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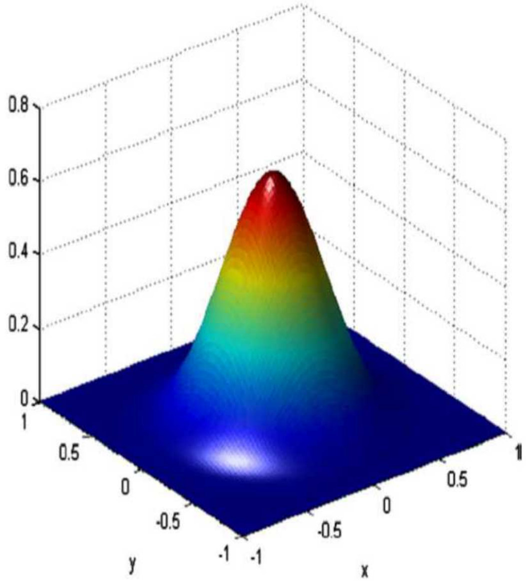

Verification and Large Deformation Analysis Using the Reproducing Kernel Particle Method

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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What is RKPM?

- Method where approximation functions, $u_i^h(\boldsymbol{x}, t)$, are constructed directly at nodal points without the need for element connectivity
  - High order of continuity or "smoothness" in approximation can be achieved
  - Reproducing conditions ensure the approximation space is "complete" to polynomials of arbitrary order
    - $\sum_{I=1}^{NP} \psi_I(\boldsymbol{x}_I) x_{1I}^i x_{2I}^j = x_1^i x_2^j \quad \forall \quad i + j \leq n$
- Employed in the Principle of Virtual Work to semi-discretize a domain in space
- Principle of Virtual Work:
  - $\int_{\Omega_x} \delta u_i^h \rho \ddot{u}_i^h d\Omega + \int_{\Omega_x} \delta u_{(i,j)}^h \sigma_{ij}^h d\Omega = \int_{\Omega_x} \delta u_i^h b_i d\Omega + \int_{\partial\Omega_x^h} \delta u_i^h h_i d\Gamma$

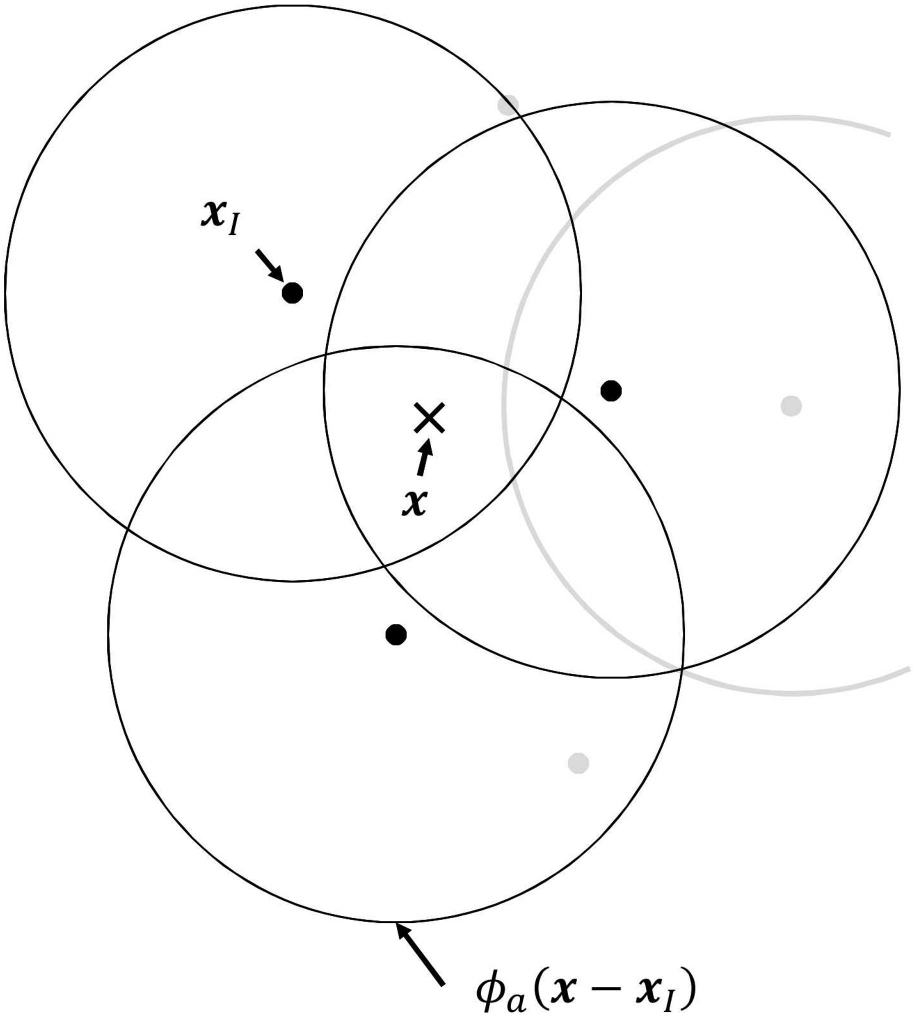
Shape Function Formulation

- Approximation
  - $\psi_I(\boldsymbol{x}) = \boldsymbol{H}^T(\boldsymbol{x} - \boldsymbol{x}_I)\boldsymbol{b}(\boldsymbol{x})\phi_a(\boldsymbol{x} - \boldsymbol{x}_I)$
    - $\boldsymbol{u}^h(\boldsymbol{x}, t) = \sum_{I=1}^{NP} \psi_I(\boldsymbol{x})\boldsymbol{u}_I^h(t)$
  - Where
    - $\boldsymbol{H}^T(\boldsymbol{x} - \boldsymbol{x}_I) = [1, x_1 - x_{1I}, x_2 - x_{2I}, \dots, (x_1 - x_{1I})^i(x_2 - x_{2I})^j]$ with $i + j \leq n$
    - $\phi_a(\boldsymbol{x} - \boldsymbol{x}_I)$ represents a kernel weighting function with support size, a
- Reproducing Condition
  - $\sum_{I=1}^{NP} \psi_I(\boldsymbol{x})(x_1 - x_{1I})^i(x_2 - x_{2I})^j = \delta_{i0}\delta_{j0}$
    - $\sum_{I=1}^{NP} \psi_I(\boldsymbol{x})\, \boldsymbol{H}(\boldsymbol{x} - \boldsymbol{x}_I) = \boldsymbol{H}(\boldsymbol{0})$
  - $\boldsymbol{M}(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x}) = \boldsymbol{H}(\boldsymbol{0})$
  - $\boldsymbol{M}(\boldsymbol{x}) = \sum_{I=1}^{NP} \boldsymbol{H}(\boldsymbol{x} - \boldsymbol{x}_I)\boldsymbol{H}^T(\boldsymbol{x} - \boldsymbol{x}_I)\phi_a(\boldsymbol{x} - \boldsymbol{x}_I)$
  - $\psi_I(\boldsymbol{x}) = \boldsymbol{H}^T(\boldsymbol{0})\boldsymbol{M}^{-1}(\boldsymbol{x})\boldsymbol{H}(\boldsymbol{x} - \boldsymbol{x}_I)\phi_a(\boldsymbol{x} - \boldsymbol{x}_I)$

J. S. Chen, C. Pan, C. T. Wu, W. K. Liu, *Reproducing kernel particle methods for large deformation analysis of nonlinear structures*, Computational Methods in Applied Mechanics and Engineering, 1996

Shape Function Formulation

- Sufficient cover required at $\boldsymbol{x}$ in order for $\boldsymbol{M}(\boldsymbol{x})$ to be invertible
  - $NP_x \geq (n+d)!/(n!\,d!)$
    - $n =$ Basis order
    - $d =$ Dimension of space
  - Each non-collinear node contributes 1 rank to $\boldsymbol{M}(\boldsymbol{x})$



