Game-Theoretic Approach for Electricity Pricing between Distribution System Operator and Load Aggregators

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***Abstract*—Transactive energy has emerged and gained more and more attention in recent years. Electricity pricing strategies play a critical role in influencing and shaping the customers' energy load profile. In this paper, we investigate the electricity pricing strategy between a distribution system operator (DSO) and load aggregators (LAs) by adopting a bi-level Stackelberg game approach. With the purpose of maximizing its own operating revenue as well as better serving its customers, the upper level DSO will determine electricity prices while considering how lower level LAs will respond to it. Peak-to-average ratio of the total demand could be limited by a preferred value, and peak load is penalized at the upper optimization level. Numerical results demonstrate the effectiveness of the proposed game approach in leveraging flexible demand potential to benefit both DSO and LAs.**

Keywords-smart grid; transactive energy; demand response; bi-level optimization; Stackelberg game; Nash equilibrium; distribution system operator; load aggregator

I. INTRODUCTION

Demand response (DR) is expected to bring significant economic value and environmental benefits to the future smart grid by effectively adjusting energy usage patterns of users from the demand side [1]. Electricity price signals can be an efficient market tool for several DR services, e.g. critical peak pricing, time-of-use pricing, and real-time pricing [2]. Such different pricing strategies have been introduced to better leverage response potential in load shifting and peak reduction for macro power grid, or to improve distributed renewable energy integration by energy trading in the local energy transaction market.

Game theoretic based approaches have been studied extensively to generate such pricing strategies. For example, the interaction between the electricity retailer and household customers has been modeled in [3] as one-leader, *N*-follower Stackelberg game, where the electricity retailer determines the retail price, and customers schedule their appliances in households to minimize electricity bills. A game-theoretic energy schedule method has also been proposed in [4] for the two-step game between power companies and its consumers with the objective of reducing peak-to-average ratio (PAR). Repeatedly, the power company pulls consumers in a round-robin fashion and provides them energy prices and current total consumption; each user then optimizes its own schedule and updates it to the supplier. In [5], a time-of-use (TOU) pricing between utility companies and customers is optimized by a game-theoretic approach, and a backward induction method is used to obtain Nash equilibrium. For the utility company, energy electricity delivery cost, user demand fluctuation cost, and overall user satisfaction cost are considered while customers tend to minimize their own electricity bill and satisfaction cost. Increased social welfare has been shown in multiple user types (residential, commercial, industrial) with different price elasticity.

The interactions between a utility company and multiple customers are formulated as one-leader, *N*-follower Stackelberg game in [6], which is aimed at balancing supply and demand as well as smoothing the aggregated load. An iterative demand response algorithm is proposed to derive Stackelberg equilibrium of power generation and power demand. In [7], an analytic model of a multi-leader and multi-follower Stackelberg game approach is developed for an open energy market with the objectives of optimizing total energy cost and reducing carbon emissions. To solve the resultant bi-level multi-objective optimization problem efficiently, a bi-level hybrid multi-objective evolutionary algorithm is proposed. In [8], an estimation method is proposed for the utility company to infer the energy requirement of aggregators based on an inverse optimization technique for the purpose of privacy protection.

Besides the game between utility companies and users, the local energy transactions among prosumers have also been studied. For instance, an *M*-leader and *N*-follower Stackelberg game approach is used in [9] to model the interaction in peer-to-peer energy trading among prosumers in a community. With multiple leaders and followers, there exist two separate competitions during the trading process: price competition among sellers which is modeled as a non-cooperative game, and seller selection competition among buyers which is solved by an evolutionary game algorithm. To facilitate energy sharing of multiple photovoltaic (PV) prosumers, the energy sharing provider (ESP) is assumed to be equipped with an energy storage in [10]. Via stochastic programming, day-ahead scheduling model of the ESP is built to improve the net power profile of an energy-sharing network with uncertainties of PV energy, electricity prices, and prosumers' load.

