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"A selection rule for flyer plate, shock sensitivity, and margin of performance"

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Abstract. A reexamination was made of optimizing detonation initiation by impacting thin flyer plates through shock-impedance selection. In this study threshold functions of the extended James type were used to gain new insights into the optimization of detonation initiation. One outcome of interest is the emergence of non-dimensional groups associated with the threshold functions. These groups served as an inspiration to introduce a non-dimensional loading space and define a "distance" to develop a new measure of shock sensitivity as well as a performance margin. The new framework is reviewed using experimental results for pressed HMX powders.

Introduction

Shock initiation of high explosives has been studied intensely for many decades. One of the well-controlled studies is to employ shocks produced by high-velocity impact of a thin flyer plate. The purpose of this paper is to revisit Hayes' idea[1,2] of flyer plate design, using contemporary initiation threshold functions that are said to work better than the one used by Hayes for a certain group of explosives[3]. One interesting outcome of this study is the emergence of self-consistent dimensionless groups that are suggestive of re-examining the threshold functions in a non-dimensional space. Such a space enables one to introduce a distance to define a measure of shock sensitivity that unifies the parameters associated with these threshold functions.

Flyer optimization

Hayes' idea was concerned with the question of how to maximize the "insult" delivered to the target explosive through selection of an "optimum" impedance of the flyer, and a case study was made using the modified Walker-Wasley criterion[4]: $P^n\tau = \text{constant}$ where P is shock pressure, τ shock duration, and n is a constant. The term optimum is taken to mean that the maximum possible value of "$P^n\tau$" is delivered into the target explosive. He showed this could be done by maximizing the quantity G given in eqn. (1) below.

$$G = P^n\tau = 2Z_e^{n-1}mV^2\left[\frac{\phi^{n-1}}{((1+\phi)^n)}\right] \quad (1)$$

where Z_e is the shock impedance of target explosive, "n" is the exponent of P in the Walker-Wasley criterion, m areal mass of the flyer, V flyer velocity, $\phi = Z_f/Z_e$, and Z_f shock impedance of the flyer material. Shock impedance of a material is defined by $\rho_{(.)}C_{(.)}$ where $\rho_{(.)}$ is initial density and $C_{(.)}$ is shock speed of the material. Since Z_e, m, and V are not affected by shock properties of the flyer material, he was able to show that there is an

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optimum selection of ϕ to maximize G and the value is given by

$$\phi_{max} = n - 1 \quad (2)$$

If n is two as in the case of Walker-Wasley criterion, Hayes showed further that this selection is equivalent to maximizing the transmitted kinetic energy.

Application to new threshold functions

The purpose of the present investigation is to extend Hayes' idea to the following criteria that are said to describe experimental data better for a certain group of explosives[3,5].

$$\frac{1}{J} = \frac{E_c}{E} + \frac{\Sigma_c}{\Sigma} \qquad \text{(James}^{(3)}\text{)} \quad (3)$$

$$\frac{1}{J_W} = \frac{E_c}{E} + \frac{\pi_c}{\pi} \qquad \text{(Welle}^{(5)}\text{)} \quad (4)$$

where $E = Pu\tau$, $\Sigma = \frac{1}{2}u^2$, and $\pi = Pu$. E_c, Σ_c, and π_c, are constant. P is shock pressure, u is particle velocity in the target explosive, and τ is shock duration. Here, τ is assumed to be twice of the shock traverse time over the distance of flyer thickness. Variables J and J_w are equivalent to G in Hayes' analysis[7].

Fig. 1 illustrates the use of Welle function to fit experimental threshold data on Class 3 and Class 5 HMX pressed powders[5,7]. These data will be used to review the new ideas proposed in this paper.

