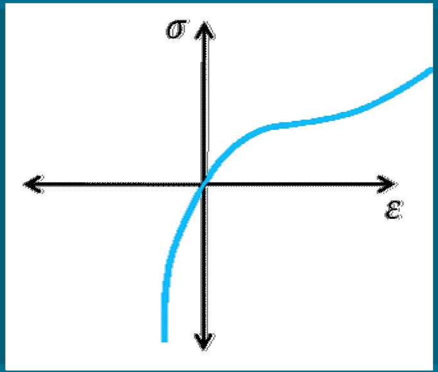
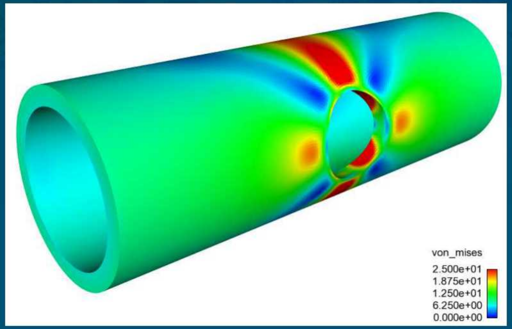
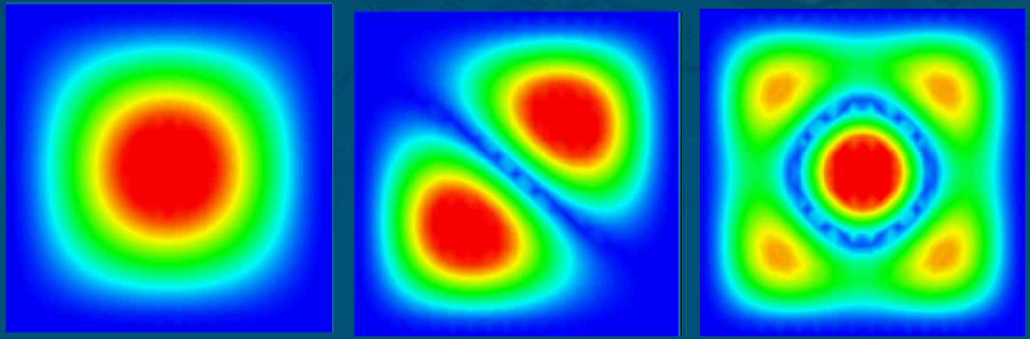
SAND2018-9923PE

Model reduction with applications to material nonlinearities







PRESENTED BY

Robert J. Kuether, Sandia National Laboratories

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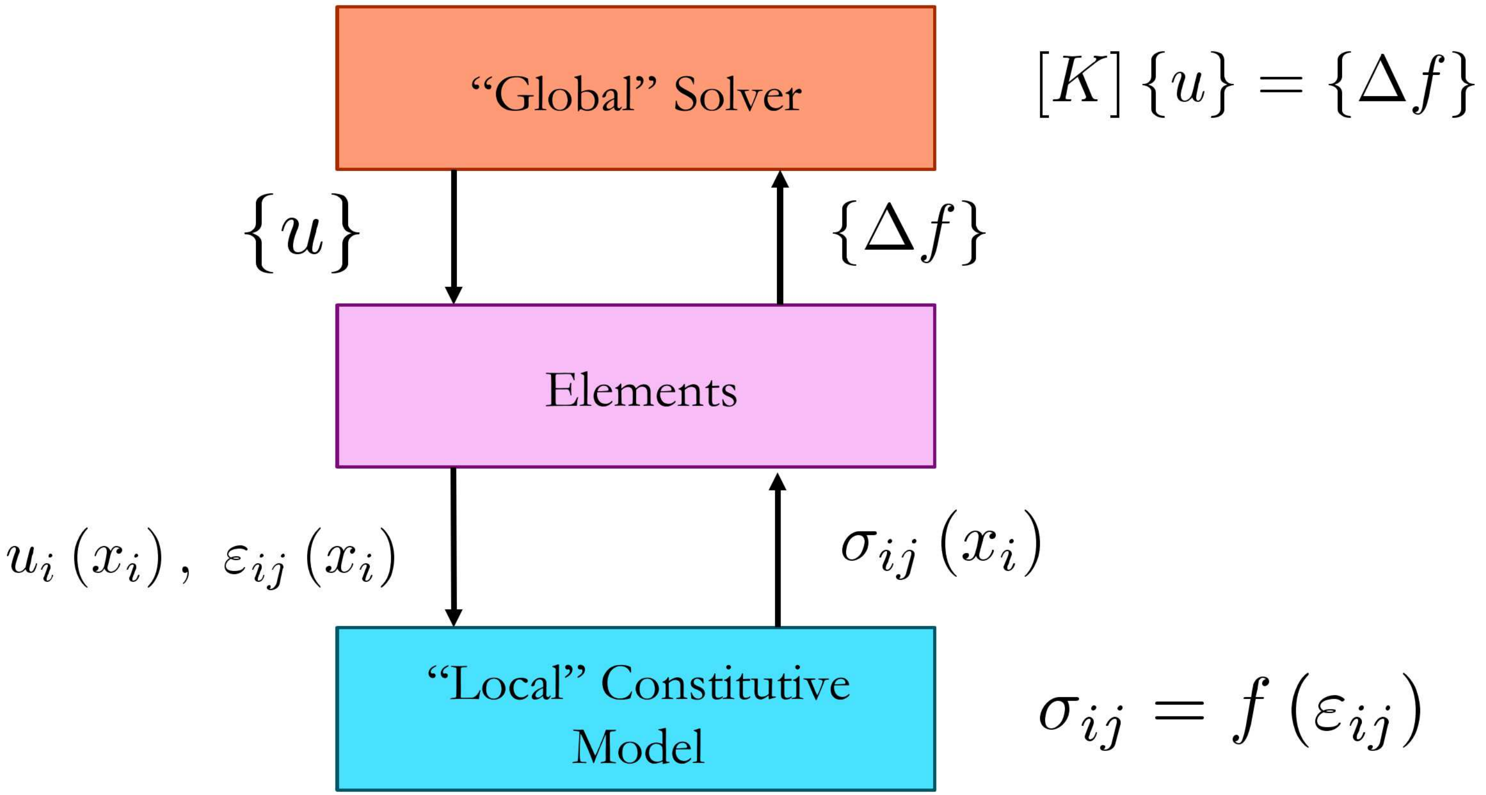
Presented at the 2nd NROM Workshop on September 13, 2018

