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Global Optimization for AC Optimal Power Flow Applications

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Collaborators:

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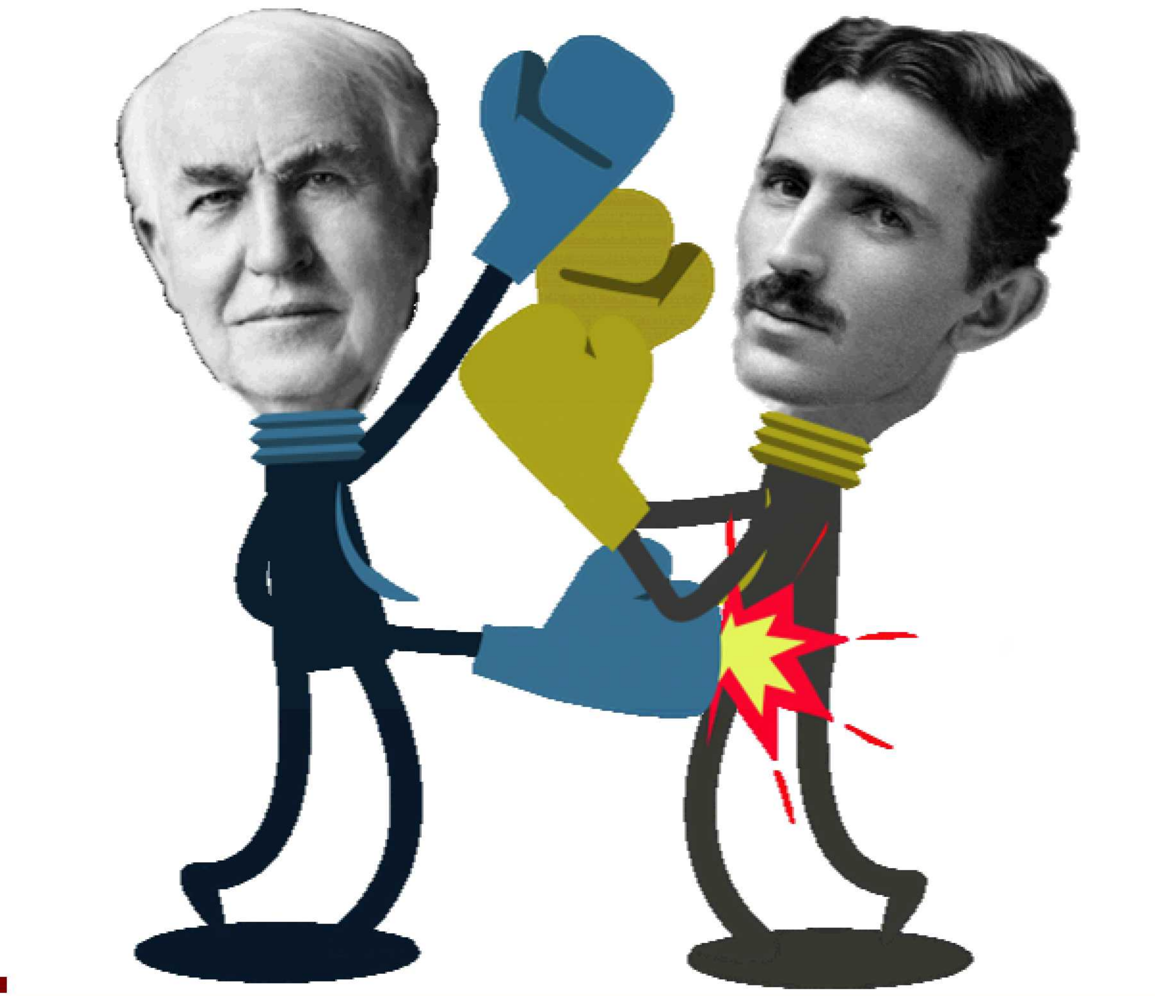
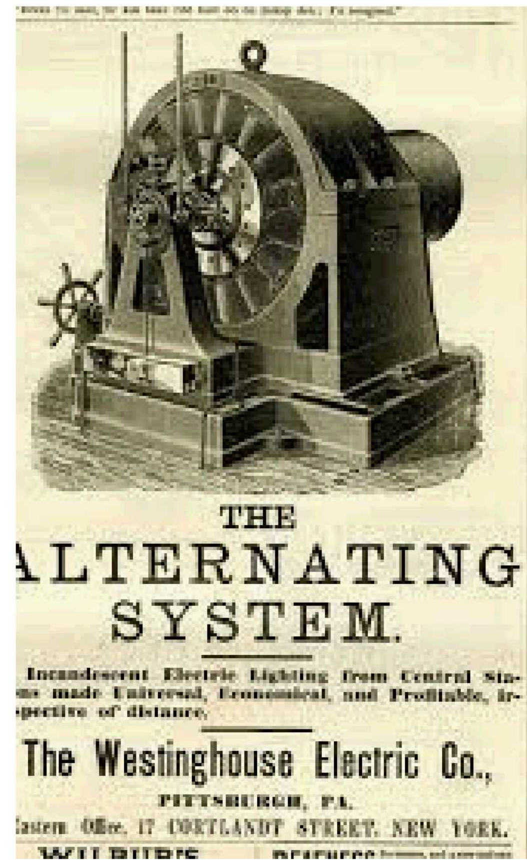
Overview

