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Noise and Noise Figure for Radar Receivers

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Abstract

An important characteristic of a radar receiver is the noise level within the receiver chain. The common parameter that specifies this is the System Noise Factor, which depends on system design and may vary with gain settings, temperature, and other factors. A modified Y-factor technique is detailed to calculate System Noise Factor for a radar receiver. Error sources are also detailed.

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Contents

Foreword

This report details the results of an academic study. It does not presently exemplify any modes, methodologies, or techniques employed by any operational system known to the author.

Classification

The specific mathematics and algorithms presented herein do not bear any release restrictions or distribution limitations.

The distribution limitations of this report are in accordance with the classification guidance detailed in the memorandum “Classification Guidance Recommendations for Sandia Radar Testbed Research and Development”, DRAFT memorandum from Brett Remund (Deputy Director, RF Remote Sensing Systems, Electronic Systems Center) to Randy Bell (US Department of Energy, NA-22), February 23, 2004. Sandia has adopted this guidance where otherwise none has been given.

This report formalizes preexisting informal notes and other documentation on the subject matter herein.

1 Introduction

A fundamental performance-limiting factor for any remote sensing system is noise; the random signal errors that are inevitable in real systems. Were it not for noise, indeed much of remote sensing engineering would be trivial, with amazing performance from simple, inexpensive, and rudimentary sensors. Detections would be certain and false alarms would be nonexistent.

In radar systems, noise sets thresholds below which desired target echoes are obscured. Much of the signal processing is designed to mitigate or otherwise overcome the debilitating effects of noise. Consequently, a thorough understanding of noise, including accurate measurements and predictions of noise characteristics, is crucial to maximizing the utility of radar systems.

Sources of noise to a radar system might generally include the following:

1. Thermal emissions of the target scene
2. Random currents in any and all components, including semiconductor shot noise
3. Data quantization effects
4. Purposeful random dithering signals
5. Signal processing artifacts including sidelobes and processing errors
6. Atmospheric phenomena
7. Cosmic sources including cosmic background radiation
8. Interference both intentional and unintentional, external and internal
9. Shot noise (Poisson noise) due to the discrete nature of electric charge
10. Flicker noise (1/f noise, pink noise) in active devices
11. Plasma Noise due to random motion of charges in an ionized gas
12. Quantum Noise due to random currents in conjunction with motion of discrete charges

Noise can generally be broadband, or narrowband, and might be frequency-dependent. Statistics may or may not be stationary with time or space. Specific applications might make some of these more important than others.

Of those listed, in this report we will concern ourselves mainly with the first four.

We have attempted herein to make the analysis generic, that is, applicable to microwave radio/radar receivers in general. Nevertheless, out principle concern for writing this report is range-Doppler imaging radars, including Synthetic Aperture Radar (SAR), and Ground Moving Target Indicator (GMTI) radar systems' intermediate products. The intended audience includes those engaged in radar system analysis and design.

There is nothing particularly original in this report; essentially simple mathematics applied to well-known and well-documented knowledge, but collected into a single document. We offer the following references as useful background information for the material in this report. These are principally concerned with Noise Factor/Figure measurement.

> Agilent Technologies offers an excellent series of application notes dealing with noise and Noise Factor measurements.[1,2,3,4]
>
> Rohde & Schwarz offer their own application notes on the topic.[5,6]
>
> Tektronix also offers a whitepaper on Noise Figure measurement.[7]

In addition, useful general background material might be found in the following references.

> Basic microwave engineering principles are discussed in the classic text by Pozar.[8]
>
> Microwave circuit design is discussed by Gonzalez.[9]
>
> Gentili[10] presents a number of aspects of microwave circuit analysis and design.
>
> Ha[11] similarly broadly discusses microwave amplifier design.

We further observe that many texts and other papers and application notes also discuss the topic. There is no shortage of material with which to educate oneself.

Other relevant publications will also be cited later in this report as warranted.

2 Whence Noise

This type of noise we discuss here was first measured by John B. Johnson at Bell Labs in 1926.[12] A theoretical exploration of his results was given by Harry Nyquist, also at Bell Labs.[13]

2.1 Basic Source and Load

Johnson and Nyquist determined that a resistive element (e.g. resister) will contain within it random charge motion whose effects can be modeled as the Thevenin circuit of Figure 1.

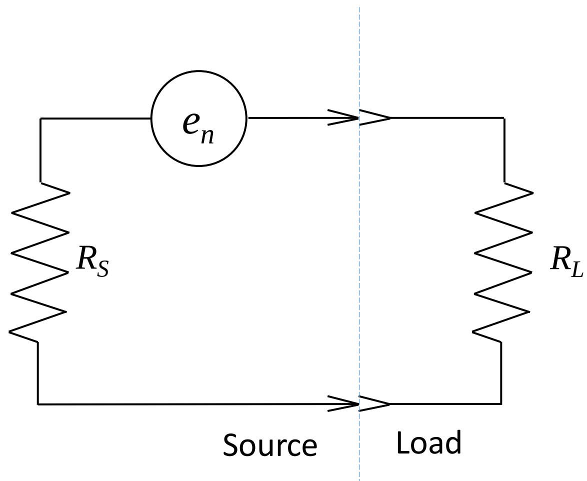

Figure 1. Thevenin equivalent circuit of thermal noise source.

The voltage source will be a random noise signal with voltage variance within some frequency band given by

$$e_n^2 = \int_B 4 R_S hf \left(e^{\frac{hf}{kT}} - 1 \right)^{-1} df \ , \tag{1}$$

where

R_S = resistance of the source element in ohms,
B = bandwidth of interest in Hz,
$h = 6.6260700481 \times 10^{-34}$ J s = Planck’s constant,
f = frequency in Hz,
$k = 1.3806485279 \times 10^{-23}$ J/K = Boltzmann’s constant,
T = absolute temperature of the resistive termination in K. (2)

This is sometimes called “thermal noise,” “Johnson noise,” “Nyquist noise,” “Johnson-Nyquist noise,” and/or “kTB noise.”

This form comes from Planck’s Law, which relates electromagnetic radiation by a black body in thermal equilibrium at some constant temperature.

This formulation assumes all frequencies are positive.

Maximum power transfer occurs when this source is connected to a load R_L with the same resistance, that is, where

$$R_L = R_S \,. \tag{3}$$

The power delivered to the load resistance is calculated to be

$$P_{noise} = \frac{1}{R_L}\left(\frac{e_n}{2}\right)^2 = \int_B hf \left(e^{\frac{hf}{kT}} - 1 \right)^{-1} df \,. \tag{4}$$

For typical microwave radar frequencies, and reasonably anticipated temperatures, we may observe

$$f << \frac{kT}{h} \approx 6 \text{ THz}, \tag{5}$$

which allows us to reasonably approximate, for any matched source/load resistance

$$P_{noise} \approx kTB \,. \tag{6}$$

We may specify the two-sided Power Spectral Density (PSD) of the transferred noise power as

$$S_{noise}(f) = \frac{1}{2} h|f| \left(e^{\frac{h|f|}{kT}} - 1 \right)^{-1} , \tag{7}$$

which for typical microwave frequencies and below can be assumed to be

$$S_{noise}(f) = \frac{1}{2} kT \,. \tag{8}$$

Note that this is a two-sided PSD, assuming both positive and negative frequencies. When band-limited by a two-sided ideal filter, the total noise power in the resistive load is again expressed as

$$P_{noise} = kTB \,, \tag{9}$$

where now we define

$$B \;= \text{one-sided bandwidth.} \tag{10}$$

The one-sided bandwidth is the positive-frequency bandwidth (same as negative-frequency bandwidth for real filters).

We note that while the source noise is not strictly "white," because of its high-frequency roll-off, nevertheless within any frequency band typically used by radar systems, the noise is for all intents and purposes white. The common assumption is that the noise in a typical microwave radar receiver is essentially band-limited white noise.

2.2 Noise Temperature

We have discussed above the noise power that is generated by a resistive source impedance (source termination) at some temperature and delivered to a matched load. The noise power so generated is proportional to the resistance temperature.

We now stipulate that equivalent noise can be generated by other sources. We now ask the question "Given a particular band-limited white noise PSD, what would be the temperature of a resistive source termination to yield the same noise power?"

The answer can be calculated as

$$T_{noise} = \frac{2S_{noise}\left(f\right)}{k} \,. \tag{11}$$

Thus, any noise PSD can be expressed as an equivalent noise temperature.