Constitutive Models in FEA

Most modern analysis performed via finite element analysis (FEA)

What role do constitutive models play in FEA?

- Displacements found via "global" solver enforcing equilibrium
- Constitutive models solve "local" problem connecting stress to displacement

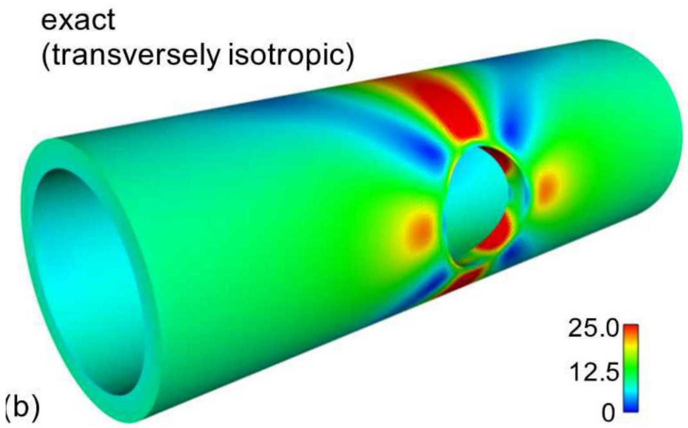


Material Modeling – Continuum Scale

Continuum scale models seek to analyze structures

Each analysis point encapsulates all of the lower scale features previously identified

Typical scale of interest to constitutive models



(Bishop and Brown, 2018, *CMAME,* Accepted)

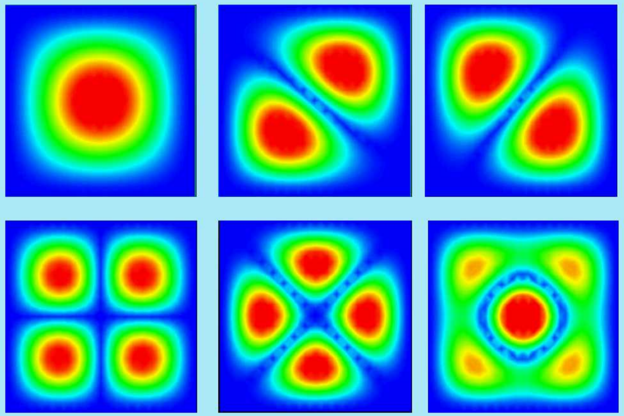
- Development of constitutive models can rely upon:
  - Upscaling/bridging the scales to homogenize lower scale results
  - Deriving mathematical relationships of phenomena of interest (phenomenological modeling)
- Mesoscale models focus on resolving microstructures
  - Polycrystalline texture, inhomogeneities
- Material modeling performed at the dislocation and atomistic scale

Galerkin-based model order reduction

FE mesh in physical coordinates

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{f}_{int}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{f}_{ext}(t)$$

Determine appropriate basis, or shape vectors based on the physical equations of motion



Project full equations of motion onto a small set of basis vectors

Related by transformation:

$$\mathbf{x}(\mathrm{t}) = \mathbf{T}\mathbf{q}(t)$$

$$^{*}\mathbf{q}(\mathrm{t}) \ll \mathbf{x}(t)$$

Solve reduced equations

$$\bar{\mathbf{M}}\ddot{\mathbf{q}} + \mathbf{L}^{\mathrm{T}}\mathbf{f}_{int}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{L}^{\mathrm{T}}\mathbf{f}_{ext}(t)$$

Model order reduction with material nonlinearities

Galerkin vs Petrov-Galerkin [1]

Direct or indirect reduction

Proper Orthogonal Decomposition vs Eigenvalue Analysis [2]

[1] Carlberg, Kevin, Charbel Bou-Mosleh, and Charbel Farhat. "Efficient non-linear model reduction via a least-squares Petrov-Galerkin projection and compressive tensor approximations." *International Journal for Numerical Methods in Engineering* 86.2 (2011): 155-181.
[2] Lülf, Fritz Adrian, Duc-Minh Tran, and Roger Ohayon. "Reduced bases for nonlinear structural dynamic systems: A comparative study." *Journal of Sound and Vibration* 332.15 (2013): 3897-3921.