- Mathematical Preface
- Motivations & Background
- Today's Practice
- Our Contributions
- Ongoing Challenges

MATHEMATICAL PREFACE

The War of Currents (circa. 1880-1890)





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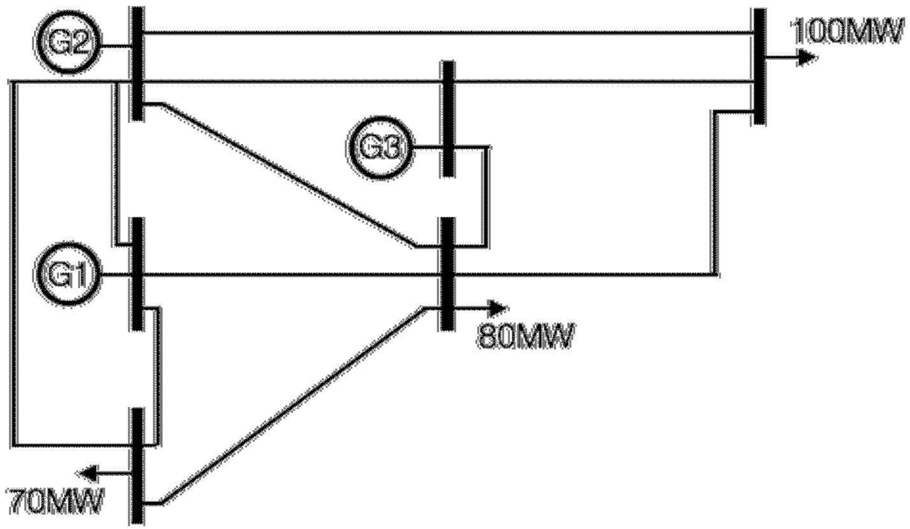
Generation & Transmission Power Network



6 Buses

3 Generators

3 Loads (250 MW)

11 Transmission Lines

What should the online generator production levels be to minimize cost?

Laws Governing Power Systems Physics

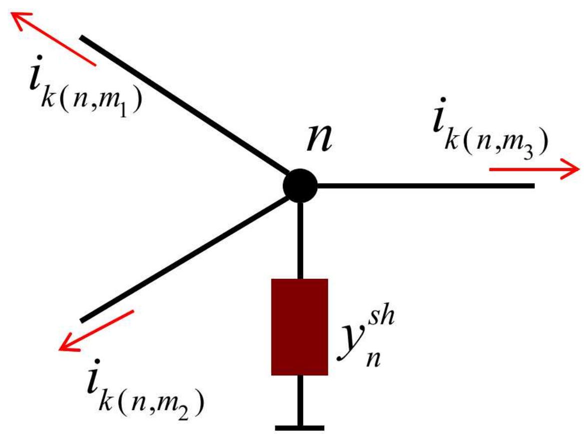
- Ohm's Law: $v = iR \qquad i = R^{-1}v$ DC Circuits

