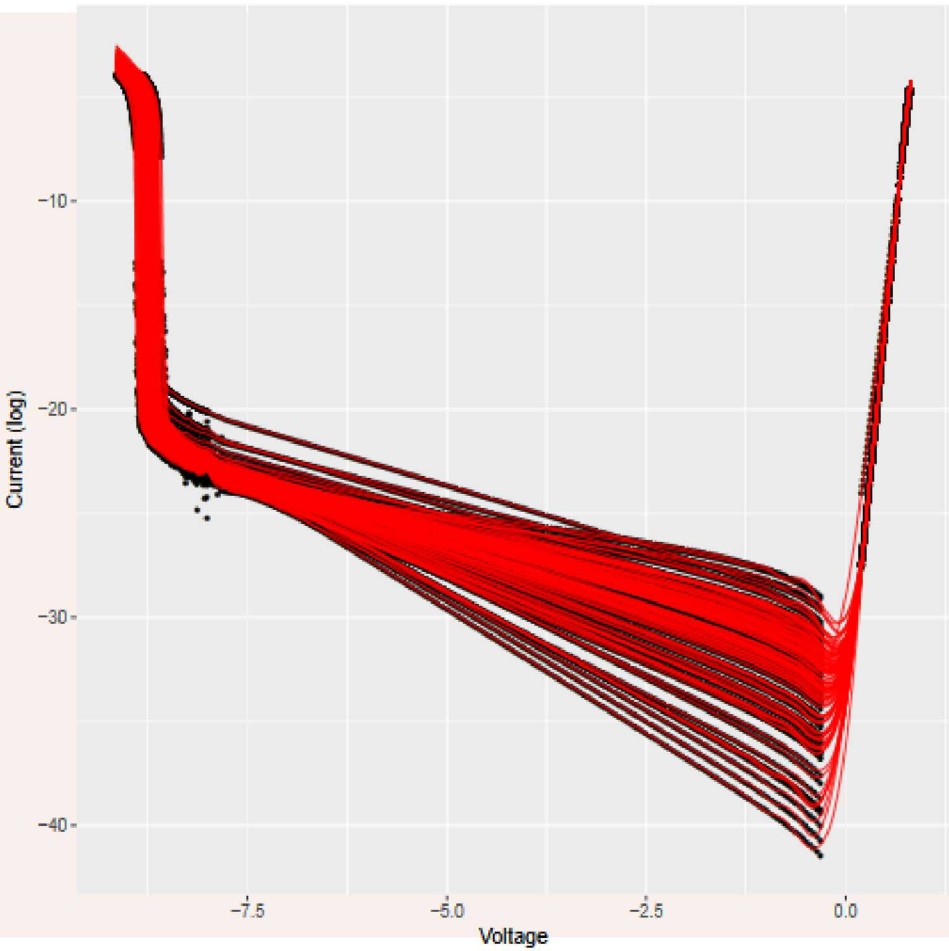
Original Scale



Log Scale

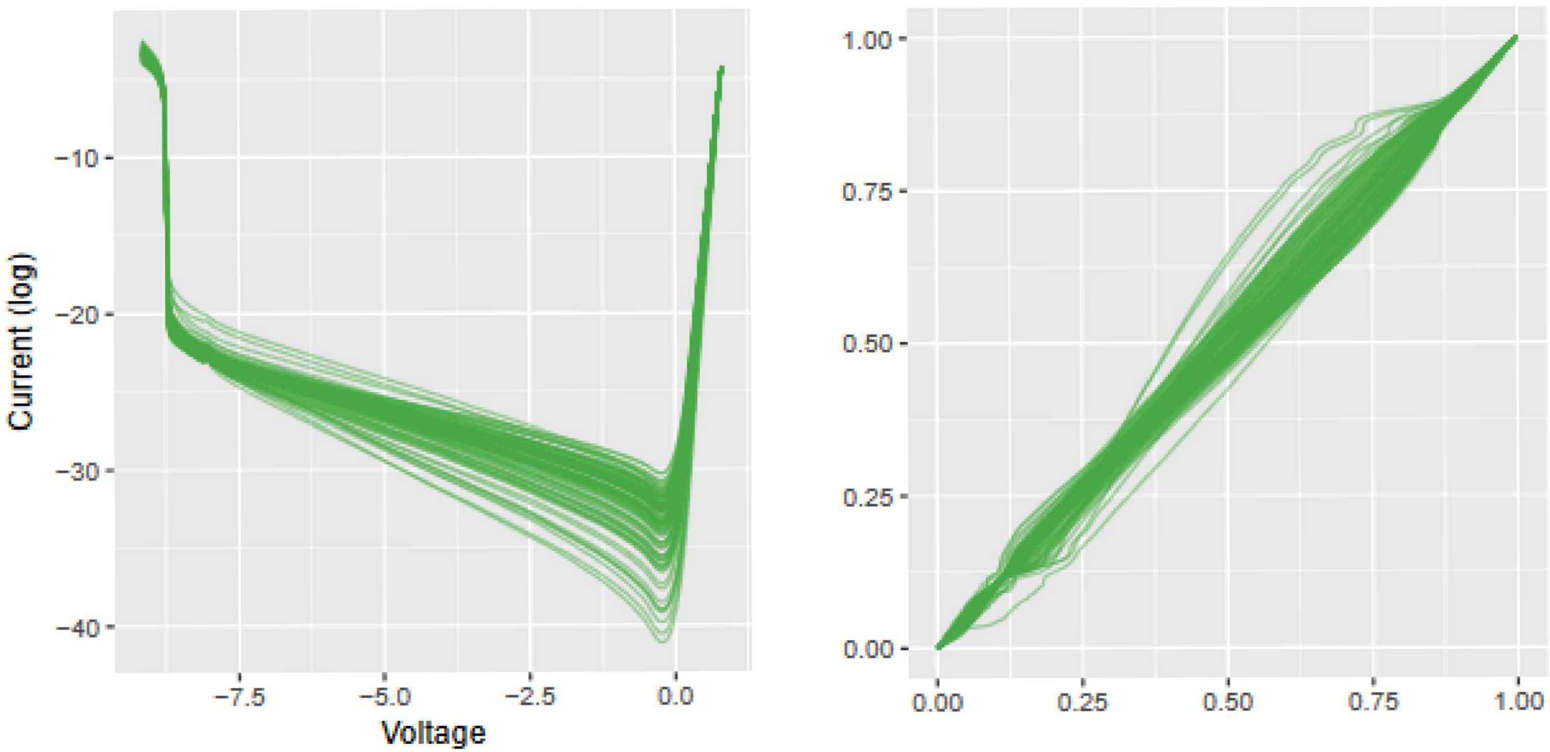


1. Smoothing & Interpolation



Smoothing spline fit to each device and then used to interpolate equally spaced values across voltage.