In [11], an improved game-theoretic demand side management framework is proposed for a neighborhood area to provide cost savings for consumers and to reduce the PAR for the neighborhood. A novel real-time price tariff model is established, and a Nash-game-theoretic-based optimization model is developed to minimize consumers' cost while maintaining an optimal comfort level and satisfying peak reduction constraints. Instead of a conventional non-cooperative game based approach, energy trading among demand aggregators and a distribution company is formulated as a bargaining based cooperative model in [12], where the energy trade amount is collaboratively decided upon. The bargaining-based model could allocate collective benefits fairly among participants and a distributed solution approach is utilized to protect privacy.

In this paper, we address and investigate the pricing game between a distribution system operator (DSO) and several load serving entities/aggregators (LAs), where the DSO broadcasts the price, and LAs send back load adjustments in response to the price. Utility functions are designed for both DSO and LAs, where electricity sales revenue, electricity generation/marginal cost, LAs' overall satisfaction, as well as preferred PAR are taken into account for the DSO, while LAs minimize their own dissatisfaction and bill payment.

The remainder of this paper is structured as follows. Section II presents the bi-level game theoretic optimization model for the transactive control electricity pricing between a DSO and multiple LAs. Simulation results are presented in Section III to demonstrate the performance of the proposed Stackelberg game approach. Finally, Section IV provides the summary and conclusion.

II. GAME MODEL AND BACKWARD INDUCTION

In this section, we will develop the Stackelberg game framework for the bi-level transactive optimization electricity pricing between a DSO and several LAs. In most electricity markets, individual customers (i.e. load) located in a distribution system do not directly participate in the electricity wholesale market to purchase electricity. Instead, a LA, which represents a group of customers, manages the electric energy transactions for customers and delivers electric power to them.

Generally, load at a given time is determined by customers and is inelastic to price except while considering DR. In some DR programs, load might be controlled (curtailed) during some periods of time by system operator or customers might adjust their demand in response to real-time electricity price. The later DR case is considered in this paper. There are several advantages to consider load aggregators instead of individual customers, for example, the dynamics of an aggregated load are slower and therefore more predictable, also, the communication burden at the distribution system will be reduced. The focus in this paper is the pricing problem between DSOs and LAs. The problem of how the committed aggregated demand response can be allocated to each individual customer will be explored in a future paper. The DSO is modeled as a leader while LAs are modeled as followers in a Stackelberg game model. All notations used in this paper are listed in Table I.

The optimization formulation for the DSO (upper level) is given by the following model, where U_0 is the total utility value to be maximized for the DSO.

$$\max_{p_t,\, l_{n,t}} U_0 = \sum_{n,t} p_t \cdot l_{n,t} - \sum_{n,t} C_t \cdot l_{n,t} + \omega \cdot \sum_{n,t} S(l_{n,t}) - \theta \cdot m \quad (1)$$

$$\text{s.t. } C_t \le p_t \le \bar{P},\ \forall t \quad (2)$$

$$m \ge \sum_n l_{n,t}\,, \forall t \quad (3)$$

The objective function (Eq. (1)) has four terms. The first term is the total revenue from sold electricity. The second term is the cost of electricity to the DSO, different cost function could be used, e.g. quadratic function [3], [13], [14]. The third term is the overall satisfaction value coming from LAs (in lower level). The fourth term is the penalty for peak demand. Note that ω and θ are weight factors for customers' satisfaction and peak demand, respectively. The constraint in Eq. (2) gives upper and lower limits to electricity price, while the constraint in Eq. (3) makes sure the peak demand $m = \max_{t\in T} \sum_n l_{n,t}$ is greater than the total load at all times. The peak-to-average ratio could be calculated by Eq. (4).

$$\text{PAR} = \frac{m}{\frac{1}{T}\cdot\sum_{n,t} l_{n,t}} \quad (4)$$

TABLE I. LIST OF PARAMETERS AND VARIABLES.