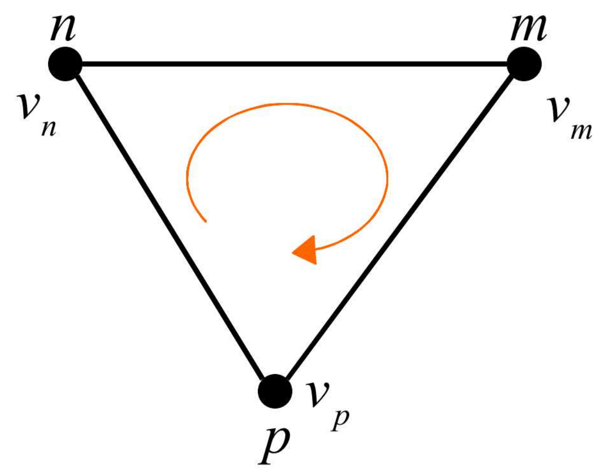
  $v = iZ \qquad i = Z^{*}v = Yv$ AC Circuits

- Joule's First Law: $p = vi$ DC Circuits

  $s = vi^{*}$ AC Circuits

- Kirchhoff's Laws:





Kirchhoff's Current Law (KCL)

$$i_n = \sum_{p\in\{1,2,3\}} i_{k(n,m_p)} + y_n^{sh} v_n$$

Kirchhoff's Voltage Law (KVL)

$$\left(v_n - v_m\right) + \left(v_m - v_p\right) + \left(v_p - v_n\right) = 0$$

The AC Optimal Power Flow (ACOPF)

$l \in \mathcal{G} :=$ set of generators
$n \in \mathcal{N} :=$ set of all nodes
$k \in \mathcal{K} :=$ set of all branches

Cost Function $\min \sum_{l \in \mathcal{G}} f_l^g \left(P_l^g \right)$

$$P_l^{\min} \leq P_l^g \leq P_l^{\max}$$
$$Q_l^{\min} \leq Q_l^g \leq Q_l^{\max}$$

Real/Reactive Generation Limits

$$V_n^{\min} \leq \left| V_n \right| \leq V_n^{\max}$$

Voltage Limits

$$V_n I_n^* = P_n + \mathbf{j} Q_n = \sum_{l(n)} P_l^g - P_n^d + \mathbf{j} \left[\sum_{l(n)} Q_l^g - Q_n^d \right]$$

Power Balancing

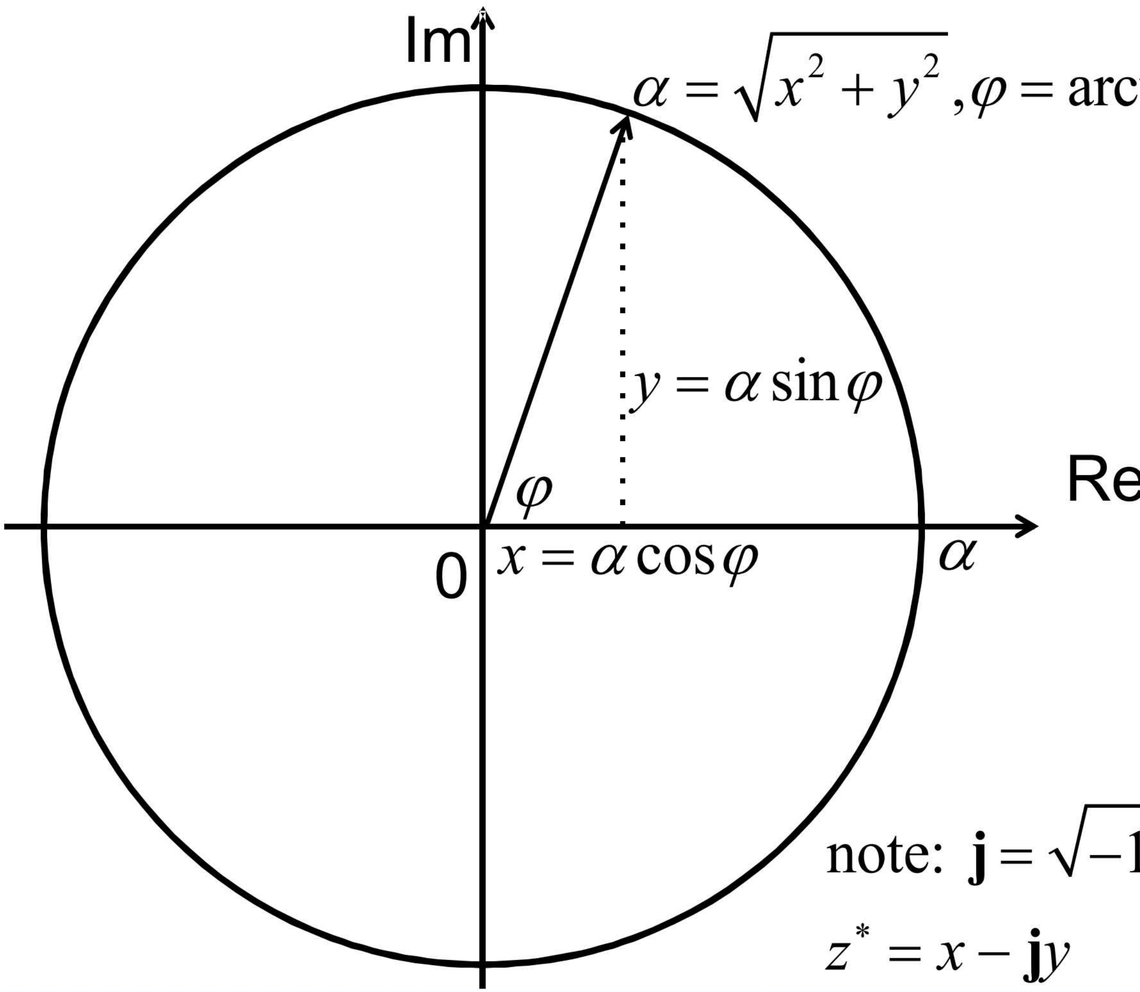
$$\sqrt{P_k^{\,2} + Q_k^{\,2}} \leq S_k^{\max}$$

