

# Parametric Optimization Problem Formulation for Connected Hybrid Electric Vehicles using Neural Network based Equivalent Model

Wanshi Hong, Indrasis Chakraborty, and Hong Wang

**Abstract**—The dynamics of powertrain control systems are complicated and involve both nonlinear plant model and control functionalities, albeit they are well defined and formulated using first principle approaches. This constitutes difficulties in exploring implementable optimal tuning rules for some selected control parameters using vehicle-to-vehicle (V2V) communications. This paper presents a way to use neural networks (NN) to represent the problem of parameter tuning for optimizing fuel consumption. For this purpose, physical modelling and validation have been firstly performed for the closed loop powertrain system of the concerned vehicle for some given driving cycles. This is then followed by the sensitivity analysis that selects most influential control parameters to optimize. Using the data generated from the obtained physical models, an equivalent NN formulation has finally been obtained that gives simple yet unified objectives and constraints ready to be used to solve the optimization problem that produces optimal tuning rules for the selected control parameters to minimize fuel consumption.

**Keywords**— Parameter optimization, V2X, Powertrain modeling, Neural Network

## I. INTRODUCTION

With the increasing concerns about limited availability of fossil energy sources and air pollution, improving the fuel efficiency and reducing harmful gas emissions have become a major concern for the vehicle research community. Hybrid Electric Vehicles (HEV) have been a good solution to this problem. To further maximize the fuel efficiency for the HEVs, there have been many approaches in recent years. Moreover, the emerging Connected Vehicle (CV) technology which utilizes present and future traffic information [1] has motivated researchers to develop optimal power management

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schemes using V2V and V2I information as an optimization input to maximize the fuel efficiency for HEVs.

Past studies have made major contributions in developing power management for HEVs to maximize the fuel efficiency. There have been two popular groups of power management strategies: *rule based methods* and *optimization based methods*. Rule based (RB) power management methods are considered as a conventional way of managing power as it is widely used by many popular vehicles such as Honda and Toyota. The control law of RB method is defined based on a set of “if-then” rules, which depends on human expertise, operation boundaries and safety considerations. The advantages of these methods are their simple mathematical calculation and the convenience to apply updates to commercially produced vehicles. In [2], the authors developed power follower control strategy, which is a well-known strategy within the rule based methods. In [3]–[9], fuzzy rule based methods have been introduced, which is very robust to various modelling errors. However, most rule based methods do not go through an optimization process to minimize the operation cost. On the other hand, optimization based power management methods minimize the operation cost over time, and this has been a trend in research in recent years. For example in [10], the authors considered optimal power control problem for HEVs when the future driving conditions are not available. Furthermore, there has been an emerging trend of using traffic information as an optimization input to the vehicle controls recent years. In [11] and [12], a dynamic programming technology has been used with optimal control schemes to predict driving cycles and design trip-based optimal power management.

To summarize, these existing studies considered the optimal control problem with respect to the controlled variables of the powertrain. Such approaches will require the construction of new control models to the vehicles. Thus it is difficult to apply such control scheme to the existing vehicles. Moreover, the optimization is not considered in most rule based methods. To address these issues, optimal parameter tuning strategies for the rule based powertrain controller can be considered to maximize fuel efficiency. However, the direct mathematical model for powertrain controller parameters to fuel efficiency is difficult to obtain. Moreover, with the emerging trend of connected vehicle, it has been a challenge to obtain a mathematical model for powertrain dynamic with V2V/V2I information. To simplify the calculation for a parameter tuning based optimization scheme, one natural approach is to use neural network equivalent model to represent the input-output relationship between powertrain controller

parameters, V2V/V2I information and fuel consumption.

From the discussion above, we formulate an equivalent neural network model for the connected HEVs. This model can be further used for parameter tuning strategy for fuel consumption optimization. The *main contributions* of this work are:

- Develop and validate a Toyota Prius hybrid system simulation model;
- Carry out sensitivity study for the powertrain control parameters with respect to change in fuel efficiency;
- Determine the driving cycle characteristics for modeling use;
- Use neural network techniques to establish algebraic equivalent models for fuel consumption, and powertrain state dynamics;
- Formulate the parameter tuning based optimization problem to maximize fuel efficiency.