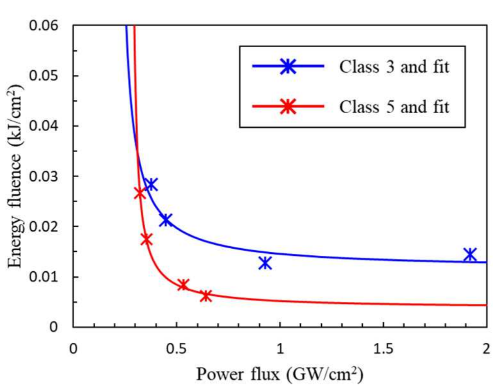

Fig. 1. Ignition thresholds for Class 3 and Class 5 HMX pressed powders (redrawn from Ref. 7).

Following Hayes' idea, we consider the variable "J or J_w" as a combined measure of insults that is comparable to the parameter "G" in his analysis. Then substituting the shock variables in eqns. (3) and (4), we obtain

$$J = \left(\frac{V^2}{2\Sigma_c}\right)\frac{\phi^2}{(1+\phi)^2(1+\alpha\phi)} \ . \quad (5)$$

$$J_w = \left(\frac{V^2 Z_e}{\pi_c}\right)\frac{\phi^2}{(1+\phi)^2(1+\beta\phi)} \ . \quad (6)$$

where $\alpha = \frac{E_c}{4m\Sigma_c}$ for the James type, and for the Welle type $\beta = \frac{Z_e E_c}{2m\pi_c}$. We note that the variable J was originally introduced to capture two types of uncertainties: experimental uncertainties such as variability in flyer performance and material related uncertainties such as inherently stochastic microstructure attributs[6, 7]. Then extending the same meaning to J_w, both of them represent a random threshold variable and can be used to form a probabilistic basis to relate the optimum design to reliability of the prediction. In this view point, J=1 or J_w=1 is said to signify 50% initiation probability.

As done in Hayes' analysis, it is straightforward to show, by evaluating the derivative of eqns. (5) and (6) with respect to ϕ and assuming that all the other variables are fixed, that the optimum value of ϕ for the James and the Welle criteria is given by the same function ϕ_{max} given below.

$$\phi_{max} = \frac{1}{2}\left[1 + \left(1 + \frac{8}{a}\right)^{1/2}\right] \quad (7)$$

where $a = \frac{E_c}{4m\Sigma_c}$ for the James type, $a = \frac{Z_e}{2m}\frac{E_c}{\pi_c}(=\beta)$ for the Welle type, $m = \rho_f l_f$ is the mass of the flyer per unit area, $\tau = \frac{2l_f}{C_f}$, and C_f is the shock speed in the flyer. we note that the parameter "a" in eqn. (7) is α or β in eqn. (5) and (6) respectively.

Test calculations of ϕ_{max} and J_w at $\phi=\phi_{max}$ are shown in Figs 2 and 3. Experimental and materials parameters used are those of eight discrete points shown in Fig. 1. Select parameters are listed in Table 1. Details are referred to the original papers[5,7].

Fig. 2 shows ϕ_{max} as a function of flyer velocity that controls the load intensity. Φ-values correspond to four experimental points obtained with either a polyimide or a parylene flyer. These four ϕ_{max} points for each class were not connected by curves to see trends, because parameter "a" (=β for the Welle type) depends on parameter "m" that represents areal mass of the flyer (= $\rho_f l_f$) and is varied, depending on the flyer thickness used[5]. Thus in contrast to the Hayes case, the optimization of loading as expressed in J_w cannot be determined as a function of flyer velocity alone unless m is fixed. However, it may be seen that as V increases, ϕ_{max} asymptotically approaches the value of 1 on an individual basis. This "trend" indicates that at high impact velocities, kinetic energy becomes the dominant component of loading. That is, repeating an argument similar to the optimization of ϕ, we can easily show that the transmitted kinetic energy is maximized when ϕ = 1.