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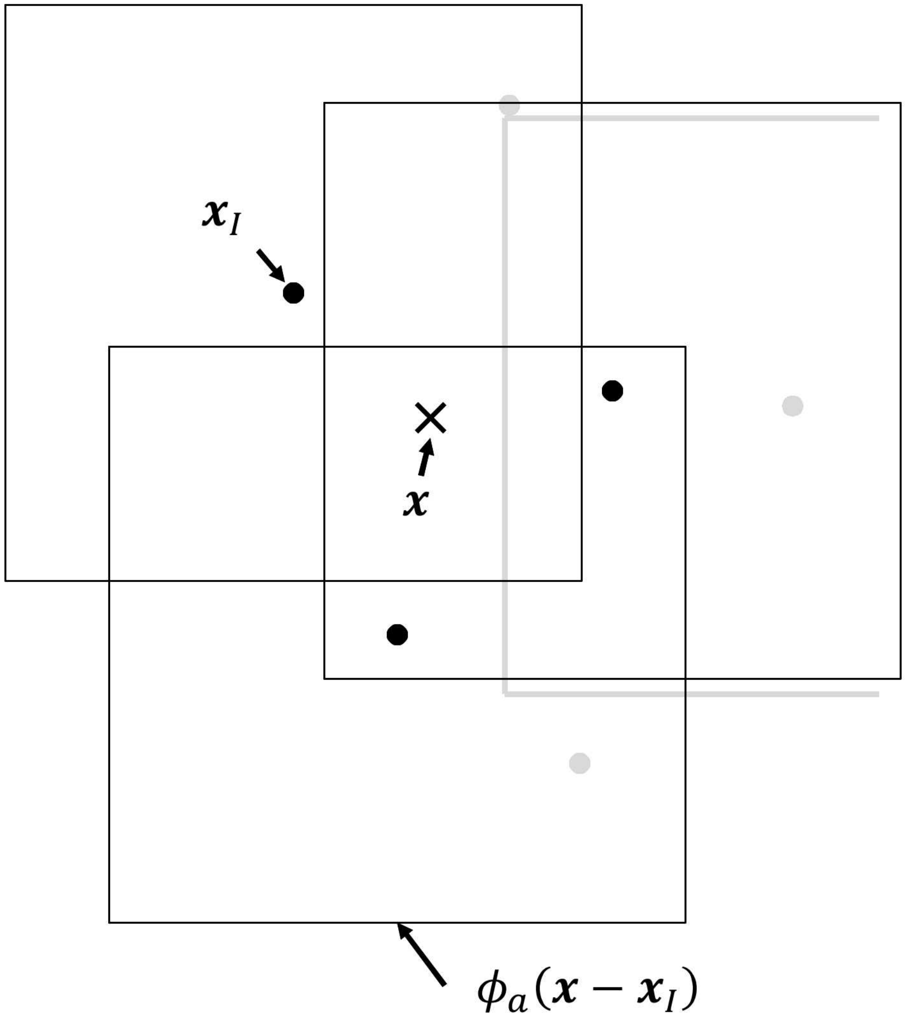
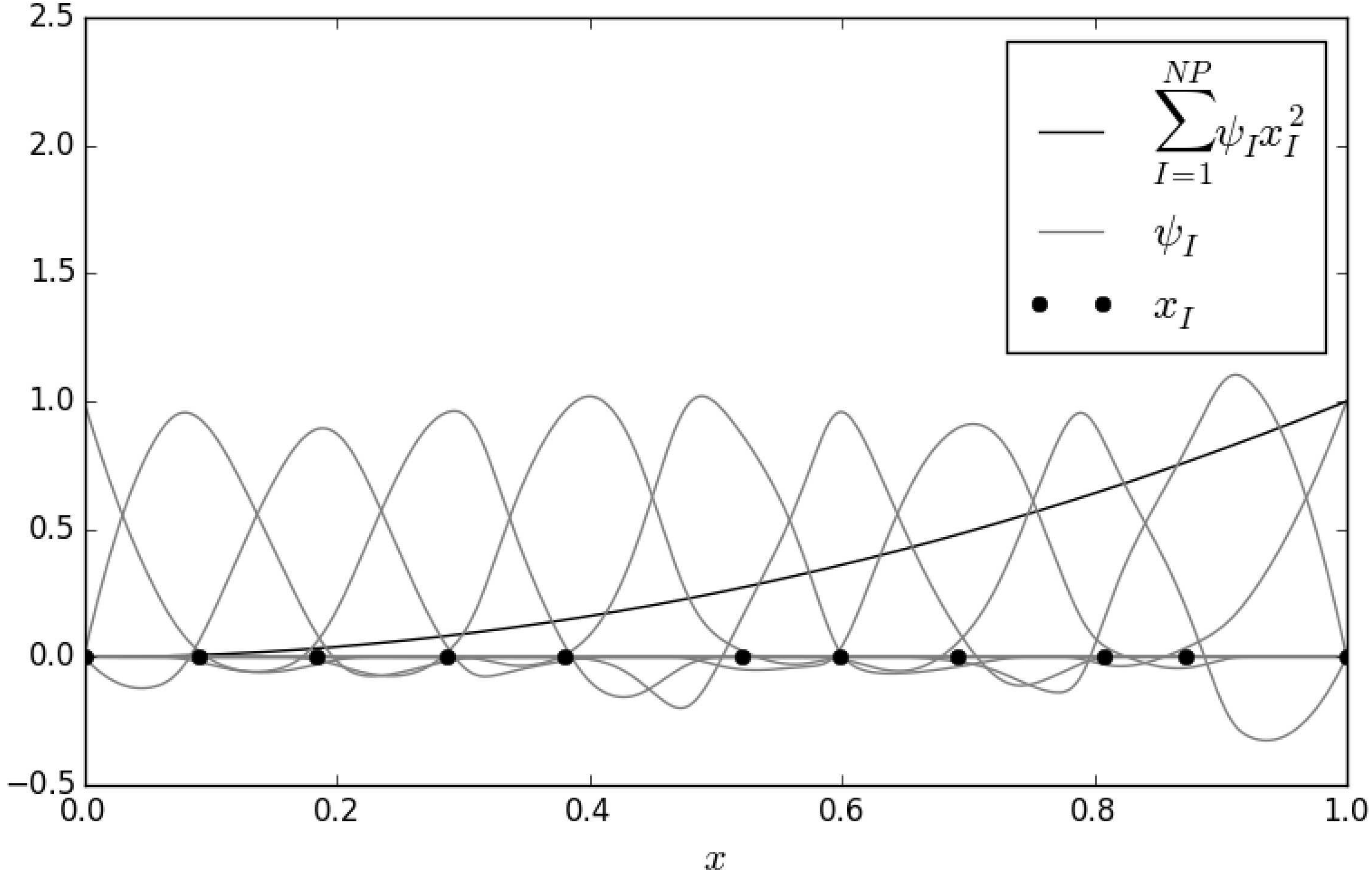
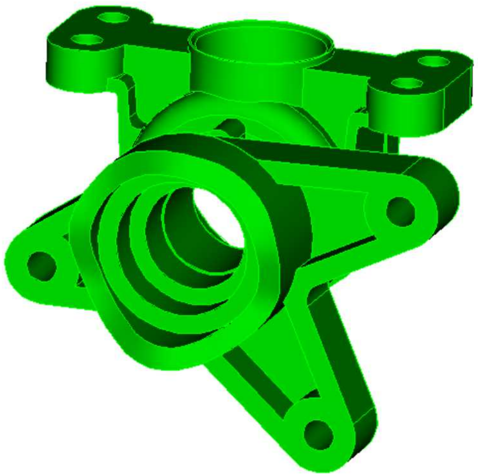