The remainder of this paper is organized as follows: Section II introduces the HEV model, the selected control parameters, and the V2V/V2I information. Section III formulates the equivalent neural network models for the system dynamics and the fuel consumption model. Section IV contains concluding remarks and discusses the future works.

## II. PROBLEM STATEMENT

This section first introduces the HEV model used in this study, then describes sensitivity study to find the parameters that fuel efficiency is most sensitive to, and characterizes V2V/V2I information with future driving cycle predictions. Finally control objectives are presented based on the definition of the parameters.

### A. Dynamic Model of HEV

In this paper, the dynamic model of a 2017 Toyota hybrid is considered. This model is derived based on previous studies in [13], [14]. The Toyota hybrid system uses a planetary gear as the power split device, which consists of a ring gear, a sun gear and the planetary carrier [15]. The engine is connected to the planetary carrier, the motor/generator MG1 is connected to the sun gear and the output shaft and motor/generator MG2 are both connected to the ring gear. The output shaft is linked to the wheels through reduction gears and the differential. The power generated by the engine is split between a mechanical path (connected to the vehicle drive axle) and an electrical path (transformed to electricity through MG1).

**Planetary gear system.** The planetary system dynamic equations by [13] are summarized as the following:

$$(I_s + I_{MG1})\dot{\omega}_{MG1} = FS - T_{MG1} \quad (1)$$

$$(I_c + I_e)\dot{\omega}_e = T_e = FR - FS \quad (2)$$

$$\begin{aligned} \left(\frac{R_{tire}}{K}m + I_{MG2}K + I_rK\right)\dot{\omega}_r &= (T_{MG2} + FR)K \\ &- T_f - mgf_r R_{tire} \\ &- 0.5\rho AC_d \left(\frac{\omega_r}{K}\right)^2 R_{tire}^3 \end{aligned} \quad (3)$$

where  $T_{MG1}$ ,  $T_{MG2}$  and  $T_e$  are the torques generated by two motor/generator sets and the engine;  $I_r$ ,  $I_s$  and  $I_c$  are inertias of the ring gear, sun gear, and carrier gear,  $I_{MG1}$  and  $I_{MG2}$  and  $I_e$  are the inertias of the power sources,  $F$  represents the internal force on the pinion gears and  $m$  is the vehicle mass,  $T_f$  is the brake torque,  $K$  is the final drive ratio,  $f_r$  is the rolling resistance coefficient, and  $0.5\rho AC_d$  is the aerodynamic drag resistance.

**Battery system.** The battery energy status is reflected by the battery state of charge (SOC). The battery SOC satisfies the following equation:

$$\dot{SOC} = -\frac{I_b}{Q_{max}} \quad (4)$$

where  $I_b$  is the battery current and  $Q_{max}$  is the battery capacity. The battery power to supply the electrical machines satisfies the following relationship

$$P_b = (T_{MG1}\omega_{MG1}\eta_{MG1}^k + T_{MG2}\omega_{MG2}\eta_{MG2}^k) \quad (5)$$

where  $\eta_{MG1}^k$  and  $\eta_{MG2}^k$  are the electric machine efficiency, and  $\eta_{i1}^k$  and  $\eta_{i2}^k$  are the inverter efficiency.  $k = 1$  when the battery is charging and  $k = -1$  when the battery is discharging. The sign of  $P_b$  will indicate whether the battery is charging ( $P_b > 0$  when the battery is discharging). Based on (5) and the battery internal resistance model  $P_b = V_o I_b - I_b^2 R_b$ , the state of charge model can be rewritten as

$$\dot{SOC} = -V_o - \sqrt{\frac{-V_o^2 - 4P_b R_b}{2R_b Q_{max}}} \quad (6)$$

where  $V_o$  is the open circuit battery voltage,  $R_b$  is the battery internal resistance.