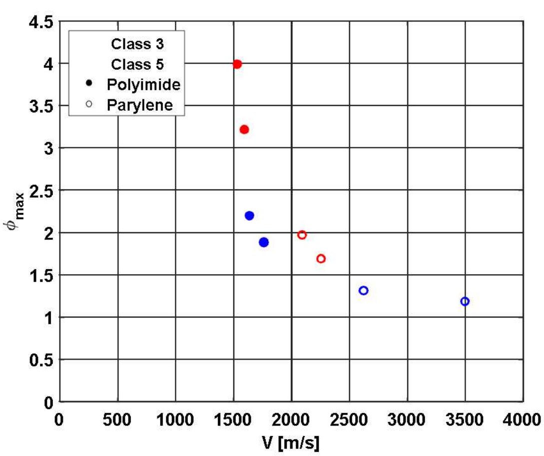

Fig. 2. ϕ_{max} is calculated as a function of flyer velocity for four experimental points in refs. 6 and 7 where polyimide and parylene flyer each impact Class 3 and Class5 HMX pressed powders.

Additionally we note that eqn. (4) implies that there exists a minimum velocity V, because $\pi \geq \pi_c$. At the critical point,

$$\rho_e(C_e + s_e u)u^2 = \pi_c \, . \tag{8}$$

And flyer velocity V is related to u through the equality of pressures across the impact interface. That is,

$$\rho_e(C_e + s_e u)u = \rho_f\big(C_f + s_f(V-u)\big)(V-u) \, . \tag{9}$$

Rearranging terms, we obtain

$$V^2 + \left(\frac{C_f}{s_f} - 2u\right)V + \left(1 - \frac{\rho_e s_e}{\rho_f s_f}\right)u^2 - \left(\frac{C_f}{s_f} + \frac{\rho_e C_e}{\rho_f s_f}\right)u = 0. \tag{10}$$

Substituting the material parameters listed on Table 1, we find that for the test samples of Class 3 and Class 5 HMX, the critical flyer velocities in km/s are (1.261, 1.403) and (1.434, 1.593), depending on flyers of polyimide and parylene, respectively. These values show that connecting data for different flyer materials may result in creating incorrect picture of limiting trends. Unfortunately, there is not enough data available to assess the errors quantitatively. Plus, the threshold parameters E_c and π_c in Table 1 are also obtained by combining the data with different flyer materials.

Fig. 3 shows J_w as a function of flyer velocity and ϕ at ϕ_{max}. Again, the calculated points were not connected for the reason explained on Fig. 2. However, with one exception, as the flyer velocity increases, J_w appears to decrease toward the value of one, signifying 50 % ignition probability. But this may not be a trend as explained for the data on Fig. 2. One data point for Class 5 with parelene flyer at V of 3,500 m/s may appear anomalous, but again we cannot connect these data points to infer it as an anomaly for the same reason as that for decreasing J_w as a function of V until we have additional date based on fixed m.

A real significance of results shown in Fig. 3 is not self-evident without a full probabilistic information on ignition boundary as a function of J or J_w. Unfortunately such information is not always available due to the cost of producing the full statistics. But in the present case this information is available and shown in Fig. 4[7]. We may now clearly see the benefit of adjusting the value of ϕ to its optimum value, by mapping J_w onto Fig. 4 and interpreting its implication in terms of cumulative ignition probability. That is, 100%

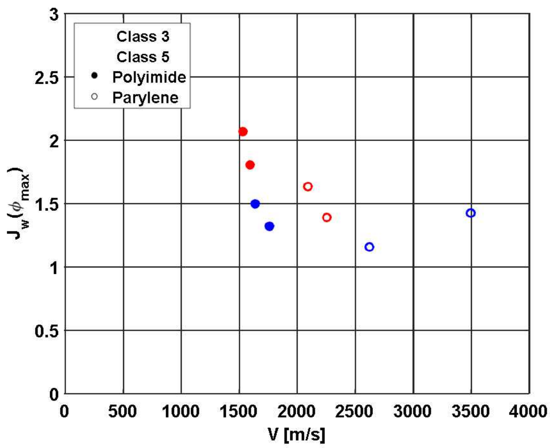

Fig. 3. James number at $\phi = \phi_{max}$ for the eight experimental points shown in Fig. 1.

ignition can be reliably obtained by a relatively minor adjustment of the flyer impedance. This approach may be an appealing alternative to increasing flyer velocity to achieve the same level of ignition probability.