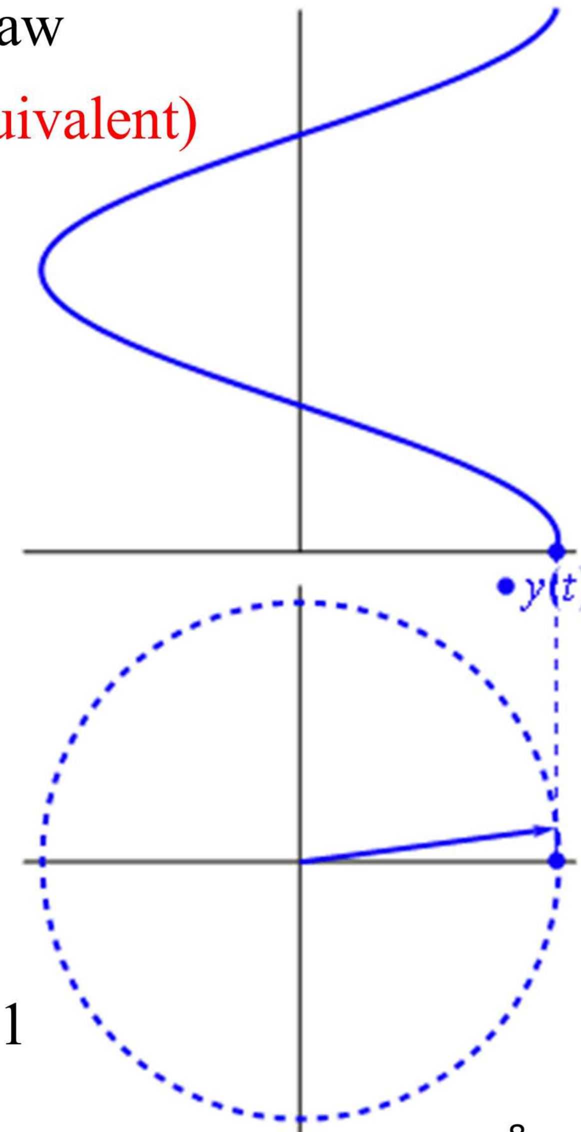
Network Constraints

Complex Number Conversion

Polar: $z = \alpha\angle\varphi = \alpha e^{\mathbf{j}\varphi} = \alpha(\cos\varphi + \mathbf{j}\sin\varphi)$ Euler's Law

Rectangular: $z = x + \mathbf{j}y$ Polar $\leftrightarrow$ Rectangular (Equivalent)





note: $\mathbf{j} = \sqrt{-1}, \mathbf{j}^2 = -1$

$z^* = x - \mathbf{j}y$

MOTIVATIONS & BACKGROUND

R&D Perspectives

Background

- 1962: Carpentier formulates the ACOPF based upon KKT conditions
- 1960's to present day: Trends with algorithmic advancements in OR
- 21st century: Global convergence methods (Phan, Jabr, Bai, Lavaei)