### B. Hybrid Vehicle Simulation Model Validation

A first principle model based on Matlab/Simulink is used to represent the dynamics of the powertrain and aftertreatment system of a 2017 Toyota Prius Hybrid, based on (1-3) and (6). Aftertreatment modeling is performed using ADVISOR (a MATLAB based package developed by National Renewable Energy Laboratory, US Department of Energy). To further validate the Simulink model for the 2017 Toyota Prius hybrid, we conducted an experiment on an actual vehicle with our collaborator at the University of Michigan. Measured data for several key operating parameters that characterize the vehicle and powertrain behavior were compared to the Simulink model predictions. The driving route evaluated for this validation is shown in Figure 1. This integrated model shown in Figure 2 which is validated using the measurement collected from several test drives of a 2017 Toyota Prius Hybrid.

Figure 3 shows the powertrain validation performances of our developed model. The red colored curve shows measured data and the blue curve gives simulated data from the integrated model. In the figure, engine and vehicle speed show accurate validation performances when compared to real measured data. Engine and generator torque also indicate satisfactory performance from validation. The aftertreatment

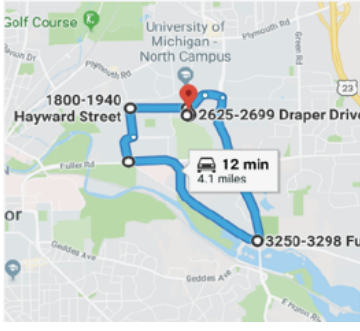


Fig. 1: Driving path for powertrain model validation.

system is also well validated by validating the mass flow rate, coolant temperature and emission rate of NOx, HC and CO. Due to the page limitation, the results are not shown here. Based on the validation results, we can conclude that the Simulink model can well represent the actual Toyota Prius hybrid vehicle system.

### C. Controller Parameter Selection

From the first principle based Matlab/Simulink model in Fig. 2, it can be seen that there are a lot of control parameters involved. These control parameters contribute to the level of the fuel consumption with different degree of impact. To realize the optimal tuning of these parameters using V2V and V2I information, it is imperative to identify what level of impact these parameters have on the fuel consumption. This requires us to carry out a sensitivity analysis of these control parameters with respect to the fuel consumption. This will allow us to focus on those control parameters which have the most significant impact on the fuel consumption, and subsequently develop relevant V2V/V2I based parameter tuning mechanisms so as to minimize the fuel consumption. In the following some fundamental aspects of the parameter sensitivity analysis will be described.

1) *Bayesian definition of sensitivity*: For a system with input  $X$  and output  $Y$ , sensitivity in Bayesian notation can be expressed as

$$S = \frac{\text{var}_X[E(Y|X)]}{\text{var}(Y)}, \quad (7)$$

where  $Y$  indicates the fuel consumption and  $X$  indicates the control system parameters in the powertrain,  $E(Y|X)$  denotes the expectation of  $Y$  conditional on a fixed value on  $X$ , and the variance  $\text{var}_X$  is taken over all the possible values of  $X$ .

TABLE I: Three control parameters selected for the sensitivity analysis.

Parameter Name	Physical Meaning	Nominal Value	Range used
$\theta_1$	Generator speed controller proportional gain	0.9	[0.5, 1]
$\theta_2$	Generator speed integral gain	0.005	[0.0001, 0.05]
$\theta_3$	Charging Controller gain	15000	[13000, 17000]

2) *FAST Method*: We have calculated the first order sensitivity index using FAST method [16]. Based on the first order sensitivity index definition with respect to the total fuel consumption, Table I tabulates the indices along with the nominal and acceptable range of operations for the selected control variables. These parameters are from the generator speed controller of motor/generator set one (MG1) and battery charging controller. Denote as  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ , respectively.