Parameters	Definition
T	Total hours with index t
N	Total aggregators with index n
$D_{n,t}$	Nominal demand of LA n
$\underline{L}_{n,t}$	Minimum demand limit for LA n
$\bar{L}_{n,t}$	Maximum demand limit for LA n
C_t	Incremental marginal cost of electricity
$\bar{P}$	Maximum possible price reference
$\alpha_{n,t}$	Preference satisfaction coefficient for LA n
Variables	**Definition**
p_t	Unit sale price of electricity
$l_{n,t}$	Actual load for LA n in response to price

The optimization formulation of each aggregator (lower level) is modeled as in Eq. (5) and Eq. (6), where U_n is the total utility value to be maximized for LAs.

$$\max_{l_{n,t}} U_n = \sum_t S(l_{n,t}) - \sum_t p_t \cdot l_{n,t} \tag{5}$$

$$\text{s.t.} \quad \underline{L}_{n,t} \leq l_{n,t} \leq \bar{L}_{n,t}, \ \forall n \tag{6}$$

where the final actual load $l_{n,t}$ will be restricted between minimum and maximum demand limits. $S(l_{n,t}) = \bar{L}_{n,t} \cdot \bar{P} \cdot (1 - e^{-\alpha_{n,t} \cdot (\frac{l_{n,t}}{D_{n,t}})})$ is used to represent monetary value of satisfaction, $\bar{P}$ is a constant price reference as a price upper limit. This exponential utility function $f(x) = 1 - e^{-\alpha \cdot x}$ is a concave increasing function which is commonly adopted in utility theory to model users' preference [4]. Its value trend is illustrated in Fig. 1 with different preference coefficients $\alpha \in [0.5, 4]$. The parameter α could be treated as sensitivity towards energy consumption curtailment, for instance, the utility value of a relatively large value of $\alpha = 4$ reaches about 0.8 when $l_{n,t} = 0.5 \cdot D_{n,t}$, while it is only 0.2 for $\alpha = 1$. $D_{n,t}$ is the nominal demand of LA n, which can be obtained as a reference from historical energy load profiles. Note that we use satisfaction here as a positive gain, *i.e.*, the larger $S(l_{n,t})$ is, the more satisfaction is achieved. While different satisfaction cost representations can also be found in [5], [15], where the less the satisfaction cost is, the more satisfaction is achieved. Both satisfaction representations are acceptable as long as several properties are fulfilled.

A classical backward induction method is used to solve for the equilibrium solution of the bi-level Stackelberg game (Eqs. (1)-(6)) by following the next two steps:

1. Derive optimal demand response to price.

Assume the electricity price is provided as a parameter from DSO, then the best load response $l^*_{n,t}$ can be obtained by the first-order derivative of LAs' objective functions.

$$\frac{dU_n}{dl} = -p_t + \alpha_{n,t} \cdot \bar{P} \cdot e^{-\alpha_{n,t} \cdot \frac{l_{n,t}}{D_{n,t}}} = 0 \tag{7}$$

$$p_t = \alpha_{n,t} \cdot \bar{P} \cdot e^{-\alpha_{n,t} \cdot \frac{l_{n,t}}{D_{n,t}}} \tag{8}$$

$$l^*_{n,h} = \frac{D_{n,t}}{\alpha_{n,t}} \cdot \ln \frac{\alpha_{n,t} \cdot \bar{P}}{p_t} \tag{9}$$

2. Derive optimal price based on user response.

After the optimal load is obtained, it can then be plugged into the optimization model of the DSO. The pricing problem then becomes the following model (Eqs. (10)-(14)). Since Eq. (8) is a decreasing function of $l_{n,t}$, the constraints of Eq. (6) can be written as in constraint Eq. (12).

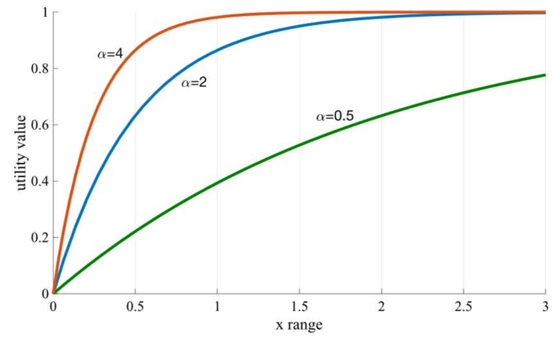

Figure 1. Utility value for different user preferences.