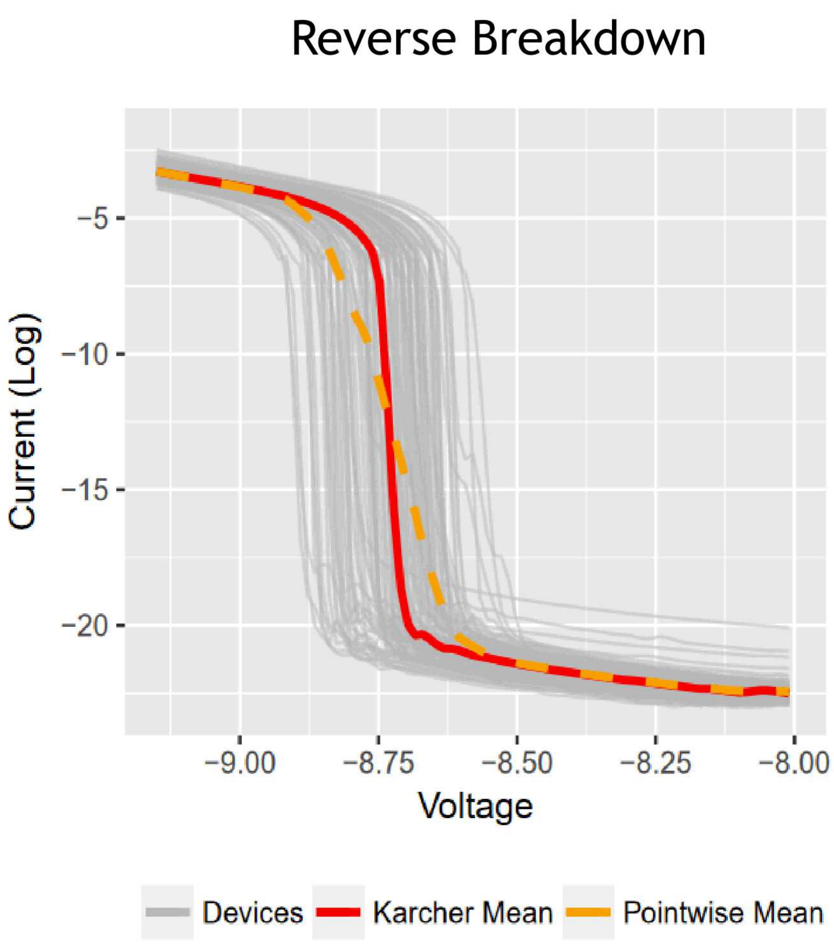
2. Data Warping

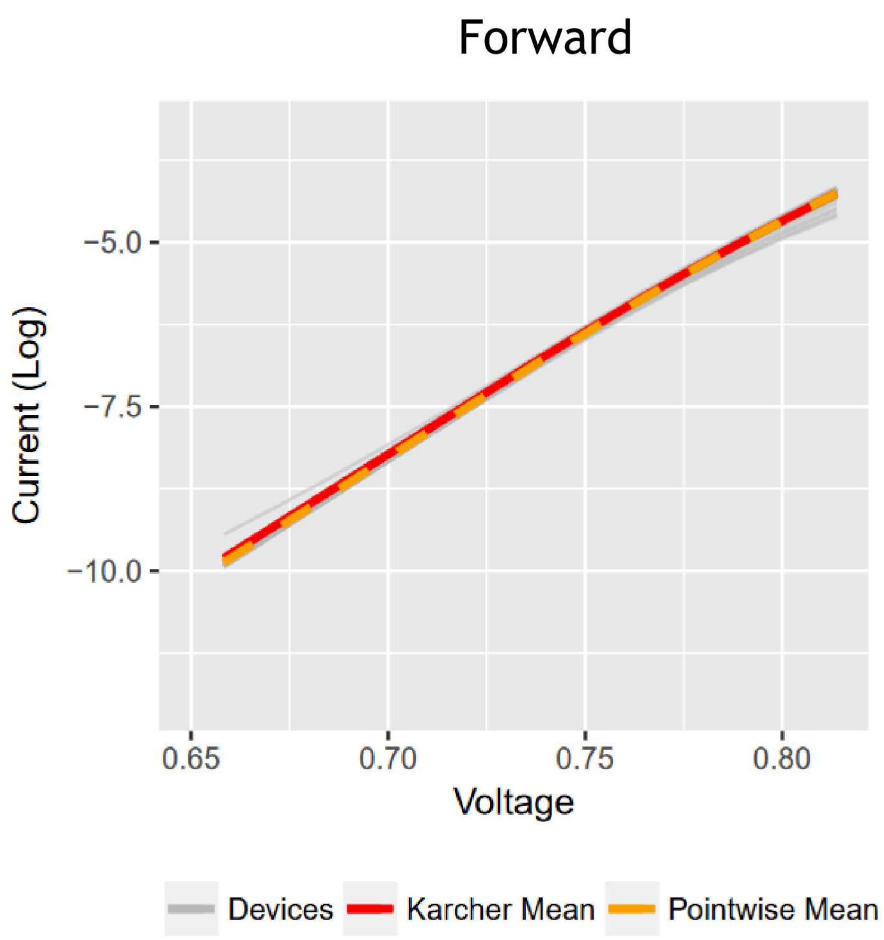


(a) Warped (Aligned) Functions

(b) Warping Functions

3. Mean Estimation





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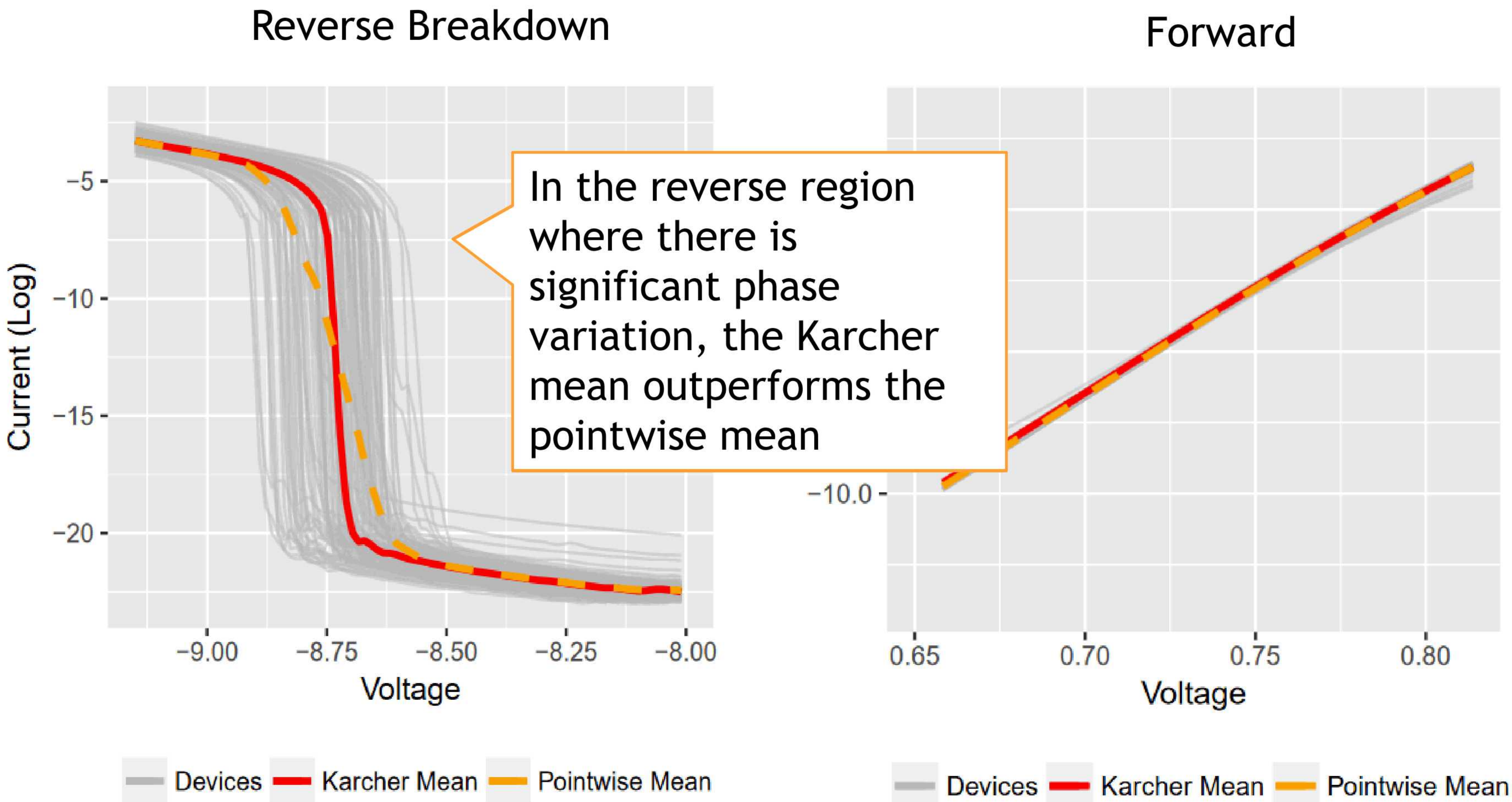
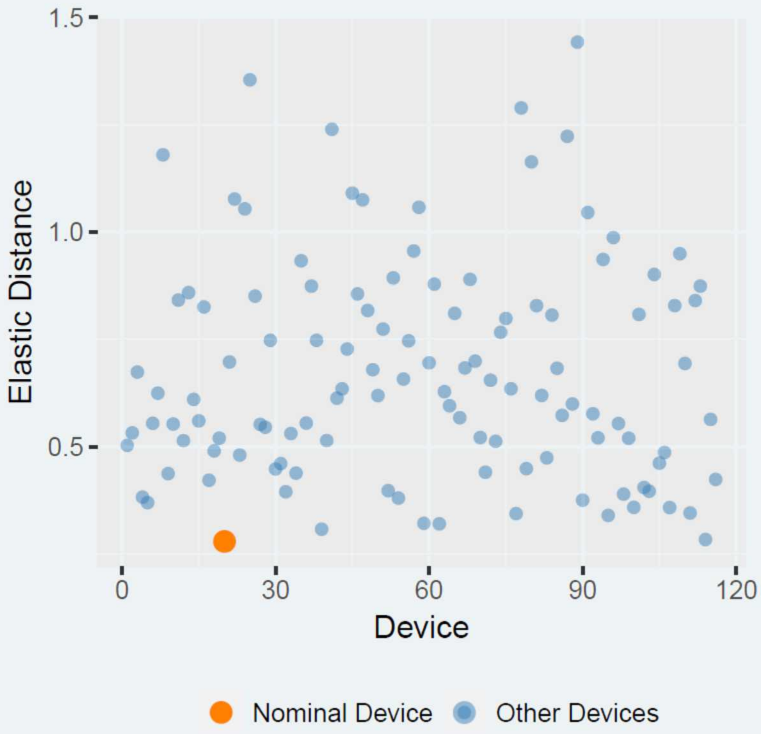
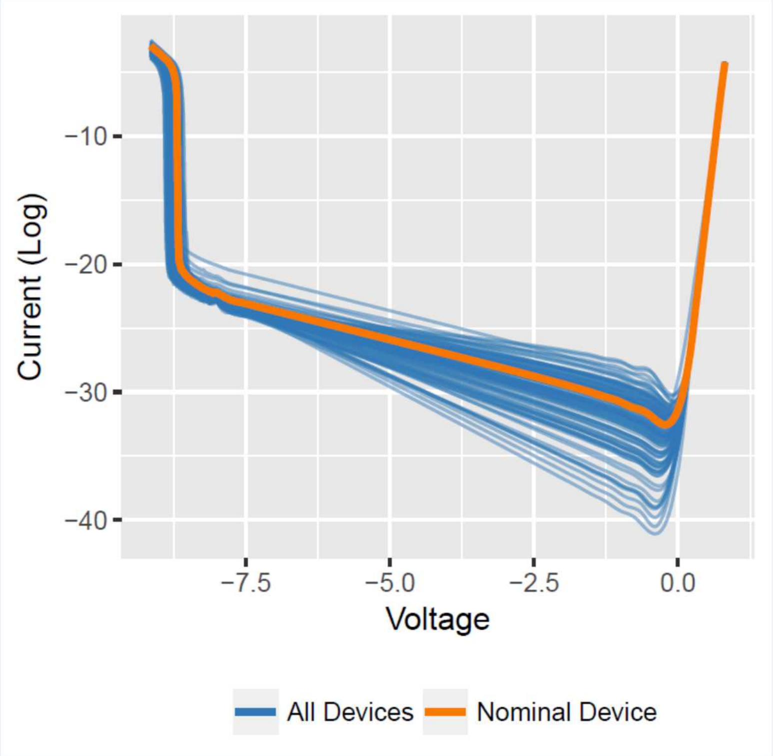


In the reverse region where there is significant phase variation, the Karcher mean outperforms the pointwise mean

4. Characterization of Variability & Selection of Nominal Device





Summary & Future Work

- **FDA approach** was used to warp data, calculate a Karcher mean, and assess elastic distances to identify a nominal device.

- This approach was able to more **accurately define a mean function** compared to a point-wise estimate, particularly in the critical reverse breakdown region.

- This provides an **objective method** to chose a representative device from a sample of devices, which is extremely useful in the first phase of parameter calibration for compact models in electronic circuit design.

Next steps include:

- Incorporating tolerance bounds for functional data.
- Propagating uncertainty in the devices to the calibration parameters.

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