**MG1 Generator Speed Controller**. The control objective of the MG1 speed controller is to have the generator speed tracks a given reference optimal generator speed. The tracking control is achieved via a classic PI control. The controller structure is

$$u_{MG1}(t) = K_{p1}(\omega_{opt}(t) - \omega_{MG1}(t)) + K_{i1} \int_0^t (\omega_{opt}(\tau) - \omega_{MG1}(\tau)) d\tau \quad (8)$$

where  $u_{MG1} = T_{MG1}$  is the control input indicating the generator torque,  $\omega_{opt}$  is the referenced optimal generator speed,  $\omega_{MG1}$  is the generator speed,  $K_{p1} = \theta_1$ ,  $K_{i1} = \theta_2$  are the proportional and integral gain of the PI controller selected for parameter optimization.

**Battery Charging Controller**. The objective of the battery charging controller is to have the battery state of charge (SOC) follow a given reference optimal SOC, which is achieved by proportional control with saturation. The controller structure is

$$u_p(t) = \begin{cases} u_{p1}(t), & u_{p1}(t) < u_{pmax} \\ u_{pmax}, & u_{p1}(t) \geq u_{pmax} \end{cases} \quad (9)$$

$$u_{p1}(t) = K_{p2}(SOC_{opt}(t) - SOC(t)) \quad (10)$$

where  $u_p = P_b$  is the control input indicating the battery charging power,  $u_{pmax}$  is the upper bound of charging power,  $SOC_{opt}$  is the given optimal state of charge.  $K_{p2} = \theta_3$  is the proportional gain of the P controller selected for parameter optimization.

To tune the selected parameters, one can see that it is difficult to directly derive optimization scheme based on the dynamic system in (1-3) and (6). Thus, it is necessary to estimate the direct input-output model to represent the dynamic for these parameters.

### D. Use of V2V/V2I Information

In this work, we use V2V/V2I information as an optimization input. The objective is to find the influence of future driving condition on the controller parameter to achieve maximum fuel saving, i.e., find the function which represent driving condition and parameter to fuel saving. To this end, we need to characterize the real-time signals into mathematical term to be used in optimization design. The V2I information mainly contains the traffic signal  $S_T$ , and the V2V information includes the speed and the acceleration of the preceding vehicle and the front vehicles on left/right lanes, denoted by  $v_p, a_p, v_l, a_l, v_r, a_r$  respectively. In this

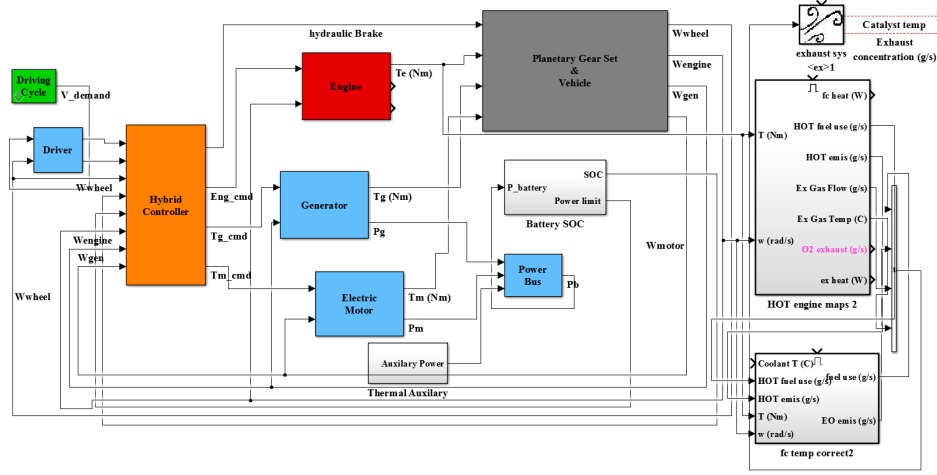


Fig. 2: Integrated powertrain and aftertreatment Simulink model.