$$\max_{p_t, l_{n,t}} U_0 = \sum_{n,t} p_t \cdot l^*_{n,t} - \sum_{n,t} C_t \cdot l^*_{n,t} + \omega \cdot \sum_{n,t} S(l^*_{n,t}) - \theta \cdot m \tag{10}$$

$$\text{s.t. } C_t \leq p_t \leq \bar{P}, \forall t \tag{11}$$

$$\max_{n \in N} (\alpha_{n,t} \cdot \bar{P} \cdot e^{-\alpha_{n,t} \cdot \frac{\bar{L}_{n,t}}{D_{n,t}}}) \leq p_t \leq \max_{n \in N} (\alpha_{n,t} \cdot \bar{P} \cdot e^{-\alpha_{n,t} \cdot \frac{\underline{L}_{n,t}}{D_{n,t}}}), \forall t \tag{12}$$

$$\text{PAR} = \frac{m}{\frac{1}{T} \sum_{n,t} l^*_{n,t}} \tag{13}$$

$$m \geq \sum_n l^*_{n,t}, \forall t \tag{14}$$

where $l^*_{n,t}$ is a function of price (Eq. (9)); this nonlinear model has only variable p_h, and it can be solved by any general nonlinear solution method. In addition, if fixed price structure is adopted, an additional constraint $p_{t-1} = p_t, t \geq 2$ should be added to the above model.

III. Numerical Experiment

A. Data Setting

In this experiment, a day-ahead hourly optimization is considered with three aggregators (n1, n2, n3). The nominal demand $D_{n,t}$, $\alpha_{n,t}$ in the satisfaction function, and marginal cost C_t are plotted in Figs. 2-4, respectively. And the parameter $\bar{P}$ is assumed to be constant equals to 30.

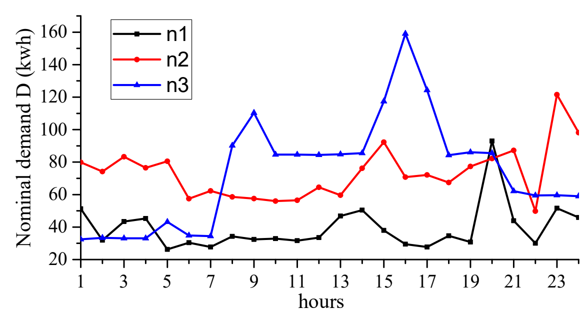

Figure 2. Nominal demand of aggregators.

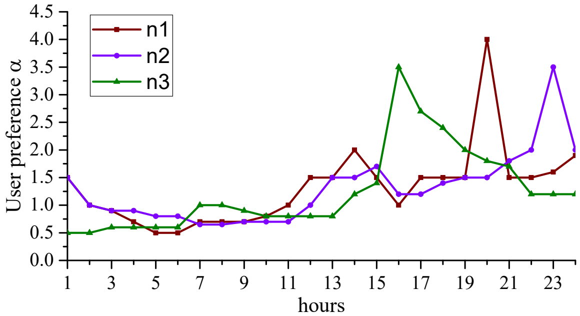

Figure 3. User preference in satisfaction.

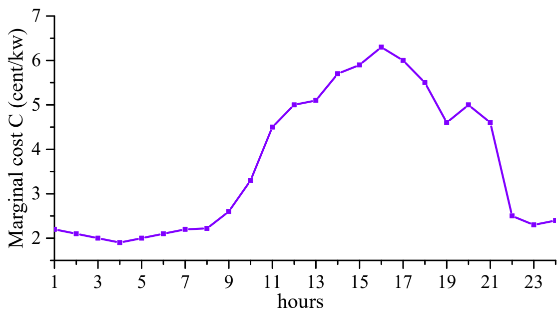

Figure 4. Incremental marginal cost of electricity.

B. Experimental Results

The developed model is solved by a non-commercial nonlinear solver called Solving Constraint Integer Programs (SCIP) [16]. Several groups of experiments are conducted for different price structures (fixed price and TOU price) and weight combinations, relative gap of 0.01 is set for all experiments. Here, we assume all aggregators have the same price determined by the DSO.