2.3 Noise Factor

Similarly, a particular noise temperature can be expressed as some scaled version of some reference temperature. That is

$$T_{noise} = F_N T_{ref} \,, \tag{12}$$

where

T_{ref} = reference noise temperature, typically 290 K, and

F_N = Noise Factor. (13)

This suggests that we may equate for any particular band-limited white noise PSD

$$S_{noise}(f) = \frac{1}{2} k T_{ref} F_N \,. \qquad (14)$$

Integrated over some frequency band this becomes

$$P_{noise} = k T_{ref} F_N B \,. \qquad (15)$$

We have termed the factor F_N as the "Noise Factor." Sometimes this is called the "Noise Figure." We will adopt the convention in this report that the factor as expressed in the preceding equations be termed the "Noise Factor," and reserve the term "Noise Figure" for an expression of this factor in units of dB. That is

$$10 \log_{10} F_N = F_{N,dB} = \text{Noise Figure.} \qquad (16)$$

2.4 Noise from an Antenna

An antenna is a transducer that will deliver to a matched load some noise power with 2-sided PSD given by

$$S_{antenna\ noise}(f) = \frac{1}{2} k T_{antenna} \,, \qquad (17)$$

where

$T_{antenna}$ = effective noise temperature provided by antenna. (18)

As a transducer, the antenna noise temperature is a function of both its physical temperature, and the noise temperature of the scene to which it is pointed, that is

$$T_{antenna} = (1-\eta) T_{antenna,physical} + \eta \xi T_{scene} \,, \qquad (19)$$

where

$T_{antenna,physical}$ = physical temperature of the antenna component,

T_{scene} = the actual temperature of the target scene, and

η = radiation efficiency of the antenna, and
ξ = emissivity of the target scene. (20)

We generally wish for the maximum efficiency of the antenna.

Ulaby, et al.,[14] state that the range of emissivity values for land surfaces "is rarely < 0.3 and is often > 0.7." Karbou, et al.,[15] show that the microwave emissivity of most land types are above 0.9. Prigent, et al.,[16] corroborate this but also show that is even more so for vertical polarization. However, for water surfaces, the emissivity may be less than 0.4, with horizontal polarization less than vertical polarization, as presented by Wilheit.[17]

We note that target scene temperature and emissivity measurements are the domain of "radiometry," such as is employed by radio astronomy. In any case, it is generally presumed for terrestrial imaging microwave radar receivers that the antenna noise temperature is simply a reference value, namely

$$T_{antenna} = T_{ref} = 290 \text{ K.} \tag{21}$$

2.5 General Source and Load Impedances

The foregoing analysis assumed matched source and load resistive impedances, which is generally a goal for circuit design. We do note that we can indeed analyze a case for unmatched source and load impedances. Consider the case of general impedances as illustrated in Figure 2.

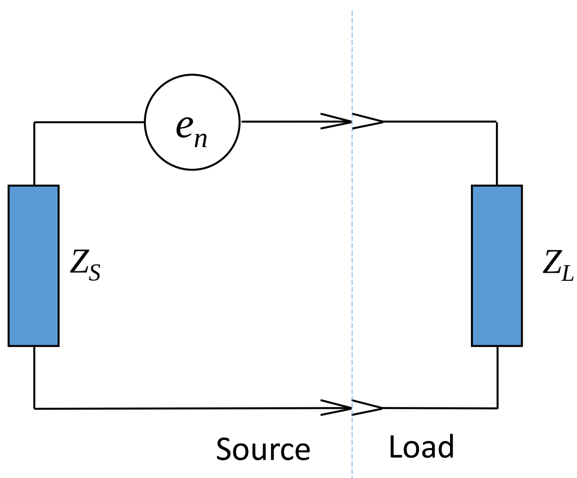

Figure 2. Thevenin equivalent circuit of thermal noise source, using general impedances.

We further identify the general impedances

$Z_S = R_S + jX_S$ = complex source impedance, and
$Z_L = R_L + jX_L$ = complex load impedance, (22)

where

$$R_S = \mathrm{Re}\{Z_S\} \text{ = resistance of source impedance,}$$
$$X_S = \mathrm{Im}\{Z_S\} \text{ = reactance of source impedance,}$$
$$R_L = \mathrm{Re}\{Z_L\} \text{ = resistance of load impedance, and}$$
$$X_L = \mathrm{Im}\{Z_L\} \text{ = reactance of load impedance.} \tag{23}$$

We stipulate that a pure reactance merely stores energy, but does not generate thermal noise, nor does it dissipate power. As a consequence, noise power delivered to the load impedance is

$$P_{noise} = \frac{1}{R_L}\left|\frac{e_n R_L}{R_S + jX_S + R_L + jX_S}\right|^2 = \frac{e_n{}^2 R_L}{\left(R_S + R_L\right)^2 + \left(X_S + X_S\right)^2} . \tag{24}$$

This can be simplified for typical microwave frequencies to

$$P_{noise} \approx \frac{4 R_S R_L kTB}{\left(R_S + R_L\right)^2 + \left(X_S + X_S\right)^2} . \tag{25}$$

Clearly, the criteria for maximum power transfer to the load impedance requires

1. Source and load reactance need to cancel, and

2. Source and load resistance need to be equal.

These criteria can be combined with the single criterion that source and load impedances need to be complex conjugates of each other, that is

$$Z_L = Z_S^* . \tag{26}$$

While this is the criterion for maximum power transfer to the load, we observe from the discussion in Appendix A that in general the conditions for maximum power transfer are not necessarily the same conditions for minimum noise factor.

However, for much of the subsequent analysis we will make the convenient assumption of matched sources and loads.

2.6 Comments

Whether we express noise in terms of a Noise Factor/Figure versus a Noise Temperature becomes a personal preference or convenience. We do note that different engineering communities exhibit different preferences. For example, terrestrial and airborne systems tend to use Noise Factor/Figure, whereas satellite and extraterrestrial systems tend to use Noise Temperature. In any case, analysis using either is equivalent to the other.

"The world is noisy and messy. You need to deal with the noise and uncertainty."
-- Daphne Koller

3 Component and System Noise Model

We reiterate that unless otherwise noted we will make the general assumption throughout the remainder of this report of matched sources and loads.

3.1 Single Component Noise Model

We begin with the single component gain-block model in Figure 3.

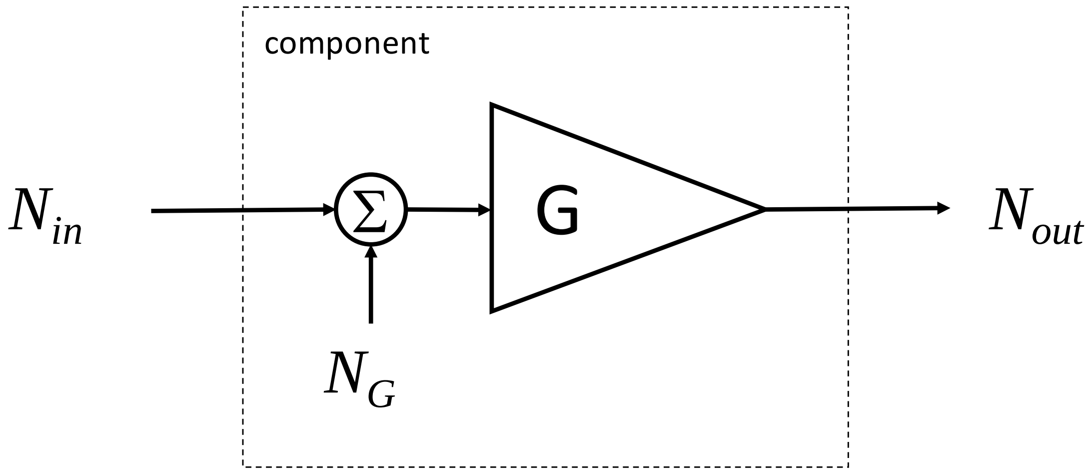

Figure 3. Model component.

In this model, we assume the following parameters.

$$
\begin{aligned}
N_{in} &= \text{noise input to the component,} \\
N_{out} &= \text{total output noise from the component,} \\
N_G &= \text{equivalent noise added by component, modeled as additional input noise,} \\
G &= \text{power gain of the component, assumed positive and non-zero.}
\end{aligned} \quad (27)
$$

It is customary to model any added noise by a component as an equivalent noise added to the input of the otherwise ideal component. We will assume all gains are constant with frequency, and all noise is band-limited white Gaussian noise, over some arbitrary bandwidth *B*.

Output noise power is calculated as

$$N_{out} = G\left(N_{in} + N_G\right). \quad (28)$$

In terms of corresponding noise temperatures, we may calculate

$$T_{out} = G\left(T_{in} + T_G\right), \quad (29)$$

where

T_{in} = noise temperature input to the component,
T_{out} = total output noise temperature from the component,
T_G = equivalent noise temperature added by component. (30)

It is customary to specify the amount of noise added by a component based on an input reference noise level, typically 290 K.[18] This means we will assume

$$T_{in} = T_{ref} = 290 \text{ K}. \tag{31}$$

We observe that in other units,

$$290 \text{ K} = 16.85 \text{ C} = 62.33 \text{ F}. \tag{32}$$

This temperature is close to room temperature and near earth long-term averages.