Common Constitutive Models

Literature is replete with examples of constitutive models

- Posed for different phenomena/combinations of phenomena
- Often multiple models exist for same phenomena

(Some) common classes of constitutive models

- Elastic
- Viscoelastic
- Plastic
- Hyperelastic

Each class may have multiple variants/types

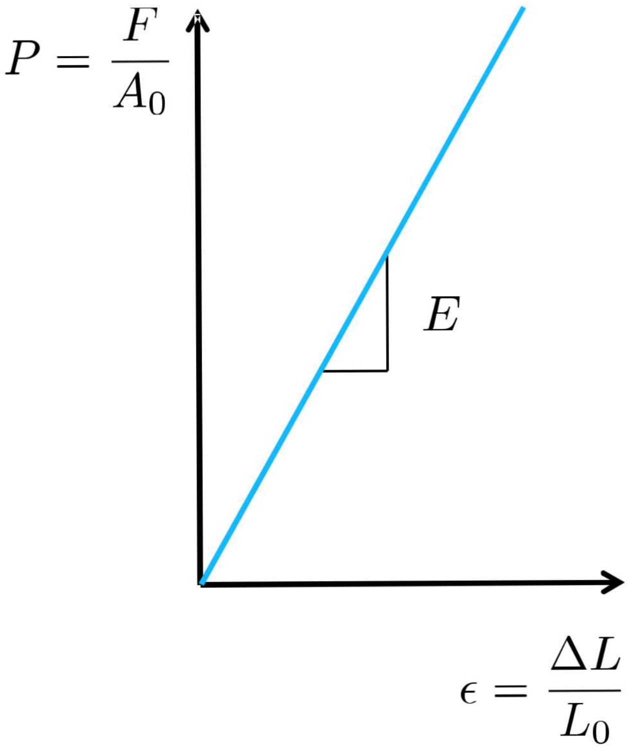
- Finite vs. infinitesimal deformations
- Isotropic vs anisotropic
- Etc…

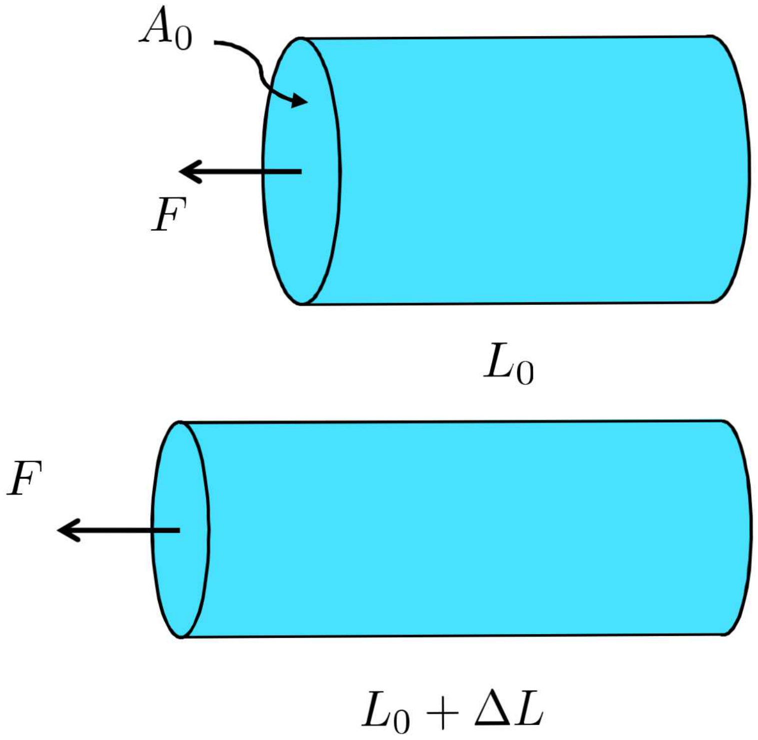
Elastic

$$\sigma_{ij} = \mathbb{C}_{ijkl}\varepsilon_{kl}$$

$$\mathbb{C}_{ijkl} = \underbrace{K\delta_{ik}\delta_{jl}}_{\text{Bulk}} + \underbrace{G\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right)}_{\text{Shear}}$$