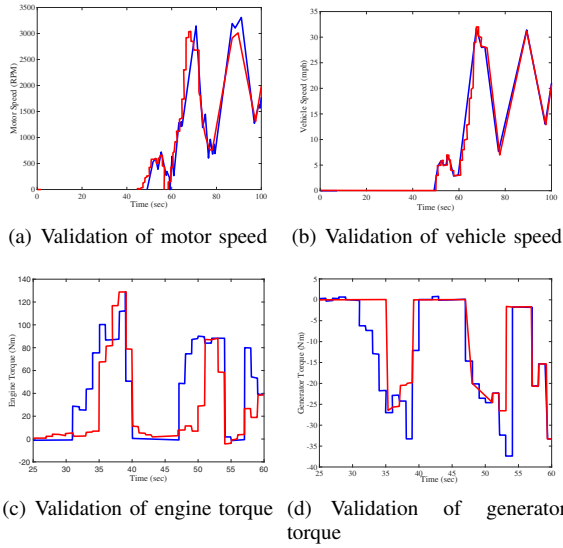


Fig. 3: Powertrain validation results (red denotes simulation and blue denotes measured data).

study, we consider the effect of V2V/V2I information and how it can be used to predict the future vehicle speed  $v_f$  for minimized fuel consumption:

$$v_f = W(S_T, v_p, a_p, v_l, a_l, v_r, a_r). \quad (11)$$

### III. EQUIVALENT NEURAL NETWORK MODEL FORMULATION

This section forms the equivalent input-output neural network (NN) model of HEV system for further optimization control design. First the principle of neural network is introduced. Then the fuel consumption dynamics and state dynamics are represented using neural network. Finally the optimization problem is formulated based on the equivalent NN model.

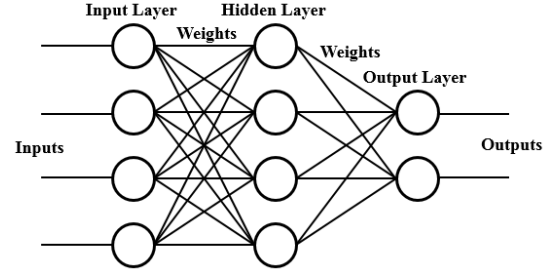


Fig. 4: Multi-layered neural network example [17].

#### A. Equivalent Neural Network Models for Optimization

As described in Subsection II-B, a MATLAB Simulink model is used in this study to characterize the powertrain system dynamics (1-3) and (6). The accuracy of this model is verified through the validation process. Although the optimization problem can be directly constructed based on the first principle model used to build this Simulink model, the amount of calculation required is computationally significant as there are 3 parameters to be optimized in total and they are interdependent. As a result, this work proposes to first run the Simulink model and collect data for the input (sensitive parameters, driving cycle information, state information: torque and speed state of different powertrain components) and output (fuel consumption, state prediction) variables we need to broadly characterize the Simulink model response. The model is run many times, then we train the function for the input/output relationship using neural networks so as to obtain a simple yet easy-to-use equivalent model for the powertrain and fuel consumption rate. The objective using the equivalent Neural Network model is to directly find the relationship between the three powertrain control parameters and the fuel efficiency powertrain system output. Fig. 4 is an example of a MLF neural network, which is trained using back-propagation training algorithms.

## B. Fuel Consumption Dynamics

The fuel consumption can be considered as the cost function for formulating an optimization problem to minimize fuel consumption based on knowledge of future speeds. The input-output model between the control parameters, V2V/V2I information and fuel consumption is

$$J = \sum_{i=1}^n \Delta f_i(\theta, cyc_i) \quad (12)$$

where  $i$  is the step size,  $n$  is the control horizon,  $\theta = (\theta_1, \theta_2, \theta_3)$  are the three parameters that fuel consumption is most sensitive to, based on the sensitivity study described in Subsection II-C,  $cyc_i$  indicates the V2V/V2I information characterizing future vehicle speed information at the  $i$ th step.  $\Delta f_i$  is the amount of fuel consumption at  $i$ th step. The physical meaning of the cost function  $J$  is the amount of fuel used within a driving cycle. The goal of the optimization problem is to find the optimized parameters  $\theta$ , which minimize  $J$  for a given driving cycle  $cyc$ . Note that since all the sampled data are generated based on the simulation of an actual driving cycle, there is an explicit constraint that the driving pattern for each driving cycle is fixed. Therefore, the driving trajectory and speed are fixed, and one can only minimize  $J$  by tuning the sensitivity parameters. A neural network is used to predict the fuel consumption function  $\Delta f_i$ .

