

# Unsorted Sparse Matrix Addition using Kokkos

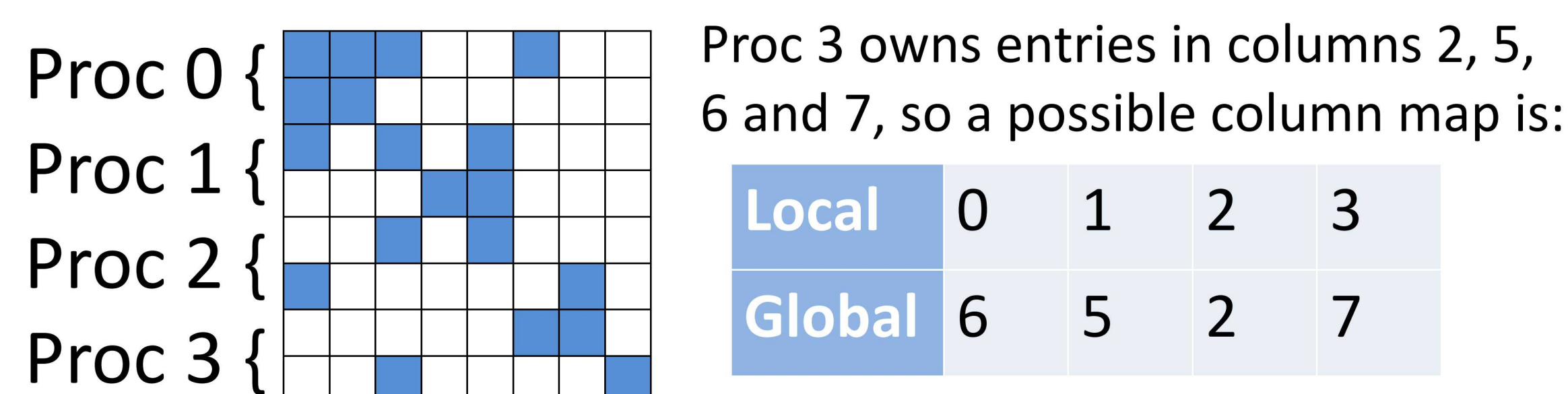
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## Problem

**Tpetra** (part of Trilinos) implements distributed (row-partitioned) sparse matrices in CRS format:



- Memory requirements reduced by storing only local column indices on each processor
- “column map” records the local to global column mappings on each processor
- Adding matrices requires temporary conversion to global indices, which **may not preserve ascending column order**

## Approach

All rows computed independently in CRS format. Example for one row of  $C = A+B$ :

A values: 

|   |    |   |
|---|----|---|
| 9 | -6 | 3 |
|---|----|---|

 A columns: 

|   |   |   |
|---|---|---|
| 3 | 0 | 4 |
|---|---|---|

B values: 

|   |   |    |   |
|---|---|----|---|
| 5 | 2 | -1 | 4 |
|---|---|----|---|

 B columns: 

|   |   |   |   |
|---|---|---|---|
| 2 | 5 | 3 | 4 |
|---|---|---|---|

Symbolic Phase: sort and merge columns

Concatenate columns: 

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 3 | 0 | 4 | 2 | 5 | 3 | 4 |
|---|---|---|---|---|---|---|

  
A/B entry permutation: 

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 0A | 1A | 2A | 0B | 1B | 2B | 3B |
|----|----|----|----|----|----|----|

Radix sort by column, updating permutation: 

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 0  | 2  | 3  | 3  | 4  | 4  | 5  |
| 1A | 0B | 0A | 2B | 2A | 3B | 1B |

Merge to get C columns: 

|    |    |        |        |    |
|----|----|--------|--------|----|
| 0  | 2  | 3      | 4      | 5  |
| 1A | 0B | 0A, 2B | 2A, 3B | 1B |

Record where original A, B columns belong in C CRS arrays:

A columns: 

|   |   |   |
|---|---|---|
| 3 | 0 | 4 |
|---|---|---|

 B columns: 

|   |   |   |   |
|---|---|---|---|
| 2 | 5 | 3 | 4 |
|---|---|---|---|

  
Apos: 

|   |   |   |
|---|---|---|
| 2 | 0 | 3 |
|---|---|---|

 Bpos: 

|   |   |   |   |
|---|---|---|---|
| 1 | 4 | 2 | 3 |
|---|---|---|---|

Numeric Phase: scatter values directly into C

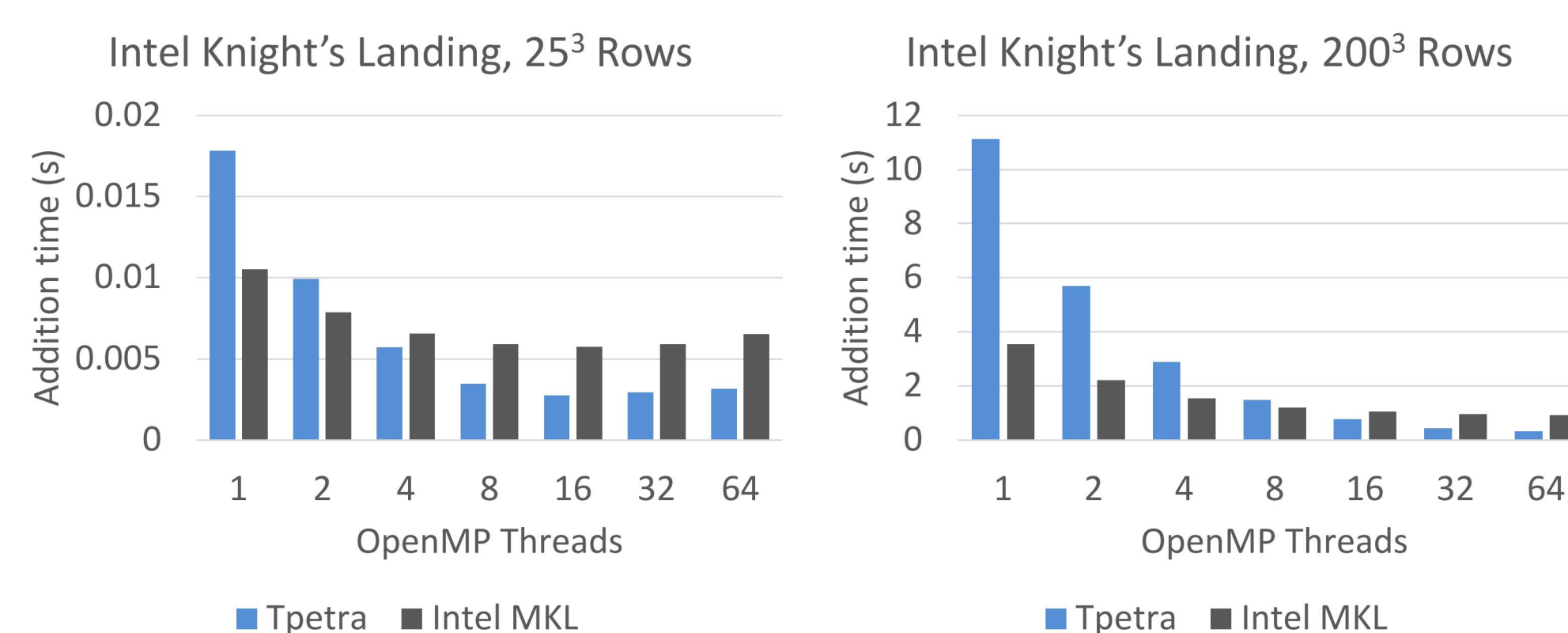
For  $i = 0 \dots A_{\text{length}}$ :  $C_{\text{vals}}[A_{\text{pos}}[i]] += A_{\text{vals}}[i]$ ;

For  $i = 0 \dots B_{\text{length}}$ :  $C_{\text{vals}}[B_{\text{pos}}[i]] += B_{\text{vals}}[i]$ ;

C values: 

|    |   |   |   |   |
|----|---|---|---|---|
| -6 | 5 | 8 | 7 | 2 |
|----|---|---|---|---|

## Results



This matrix addition arises in the Jacobi smoother used by algebraic multigrid (MueLu) to interpolate from a coarse grid to a fine grid.

In this case, multigrid is being run on a 3D Laplacian grid with  $25^3$  vertices (left) and  $200^3$  vertices (right).

Although Intel's MKL library is faster than Tpetra for low thread counts, Tpetra has much better scaling and is faster for high thread counts (the intended use case for the many-core KNL architecture).

## Significance

The new, Kokkos-based matrix addition in Tpetra saves a significant amount of time in real-world finite element problems (compared to the previous implementation).

Drekar (a multiphysics application) saw an overall speedup of almost 20% in a small ( $20^3$  element) plasma fluid dynamics simulation. The proportion of time spent in matrix addition was reduced from about 20% to under 5%. This test was run on 1 KNL node.