In 1D, axial *stress* is proportional to axial *strain*



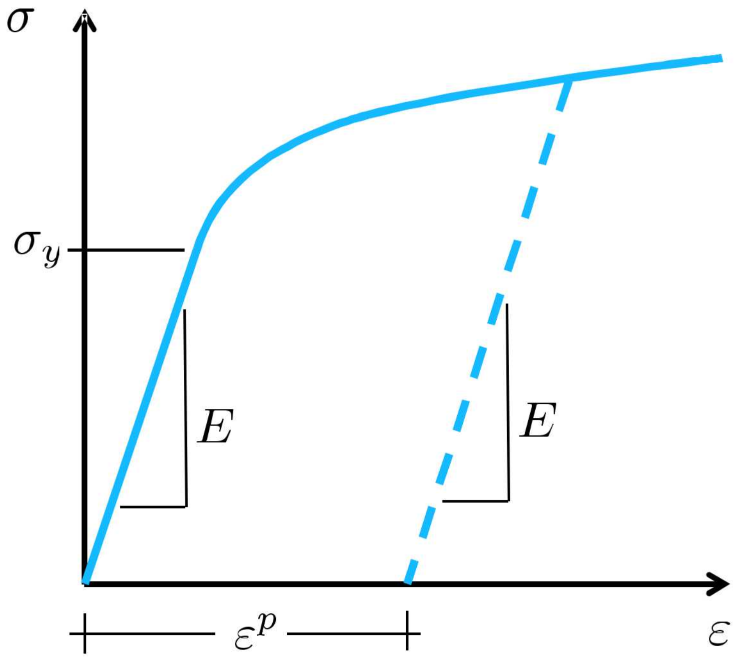


Elastic-Plastic

Permanent deformations arising from dislocation motion (slip)

Common for structural metals such as steel, aluminum and titanium

One of most popular/common class of material models



- Material yields at yield stress, σ_y
- Material unloads elastically before AND after yield
- Yielding produces permanent deformation after unload

Viscoelastic

Viscoelastic materials exhibit history dependence

- Stress/strain functions of *time*
- Linear viscoelasticity: stress proportional to strain rate

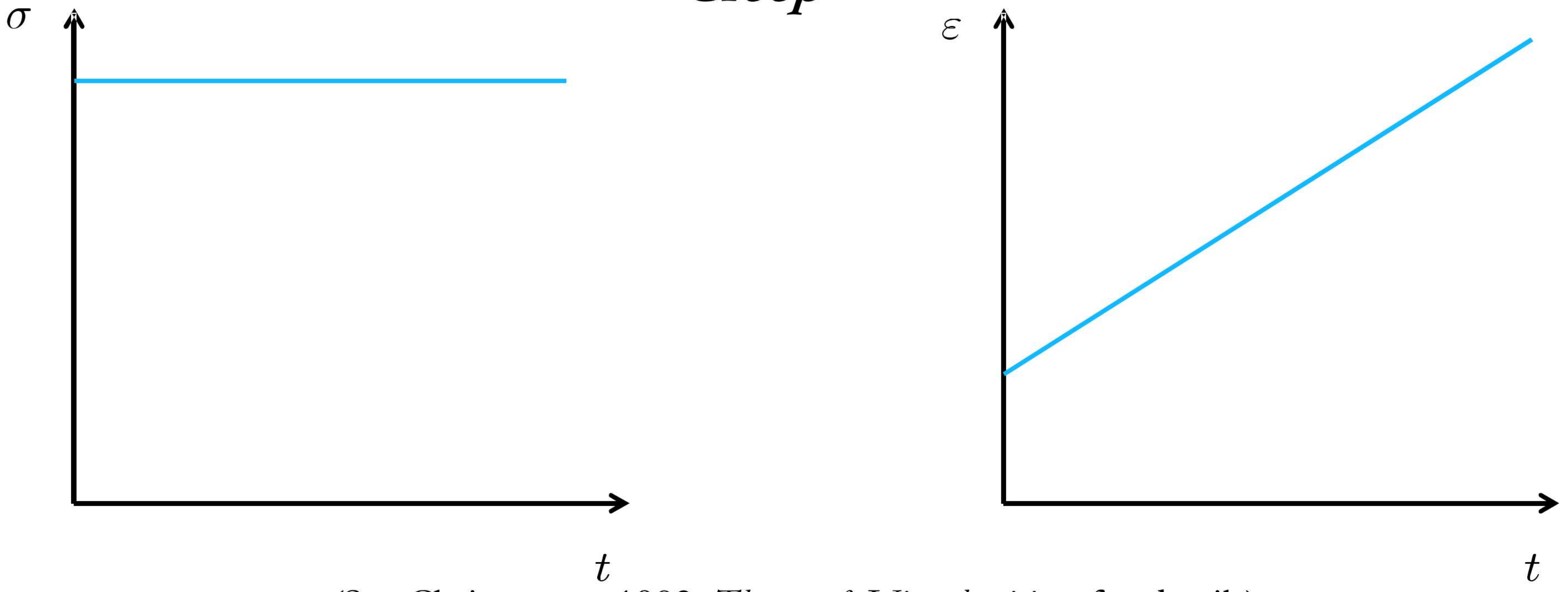
$$\sigma_{ij}(t) = \int_0^t 3K(t-\tau)\left(\frac{1}{3}\delta_{ij}\frac{d\varepsilon_{kk}}{d\tau}\right)d\tau + \int_0^t 2G(t-\tau)\left(\frac{d\varepsilon_{ij}}{d\tau} - \frac{1}{3}\delta_{ij}\frac{d\varepsilon_{kk}}{d\tau}\right)d\tau$$