Shape Function Illustration



Motivation

- Rapid development of a model
  - Development of a quality mesh is difficult and time-consuming for FEM
- Large-Deformation Analysis where FEM fails
  - Elimination of a mesh obviates issues with mesh entanglement
- Fragmentation can be captured using Semi-Lagrangian formulation

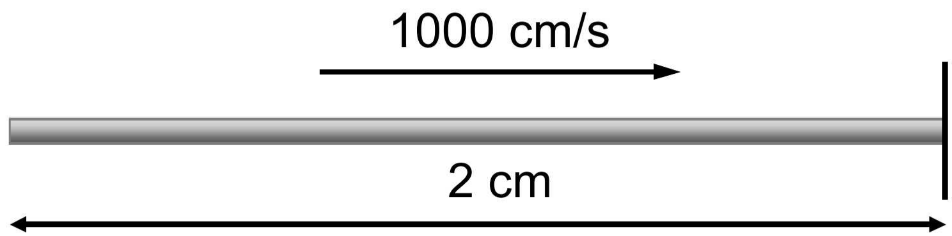


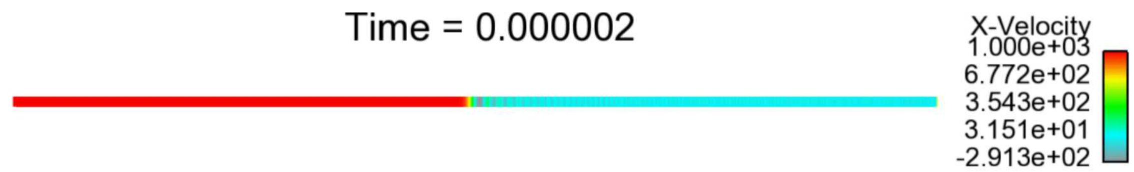
Verification & Validation

- Convergence and Scaling studies over the following:
  - Wave propagation in bar
  - Taylor bar impact model
- Demonstrate large-deformation capability in problems where FEM experiences severe mesh distortion
  - Foam crush in aluminum case

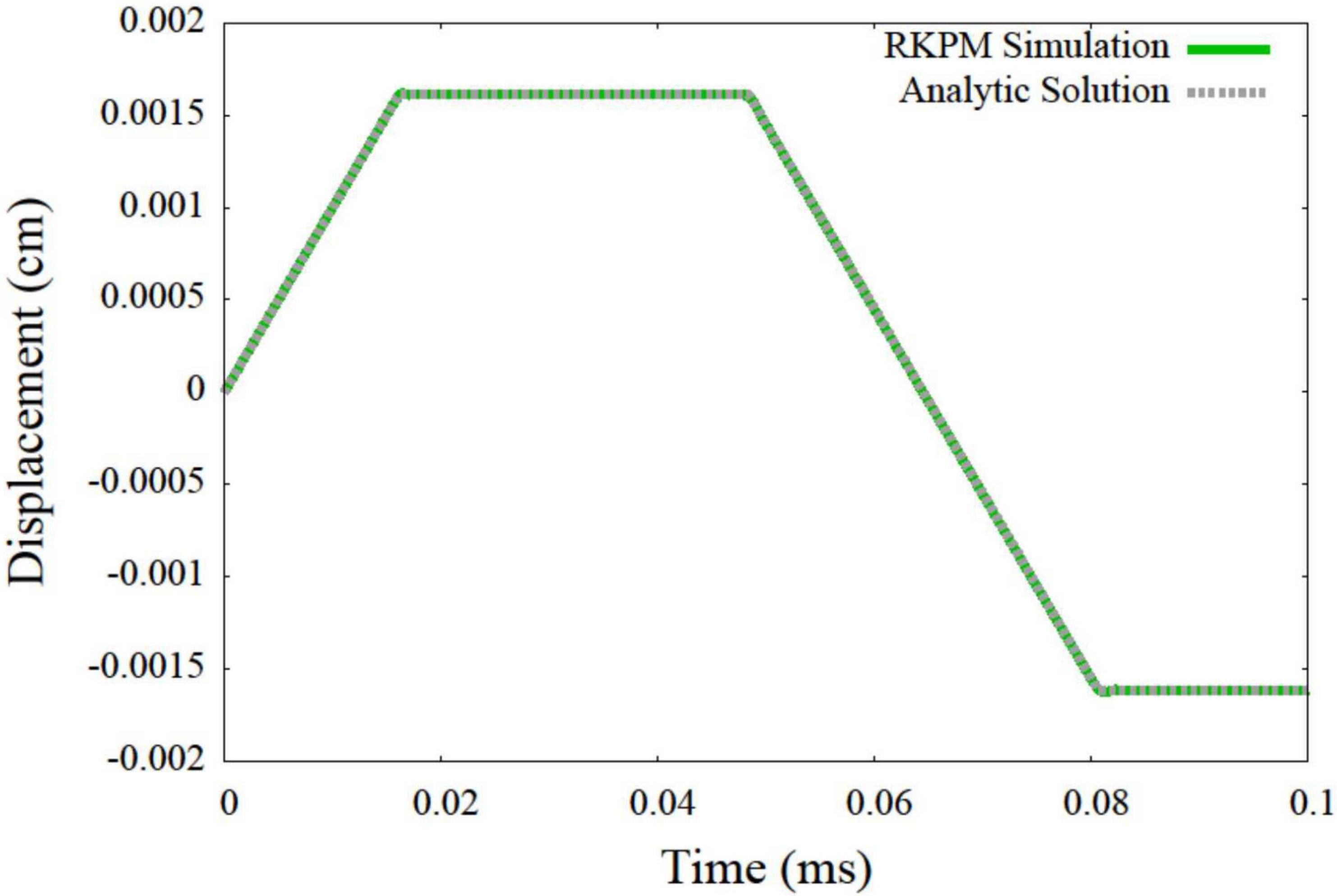
Wave Propagation Model

- Linear elastic model
  - $E = 300$ GPa
  - $\nu = 0$
  - $\rho = 7.8$ g/cc³
- $v_0 = 1000$ cm/s
- Fixed BC on right side
- Exact Solution:
- $u(x,t) = \sum_{n=1}^{\infty} A_n \sin(\omega_n t) \sin\left(\frac{(2n-1)\pi}{2L} x\right)$
- $A_n = \frac{8 v_0 L}{(2n-1)^2 \pi^2} \sqrt{\frac{\rho}{E}},\ \omega_n = \frac{(2n-1)\pi}{2L} \sqrt{\frac{E}{\rho}}$





Fine discretization (31,409 nodes)