Nevertheless, the total output noise temperature can then be written as

$$T_{out} = GT_{ref}\left(1 + \frac{T_G}{T_{ref}}\right). \tag{33}$$

This means the output noise is the input noise scaled by component gain, and additionally scaled by a "noise factor" written as

$$T_{out} = GT_{ref} F_N , \tag{34}$$

where the noise factor is calculated as

$$F_N = \left(1 + \frac{T_G}{T_{ref}}\right). \tag{35}$$

Likewise, the component noise temperature can be calculated from its component noise factor by

$$T_G = T_{ref}\left(F_N - 1\right). \tag{36}$$

<u>Alternate Development of Noise Factor</u>

The first references to noise factor defined it as a ratio of output Signal-to-Noise Ratio (SNR) to input SNR. That is

$$F_N = \frac{S_{in}/N_{in}}{S_{out}/N_{out}}, \quad (37)$$

where

S_{in} = input signal power, and
S_{out} = output signal power. (38)

Recognizing that the component signal gain is

$$G = \frac{S_{out}}{S_{in}}, \quad (39)$$

the expression for noise factor can be expanded to

$$F_N = \frac{N_{out}}{G\, N_{in}} = \frac{N_{in} + N_G}{N_{in}} = 1 + \frac{T_G}{T_{in}}. \quad (40)$$

If the input noise temperature is set equal to the reference temperature T_{ref} , then this expression reduces to the same as the previous development of Eq. (35).

3.2 Two Component Noise Model

We now consider two gain-blocks concatenated as in Figure 4.

In this model, we assume the following parameters.

N_{G1} = equivalent noise added by first component,
G_1 = power gain of the first component,
N_{G2} = equivalent noise added by second component, and
G_2 = power gain of the second component. (41)

We may additionally specify

T_{G1} = equivalent noise temperature added by first component, and
T_{G2} = equivalent noise temperature added by second component. (42)

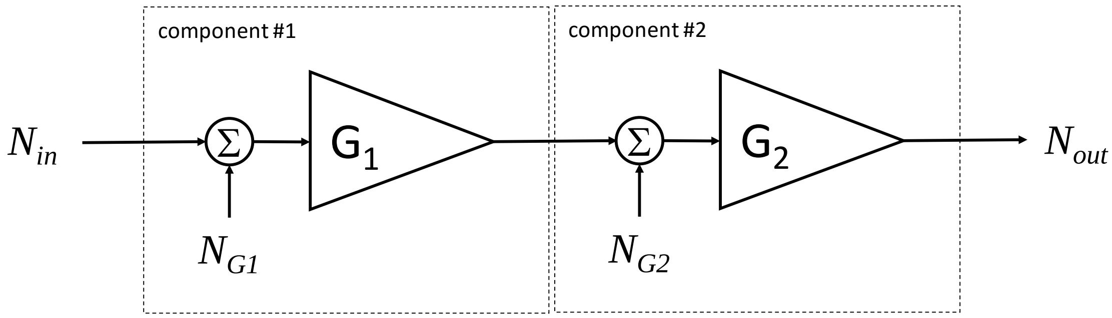

Figure 4. Two component noise model.

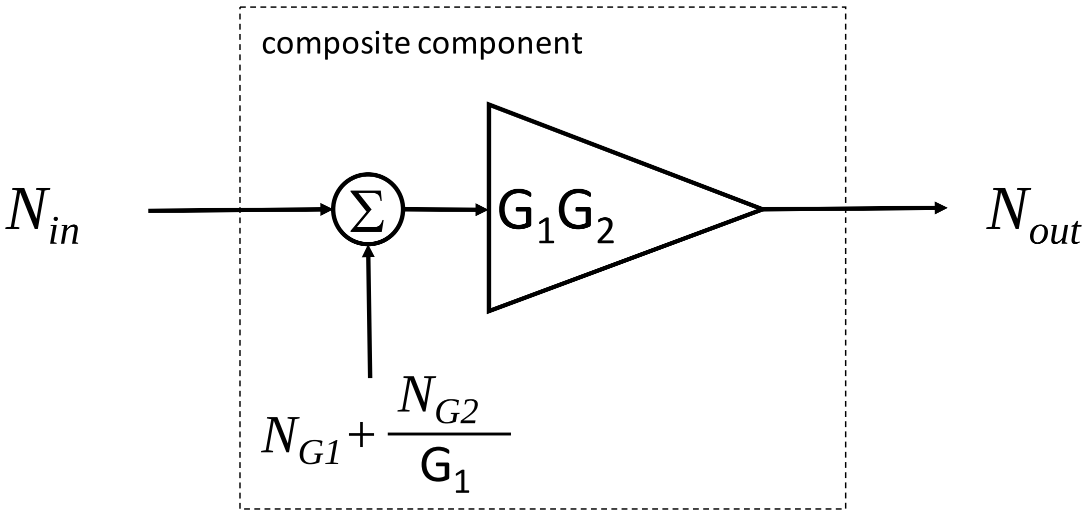

Figure 5. Equivalent two component noise model.

We may then calculate the output noise temperature as

$$T_{out} = G_2\left(G_1\left(T_{in}+T_{G1}\right)+T_{G2}\right) = G_2G_1\left(T_{in}+T_{G1}+\frac{T_{G2}}{G_1}\right). \tag{43}$$

In terms of noise powers, this becomes

$$N_{out} = G_2G_1\left(N_{in}+N_{G1}+\frac{N_{G2}}{G_1}\right). \tag{44}$$

This allows us to substitute the model of Figure 4 with the model of Figure 5.

The output noise temperature, when input with a reference noise temperature, may be written as

$$T_{out} = G_2 G_1 T_{ref}\left(1+\frac{T_{G1}}{T_{ref}}+\frac{T_{G2}}{T_{ref}G_1}\right). \tag{45}$$

From this we may identify an equivalent circuit, or system, noise temperature as

$$T_N = T_{G1}+\frac{T_{G2}}{G_1}. \tag{46}$$

From this we may also identify an overall system composite, or net, noise factor as

$$F_N = \left(1+\frac{T_N}{T_{ref}}\right) = \left(1+\frac{T_{G1}}{T_{ref}}+\frac{T_{G2}}{T_{ref}G_1}\right). \tag{47}$$

This may also be written as a function of the individual components' noise factors as

$$F_N = F_{N1}+\frac{F_{N2}-1}{G_1}, \tag{48}$$

where the individual noise figures are calculated as

$$F_{N1} = 1+\frac{T_{G1}}{T_{ref}}, \text{ and}$$
$$F_{N2} = \left(1+\frac{T_{G2}}{T_{ref}}\right). \tag{49}$$

From this we make the following important observations.

- The overall noise factor of the two component system is a function of both individual components' noise factors.
- If the first component has substantial gain, then the influence of the second component's noise factor is substantially diminished by the gain.
- To keep the overall noise factor as low as possible, the first gain component should exhibit low noise and high gain. An amplifier component that exhibits this characteristic is termed a Low-Noise Amplifier (LNA). Some amplifiers are specifically designed for this function.[19]

Passive Lossy Components

Consider the two-component model of Figure 4 where the first component is a passive and lossy component. As such, its gain is less than unity. In fact, it can be treated as a resistor network. This means that its output noise temperature is that of an equivalent resistor. Assuming all temperatures are at the reference temperature, this means that the passive component's output temperature is the same as the input reference temperature, namely

$$T_{out1} = G_1 T_{ref} F_{N1} = T_{ref} \ . \tag{50}$$

This can only be true if the noise factor of the passive lossy network is the inverse of its gain, which we also define as its loss. That is

$$F_{N1} = L_1 , \tag{51}$$

where

$$L_1 = \frac{1}{G_1} \ \text{= loss factor of the passive loss network.} \tag{52}$$

Under these circumstances, the net composite noise factor can then be written as

$$F_N = L_1 F_{N2} , \tag{53}$$

From this we make the following important further observations.

- Any loss ahead of the first substantial gain in the system directly multiplies the composite system noise factor.
- This means, for an overall low-noise system, we need to minimize any passive losses ahead of the LNA, recognizing that some loss is practically unavoidable.
- The system noise factor cannot be lower than that of the LNA component.
- This analysis has assumed that the passive components were at the reference temperature. This may not always be the case. More on this later.

3.3 General Cascaded Circuit Noise Model

We may generalize the results to two or more cascaded stages. The resulting equations are known as "Friis' formula" for noise temperature or noise factor/figure.[20]

System noise temperature for M cascaded stages may be calculated as

$$T_N = T_{G1} + \frac{T_{G2}}{G_1} + \frac{T_{G3}}{G_1 G_2} + \frac{T_{G4}}{G_1 G_2 G_3} + ... + \frac{T_{GM}}{G_1 G_2 G_3 ... G_{M-1}} \ . \tag{54}$$

System noise factor for M cascaded stages may be calculated as

$$F_N = F_{G1} + \frac{F_{G2}-1}{G_1} + \frac{F_{G3}-1}{G_1 G_2} + \frac{F_{G4}-1}{G_1 G_2 G_3} + ... + \frac{F_{GM}-1}{G_1 G_2 G_3 ... G_{M-1}} . \quad (55)$$

Some important comments include the following.

- As gain accumulates along the cascaded stages, the influence of subsequent components' noise factors diminishes, but does not go to zero.
- A particularly strong late-stage noise factor can still have significant influence on the overall system noise factor.
- If some of the stages need to be losses instead of gains (so as to achieve an overall desired gain), then it makes a difference which stages are made losses versus gains. Generally, for best Noise Factor, gains need to be earlier. However, this might interfere with system linearity requirements, necessitating a more comprehensive system analysis. This is one reason why RF engineers get the big bucks.
- A system noise factor can be expected to be system gain dependent.