Bulk Shear

Common for amorphous materials (e.g. polymers, glasses)

Two common characteristic responses

Creep



(See Christensen, 1982, *Theory of Viscoelasticity* , for details)

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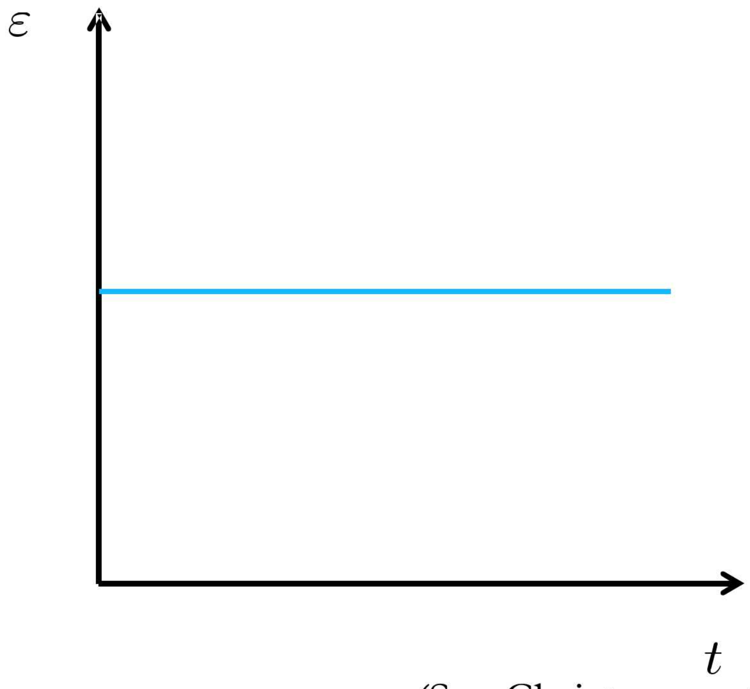
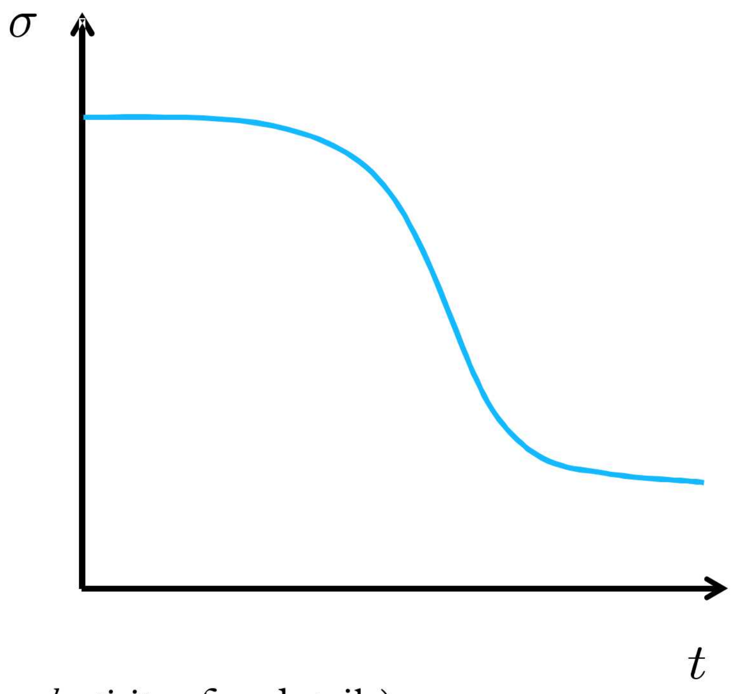
Stress Relaxation