Motivations

- Co-optimizes Real and Reactive Power Injections
- Changing Energy Landscape
  - More utility-scale renewables
  - More distributed resources
- Co-Optimize for Market Efficiency and Security
- Practical Application

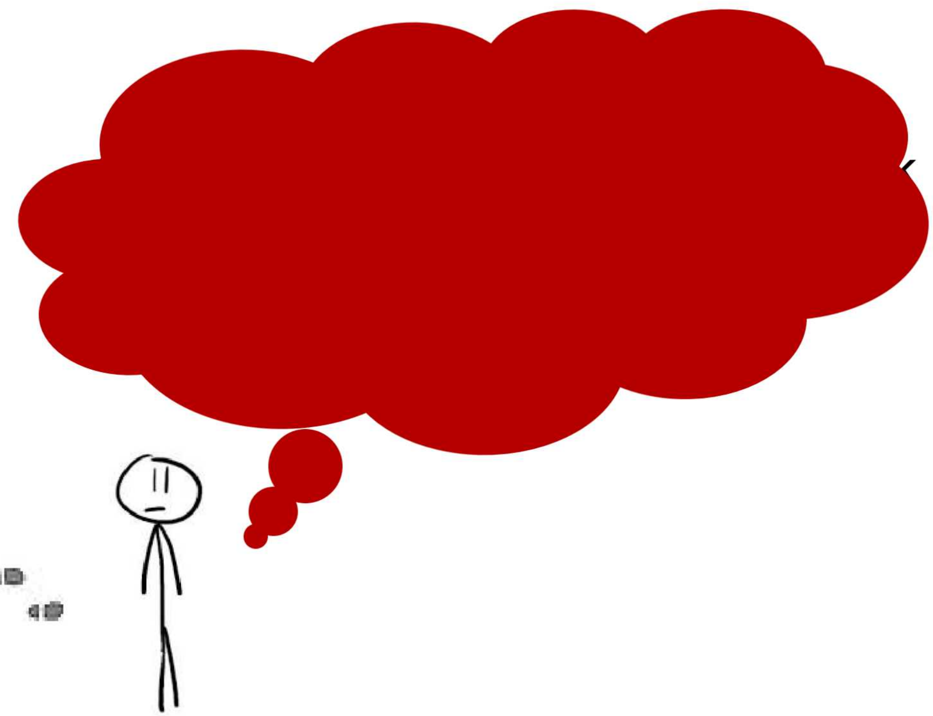
Where are we going?

- In Industry?
  - “[redacted]”

- In Academia?
  - Conventions/Assumptions
  - Toy Networks (IEEE, MATPOWER)
  - Little Knowledge of Practice

The Hype:

- N-1, N-1-1, N-k SCOPF
- ________ + ACOPF

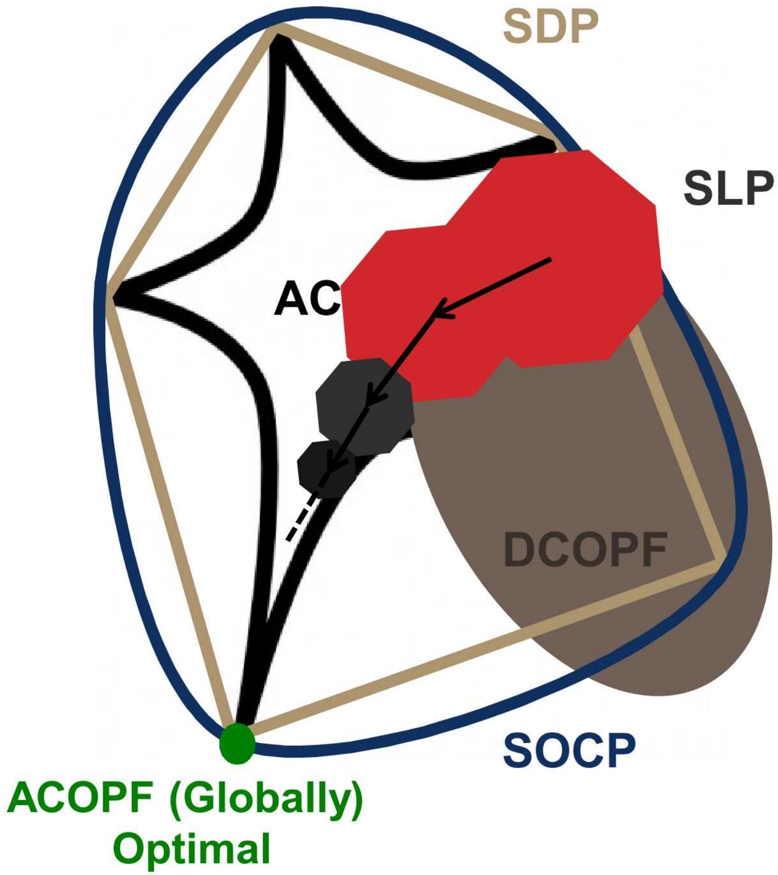


The Bottleneck:

- ACOPF

Solution Techniques

- **DCOPF (Linearized Transmission Models)**
  - Linear B-Theta, PTDF
  - Extensions with losses
- **ACOPF Convex Relaxations**
  - SDP (Bai et al., Lavaei and Low)
  - SOCP (Jabr, Kocuk et al.)
  - QCP (Coffrin et al., Hijazi et al.)
- **ACOPF Approximations**
  - Decoupled Methods
  - Iterative Methods (e.g., SLP)

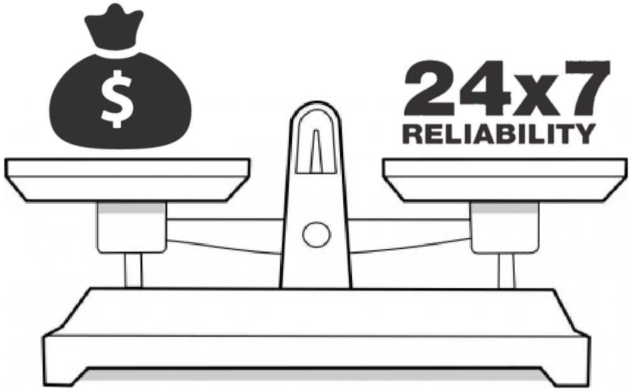


TODAY'S PRACTICE

System Operator Goals

1. Ensure Just and Reasonable Rates, Terms, and Conditions

2. Promote Safe, Reliable, Secure, and Efficient Infrastructure



… Deliver reliable and affordable electric power

Wholesale Market (Sale for Resale)

- Federal Energy Regulatory Commission (FERC)

  Regulatory Authority:

  - Federal Power Act (FPA)
  - Public Utility Regulatory Policies Act (PURPA)

Retail Market (Sale for Use)

- State Regulatory Commissions

  Regulatory authority varies by state statute

Real-World OPF Applications

Day-Ahead Market | Residual Unit Commitment | Real-Time Market Look-Ahead | Real-Time Economic Dispatch

Operations and Pricing (Market Layer)

DCOPF | DCOPF with Losses | Decoupled OPF | AC Feasibility | ACOPF

Mathematical Representation of Operational/Physical Constraints (Software Layer)

Transmission Elements | Generation Resources | Controllable Devices | Operational and Thermal Limits | Demand

Power Grid (Physical Layer)

The ACOPF and its Approximations in Practice

	DCOPF (+AC Feasibility)	Decoupled OPF	ACOPF
Strengths	Linear: $\theta = -B^{-1}P$ (susceptance matrix B) Reasonably accurate for small θ Reliable and fast performance	Solves for both real (P) and reactive (Q) power Linear: Strong decoupling between P-θ and Q-V subproblems	Co-optimizes real (P) and reactive (Q) power Internalizes losses Highly accurate, efficient dispatch
Weaknesses	Ignores reactive power dispatch (Q = 0) and voltage support (V = 1) Network perfectly efficient (lossless) Invalid assumptions on low voltage networks	V restricts P dispatch, or θ restricts Q dispatch High physical P-Q coupling Estimates losses	Nonlinear, nonconvex Complex numbers Market software leverages LP/MIP solvers (not NLP solvers)