Acknowledging that different communities have their preferences with respect to using noise factor versus noise temperature, in fact either can be used in with equal effectiveness in system design and analysis. Hereafter we will converge to subsequent analysis using Noise Factor unless convenience might suggest otherwise.

3.4 Analog to Digital Converters

Particularly in modern radar systems, the received signals are eventually sampled and quantized by an Analog to Digital Converter (ADC). The quantization operation introduces an error in the signal that we generally characterize as uncorrelated with other samples, and hence white, but modelled with a zero-mean Uniform distribution instead of a Gaussian model. We do note that any bandpass filtering with bandwidth less than the ADC sampling frequency will in fact cause some degree of correlation in the fast-time samples (samples from the same pulse echo), but this is typically very slight. The noise model is thus typically better characterized as band-limited white Uniform noise.

Nevertheless, any subsequent linear processing of the data will combine multiple Uniformly distributed samples, and via the Central Limit Theorem, cause the result to trend towards a Gaussian distribution. Consequently, the Uniform distribution nature of the quantization noise gets washed out by the processing anyway, and no other statistics beyond mean and variance are important to the resulting output.

Thermal noise from preceding stages can play an important role in a radar receiver, namely that of serving as a dithering source to allow observability for extremely low signals. In short, coherent signal levels below the quantization step size of the ADC will not be observable unless some dithering is added to the signal (and later removed or otherwise minimized). Thermal noise can serve as such a dithering mechanism, with the signal essentially serving to provide a non-zero (albeit generally varying) carrier to the thermal noise, and the combination essentially whitening the quantization noise, allowing subsequent coherent processing to discriminate between signal and all noise. The usual criterion is to provide enough system gain to allow an RMS noise level at the ADC to equal the voltage quantization step size. Greater amounts of noise may also sometimes be useful to help mitigate the effects of spurs, I/Q imbalance, etc.[21, 22, 23, 24]

Absent adequate thermal noise, sometimes an additional dithering signal (or noise) can be added to the signal path prior to the ADC. Any in-band noise used for this purpose must also be included in any noise factor calculation.

In any case, the ADC quantization noise can be a significant contributor to the total noise in the digital data, and hence must typically be included in noise factor calculations relevant to the digital data.

However, for any noise calculations in the analog circuits prior to the ADC, the quantization noise is obviously irrelevant. Therefore, a system configuration may need to keep track of two different noise factors; 1) an overall system noise factor for the data, and 2) an analog system noise factor for noise measures prior to the ADC. These might also be referred to as “pre-ADC” and “post-ADC” noise factors.

3.5 Noise Reference Location

Clearly, while individual components can be characterized with a noise factor, so too can entire systems comprised of cascaded components be characterized with a noise factor. The noise factor of such a system is referred to as by the less-than-cryptic term “System Noise Factor.”

However, for a system noise factor to be unambiguous, we also need to specify precisely where in the cascaded signal chain the system noise factor applies. While in principle it may apply to anywhere in the chain, there are two points in common use:

1. The receiver antenna port, and
2. The input to the first receiver amplifier, typically the LNA.

These are related to each other by any losses between the antenna port and the amplifier input. See section 3.2 above. Hereafter we will use the first-listed point; the receiver antenna port.

4 Noise Factor/Figure Measurement

There are a number of techniques with which to measure the system noise factor/figure of a radar receiver.[2,7] We will herein discuss the Y-factor measurement technique.

4.1 Basic Y-Factor Technique

We begin with the model of Figure 3, and assume that it applies to the whole system. The basic technique is to make output noise N_{out} measurements for two different, but each a precise, input noise level N_{in}. A calculation will result in the added noise N_G.

Towards that end, we define two input noise levels as

$$N_{in,hot} = \text{"hot" input noise level, and}$$
$$N_{in,cold} = \text{"cold" input noise level.} \tag{56}$$

Each noise level is precisely known, with

$$N_{in,hot} > N_{in,cold} . \tag{57}$$

Corresponding output noise levels are

$$N_{out,hot} = G\left(N_{in,hot} + N_G\right) = \text{"hot" output noise level, and}$$
$$N_{out,cold} = G\left(N_{in,cold} + N_G\right) = \text{"cold" input noise level.} \tag{58}$$

The task is to calculate N_G based on only the output noise measurements, but with knowledge of the input noise levels. First, we define

$$Y = \frac{N_{out,hot}}{N_{out,cold}} = \text{"Y" factor.} \tag{59}$$

We may then calculate the noise added by the system as

$$N_G = \frac{N_{in,hot} - YN_{in,cold}}{Y - 1} . \tag{60}$$

In terms of noise temperatures, this may be written as

$$T_G = \frac{T_{in,hot} - Y\,T_{in,cold}}{Y - 1} , \tag{61}$$

where

$T_{in,hot}$ = "hot" output noise temperature, and

$T_{in,cold}$ = "cold" output noise temperature. (62)

These are both presumed to be precisely known.

With the system noise temperature so calculated, we may further calculate the system noise factor as

$$F_N = \left(1 + \frac{T_G}{T_{ref}}\right). \tag{63}$$

This technique works best for Y-factors not close to one.

Noise Sources

Noise sources are often characterized by "Excess Noise Ratio" (ENR) in units of dB, where

$$ENR_{dB} = 10\log_{10}\left(\frac{T_{source}}{T_{ref}} - 1\right), \tag{64}$$

where

T_{source} = noise source noise temperature. (65)

Very common for radar noise factor measurements, we use a precision source only for the hot measurement, giving us

$$T_{in,hot} = T_{source} = T_{ref}\left(1 + 10^{\frac{ENR_{dB}}{10}}\right), \text{ and}$$

$$T_{in,cold} = T_{ref}\ . \tag{66}$$

We note that the hot noise temperature is for the "on" state of the noise source, and we have presumed that the cold temperature is for the "off" state of the noise source, which we also presume is the reference temperature. It is not always true for all noise sources that the "off" temperature of the noise source is the reference temperature. Nevertheless, for this report we will assume this anyway.

Gratuitous Observation

Suppose we have a hot noise source with ENR_{dB} = 30 dB, which we attenuate by 25 dB prior to any connection to the system. The question is “What is the equivalent ENR_{dB} at the output of the attenuator?”

We calculate the correct answer as follows. The hot noise source provides a noise temperature of

$$T_{in,hot} = T_{ref}\left(1 + 10^{\frac{ENR_{dB}}{10}}\right). \tag{67}$$

The attenuated hot noise source has noise temperature that must also include the noise contribution of the attenuator itself, so that

$$T_{in,hot,atten} = \frac{T_{in,hot}}{L_{hot}} + \frac{\left(L_{hot} - 1\right)T_{ref}}{L_{hot}}, \tag{68}$$

which can be expanded and simplified to

$$T_{in,hot,atten} = T_{ref}\left(1 + \frac{10^{\frac{ENR_{dB}}{10}}}{L_{hot}}\right), \tag{69}$$

This is the same power provided by an equivalent hot noise source with equivalent ENR

$$ENR_{dB,equivalent} = ENR_{dB} - 10\log_{10}\left(L_{hot}\right), \tag{70}$$

or perhaps

$$ENR_{dB,equivalent} = ENR_{dB} - L_{hot,dB}, \tag{71}$$

where

$$L_{hot,dB} = 10\log_{10}\left(L_{hot}\right). \tag{72}$$

The moral of the story is that the equivalent ENR of an attenuated a hot noise source can be calculated by just scaling the ENR by the loss, assuming the attenuator is at the reference temperature.

4.2 Modified Y-Factor Technique

An issue arises when the hot and cold noise sources cannot be injected at the point where the system noise factor is desired to be known. In fact, hot and cold noise sources may not even be injected to the same common point, and that point may be different yet than to where the system noise factor is desired. This requires us to modify the Y-factor method somewhat.

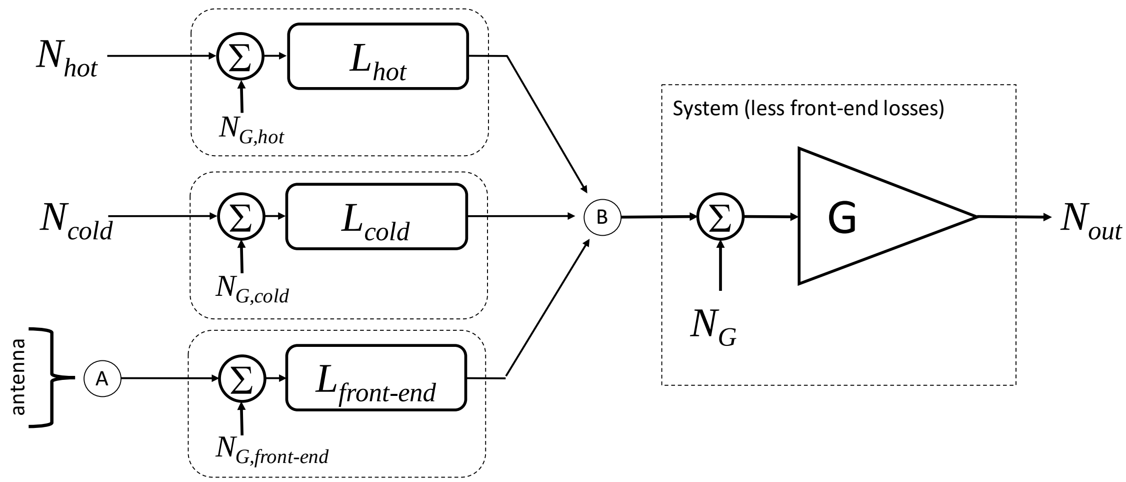

Figure 6. Modified Y-factor system model.