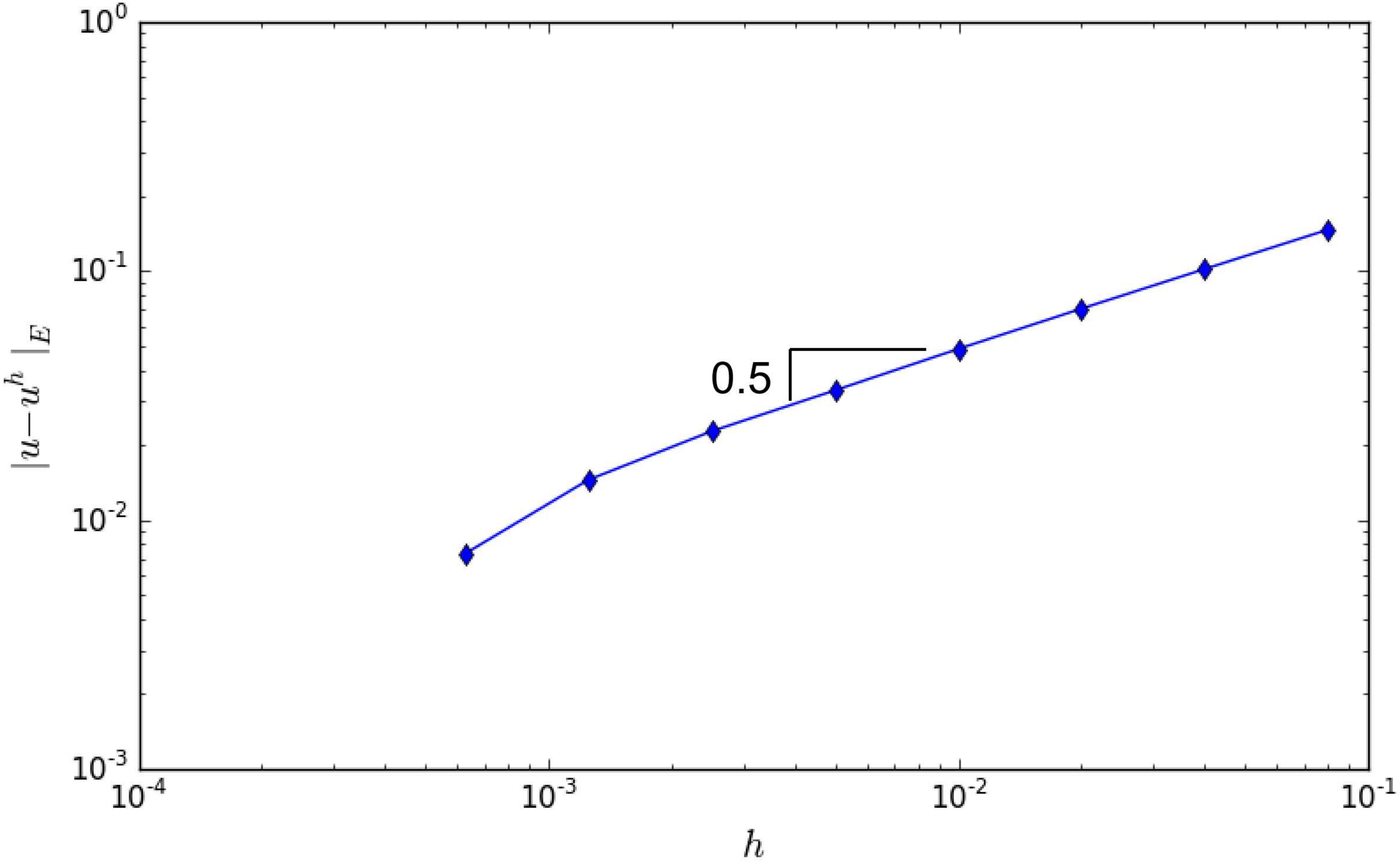


D. Littlewood, F. Beckwith, et. al., *Implementation and verification of RKPM in the Sierra/SolidMechanics analysis code*, International Mechanical Engineering Congress & Exposition, (Accepted)

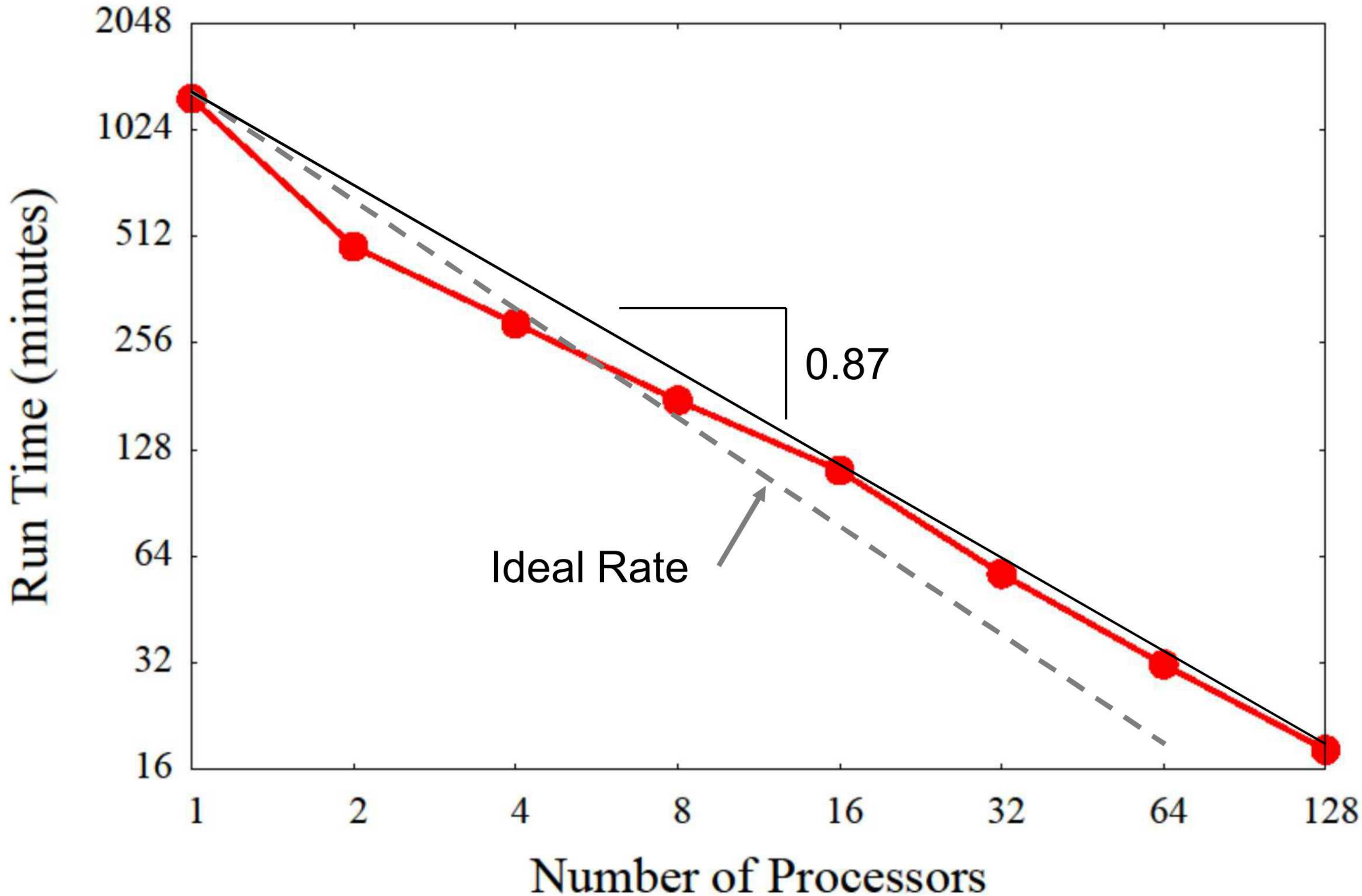
Convergence Rate

- Internal Energy
  - Exact: $a(u,u) = \frac{1}{2}\rho x_{wave} A v_0^2$
  - Approximate: $a(u^h,u^h) = \frac{1}{2}\sum_{I=1}^{NP} \rho \boldsymbol{E}_I^h : \boldsymbol{S}_I^h V_I$
  - Error: $a(e,e) = a(u,u) - a(u^h,u^h)$
  - Error Norm: $\left\| u - u^h \right\|_E = \sqrt{a(e,e)}$
    - ρ = Density
    - A = Area of bar
    - v_0 = Initial Velocity
    - x_{wave} = Position of strain wave front
    - $\boldsymbol{E}_I^h$ = Approximate Green-Lagrange strain at node I
    - $\boldsymbol{S}_I^h$ = Approximate Second-Piola Kirchhoff stress at node I
    - V_I = Initial volume of integration cell at node I