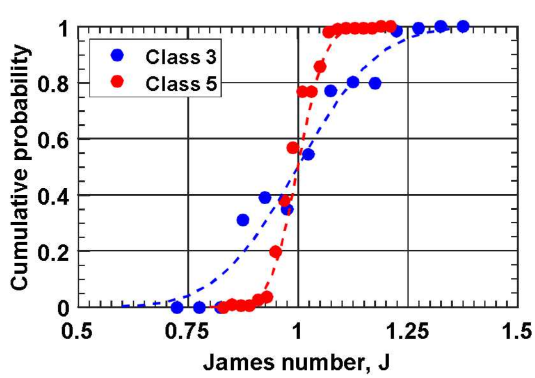

Fig. 4. Cumulative ignition probability of Class 3 and Class 5, pressed HMX powder. Broken lines are truncated Gauss error functions. Standard deviations for class 3 and class 5 are 0.143 and 0.048, respectively[7].

Dimensionless loading space

One of the interesting byproducts of introducing James number (J or J_w) through eqns.(2) and (3) is that, as shown in eqns. (5) and (6), it generates self-consistent non-dimensional groups. They are $\frac{V^2}{2\Sigma_c}$, $\frac{V^2 Z_e}{\pi_c}$, $\frac{E_c}{4m\Sigma_c}$, and $\frac{Z_e E_c}{2m\pi_c}$. That is, Σ_c, π_c, and τ_c ($=E_c/\pi_c$) can be nondimensionalized by $V^2/2$, $V^2 Z_e$, and $2m/Z_e$, respectively. These groups suggest that we can also non-dimensionalize the loading space as well as the threshold functions by these groups in a self-consistent manner. A significance of the non-dimensional space is as follows. The space defined by either (E, Σ) or (E, π) is spanned by the coordinates that have different dimensions, so we cannot introduce the concept of "distance" in these spaces. But if there is a non-dimensional space we can measure separation between two points as in a Cartesian space. One distance of particular interest in the threshold space, is the distance between the origin (ambient state) and a state on the threshold boundary, (*e.g.* 50% ignition). Additionally, the closest distance from the ambient state to 50% ignition boundary is another quantity of interest, because it may be used as an integrated measure of impact shock sensitivity. Then the coefficient of its variation will be a measure of the reliability of the central tendency to initiate.

In this study, however, we like to show the usefulness of a non-dimensional space, using a simple rearrangement of the threshold function itself, recognizing the fact that J or J_w is a non-dimensional variable. That is,

$$\frac{1}{J} = \frac{1}{\frac{E}{E_c}} + \frac{1}{\frac{\Sigma}{\Sigma_c}} \qquad \text{(James)} \ . \qquad (11)$$

$$\frac{1}{J_w} = \frac{1}{\frac{E}{E_c}} + \frac{1}{\frac{\pi}{\pi_c}} \qquad \text{(Welle)} \ . \qquad (12)$$

Selecting a special case of $J=J_w=1$ for illustration, we find that these two boundary functions can be expressed by a single hyperbolic function,

$$1 = \frac{1}{\frac{E}{E_c}} + \frac{1}{\frac{\pi}{\pi_c}} \equiv \frac{1}{Y} + \frac{1}{X} \ . \qquad (13)$$

It is trivial now to show that the minimum "distance" between the ambient state and the ignition threshold boundary is found at X=Y=2, provided that X and Y are independent. In the Welle space, although Y can be replaced by the product of X and another variable, say T ($=\tau \frac{\pi_c}{E_c}$), we can show that the minimum point is still found at X=Y=2. In the original variables, the minimum point is specified as follows.