Consider the system model of Figure 6.

The essence of this model is that normal operation is for the antenna to supply signals to its port at point A that undergo some front-end losses before being applied to the LNA at point B. We desire to know the system noise factor at the antenna port at point A.

However, the system architecture is such that we don't have access to point A, and must apply noise sources via slightly different paths, with possibly different losses before the LNA. We will allow for different losses for the hot and cold noise sources, and that these are different yet from the front-end losses of the normal signal path. We will presume that all losses are precisely known.

We will begin by finding the system noise factor at point B, the input to the LNA. Once found, it is a simple matter to calculate the system noise factor at point A.

We specifically define the various loss terms as

$$\begin{aligned} L_{hot} &= \text{Loss between hot source and LNA,} \\ L_{cold} &= \text{Loss between cold source and LNA, and} \\ L_{front-end} &= \text{Loss between antenna terminals and LNA.} \end{aligned} \tag{73}$$

All losses are the inverse of gains, with losses no less than one. Recall also that these losses also have their own noise factor components.

As before, we define the Y-factor as

$$Y = \frac{N_{out,hot}}{N_{out,cold}} = \text{"Y" factor.} \tag{74}$$

We may then calculate the noise added by the system at the LNA as an equivalent

$$N'_G = \frac{\frac{N_{in,hot}}{L_{hot}} + \frac{N_{G,hot}}{L_{hot}} - Y\left(\frac{N_{in,cold}}{L_{cold}} + \frac{N_{G,cold}}{L_{cold}}\right)}{Y-1}. \tag{75}$$

In terms of noise temperatures this may be written as

$$T'_G = \frac{\frac{T_{in,hot}}{L_{hot}} + \frac{T_{G,hot}}{L_{hot}} - Y\frac{T_{in,cold}}{L_{cold}} - Y\frac{T_{G,cold}}{L_{cold}}}{Y-1}, \tag{76}$$

or in terms of what we know is the noise added by resistive attenuators,

$$T'_G = \frac{\frac{T_{in,hot}}{L_{hot}} + \frac{T_{ref}\left(L_{hot}-1\right)}{L_{hot}} - Y\frac{T_{in,cold}}{L_{cold}} - Y\frac{T_{ref}\left(L_{cold}-1\right)}{L_{cold}}}{Y-1}, \tag{77}$$

where it remains true that

$$T_{in,hot} = T_{ref}\left(1+10^{\frac{ENR_{dB}}{10}}\right). \tag{78}$$

Note again that this is *at the LNA*, identified as point B in Figure 6. The system noise factor at the antenna port (point A) has to account for the front-end losses between antenna and LNA. It is then calculated as

$$F_N = L_{front-end}\left(1+\frac{T'_G}{T_{ref}}\right) = \text{system noise factor at antenna port.} \tag{79}$$

We make the following observations:

- If any part of L_{hot} is used to adjust ENR of the hot noise source, then it must be done intelligently. Whatever scales the ENR cannot be used again in the subsequent noise factor calculations. We must avoid double-counting.

- It is quite common to select $T_{in,cold} = T_{ref}$ = 290 K.

- If indeed $T_{in,cold} = T_{ref}$, then we may also simply assume $L_{cold} = 1$, regardless of what the loss actually is. It just doesn't matter.

If we may assume $T_{in,cold} = T_{ref}$, then the system noise temperature may be calculated with the simplified expression

$$T_G' = \frac{\dfrac{T_{in,hot}}{L_{hot}} + \dfrac{T_{ref}\left(L_{hot} - 1\right)}{L_{hot}} - Y\,T_{ref}}{Y - 1} . \tag{80}$$

It remains true for the system noise factor to be calculated as

$$F_N = L_{front-end}\left(1 + \frac{T_G'}{T_{ref}}\right) = \text{system noise factor at antenna port.} \tag{81}$$

These might be combined into the single calculation

$$F_N = \frac{L_{front-end}}{L_{hot}}\left(\frac{\dfrac{T_{in,hot}}{T_{ref}} - 1}{\left(Y - 1\right)}\right) = \frac{L_{front-end}}{L_{hot}}\left(\frac{10^{\frac{ENR_{dB}}{10}}}{\left(Y - 1\right)}\right). \tag{82}$$

Let us now further assume the specific testing architecture where in Figure 6, point A is the same location as point B, that is, where front-end losses are absorbed into the system model. Otherwise said, a model with no overt front-end system losses outside of the system model. In this case $L_{front-end} = 1$, and the noise factor may be calculated as

$$F_N = \left(\frac{10^{\frac{ENR_{dB} - L_{hot,dB}}{10}}}{\left(Y - 1\right)}\right) = \left(\frac{10^{\frac{ENR_{dB,equivalent}}{10}}}{\left(Y - 1\right)}\right). \tag{83}$$

4.3 Sources of Error and Uncertainty

There are many factors that can influence the accuracy and precision of noise factor measurements and calculations. These might include[4]

1. Nonlinearity – The Y-factor equations assume linear gain. Any nonlinearities in the gain will perturb the relative measurements and result in errors in the noise factor result calculation.
2. Instrumentation uncertainty – The actual measurement instrument needs to be linear for good measurements. This implies that any sampling by an ADC needs to be such that the measurements yield an accurate and precise power measure.
3. ENR uncertainty – The ENR of especially the hot noise source needs to be precise. Furthermore, the general presumption is that the ENR is white; with flat PSD over the band of interest. Any departure from the presumed value or spectral flatness assumption will render an inaccurate noise factor calculation. An ENR flatness specification of ±1 dB over the frequency band of interest may mean commensurate uncertainty of a resulting noise factor measurement; more problematic for off-frequency, or broadband measurements. In addition, errors in the cold noise source will do the same. We note that any hot or cold noise sources will generally also be somewhat temperature dependent, which implies errors in assumed temperature will provide an error source.
4. Mismatch uncertainty – The assumption herein has been that all stages and interfaces are well-matched, including between hot/cold noise sources and the respective inputs. In fact the ENR_{dB} of a hot noise source typically assumes perfect match. This means that the noise power delivered by noise sources generally presume that the total available power is delivered to their loads. Imperfect impedance matches mean diminishment of delivered power, and not accounting for this means errors in noise factor calculations. Uncertainty of the match means uncertainty of the calculation. In addition, we note that impedance match is also often frequency-dependent, affecting broadband performance.
5. System architecture uncertainty – The relevant losses, especially L_{hot} and $L_{front-end}$ need to be known very accurately to the point that noise factor is measured. Noise factor is directly proportional to $L_{front-end}$, and nearly inversely proportional to L_{hot}. In terms of dB, proportional means a one-for-one dB-per-dB error. Furthermore, the presumption is that the passbands of any filters, attenuators, or couplers are flat over the bandwidth of any noise used for noise factor measurement. Otherwise, noise factor measurements become bandwidth dependent, the compensation of which adds unnecessary complication, and hence uncertainty.

We readily concede that other sources of errors and/or uncertainties are also reported in the literature.[4]

4.4 Using Radar Data

We observe in the preceding analysis that the actual system gain is not used in any calculations for system noise factor. In fact, it is divided out during the derivation of the relevant equations. Everything depends on a relative measurement between employing hot and cold noise sources.

This suggests that anywhere in the signal processing chain where the noise levels are observable can be used to calculate system noise factor, as long as any differences in system gains or data scale factors are known.

Consider two laboratory Synthetic Aperture Radar (SAR) images; one with the hot noise source enabled, and the other with the hot noise source disabled so that the cold noise temperature is the nominal reference temperature. Since we are interested in noise measurements, any other content is not material to our measurements. In fact, we need image areas for which direct echoes are absent, such as shadow areas or other zero-backscatter areas. In a laboratory, this might be an image with transmitter disabled.

The noise levels should be observable in both images. This means that the RMS noise level should be above the voltage quantization step-size in the image, but not so high as to exhibit excessive image saturation. We stipulate that all other radar parameters (e.g. gains, scale factors, etc.) need to be the same for both images, or at least correctable as such. Additional cautions include the following.

- If the image has any amplitude adjustments, such as compensation for antenna beam roll-off, then unless the amplitude adjustments are removed, identical regions in both images need to be used, to guarantee identical data scaling. Image area near the center of the image is preferable, as it is typically minimally affected by amplitude corrections.

- Any spurious signals, including image DC leakage needs to be avoided in the measurement region. Including these will impact the cold noise level more than the hot noise level, and therefore artificially raise the calculated noise factor.

- Often, image header (meta-data) information will include an estimate of the cold noise level, or at least information allowing the calculation of such an estimate. While such data might be employed to calculate system noise factor, we opine that this image parameter should be verified with cold-image measurements before it is used. Note that such a parameter is typically only really accurate at image center, placing limits on how hot-noise is measured.