Convergence Rate



Scaling Study (360,192 nodes)



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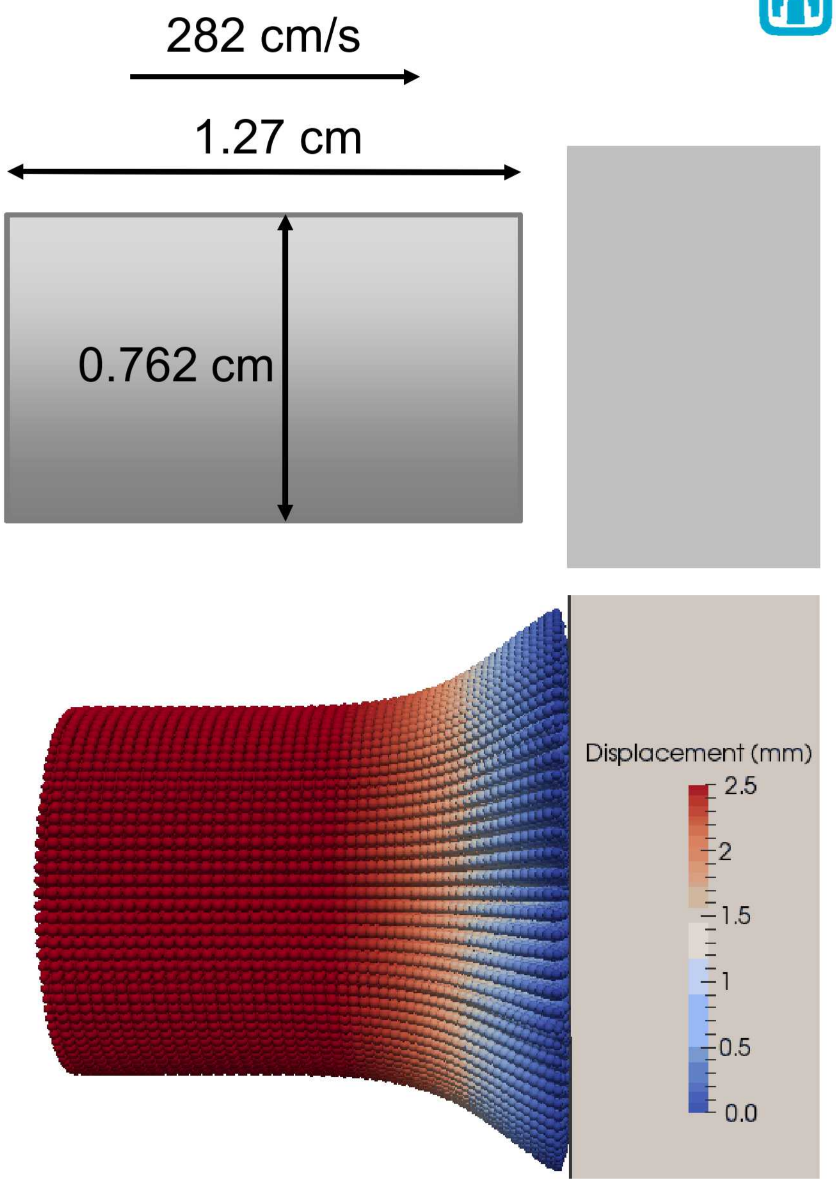
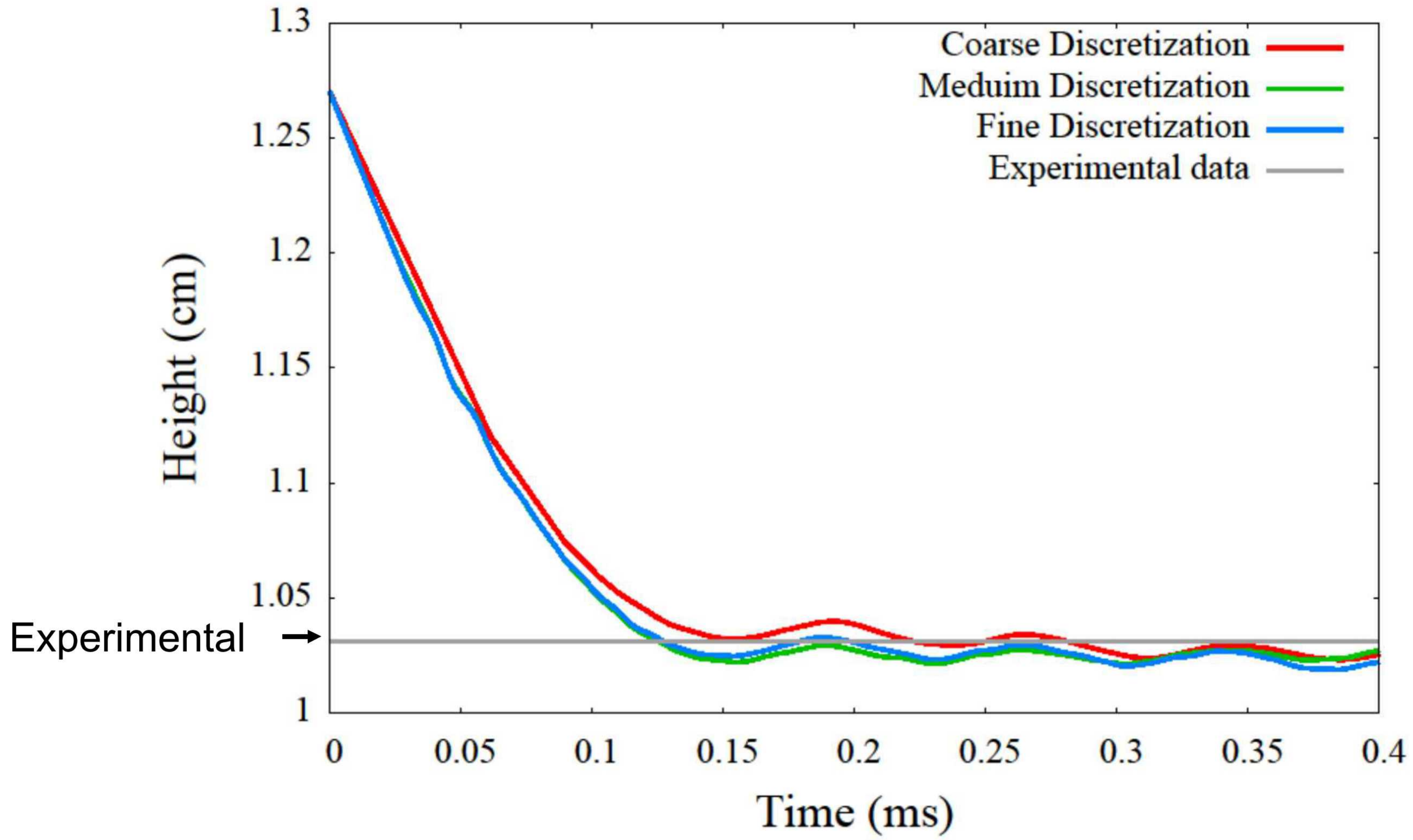
Taylor Bar Impact

- Final height recorded after deformation
- RKPM employs contact algorithm and Johnson-Cook model
  - $E = 200\ GPa$
  - $\sigma_y = 792\ MPa$
  - $\nu = 0.29$
  - $\rho = 7830\ kg/m^3$
  - Strain Hardening Coef's:
    - $B = 510\ MPa$
    - $C = 0.014$
    - $n = 0.26$