$$E = Pu\tau = 2E_c \quad (14)$$
$$\pi = Pu = 2\pi_c \quad (15)$$

Then using the definition of shock pressure ($P = \rho_e U_e u$, where U_e is shock speed), we find

$$\pi = \rho_e U_e u \cdot u = \rho_e (C_e + s_e u) u^2 = 2\pi_c \quad (16)$$

This equation shows that the minimum point in the non-dimensional space is specified independent of the flyer velocity V. In contrast eqn. (14) imposes a condition on shock duration (τ) and flyer thickness (l_f). That is,

$$\tau = \frac{E}{\pi} = \frac{E_c}{\pi_c} = \frac{2l_f}{U_f} \ . \quad (17)$$

$$l_f = \frac{E_c}{\pi_c}\left(\frac{U_f}{2}\right) = \left(\frac{E_c}{2\pi_c}\right)\left[C_f + s_f(V-u)\right] . \quad (18)$$

Similarly, eqn. (15) imposes a condition on V through the equality of pressure across the impact interface. That is,

$$\rho_e (C_e + s_e u) u = \rho_f \left(C_f + s_f (V-u)\right)(V-u) . \quad (19)$$

This reduces to

$$V^2 + \left(\frac{C_f}{s_f} - 2u\right) V + \left(1 - \frac{\rho_e s_e}{\rho_f s_f}\right) u^2 - \left(\frac{C_f}{s_f} + \frac{\rho_e C_e}{\rho_f s_f}\right) u = 0. \quad (20)$$

So, given the solution to eqn. (16), we can solve eqn. (20) for V at the minimum point. Since C_e and C_f are bigger than u, there is only one positive root for eqn. (20). We shall come back to this equation in the next section where we introduce the notion of "distance" in the non-dimensional space.

Distance in the non-dimensional loading space

In the coordinate space that defines either the James or Welle ignition boundary, each of the coordinates has a different dimension. So, it is not possible to measure a distance (or separation) unless we are moving in a direction parallel to a coordinate axis. For example, a shock will take us from the ambient state to a shocked state (Pu) along the π-axis, and then move vertically up, parallel to the E-axis, depending on the duration of the shock (τ). But we cannot draw a line from the origin to the terminal state (E, π) and quantify the distance between these two points because of dimensionality. However, this will not be an issue with the non-dimensional X and Y coordinates and we can introduce a Cartesian distance (L) defined by

$$L \equiv \sqrt{X^2 + Y^2} \, . \quad (21)$$

An advantage of this space is that we can now define a minimum distance between the ambient state and any point on the ignition boundary (see Fig. 4), and consider it as a measure of impact shock sensitivity: the shorter the distance, the more sensitive the explosive is.

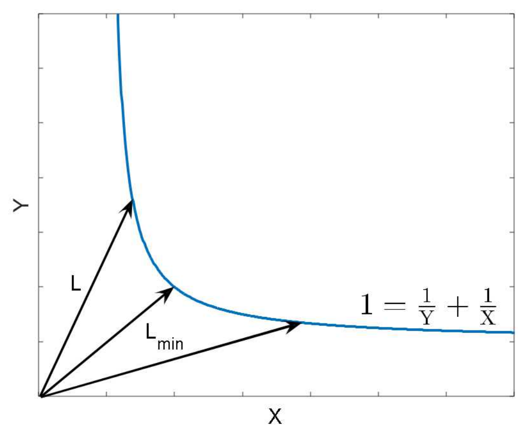

Fig. 4. Generalized plot of non-dimensional hyperbolic threshold space, with distances L to the boundary. L_{min} is the minimum distance to the threshold boundary.