We may measure noise variance in both images. Note that the proper noise variance measure is to take a mean of the square of the magnitude of a large set of noise pixels. A ratio of the variance of the hot image to the variance of the cold image is in fact the Y-factor. System noise factor can be calculated from this in the manner previously described.

4.5 Accuracy of the Noise Measures

Our system noise factor calculation depends on measuring the noise power both when using hot and cold noise sources. However, noise is a random process. The best we can hope for is to estimate the noise statistics we seek from a finite data set. The question we wish to answer now is "How many independent noise samples do we need?"

Let us begin by assuming that we have N independent samples of complex Gaussian distributed noise. We emphasize that we are assuming that noise samples are "independent." We define the following

$$x_n \text{ = the } n^{\text{th}} \text{ complex noise sample,}$$
$$\mu = E\{x\} = 0 \text{ = mean value of the overall noise process, and}$$
$$\sigma^2 = E\left\{|x|^2\right\} \text{ = variance of the overall noise process,} \tag{84}$$

where we define

$$E\{z\} \text{ = Expected value of } z. \tag{85}$$

We stipulate the unbiased estimators of the mean and variance from the samples to be

$$\hat{\mu} = \frac{1}{N}\sum_N x_n \text{ = estimated mean value of the noise process, and}$$
$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_N \left|x_n - \hat{\mu}\right|^2 \text{ = estimated variance of the noise process.} \tag{86}$$

With respect to the estimate of the mean, we calculate

$$E\{\hat{\mu}\} = \mu \text{ = expected value of the estimate of the mean, and}$$
$$E\left\{|\hat{\mu} - \mu|^2\right\} = \frac{\sigma^2}{N} \text{ = variance of the estimate of the mean.} \tag{87}$$

With respect to the estimate of the variance, we calculate

$$E\left\{\hat{\sigma}^2\right\} = \sigma^2 \text{ = expected value of the estimate of the variance, and}$$
$$E\left\{\left(\hat{\sigma}^2 - \sigma^2\right)^2\right\} \xrightarrow[\text{large } N]{} \frac{2\sigma^4}{N} \text{ = variance of estimate of the variance.} \tag{88}$$

At this time, we state that we wish the RMS error in the estimate of the variance to be less than some small fraction of the variance, that is, we desire

$$\sqrt{E\left\{\left(\hat{\sigma}^2-\sigma^2\right)^2\right\}}<\varepsilon\,\sigma^2, \tag{89}$$

where

ε = the allowable fraction of noise power for the error in the estimate. (90)

This transmogrifies to the bound on N as

$$N>\frac{2}{\varepsilon^2}. \tag{91}$$

This means we will tolerate an estimate of the variance such that

$$(1-\varepsilon)\sigma^2<\hat{\sigma}^2<(1+\varepsilon)\sigma^2. \tag{92}$$

For example, for a 0.1 dB error, we calculate $\varepsilon \approx 0.02$, which indicates $N \geq 5000$ independent samples are required to achieve this accuracy and precision.

We offer a quick note about quantization noise from the ADC. As previously stated, ADC quantization noise can be a significant contributor to system noise factor. However, for meaningful noise measurements to be made that include quantization noise, the quantization noise needs to be observable, and in fact whitened; made sufficiently uncorrelated over the sample data set. This requires a dithering signal of some sort. Such a dithering signal can be front-end thermal noise, an additional dithering noise source, or may be some other signal that sufficiently randomizes the quantization noise.

To be sure, the quantization noise can be relatively accurately calculated, but its contribution to the system noise factor will depend on the net gain of prior analog stages. We do anticipate that quantization noise will be most significant at low net gain levels that correspond to fairly strong expected radar echo signals. Consequently, in practice, we expect natural dithering to occur. However, this is not guaranteed in calibration modes where system noise figure is determined.

4.6 Example Procedure for Noise Factor Measurement

We summarize the previous sections with a notional procedure for determining the system noise factor for a radar receiver at the antenna port, as manifest in the data.

Preliminary Measurements/Calculations

We begin by determining appropriate pre-LNA loss factors, namely

L_{hot} = Loss between hot source and LNA,
L_{cold} = Loss between cold source and LNA, and
$L_{front-end}$ = Loss between antenna terminals and LNA. (93)

We then calculate appropriate hot and cold noise temperatures as

$$T_{in,hot} = T_{source} = T_{ref}\left(1+10^{\frac{ENR_{dB}}{10}}\right), \text{ and}$$

$$T_{in,cold} = T_{ref}\ . \tag{94}$$

With this definition for $T_{in,cold}$, we can assume that $L_{cold} = 1$.

These parameters are constants for the actual calculations.

Actual Noise Factor Measurement/Calculation

The following procedure needs to be followed at each gain setting.

1. Measure $N_{out,hot}$ and $N_{out,cold}$ with hot/cold noise sources respectively. This needs to be done with appropriate certainty, e.g. 5000 independent noise samples for 0.1 dB RMS accuracy/precision. If dithering is inadequate, see subsequent discussion for substitutions.

   We make the tacit assumption that these noise levels are in fact measurable. Should they not be measurable, then we need to determine system noise factor somewhat differently.

2. Calculate the Y-factor as

$$Y = \frac{N_{out,hot}}{N_{out,cold}} = \text{“Y” factor.} \tag{95}$$

3. Calculate the system noise temperature as

$$T'_G = \frac{\dfrac{T_{in,hot}}{L_{hot}} + \dfrac{T_{ref}\left(L_{hot}-1\right)}{L_{hot}} - Y\,T_{ref}}{Y-1}. \tag{96}$$

4. Calculate the corresponding noise factor as

$$F_N = L_{front-end}\left(1+\frac{T'_G}{T_{ref}}\right) \text{ = system noise factor at antenna port,} \tag{97}$$

and the corresponding noise figure as

$$F_{N,dB} = 10\log_{10} F_N \text{ = system noise figure.} \tag{98}$$

We stipulate that simpler versions of these equations might be appropriate, if conditions so warrant. For example, we might make use of the direct calculation in Eq. (83).

We further stipulate that these are the correct calculations for calculating the noise in the radar data, *after* the ADC. This is *not* necessarily the right number to use for calculating noise prior to the ADC, which must omit any contribution from quantization noise. While straightforward, we nevertheless defer further discussion of this as beyond the scope of this report.

<u>Inadequate Dithering of Quantization Noise</u>

We have presumed in the above procedure that we can in fact adequately measure individual samples of $N_{out,hot}$ and $N_{out,cold}$ with which we can determine necessary statistics. This means that the statistics are embodied in the digital data collected by the ADC. This also means that there is adequate noise power to overcome the quantization by the ADC, so that we can make a reasonable measurement.

Now suppose that the noise signal sampled by the ADC is inadequate; does not exhibit amplitudes that transcend a single quantization step. This means that the noise by itself cannot be measured, and using the corresponding data will provide bogus results in the noise factor calculations. We note that the first measurement that runs into the quantization problem is $N_{out,cold}$, and as $N_{out,cold} \to 0$, albeit erroneously due to quantization, we have $Y \to \infty$, and ultimately $F_N \to 0$, which is nonsense. We can avoid this by setting a floor to $N_{out,cold}$ equal to the known quantization noise if the signal were properly dithered. We will insist on the floor

$$N_{out,cold} \geq N_{quantization}, \tag{99}$$

where

$$N_{quantization} \text{ = quantization noise level.} \tag{100}$$

We note that the quantization noise variance is 1/12 of the quantization step-size squared, consistent with a uniform distribution. Any $N_{out,cold}$ measurements less than this is erroneous and should be substituted with this.

Now, in addition to the floor for $N_{out,cold}$, as we reduce $N_{out,hot} \to N_{quantization}$, this will cause $Y \to 1$, which will cause $F_N \to \infty$, which is in fact real but also untenable. We opine that at gain settings where this might be possible, we are probably not too interested in noise levels anyway, so don't need to be particularly accurate with system noise factor. We might consider just limiting values to something arbitrary and convenient, just to keep calculations from blowing up. For example, we might arbitrarily limit

$$N_{out,hot} \geq 2N_{quantization} . \tag{101}$$

With these floors in place, the Y-factor calculations for ultimately the system noise factor may proceed as presented above.

4.7 Temperature Effects

As a practical matter, neither the radar system components, nor the system as a whole, will operate at the standard reference temperature T_{ref}. This means that component noise factors as well as system noise factors will manifest with temperature dependencies. Essentially, we cannot count on noise levels to stay constant with temperature. The best we can do is approximate it, and compensate any variability as accuracy and precision requirements dictate.

One way to deal with this is just presume the noise level accuracy is "good enough" based on reference temperature measurements, or even room temperature measurements. Another way to deal with this is to incorporate temperature dependencies into the noise calculations. Another way yet is to perform system noise factor calibrations at different temperatures.

All have their pros and cons, and none of these is fool-proof for predicting noise levels in radar data. In any case, it is reasonable for the radar system engineer to at least understand the temperature dependencies of various component noise factors. To that end, we now discuss some attributes of various component classes.

We note that with respect to the reference temperature of 290 K, a 1 dB change in noise temperature will result by either increasing noise temperature by 75 K, or decreasing noise temperature by 60 K.