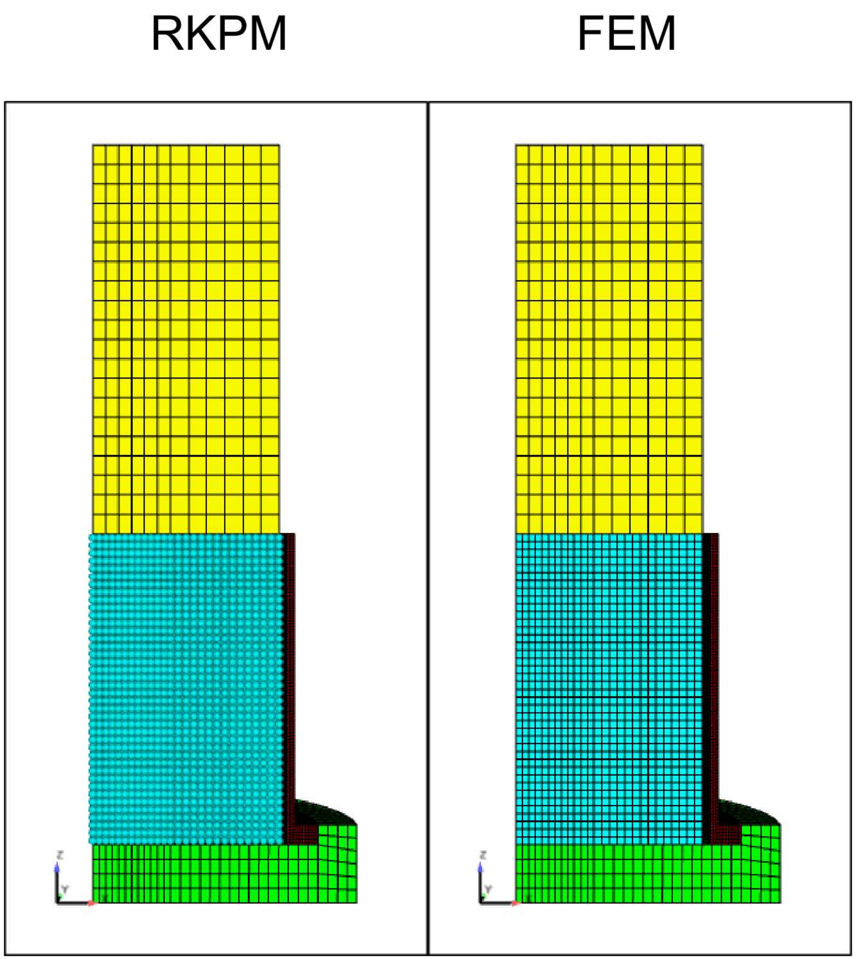
Simulation Comparison



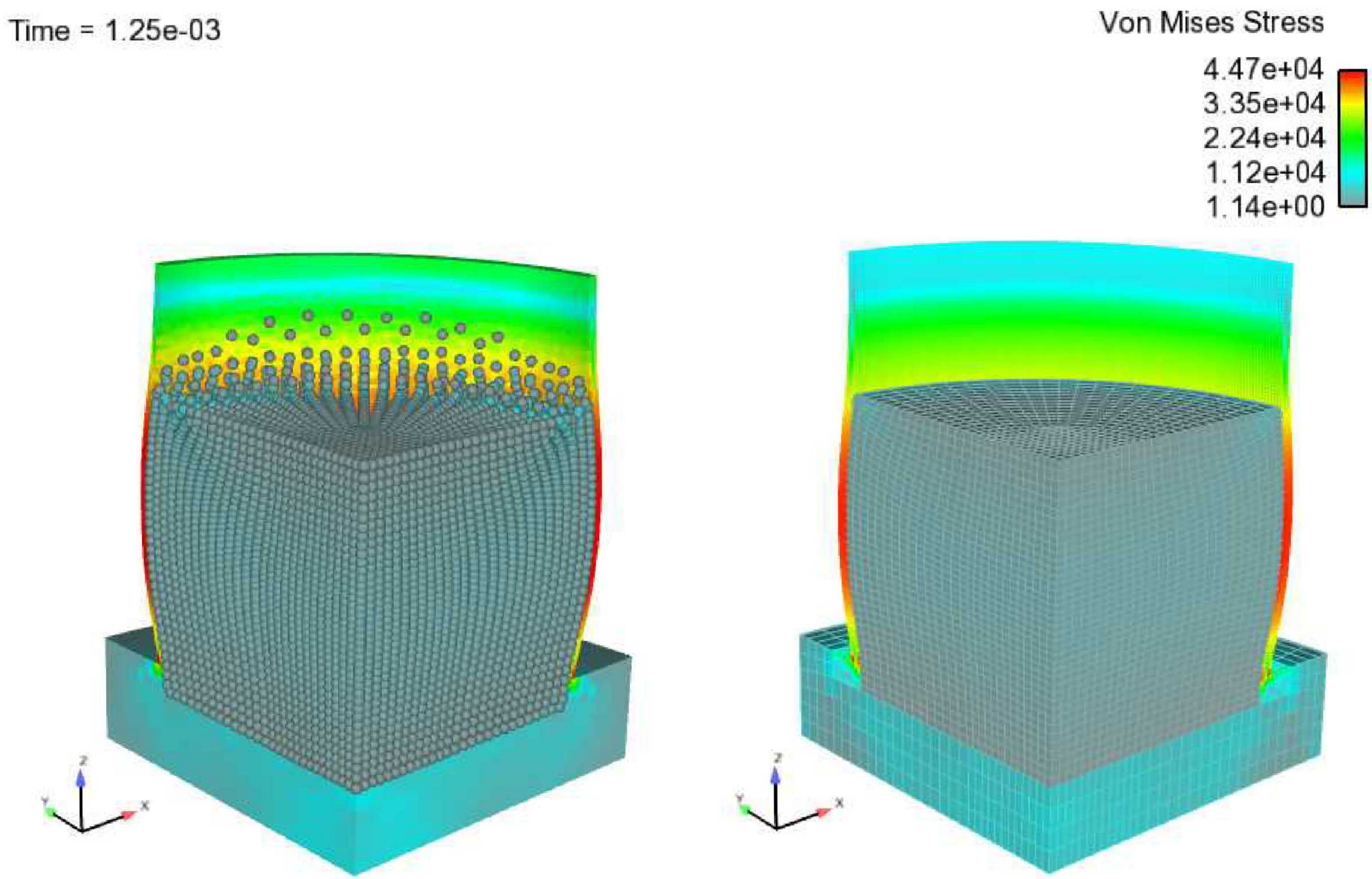
D. Littlewood, F. Beckwith, et. al., *Implementation and verification of RKPM in the Sierra/SolidMechanics analysis code*, International Mechanical Engineering Congress & Exposition, (Accepted)