The predicted cost function is

$$\tilde{J} = \sum_{i=1}^n \Delta \tilde{f}_i(\theta, cyc_i), \quad (13)$$

where  $\Delta \tilde{f}_i$  is obtained by training the neural network. The neural network expression is given by

$$\Delta \tilde{f}_i = w_{c2}(\tanh(w_{c1}z_c + b_{c1}) + b_{c2}), \quad (14)$$

which indicate a neural network with 1 input layer  $z_c = [\theta, cyc_i]^T$  (6 inputs, 3 controller parameters  $\theta \in R^{1 \times 3}$  and 3 steps of future driving speed information ahead  $cyc_i \in R^{1 \times 3}$ ), 1 hidden layer (10 neurons), 1 output layer (1 output: fuel consumption), and  $w_{c1} \in R^{10 \times 6}$ ,  $w_{c2} \in R^{1 \times 10}$  are the weights,  $b_{c1} \in R^{10 \times 1}$ ,  $b_{c2} \in R$  are the biases. The prediction results are shown in Fig. 5, where the blue curve shows the actual fuel consumption in different driving cycles and the red curve is the fuel consumption predictions. Based on the error histogram and the prediction of fuel consumption function  $\tilde{J}$  for each driving cycle in comparison to the actual fuel consumption  $J$ , we can conclude that the neural network represented by (14) can accurately predict the fuel consumption trend, and sufficiently predicts the cost function. The neural network training characteristics are shown in Fig. 6, including the training error histogram and the validation performance.

## C. Powertrain State Dynamics

The powertrain state dynamic with the change of future driving information controller parameter is a necessary con-

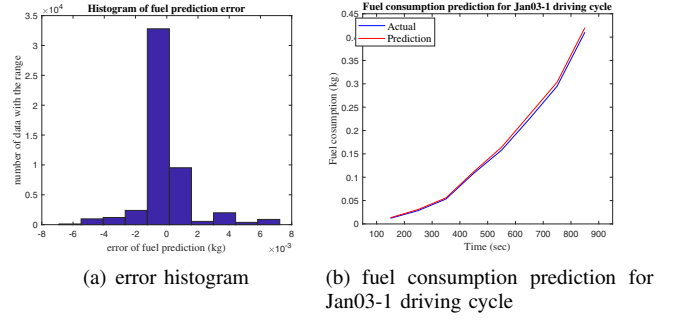


Fig. 5: Neural network fuel prediction results.

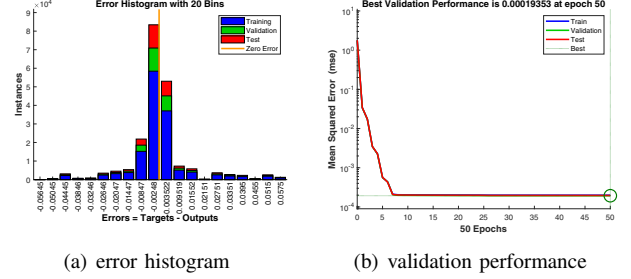


Fig. 6: Neural network training performances for fuel consumption predictions.

straint for the optimization problem. The equivalent powertrain state dynamic model is

$$x(k+1) = g(x(k), \theta, cyc), \quad (15)$$

where  $k$  is the step size,  $x = [T_e, T_g, T_m, \omega_e, \omega_g, \omega_m]^T$  which are engine torque, generator torque, motor torque, engine speed, generator speed, motor speed, respectively. We construct a neural network model  $g$ . For this purpose, the inputs to the neural network are the state at  $k$ th step, the sensitive parameters and the driving cycle information. The output of the neural network is the state at the  $(k+1)$ th time step. The prediction of the state constraint function is expressed by the neural network