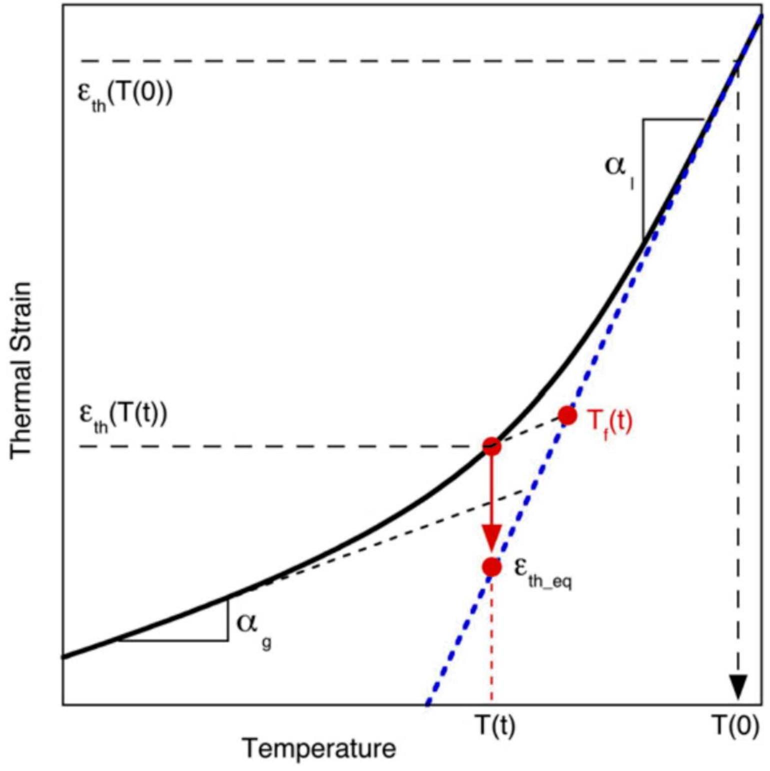


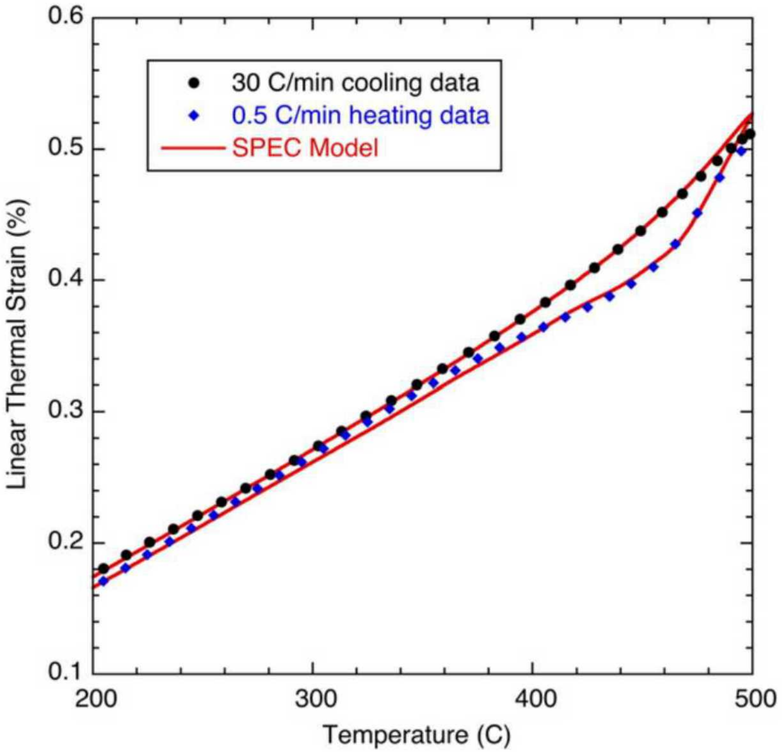
(See Christensen, 1982, *Theory of Viscoelasticity*, for details)

Nonlinear Viscoelastic

Nonlinear theories of viscoelasticity exist in literature

- Capture complex, large-deformation behavior
- Material changes under deformation
- E.g., "Potential Energy Clock" Model of Caruthers, Adolf, Chambers, and Shrikhande, 2004, *Polymer*, **45**, pp. 4577-4597





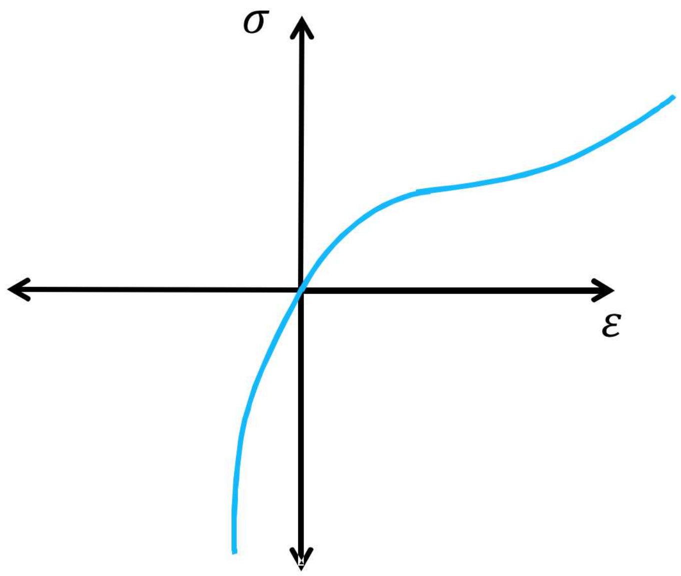
(Figures from Chambers *et al.*, 2016, *Jrnl. Non-Crystl. Solids*, **432**, pp. 545-555)