Foam Crush Introduction

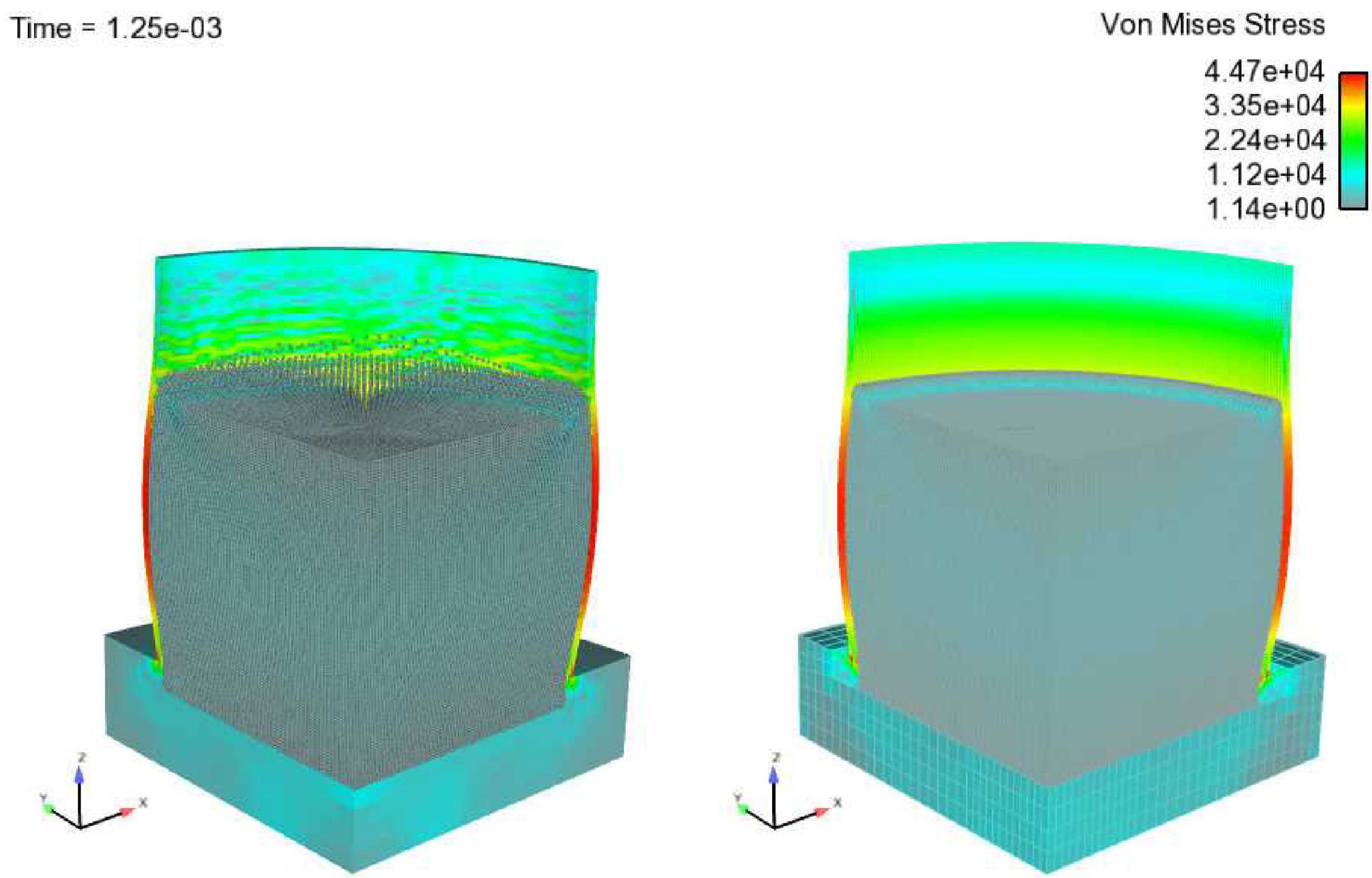
- Nick Kerschen under Dr. Chris Hammeter
- Impact piston striking block of “Tuf Foam” inside can
  - `FOAM_PLASTICITY` used for foam
  - J_2 plasticity used for aluminum casing
- FEM experiences large mesh distortion at edge of impactor