Two sets of prices as well as different weight preferences on customers' satisfaction cost and peak load cost are tested, see Table II. All results of prices and demand responses are shown in Figs. 5-10. In Fig. 5, the resulted fixed price slightly increases from structure F1 to F2 as a larger penalty is put on aggregated peak load, and then drops to F3 due to a higher weight on user's satisfaction value. It can also be observed that under TOU pricing in Fig. 6, the price follows the same pattern as in Fig. 5, P3 is the lowest with a higher weight on overall satisfaction. It is noted that P1 overlaps with P3 before time point 11 and then overlaps with P2 for the rest of time. When $\theta = 15$ in TOU price P2, the resulted peak demand for each aggregator has been shaved or shifted in Figs. 7-9, and for the aggregated demand in Fig. 10. On contrast, when $\omega = 5$ in TOU price P3 with minimum penalty on peak demand and maximum weight on satisfaction value, the demand of each aggregator tends to increase following the nominal demand patterns, which results in higher energy consumption. The detailed revenue and cost data for the different pricing schemes are compared and summarized in Table III.

TABLE II. DIFFERENT PRICE STRUCTURE AND WEIGHT PREFERENCES

F1: Fixed price, $\omega = 0.1, \theta = 0.1$	**P1**: TOU price, $\omega = 0.1, \theta = 0.1$
F2: Fixed price, $\omega = 0.1, \theta = 15$	**P2**: TOU price, $\omega = 0.1, \theta = 15$
F3: Fixed price, $\omega = 5, \theta = 0.1$	**P3**: TOU price, $\omega = 5, \theta = 0.1$

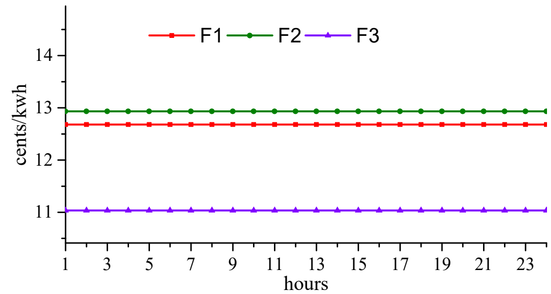

Figure 5. Fixed prices for different preferences.

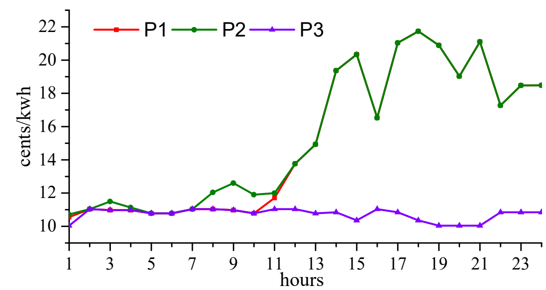

Figure 6. TOU prices for different preferences.

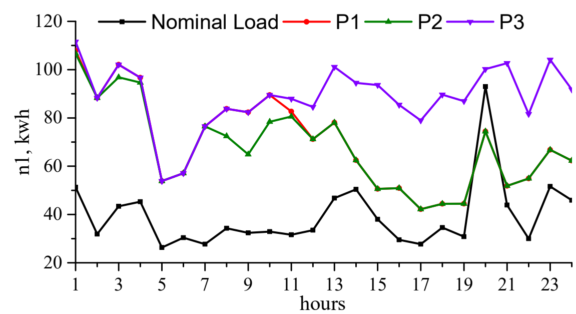

Figure 7. Demand response of n1 under TOU price.

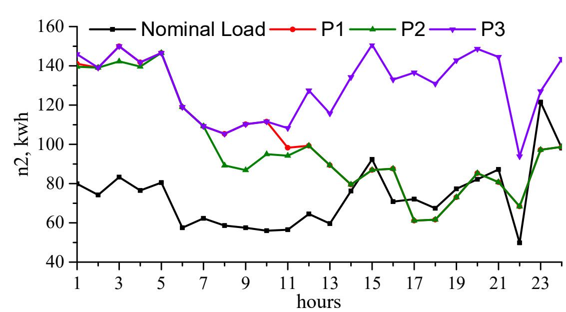

Figure 8. Demand response of n2 under TOU price.

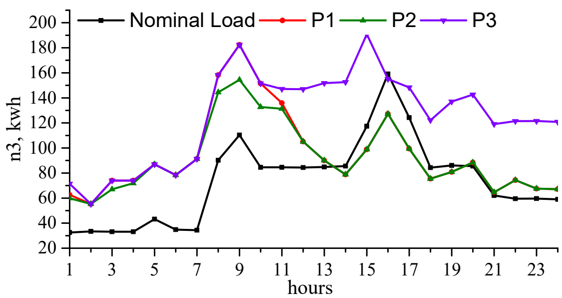

Figure 9. Demand response of n3 under TOU price.