Passive Lossy Components

Recall from the discussion on passive lossy components in section 3.2 that when a passive lossy component is at the nominal reference temperature, the noise factor is equal to the loss. This means that with respect to the model in Figure 3, we would calculate

$$T_G = T_{ref}\left(L-1\right), \tag{102}$$

where

L = the signal loss of the component. (103)

Even if the loss were to remain constant, we would expect the noise generated by the component to increase with temperature such that contributed noise temperature is

$$T_G = T_{actual}\left(L-1\right), \tag{104}$$

where

T_{actual} = the actual physical temperature of the lossy component. (105)

However, this noise temperature can be related to a noise factor such that

$$T_{actual}\left(L-1\right) = T_{ref}\left(F_N-1\right). \tag{106}$$

This might be rearranged to solve for the new noise factor, giving

$$F_N = \frac{T_{actual}}{T_{ref}}\left(L-1\right)+1. \tag{107}$$

The component output noise temperature can be then calculated as

$$T_{out} = T_{actual} + \frac{T_{in}-T_{actual}}{L}. \tag{108}$$

Clearly, the actual lossy component temperature has an impact on the component's noise factor, and output noise level. Higher actual component temperature means greater noise factor, and more noise. In addition, there is no reason to expect the loss itself to remain temperature independent.

Nevertheless, for a physical temperature range of perhaps –40 C to +40 C, for high-loss components, this might mean a Noise Factor variation of –0.95 dB to +0.33 dB.

Active Gain Stages

Noise generated by active electronics is very typically temperature dependent, with higher component physical temperatures generating more component noise than lower temperatures. Table 1 shows measurements on one popular commercial LNA at Ku-band, which indicates a temperature coefficient for noise factor of approximately 0.0028 K^{-1} near the reference temperature.

Table 1. Anecdotal LNA noise figure dependence on physical operating temperature at Ku-band.

Operating Temperature	*Noise Figure*	*Noise Factor*	*Noise Temperature*
–30 C (243.15 K)	0.93 dB	1.24	69 K
–10 C (263.15 K)	1.05 dB	1.27	79 K
+10 C (283.15 K)	1.23 dB	1.33	95 K
+30 C (303.15 K)	1.41 dB	1.38	111 K
+50 C (323.15 K)	1.62 dB	1.45	131 K

These kinds of temperature measurements or specifications are not common in data sheets, but some vendors do include bounds for temperature extremes as well as at room temperature. Temperature coefficients can sometimes be estimated from these.

For example, the data sheet for a M/ACOM MAAL-010528 X-Band Low Noise Amplifier[25] suggests a temperature dependence at 10 GHZ of approximately 0.003 K^{-1}, with a room temperature noise figure of about 1.6 dB.

As another example, the data sheets for the MiniCircuits AVA-24+ Wideband, Microwave Monolithic Amplifier[26] suggests a temperature dependence at 16 GHZ of approximately 0.025 K^{-1}, with a room temperature noise figure of about 6 dB. This temperature coefficient is more than 8 times the M/ACOM example.

We would also be remiss in not mentioning that LNA gain itself is also often quite temperature dependent. This adds additional temperature variations to the system noise factor.

Antenna

From the previous discussion in section 2.4, the noise contributed by an antenna is

$$T_{antenna} = (1-\eta) T_{antenna,physical} + \eta \xi T_{scene} , \quad (109)$$

where the parameters were previously identified. Since we do our best to build high-efficiency antennas, the dominant contributor to antenna noise for terrestrial imaging radars is the target scene itself. That is, we may reasonably approximate

$$T_{antenna} \approx \xi T_{scene} . \quad (110)$$

Now consider a temperature range for target scenes of interest between perhaps −40 C and +40 C, with emissivity ranging between 0.9 and 1.0 over that range, but allowing for an emissivity of 0.4 at 0 C. This allows us to calculate a noise temperature range for the antenna of approximately

$$109 \text{ K} < T_{antenna} < 313 \text{ K} . \quad (111)$$

With respect to the reference temperature of 290 K, this represents variations from −4.2 dB to +0.33 dB. The lopsided variations are due to the low emissivity assumed for sea water at 0 C. We note that this analysis is for emissivity only, and does not include any reflections from other noise sources like, say, the sun.

Nevertheless, higher physical temperatures for the target scene can be expected to mean higher noise levels from the antenna.

<u>Net Effects on System Noise Factor</u>

While not absolutely guaranteed for all individual components, we might nevertheless expect that the system noise factor will generally increase with increasing hardware temperature.

If a precise system noise factor is required/desired over all operating conditions, then the system needs to be calibrated to some extent over temperature. Once calibrated, to apply any temperature calibration of course means that the system temperature needs to be sensed. Since the dominant source of noise is often likely still the first LNA and prior lossy components, these components' temperatures are consequently the most crucial to sense.

While the system noise factor can so be calibrated, the actual noise input to the system will also depend on the temperature of the scene to which the antenna is pointed. We note that the target scene temperature is not necessarily coupled to the component or system physical temperatures. To remove this variability from noise measurements requires switching the input to the radar receiver from the antenna to a known termination, with known noise temperature.

<u>A Note About Receiver Gain</u>

Recall that noise factor is about equivalent "input" noise. To predict the noise level in radar data, we need to know not only the noise factor of the system, but also the gain of the receiver. That is, actual noise in the radar data needs accurate receiver gain knowledge in addition to the system noise factor.

More generally, any absolute calibration of received power in the data, including radar echo signals, needs accurate receiver gain knowledge. Gain itself is a measure of output power versus input power, usually signal power, but could be noise power as well.

Measuring gain is typically about providing a known input signal, measuring the output signal in the data, and calculating the power ratio. Consequently, such a gain measure is only as accurate as the knowledge of the input signal level, or noise level if noise is used for this purpose. The input must be at a known reference power level, or "peg-point" for gain calculations.

Should our reference input power vary, say, due to temperature, aging, or other factors, and if the input power variation is unknown to the receiver, then we have an unknown error in the input power, and any gain calculation will also have a corresponding error. We recall that with respect to noise, even the scene within the received antenna beam will offer variations in input noise. Consequently, if input noise is to be used as an absolute reference for receiver gain measurements then we must address the following.

1. The antenna should be disconnected from the receiver, and substituted with a known load, with known noise temperature.

2. The temperature characteristics of the front-end receiver components must be known.

3. The temperature of the front-end receiver components must be measured and supplied to the radar control software, for proper corrections to be calculated and applied.

4. If the input noise becomes unmeasurable, say, due to excessive receiver attenuation, then another signal needs to be substituted for the input noise, provided that the new signal is also observable in the data. In the absence of such a signal, then using nominal attenuator settings might be warranted.

Absent any compensation for these variations, an absolute gain calibration must contend with corresponding absolute errors. However, "relative" gain might still be reasonably measured.

“Although our intellect always longs for clarity and certainty,
our nature often finds uncertainty fascinating.”
-- Carl von Clausewitz

5 Conclusions

We reiterate the following points.

- Proper noise factor measurement requires proper accounting for differences in losses between signal paths used by hot noise source, cold noise source, and actual front-end signal path.

- Losses need to be known with accuracy and precision of the desired final noise factor.

- System noise factor and system noise temperature are straightforward functions of each other, and can be used with equivalent accuracy and precision in calculating system noise levels.

- A modified Y-factor technique was detailed to calculate system noise factor.

- Numerous factors can contribute to errors and uncertainty to a noise factor measurement.

"I used to be scared of uncertainty; now I get a high out of it."
-- *Jensen Ackles*

Appendix A – General Two-Port Analysis

Here we discuss a general two-port network, as presented in Figure 7. Our interest is in both gain and noise factor. Generally, a noisy two-port network can be modelled as a noise-free two-port network with noise sources externally added to each port.

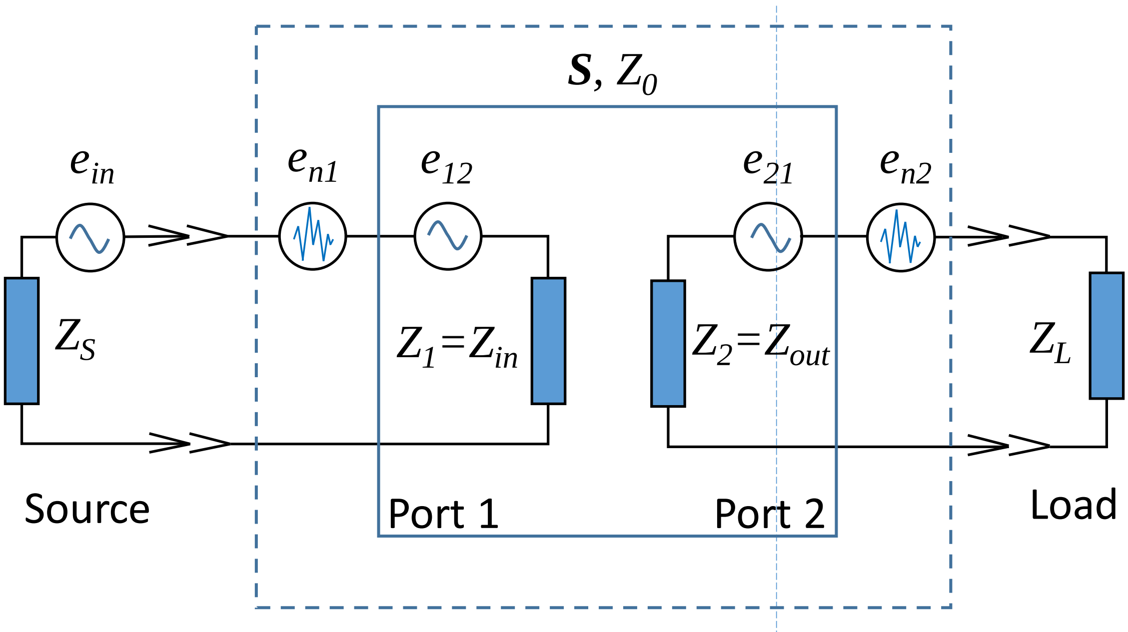

Figure 7. General Two-Port network.