Fig. 5 (top) illustrates the variation of distance L as a function of flyer velocity V, as well as the locations of L_{min} for the experimental data of Class 3 and Class 5 HMX powders discussed earlier. V at L_{min} can be analytically obtained by solving eqn. (20) for particle velocities that satisfy eqn. (16). These values in km/s are 1.707 and 1.885 for Class 3 and 1.935 and 2.127 for Class 5, depending on flyers of polyimide and parylene, respectively. If V at L_{min} depicts a measure of "impact shock sensitivity", Class 3 is more "sensitive" than Class 5.

The above described result may look contradictory to the experimental data shown on Fig. 1. But it can be seen that it is not so by calculating the corresponding minimum thickness required for ignition by use of eqn. (18). The results are 0.1133 mm and 0.1049 mm for Class 3 and 0.02875 mm and 0.02683 mm for Class 5, depending on the flyer material; polyimide and parylene respectively. That is, regardless of the flyer, Class 3 requires about four times thicker flyers than Class 5. This explains why the reversal of the relative position of ignition boundaries happens in the original (E, π). The difference in Vmin, which is on the order of 20% is overshadowed by the requirement for thickness (longer duration for Class 3). These results show how the ignition thresholds that are described by either James or Welle function depend sensitively on the flyer through its shock impedance, impact velocity, and thickness. These ignition functions represent a composite picture of initiation behavior. On the other hand, V_{min} and (min l_f) posit the minimum condition on impact velocity and thickness, which could be considered as a new measure of sensitivity, depending on the shock impedances of the flyer and the explosive. Vmin may be considered a kind of activation energy, representing impact sensitivity.

There are other noteworthy points to observe on Fig.5 (top). First is that the distance curves depend on the flyer materials as well as the explosive properties. Thus, the mixing of ignition data, using different flyer materials, may not be a good idea in understanding shock sensitivity. For example, the boundary obtained by use of a polymeric flyer may not be easily compared with those of metallic flyers. To illustrate the point, Fig. 5 (bottom) shows L-curves and V at L_{min} for a flyer made of Al 1100. The decrease in V_{min} is caused by the high impedance of the aluminum flyer that in turn induces a higher shock pressure and particle velocity in the target explosive.

Interestingly, however, the relative positions of V at L_{min} for Class 3 and Class 5 HMX powders is preserved regardless of the flyer material, indicating that V at L_{min} may be used as an integrated measure of shock sensitivity. But the magnitude is still flyer dependent.

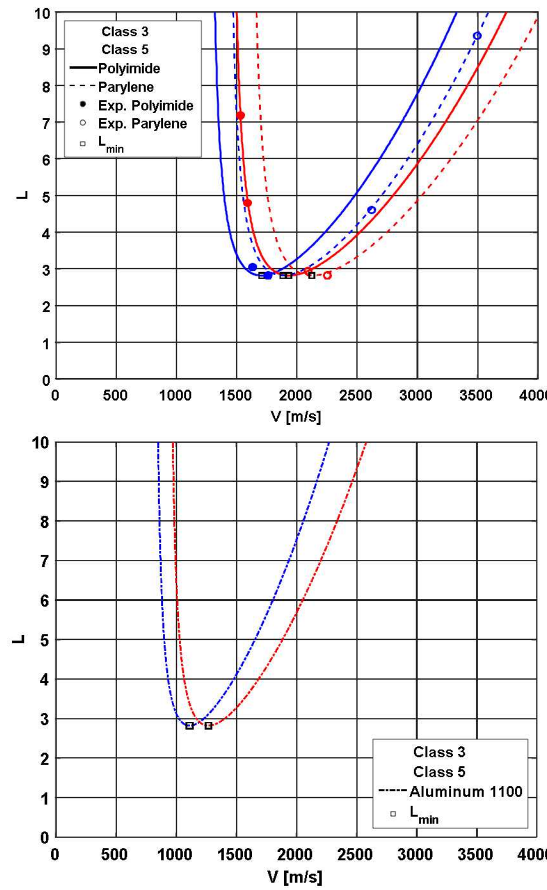

Fig. 5. Measure of the distance from the ambient state to the ignition threshold boundary as a function of flyer velocity for Class 3 and Class 5 pressed HMX powders.