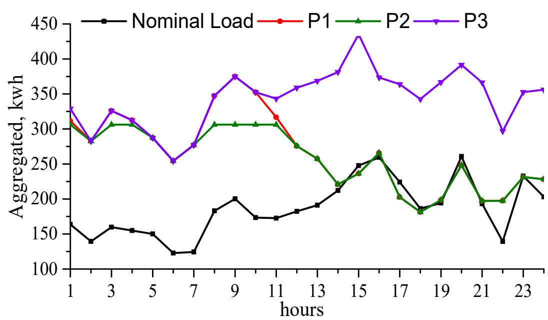

Figure 10. Total demand under TOU price.

TABLE III. PROFIT AND COST IN SUMMARY (PRICE: CENT, COST/REVENUE/SATISFACTION: $).

DSO Level	**F1**	**F2**	**F3**	**P1**	**P2**	**P3**
Peak Load	375.61	369.82	416.53	375.00	306.16	435.25
PAR Value	1.30	1.32	1.24	1.41	1.18	1.26
Total Satisfaction	2208.69	2187.79	2332.77	2165.92	2143.1	2347.69
DSO Revenue	873.40	869.63	887.39	920.35	913.46	882.58
DSO Gen. Cost	265.92	260.36	305.22	224.46	219.11	313.08
DSO Profit	607.48	609.27	582.17	695.89	694.35	569.49
Aggregator Level	**F1**	**F2**	**F3**	**P1**	**P2**	**P3**
n1 Average Price	12.68	12.93	11.03	14.29	14.62	10.70
n2 Average Price	12.68	12.93	11.03	14.23	14.54	10.70
n3 Average Price	12.68	12.93	11.03	14.69	15.11	10.72
n1 Bill Payment	225.41	224.45	228.96	239.49	237.66	227.44
n2 Bill Payment	333.86	332.95	336.05	347.56	344.53	333.45
n3 Bill Payment	314.11	312.22	322.37	333.29	331.26	321.68
n1 Satisfaction	607.44	602.43	636.16	608.78	603.66	639.10
n2 Satisfaction	816.57	809.65	858.53	792.55	783.67	863.22
n3 Satisfaction	784.68	775.71	838.08	764.59	755.77	845.37

It is observed in Table III that for a fixed price structure, PAR value and average price ($\frac{\sum_t p_t \cdot l_{n,t}^*}{\sum_t l_{n,t}^*}$) for aggregators are decreasing/increasing at the same time since the load for all hours behaves similarly due to the fixed price. In other words, the fixed price structure does not support load shifting ability. In contrast, the flexibility of TOU price structure can shave/shift the peak load and reshape the load profile, for instance, P2 has the lowest aggregated peak load of 306.16 kW and PAR of 1.18, meantime it has the highest average prices for aggregators as indicated in Fig. 6 and Table III. Comparing F1 to P1 and F2 to P2, when the weight on satisfaction is low, the profit of DSO increases while all customers pay more, and their overall satisfaction decreases. On the other hand, when satisfaction is over weighted, then customers tend to increase their demand following their nominal patterns, for example, comparing F3 to P3, DSO makes less profit and customers have the lowest average price, more consumption, and highest satisfaction value. Therefore, the tradeoff between DSO and aggregators should be balanced in actual implementation of DR.

IV. CONCLUSION

In this paper, a Stackelberg game is adopted to model the pricing game between a DSO and LAs, where different utility functions are designed for the leader and followers. In the upper level, total profit, social obligation, and PAR are optimized for the DSO, while LAs minimize their dissatisfaction and bill payment at the lower level. With different weighting combinations and different pricing structures, optimal prices and load responses are obtained. In addition, the tradeoff between DSO and LAs are analyzed. In the future, the load commitment of aggregators will be addressed by allocating the allotted energy among different buildings. In addition, more practical conditions such as larger scale analysis and distributed algorithms will be developed.

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