$$\tilde{g} = w_{s2}(\tanh(w_{s1}z_s + b_{s1}) + b_{s2}), \quad (16)$$

which indicates a neural network with 1 input layer  $z_s = [\theta, cyc, x]^T$  (12 input,  $\theta \in R^{1 \times 3}$ ,  $cyc \in R^{1 \times 3}$ ,  $x \in R^{1 \times 6}$ ), 1 hidden layer (10 neurons), 1 output layer (6 output), and  $w_{s1} \in R^{100 \times 12}$ ,  $w_{s2} \in R^{1 \times 100}$  are the weights,  $b_{s1} \in R^{100 \times 1}$ ,  $b_{s2} \in R^{6 \times 1}$  are the biases. The prediction results are shown in Fig. 7, where the blue curves are the actual states and the red curves are the state predictions. We can conclude that the neural network represented by (16) can accurately predict future state information. The neural network training characteristics are shown in Fig. 8, including the training error histogram and the validation performance.

## D. Optimization Problem Description

Based on the analysis above, the parameter optimization problem is formulated as follows:

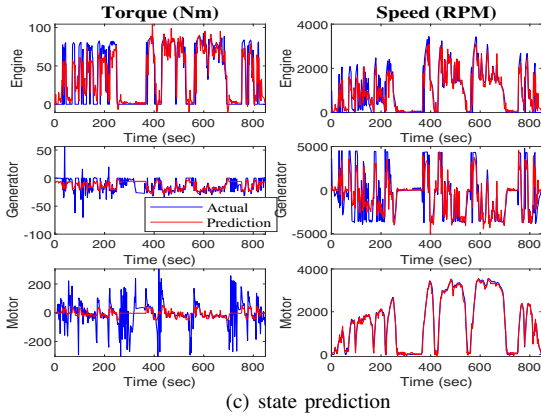
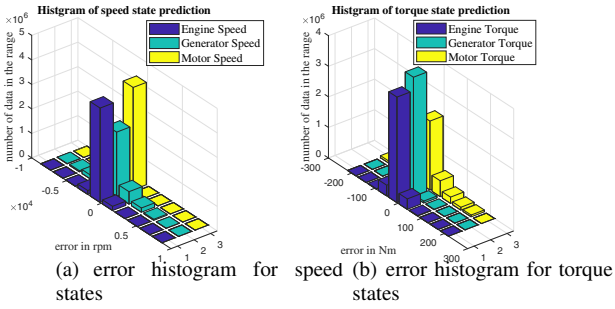


Fig. 7: Neural network prediction results for state constraint predictions.

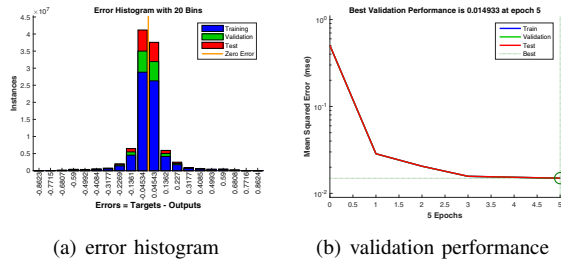


Fig. 8: Neural network training performances for state constraint predictions.

Design an optimization problem to optimize the sensitive powertrain control parameters  $\theta$  to minimize the fuel consumption, as characterized by the cost function (12). Moreover, we consider the state model (15) as constraints to the optimization problem. The optimization problem is simplified in this work by optimizing the approximated cost function (13) for the sensitive parameters  $\theta$ , with approximated state model (16) as constraints.

#### IV. CONCLUSIONS AND FUTURE RESEARCH

This paper formulated a new optimization problem model to maximize the fuel efficiency for HEVs. The cost function and the constraints are predicted using the neural network given in Section III. This optimization problem model is then verified from the physical model of the powertrain built in Simulink. The significance of this work is to build

the simplified model for parameter tuning based on an optimization scheme, which is an optimal yet easily applied approach for commercial vehicles.

The future work based on this research is to solve this optimization problem. The ultimate goal of solving this problem is to come up with an online parameter update law based on the V2V/V2I information to maximize the fuel efficiency.

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