Performance margin

If a full statistics of ignition thresholds is available, then J_w will serve, as shown in Figs. 3 and 4, not only as a measure of performance margin in a probabilistic term, but also its reliability based on its standard of deviation. But in practice very little or no statistics is available on the threshold function for reason of cost and time, so it is useful to define an alternative measure of performance margin based just one threshold function.

Table 1. Material Parameters

	Class 3	Class 5	Polyimide	Parylene
ρ (kg/m³)	1,790	1,790	1,414	1,286
C (km/s)	2.271	2.271	2.775	2.228
S	2.043	2.043	1.376	1.376
π_c (GW/cm²)	0.2072	0.2776	-	-
E_c (kJ/cm²)	0.01157	0.00377	-	-

In the non-dimensional loading space, the above goal can be simply achieved by comparing distances from the ambient state (origin) to a loading point (design point) and the point on the threshold boundary, which is the intersect of the loading line and the threshold boundary. For the margin of safety, the loading line is extended to the boundary to locate the latter point. By designating the distance to a design load by $D = \sqrt{X^2+Y^2}$ and the distance to the intersect by symbol ξ, the performance margin may be defined by D/ξ or $D-\xi$. The corresponding margin of safety will be $\xi - D$ (for no-go). Factor of safety is ξ/D (for no-go). Since ξ is the distance to the intersect of the loading line and the ignition boundary, it must satisfy both

$$Y = \left(\tau \frac{\pi_c}{E_c}\right) X\,, \qquad (21)$$

and

$$1 = \frac{1}{X} + \frac{1}{Y}\ .$$

Thus, ξ is given by

$$\xi = \left(1 + \tau\frac{\pi_c}{E_c}\right)\sqrt{\frac{1}{\left(\tau\frac{\pi_c}{E_c}\right)^2} + 1}\ . \qquad (22)$$

It is interesting to note that eqn. (22) implies the existence of a path to optimize the performance margin and this path is to choose τ such

$$\tau\frac{\pi_c}{E_c} = 1\ . \qquad (23)$$

As easily seen geometrically, the optimum path bisects the X and Y coordinates and $\xi_{min} = L_{min}$. For purpose of illustration, if we substitute the values of E_c and π_c on Table 1, we find that the "optimum" shock duration for Class 3 and Class 5 powders are 55. 8 and 13.6 ns respectively. Obviously longer or shorter pulses may be chosen, but they are not necessarily the path to gain the optimal performance margin in selecting flyer thickness.

The optimum selection of τ implies the minimum thickness of the flyer that is given by eqn. (18), in which u is a solution of either eqn. (19) or (20). Again, for purpose of illustration, if we use the highest data point on Fig. 1 for which the explosive was Class 5 and the flyer was polyimide, V=1,533 m/s and u is 698.8 m/s, we find the optimal l_f to be 0.0267mm. The actual thickness used for the data point was 0.183 mm, indicating a large margin of performance.

Conclusions

Hayes' idea of optimizing detonation initiation by the impact of thin flyer plates through shock-impedance selection was reexamined by use of two more recent ignition functions. The study led to several unexpected results. They are (1) the appearance of self-consistent, non-dimensional groups that are associated with the initiation functions, (2) the idea of non-dimensional loading space for which we can introduce the notion of "distance", and (3) the introduction of geometric distances to develop a unified measure of shock sensitivity, factor of safety, and a performance margin. The new ideas are examined using experimental results for pressed HMX powders. New findings include (1) that optimum selection of flyer impedance is an efficient way of increasing cumulative ignition probability, (2) the new sensitivity measure based on the minimum flyer velocity required for ignition depicts a different kind of sensitivity that is not visible geometrically on the ignition function, and (3) dimensionless loading space allows the introduction of a margin of performance as well as a margin of safety that has an intuitive appeal and can be calculated using a single threshold function.

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