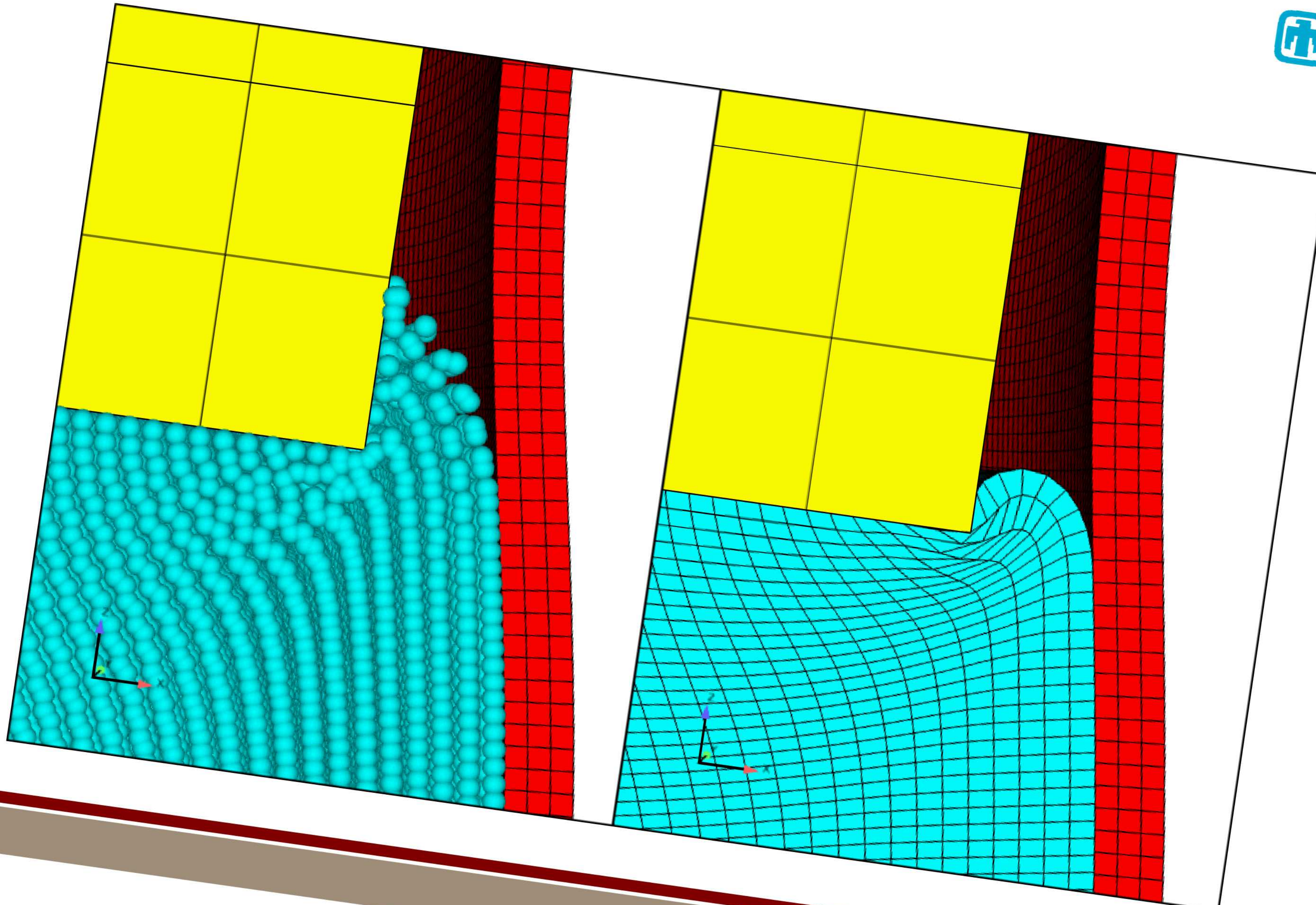
Foam Crush Comparison



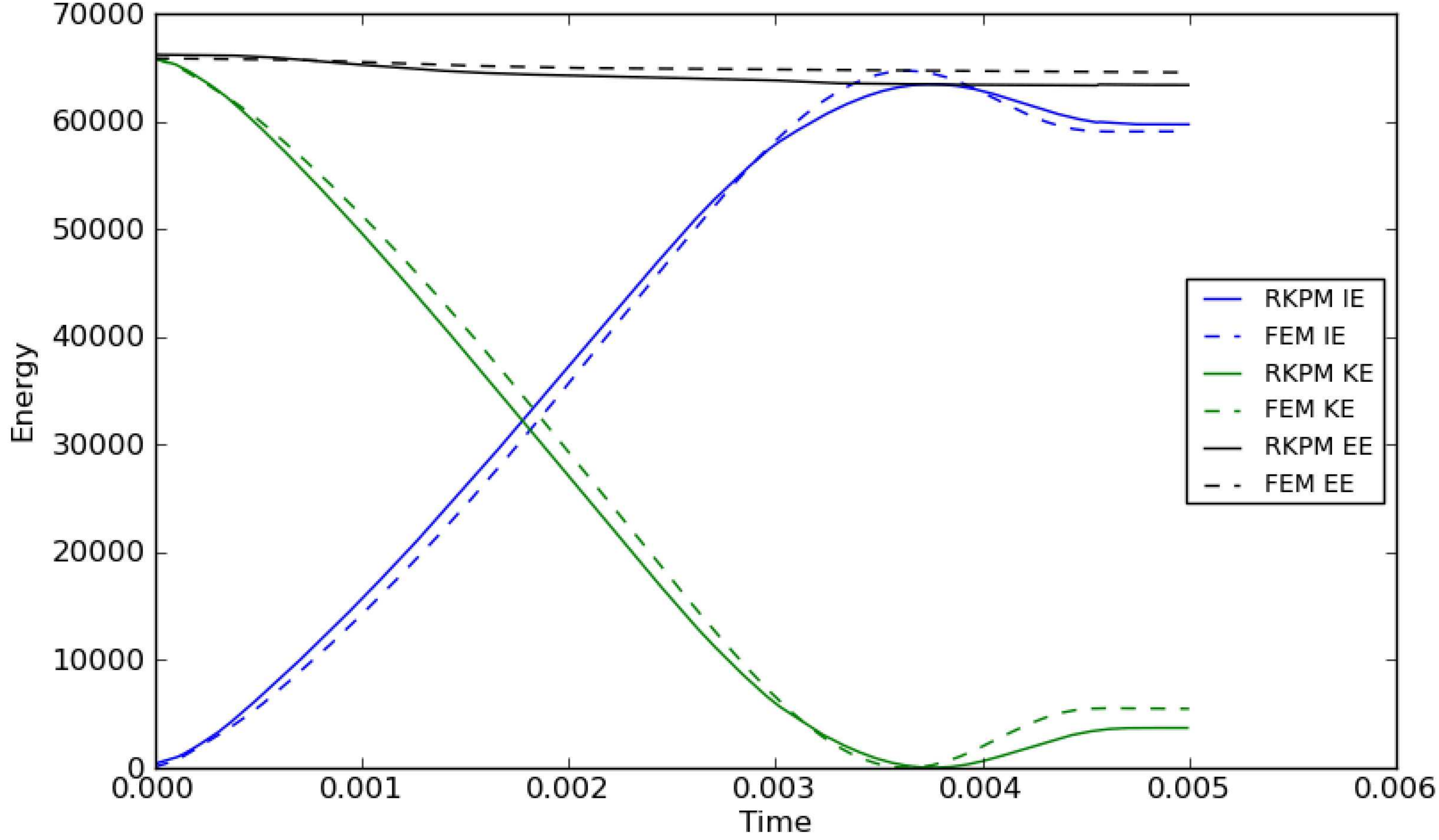
Foam Crush Comparison



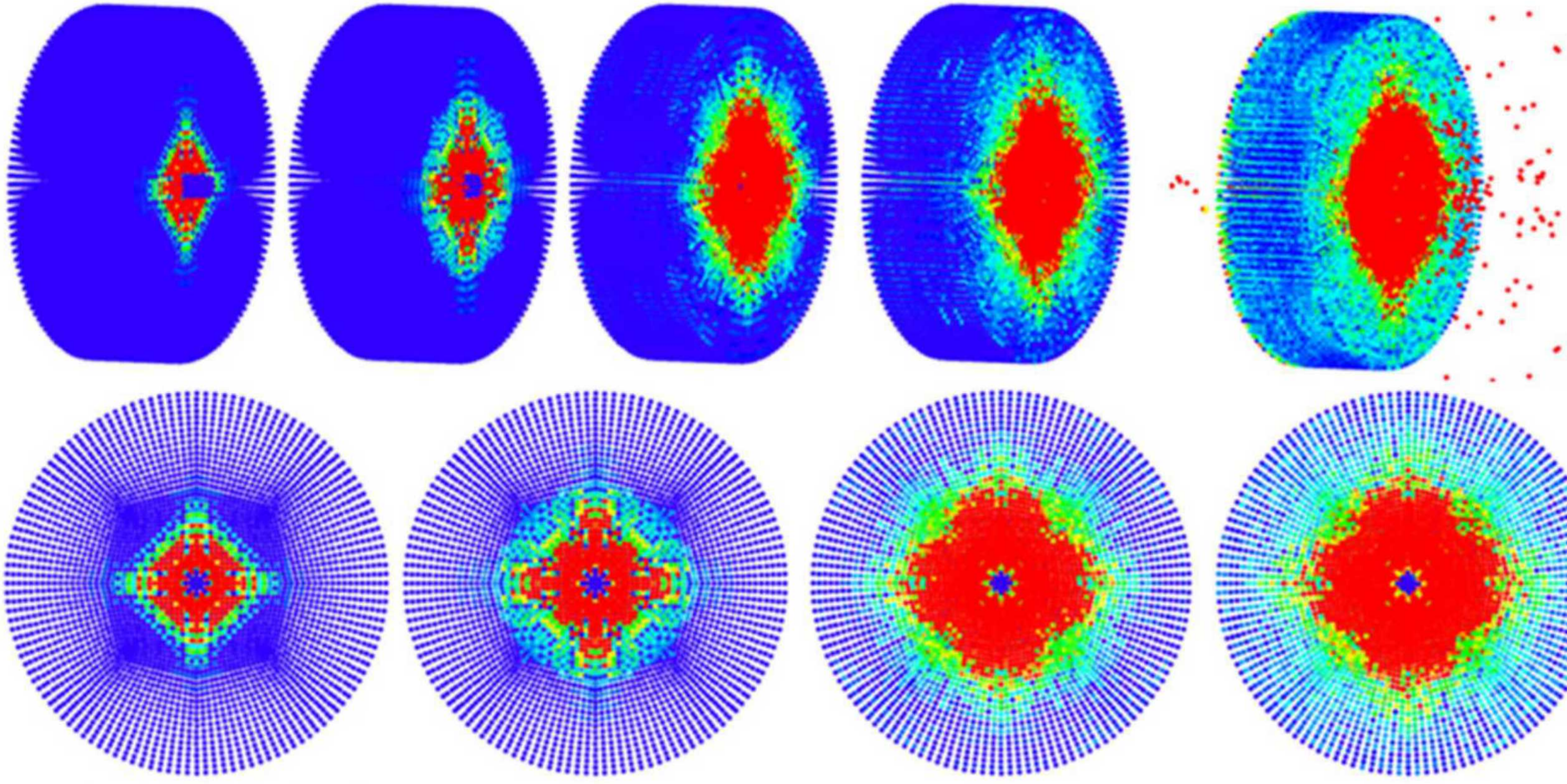
Element Distortion



Energy of Solutions



Future Goal



P.C. Guan, S. W. Chi, J. S. Chen, T. R. Slawson, M.J. Roth, *Semi-Lagrangian reproducing kernel particle method for fragment-impact problems*, International Journal of Impact Engineering, 2011

Current Issues

- Sierra/RKPM is not completely meshfree
  - Still relies on the original mesh information to create integration cells
- Removal of element connectivity discards surface definition
  - Traction BC's currently work but are not consistent with RKPM formulation
  - Contact difficult to implement due to generalized coordinate
- Lagrangian formulation currently implemented but does not handle fracture
  - Semi-Lagrangian formulation required to handle large-deformation and fracture problems
  - SCNI method of integration has some stability issues
- Different materials must be modeled with tied contact
- Performance is very slow in comparison with FEM, 10-20x
- Method consumes a lot of memory

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