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Approximating Two-Stage Chance-Constrained Programs with Classical Probability Bounds

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Chance Constraint Setting

Consider a linear Joint Chance Constraint:

$$P(x_t \leq y_t^\omega + w_t^\omega, \forall t \in T) \geq 1 - \varepsilon$$

Background:

- Two-stage stochastic program with recourse
- Possibly integer restrictions
- i.i.d. samples of uncertainty w_t^ω
- First stage decision, x_t, second-stage decision, y_t^ω

`Singh, Morton, Santoso. "An Adaptive Model with Joint Chance Constraints for a Hybrid Wind-Conventional Generator System." (Computational Management Science, 2018)`

Challenges

- CC models are computationally intractable
- A known NP-hard problem
- Existing algorithms not scalable to practical sized problems
- Feasible region is non-convex

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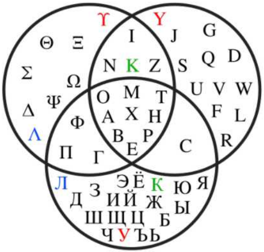
Approximations with classical probability bounds

Satisfying a JCC is an intersection of events. Failing a JCC is a union of events.



We can rewrite the JCC as follows:

$$\bigcup_{t \in T} F_t \leq \varepsilon$$

where F_t denotes the probability of "failure" at t; i.e.,
$F_t = P(x_t > y_t^{\omega} + w_t^{\omega})$.

Approximations with Classical Probability Bounds

$$\bigcup_{t \in T} F_t \leq \varepsilon$$

Consider an optimization model with a JCC with a maximization objective.

- Lower Bound (LB): Approximate the LHS using a quantity **larger** than $\bigcup_{t \in T} F_t$. Feasible region is **restricted**.
- Upper Bound (UB): Approximate the LHS using a quantity **smaller** than $\bigcup_{t \in T} F_t$. Feasible region is **enlarged**.

Approximations with Classical Probability Bounds

$S_k = \sum_{1 \le i_1 < \cdots < i_k \le |T|} F_{i_1} \cap \cdots \cap F_{i_k}$.

Bonferroni bounds:

$$\bigcup_{t \in T} F_t \le \quad S_1 \leftarrow LB \tag{1a}$$

$$\bigcup_{t \in T} F_t \ge \quad S_1 - S_2 \leftarrow UB \tag{1b}$$

Approximations with Classical Probability Bounds

$S_k = \sum_{1 \leq i_1 < \cdots < i_k \leq |T|} F_{i_1} \cap \cdots \cap F_{i_k}$.

Tighter bounds from Sathe et al. [1980]:

$$\bigcup_{t \in T} F_t \leq \quad S_1 - \frac{2}{T} S_2 \leftarrow LB \tag{2a}$$

$$\bigcup_{t \in T} F_t \geq \quad \frac{1}{T^2}(2S_2 + S_1) \leftarrow UB \tag{2b}$$

Approximations with Classical Probability Bounds

$S_k = \sum_{1 \leq i_1 < \cdots < i_k \leq |T|} F_{i_1} \cap \cdots \cap F_{i_k}.$

And more from Dawson and Sankoff [1967]:

$$\bigcup_{t \in T} F_t \geq \frac{{S_1}^2}{S_1 + 2S_2} \leftarrow UB \tag{3a}$$

can be linearized for JCC $\leq \varepsilon$:

$$2\varepsilon S_2 \geq \alpha_n S_1 + \beta_n, n = 0, 1, \ldots |N| - 1, \leftarrow UB \tag{3b}$$

Optimizing over JCCs

$u_t^\omega = 1$: failure at t in scenario ω
$v_{tt'}^\omega = 1$: failure at t and t' in scenario ω

$$x_t - y_t^\omega - w_t^\omega \leq M_t^\omega u_t^\omega, \forall t \in T, \omega \in \Omega$$

$$\text{McCormick envelope} \begin{cases} v_{t,t'}^\omega \leq u_t^\omega, (t, t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \leq u_{t'}^\omega, \forall (t, t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \geq u_t^\omega + u_{t'}^\omega - 1, \forall (t, t') \in T, t < t', \omega \in \Omega \end{cases}$$

$$u_t^\omega = \{0, 1\}, \forall t \in T, v_{t,t'}^\omega = \{0, 1\}, \forall (t, t') \in, \omega \in \Omega$$

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A case study

$$\max_{x,y} \quad \sum_{t \in T} (R_t x_t - \mathbb{E}[B_t y_t^\omega]) \tag{4a}$$