Hyperelastic

Nonlinear elastic behavior over large deformations

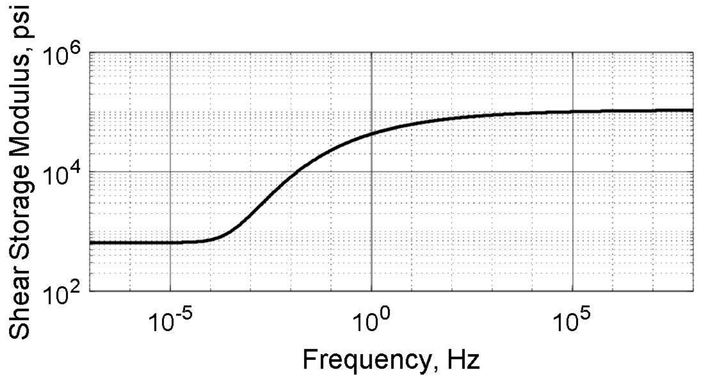
Commonly used to model elastomers, biological tissues, or polymers

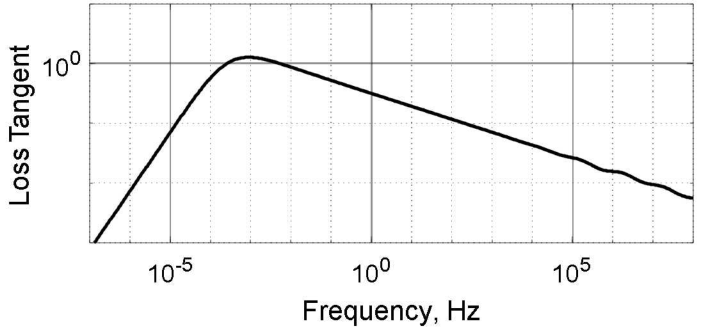
Typically rate independent and fully recoverable deformations



- Saint Venant-Kirchhoff Model
- Mooney-Rivlin
- Ogden
- Neo-Hookean
- Etc..

Example Applications





Damping layers for composite plates in aerospace applications

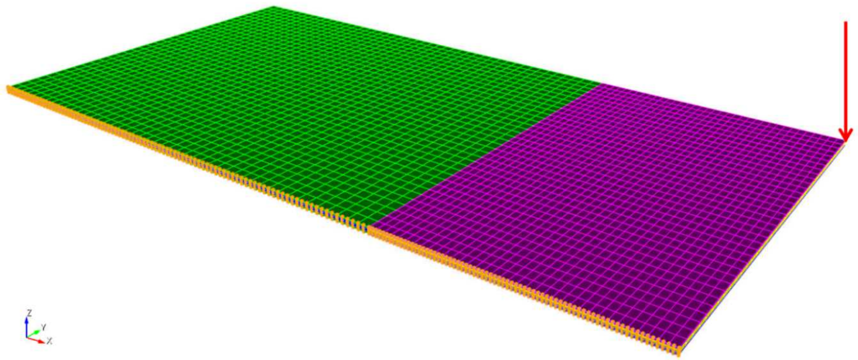
- Linear viscoelasticity: Prony Series
- E.g. low density PMDI foam, cellular silicone

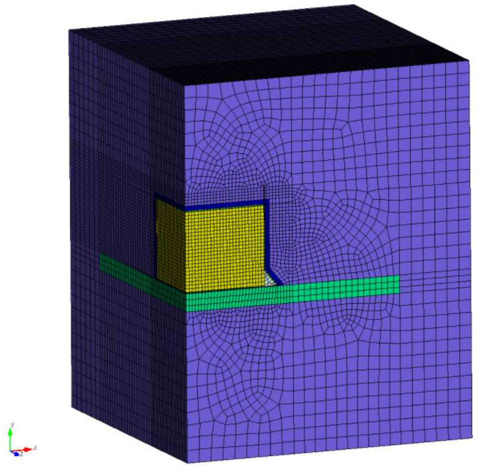
Linear constitutive law, linear system of equations, but nonlinear eigenvalue problem!

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}_K \int_0^t \zeta_K(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}_G \int_0^t \zeta_G(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}_\infty \mathbf{x} = \mathbf{f}(t)$$

$$\left(\lambda_r^2\mathbf{M} + \lambda_r\mathbf{C} + \lambda_r\mathbf{K}_K \sum_{i=1}^{N_K} \frac{\hat{K}_i}{\lambda_r + 1/\tau_{K,i}} + \lambda_r\mathbf{K}_G \sum_{i=1}^{N_G} \frac{\hat{G}_i}{\lambda_r + 1/\tau_{G,i}} + \mathbf{K}_\infty\right)\boldsymbol{\phi}_r^* = 0$$

Governing equations-of-motion reduced using Galerkin approach with a variety of modal bases [1, 2].





[1] L. Rouleau, J.-F. Deü, and A. Legay, "A comparison of model reduction techniques based on modal projection for structures with frequency-dependent damping," *Mechanical Systems and Signal Processing*, vol. 90, pp. 110-125, 2017.
[2] Kuether, R.J., "Two-tier Model Reduction of Viscoelastically Damped Finite Element Models", *Computers & Structures*, (in peer review).