The link between physics and prices

- Locational marginal pricing (LMP) is the spot price of electricity
- Dual variable/Lagrange multiplier (λ_n) to real power balancing at all buses

$$p_n - p_n^d + p_n^g = 0 \qquad (\lambda_n)$$

ACOPF
$$p_n = |v_n| \sum_{m \in \mathcal{N}} |v_m| \left(G_{nm} \cos\theta_{nm} + B_{nm} \sin\theta_{nm}\right)$$

DCOPF
$$p_n = \sum_{m \in \mathcal{N}} \left(B_{nm}\theta_{nm}\right) \approx |\tilde{v}_n| \sum_{m \in \mathcal{N}} |\tilde{v}_m| \left(B_{nm}\theta_{nm}\right)$$

DCOPF with losses
$$p_n = \sum_{m \in \mathcal{N}} \left(G_{nm} \left(\theta_{nm}\right)^2 \big/ 2 + B_{nm}\theta_{nm}\right)$$

The LMP incorporates the marginal cost of supplying the next MW of load for a given location in time; includes

1. marginal unit cost,
2. cost of network congestion (due to thermal line limits), and
3. cost of real power losses on the network

OUR CONTRIBUTIONS

ACOPF Formulations

$$\min \sum_{g \in \mathcal{G}} [A_g^2 (p_g^G)^2 + A_g^1 p_g^G + A_g^0]$$

s.t.

$$\sum_{(b,k)\in\mathcal{L}_b^{in}} p_{b,k}^t + \sum_{(b,k)\in\mathcal{L}_b^{out}} p_{b,k}^f + G_b^{sh} w_b + P_b^D - \sum_{g\in\mathcal{G}_b} p_g^G = 0 \qquad \forall\, b \in \mathcal{B}$$

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Definitions for different forms

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or

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$$\sum_{(b,k)\in\mathcal{L}_b^{in}} q_{b,k}^t + \sum_{(b,k)\in\mathcal{L}_b^{out}} q_{b,k}^f - B_b^{sh} w_b + Q_b^D - \sum_{g\in\mathcal{G}_b} q_g^G = 0 \qquad \forall\, b \in \mathcal{B}$$

$$p_{b,k}^f = G_{b,k}^{ff} w_b - G_{b,k}^{ft} c_{b,k} + B_{b,k}^{ft} s_{b,k} \qquad \forall\, (b,k) \in \mathcal{L}$$

$$q_{b,k}^f = -B_{b,k}^{ff} w_b + B_{b,k}^{ft} c_{b,k} + G_{b,k}^{ft} s_{b,k} \qquad \forall\, (b,k) \in \mathcal{L}$$

$$p_{b,k}^t = G_{b,k}^{tt} w_k - G_{b,k}^{tf} c_{k,b} + B_{b,k}^{tf} s_{k,b} \qquad \forall\, (b,k) \in \mathcal{L}$$

$$q_{b,k}^t = -B_{b,k}^{tt} w_k + B_{b,k}^{tf} c_{k,b} + G_{b,k}^{tf} s_{k,b} \qquad \forall\, (b,k) \in \mathcal{L}$$

$$(p_{b,k}^f)^2 + (q_{b,k}^f)^2 \leq (S_{b,k}^{max})^2 \qquad \forall\, (b,k) \in \mathcal{L}$$

$$(p_{b,k}^t)^2 + (q_{b,k}^t)^2 \leq (S_{b,k}^{max})^2 \qquad \forall\, (b,k) \in \mathcal{L}$$

$$(V_b^{min})^2 \leq w_b \leq (V_b^{max})^2 \qquad \forall\, b \in \mathcal{B}$$

$$P_g^{G,min} \leq p_g^G \leq P_g^{G,max} \qquad \forall\, g \in \mathcal{G}$$

$$Q_g^{G,min} \leq q_g^G \leq Q_g^{G,max} \qquad \forall\, g \in \mathcal{G}$$

Definitions for different forms

Rectangular

$$w_b := (v_b^r)^2 + (v_b^j)^2$$

$$c_{b,k} := v_b^r v_k^r + v_b^j v_k^j$$

$$s_{b,k} := v_b^j v_k^r - v_b^r v_k^j$$

or

Polar

$$w_b := v_b^2$$

$$c_{b,k} := v_b v_k \cos(\theta_b - \theta_k)$$

$$s_{b,k} := v_b v_k \sin(\theta_b - \theta_k)$$

ACOPF Relaxations

$$c_{b,k}^2 + s_{b,k}^2 = w_b w_k$$

SOCR

$$\boxed{c_{b,k}^2 + s_{b,k}^2 \leq w_b w_k}$$

[Jabr, 2006] Second order cone (SOC) relaxation (Very efficient – scales well)

[Kocuk et al. 2016] extensions to the SOC relaxation
- Cycle constraints (ensure the voltage angle differences around the cycles sum to 0)
- Semidefinite programming based cuts

[Liu et al. 2017]: global solution of ACOPF using Outer Approximation (NLP ←→ MISOCP) based on SOC relaxation with "cycle constraints" and piecewise refinement of arctans and McCormick for the other side of the SOC.