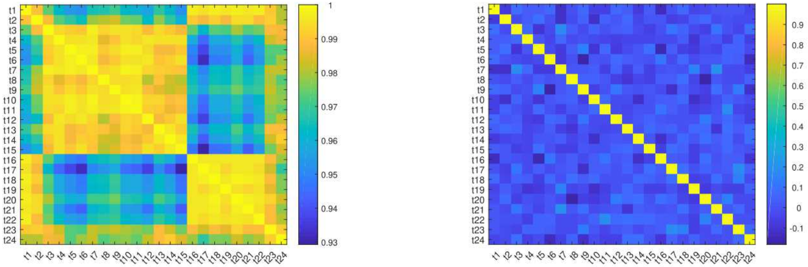
$$\text{s.t.} \quad \mathbb{P}(y_t^\omega + w_t^\omega \geq x_t, \forall t \in T) \geq 1 - \varepsilon \tag{4b}$$

$$0 \leq y_t^\omega \leq \Delta, \forall t \in T, \omega \in \Omega \tag{4c}$$

$$x_t \geq 0, \forall t \in T. \tag{4d}$$

Computational results

We compare two sampling procedures: (a) ARMA(2,2) process, and (b) normal random variables. Both samples have the same hourly means and variances.



(a) (b)

Figure: Correlation structure of w_t

Computational results: ARMA (large correlation)

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(1a)	**8,351.3**	8,351.3	0%	4	3.3%
	(1b)	**21,282.8**	21,282.8	0%	18	59.4%
	(2a)	**8,339.3**	8,410.9	0.85%	2100	3.4%
	(2b)	8,339.3	**10,682.2**	21%	2100	19.2%
	(3a)	8,339.3	**39,905**	79.1%	2100	78.4%
	(3b)	8,688.9	**8,708.9**	0.20%	2100	0.9%
0.03	(1a)	**8,374.6**	8,374.6	0%	9	8.5%
	(1b)	**22,353.2**	22,353.2	0%	95	59.0%
	(2a)	**8,339.6**	8,787.3	5.0%	2100	8.9%
	(2b)	8,339.3	**13,300.4**	37.3%	2100	31.2%
	(3a)	8,339.3	**40,975**	79.6%	2100	77.7 %
	(3b)	8610.0	**9,559.2**	9.9%	2100	4.2%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 8,351.3 and 8,708.9; true optimal value is 8,634.1
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 8,374.6 and 9,559.2; true optimal value is 9,154.9

Computational results: Gaussian (weak correlation)

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(1a)	**9,100.8**	9,100.8	0%	1	2.7%
	(1b)	**21,606.6**	21,606.6	0%	22	56.7%
	(2a)	**9,092.3**	9,106.9	0.16%	2100	2.8%
	(2b)	9092.3	**11,266.8**	19%	2100	17.0%
	(3a)	9,092.3	**40,054.2**	77.3%	2100	76.7%
	(3b)	9,428.2	**9,449.9**	0.23%	2100	1.0%
0.03	(1a)	**9,124.3**	91,24.3	0%	2	7.7%
	(1b)	**22,762.1**	22,762.1	0%	28	56.6%
	(2a)	**9,092.3**	9,174.9	0.9%	2100	8.0%
	(2b)	9,092.3	**13,981.7**	34.9%	2100	29.3%
	(3a)	9,092.3	**41,366.2**	78.0%	2100	76.1%
	(3b)	9,485.1	**9,994.8**	5.1%	2100	1.1%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 9,100.8 and 9,449.9; true optimal value is 9,353.2
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 9,124.3 and 9,994.8; true optimal value is 9,884.0

Computational results

- Bonferroni lower bound and Dawson & Sankoff upper bound consistently perform better than others
- Weaker correlation in uncertainty leads to easier-to-solve models
- MIQCP formulation of Dawson & Sankoff bound is challenging

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Future directions

Summary

- Working on algorithm development for approximating chance-constrained optimization models
- Modeling interdiction with AC optimal power flow
- Investigating other applications of chance-constrained models, such as public health

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Summary

- Working on algorithm development for approximating chance-constrained optimization models
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We seek collaborations with faculty and students!
Won a $80,000 Laboratory Directed Research and Development grant on "Chance-Constrained Optimization for Critical Infrastructure Protection".

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Possible reasons for long computation time of naive solve

- No extended variable formulation above
- Big M
- Less reliable regime, more combinations to choose from

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