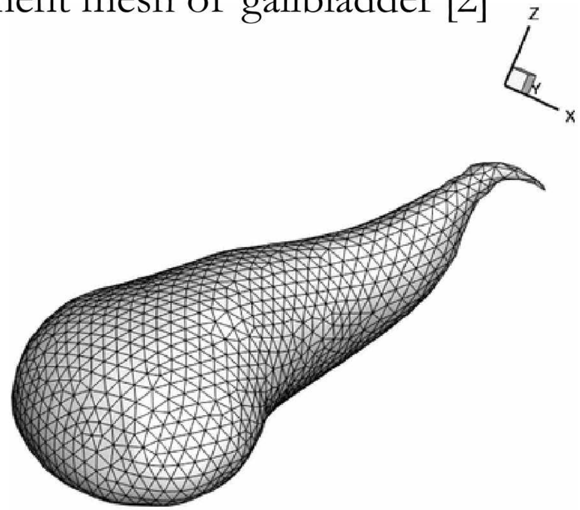
Example Applications

Real time simulations for computational surgery [1] in medical fields
- Provides practitioner with a safe, fast method to develop skills without making costly errors
- Nonlinear physics including hyperelastic materials, frictional contact and cutting

Offline costs irrelevant and can be sacrificed to improve online costs
- Explicit schemes benefit by increasing stable time step

Proper orthogonal decomposition (POD) and variants of this have been used in the literature

Proper generalized decomposition (PGD) is offline computation of general solutions and evaluated in real-time
- *Computational vademecum* that provides solution for any possible situation



(Top) Surgical simulator, (bottom) finite element mesh of gallbladder [2]

[1] Cueto, Elías, and Francisco Chinesta. "Real time simulation for computational surgery: a review." *Advanced Modeling and Simulation in Engineering Sciences* 1.1 (2014): 11.
[2] Mena, Andrés, et al. "Towards a pancreatic surgery simulator based on model order reduction." *Advanced Modeling and Simulation in Engineering Sciences* 2.1 (2015): 31.

Seed questions to provoke discussion

What challenges are faced with developing ROMs of models with nonlinear materials?

What are the application areas that can benefit from research in this area?

Other topics for discussion?

Any questions?

Contact information

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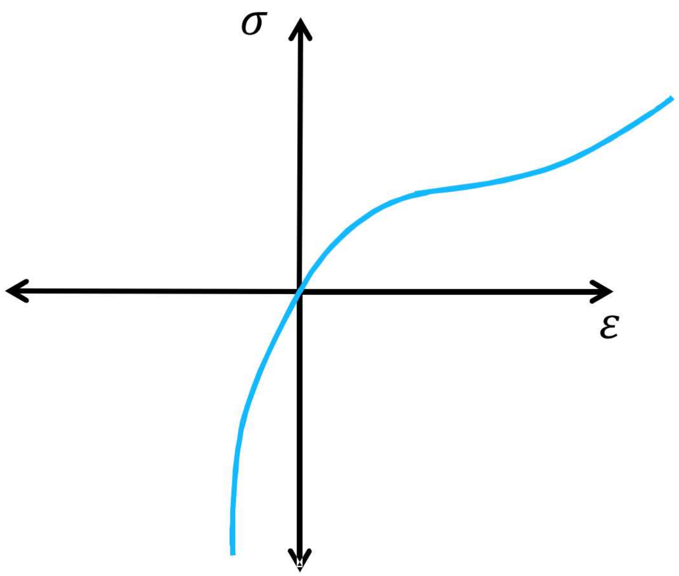

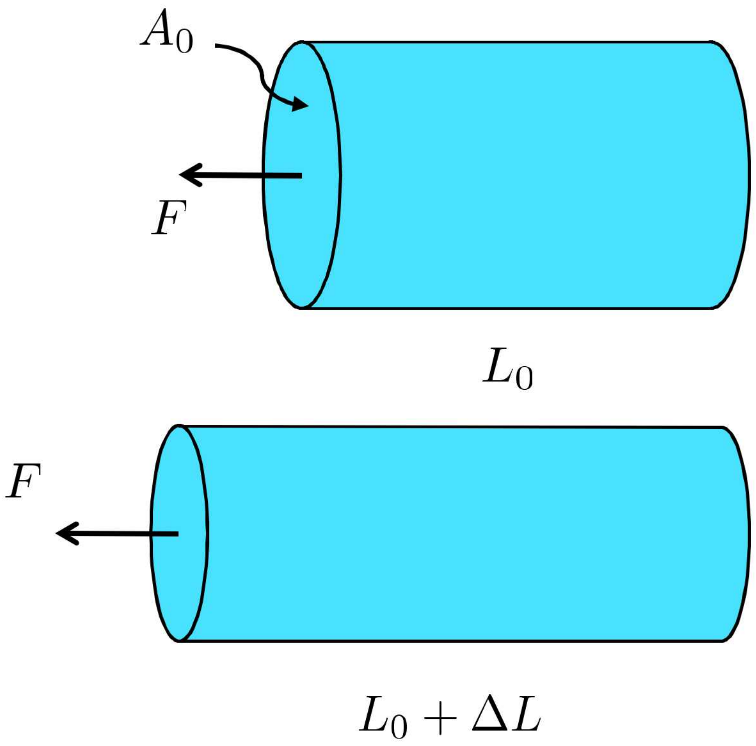

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