We will summarize the analysis presented in the literature using Scattering Parameters, or S-parameters. We accordingly identify the following parameters in Figure 7, noting that port 1 is the input port, and port 2 is the output port. The various impedances are identified as

$$\begin{aligned}
Z_S &= \text{source impedance,} \\
Z_L &= \text{load impedance,} \\
Z_{in} &= \text{network input impedance,} \\
Z_{out} &= \text{network output impedance,} \\
Z_1 &= \text{port 1 impedance} = Z_{in}\,, \\
Z_2 &= \text{port 2 impedance} = Z_{out}\,, \\
Z_0 &= \text{reference impedance for S-parameters.}
\end{aligned} \tag{112}$$

The various RMS voltages are identified as

e_{in} = source voltage,
e_{12} = voltage on port 1 generated by input to port 2,
e_{21} = voltage on port 2 generated by input to port 1,
e_{n1} = equivalent noise voltage added to port 1, and
e_{n2} = equivalent noise voltage added to port 2. (113)

Noise voltages e_{n1} and e_{n2} are equivalent noise voltages to represent internal noise sources. They are generally only partially correlated, and partially uncorrelated.

For completeness, we identify the S-parameters as

S_{11} = ratio of voltage exiting port 1 due to voltage wave entering port 1,
S_{21} = ratio of voltage exiting port 1 due to voltage wave entering port 2,
S_{12} = ratio of voltage exiting port 2 due to voltage wave entering port 1, and
S_{22} = ratio of voltage exiting port 2 due to voltage wave entering port 2. (114)

Recall that our S-parameters are with respect to a reference impedance Z_0.

We now make preliminary calculations of reflectance coefficients to be

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$ = reflection coefficient seen looking from network to load,
$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$ = reflection coefficient seen looking from network to source,
$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$ = reflection coefficient seen looking from source to network, and
$\Gamma_{out} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0}$ = reflection coefficient seen looking from load to network.
(115)

Note that Z_0 is merely a reference impedance. It is also useful to recognize that the above equations can be manipulated so that

$$Z_S = Z_0 \frac{1+\Gamma_S}{1-\Gamma_S}. \quad (116)$$

Similar expressions can be developed for the other impedances and reflectance coefficients.

In terms of S-parameters, it can readily be shown that

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\text{, and}$$
$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}. \tag{117}$$

These now allow us to calculate various power levels, and potential power levels. From Pozar[8] we may calculate the average power delivered to the network as

$$P_{in} = \left(\frac{{e_{in}}^2}{4Z_0}\right)\frac{\left|1-\Gamma_S\right|^2\left(1-\left|\Gamma_{in}\right|^2\right)}{\left|1-\Gamma_S\Gamma_{in}\right|^2} \;. \tag{118}$$

A special case of this is the maximum average power available to the network, when the input impedance of the network is conjugate-matched to the source impedance. This is calculated as

$$P_{in,av} = P_{in}\Big|_{\Gamma_{in}=\Gamma_S^*} = \left(\frac{{e_{in}}^2}{4Z_0}\right)\frac{\left|1-\Gamma_S\right|^2}{\left(1-\left|\Gamma_S\right|^2\right)} \;. \tag{119}$$

The average power delivered to the load can be expressed as

$$P_L = \left(\frac{{e_{in}}^2}{4Z_0}\right)\frac{\left|S_{21}\right|^2\left(1-\left|\Gamma_L\right|^2\right)\left|1-\Gamma_S\right|^2}{\left|1-S_{22}\Gamma_L\right|^2\left|1-\Gamma_S\Gamma_{in}\right|^2} \;. \tag{120}$$

A special case of this is the maximum power available to the load, when the load impedance is conjugate-matched to the output impedance of the network. This is calculated as

$$P_{L,av} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \left(\frac{{e_{in}}^2}{4Z_0}\right)\frac{\left|S_{21}\right|^2\left|1-\Gamma_S\right|^2}{\left|1-S_{11}\Gamma_S\right|^2\left(1-\left|\Gamma_{out}\right|^2\right)} \;. \tag{121}$$

These definitions now allow us to calculate power gains.

Two-Port Power Gain

The power gain of a network is the ratio of the output power to the input power. However, there are multiple variations in the definitions of both output and input powers, thereby yielding multiple gain definitions. Following the development in Pozar,[8] the most general gain definition is

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2 \left(1 - |\Gamma_{in}|^2\right)} = \text{power gain.} \qquad (122)$$

The available power gain is the ratio of available output power to available input power. This becomes

$$G_{av} = \frac{P_{L,av}}{P_{in,av}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{|1 - S_{11}\Gamma_S|^2 \left(1 - |\Gamma_{out}|^2\right)} = \text{available power gain.} \qquad (123)$$

Recall that this assumes both conjugate-matched source and conjugate-matched load impedances.

Other gain definitions of interest include the transducer power gain, calculated as

$$G_T = \frac{P_L}{P_{in,av}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2 \, |1 - \Gamma_S\Gamma_{in}|^2} = \text{transducer power gain.} \qquad (124)$$

If the input and output networks are matched, such that $\Gamma_S = 0$, and $\Gamma_L = 0$, then this becomes the matched transducer power gain, sometimes called insertion gain, that is

$$G_{T,matched} = G_T\Big|_{\substack{\Gamma_S=0 \\ \Gamma_L=0}} = |S_{21}|^2 = \text{matched transducer power gain.} \qquad (125)$$

If we have a system with $S_{12} = 0$, then the transducer gain becomes the unilateral transducer power gain, that is

$$G_{T,unilateral} = G_T\Big|_{S_{12}=0} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2 \, |1 - S_{11}\Gamma_S|^2}$$

$$= \text{unilateral transducer power gain.} \qquad (126)$$

Two-Port Noise Factor

Recall that noise factor generally tries to model all component internal noise as being generated at the input of the network. The basic strategy for developing the noise factor of the network then becomes one of transmogrifying e_{n2} to port 1 of the network, and relating the total network-generated noise to the input. The corresponding derivation is available in the published literature.[10,11,27]

The noise figure of a two-port amplifier can be expressed as

$$F_N = F_{N,\min} + \frac{R_N}{G_S}\left|Y_S - Y_{S,opt}\right|^2 , \tag{127}$$

where

$Y_S = 1/Z_S$ = source admittance,
$G_S = \mathrm{Re}\{Y_S\}$ = real part of source admittance,
$F_{N,\min}$ = minimum achievable noise factor,
$Y_{S,opt}$ = optimum source admittance that results in minimum noise factor, and
R_N = equivalent noise resistance. (128)

The quantities, $F_{N,\min}$, Y_{opt}, and R_N, are "noise parameters" specific to a device. They completely describe the noise characteristics of the device.

The equivalent noise resistance is another way of expressing the port 1 noise power contribution, that is

$$R_N = \frac{e_{n1}^{\ 2}}{4kT_{ref}B} . \tag{129}$$

The optimum source impedance is simply

$$Z_{S,opt} = 1/Y_{S,opt} . \tag{130}$$

Both the optimum source impedance and the minimum noise factor depend on inner details of the two-port network, including how the respective ports' noise models interact with each other; how they correlate with each other.

The noise factor can also be written in terms of reflectance coefficients as

$$F_N = F_{N,\min} + \frac{4R_N}{Z_0}\left(\frac{\left|\Gamma_{opt} - \Gamma_S\right|^2}{\left|1+\Gamma_{opt}\right|^2\left(1-\left|\Gamma_S\right|^2\right)}\right), \tag{131}$$

where

$$\Gamma_{opt} = \frac{Z_{opt} - Z_0}{Z_{opt} + Z_0}. \tag{132}$$

In this case, $F_{N,\min}$, Γ_{opt}, and R_N, are the relevant "noise parameters" specific to a device, noting that Z_0 needs to be specified as well.

We make several important observations now.

- Note that in general, the optimum source impedance is not the same impedance for maximum power transfer, that is

$$Z_{S,opt} \neq Z_{in}^{*}. \tag{133}$$

- The noise factor of the two-port network will change as the source impedance changes.
- This means that there is an inherent trade between maximizing power gain and minimizing noise factor. Said another way, maximizing power transfer is not the same as minimizing noise factor. This is a design trade.

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“So much of life, it seems to me, is determined by pure randomness.”
-- *Sidney Poitier*

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