[Liu et al. 2018]: extended this approach to global solution of unit-commitment problems with AC power flow constraints (48 hour time horizon).
- Outer Approximation approach MISOCP ←→ NLP
- NLP solved using above, MISOCP refined with integer cuts only
- Good results on the few available test cases, but not yet "operational"

Case	Upper Bound ($)	Lower Bound ($)	Optimality Gap (%)	Wall Clock Time (s)	Iteration (k)
6-bus	101,763	101,740	0.02%	8.5	2
RTS-79	895,040	894,392	0.07%	1394	6
RTS-96	886,362	885,707	0.07%	321.0	1
IEEE-118mod	835,926	833,057	0.34%	14400*	2

ACOPF Relaxations

SOCR

$$c_{b,k}^2 + s_{b,k}^2 \leq w_b w_k$$

Rectangular

$$w_b = (v_b^r)^2 + (v_b^j)^2$$
$$c_{b,k} = v_b^r v_k^r + v_b^j v_k^j$$
$$s_{b,k} = v_b^j v_k^r - v_b^r v_k^j$$

Polar

$$w_b = v_b^2$$
$$c_{b,k} = v_b v_k \cos(\theta_b - \theta_k)$$
$$s_{b,k} = v_b v_k \sin(\theta_b - \theta_k)$$

[Jabr, 2006] Second order cone (SOC) relaxation
(Very efficient – scales well)

[Kocuk et al. 2016] extensions to the SOC relaxation
- Cycle constraints (ensure the voltage angle differences around the cycles sum to 0)
- Semidefinite programming based cuts

McCormick Relaxations?
- SOCR ⊆ R-McC with particular vr, vj bounds

[Hijazi et al. 2017] convex quadratic relaxation of the polar form

ACOPF Relaxations

$$c_{b,k}^2 + s_{b,k}^2 = w_b w_k$$

SOCR

$$\boxed{c_{b,k}^2 + s_{b,k}^2 \leq w_b w_k}$$

[Jabr, 2006] Second order cone (SOC) relaxation (Very efficient – scales well)

[Kocuk et al. 2016] extensions to the SOC relaxation
- Cycle constraints (ensure the voltage angle differences around the cycles sum to 0)
- Semidefinite programming based cuts

[Liu et al. 2017]: global solution of ACOPF using Outer Approximation (NLP ←→ MISOCP) based on SOC relaxation with “cycle constraints” and piecewise refinement of arctans and McCormick for the other side of the SOC.

Table 2
Computational performance. Note that the solver CPU time does not include overhead computational costs related to data processing, model construction, and OBBT.

Case name	Upper bound	Lower bound	Gap (%)	CPU time (s)	Iteration
nesta_case3_lmbd	5812.64	5812.64	0.00	0.25	3
nesta_case5_pjm	17551.89	17536.94	0.09	50.37	16
case6ww	3143.97	3142.55	0.02	0.07	1
nesta_case6_ww	3143.97	3143.43	0.02	0.11	1
case14	8081.52	8081.10	0.01	0.10	1
nesta_case14_ieee	244.05	244.04	0.00	0.07	1
case30	576.89	576.45	0.08	33.01	6
nesta_case30_ieee	204.97	204.78	0.09	250.02	8
case39	41864.12	41862.14	0.00	2.76	1
nesta_case39_epri	96505.52	96499.21	0.01	0.72	1
case57	41737.79	41731.17	0.02	0.92	1
nesta_case57_ieee	1143.27	1143.10	0.01	0.27	1
case118	129660.69	129562.20	0.08	53.83	3
nesta_case118_ieee	3718.64	3696.81	0.59	423.13[a]	4

[a] This case failed to solve to a gap of 0.1% in the allotted time (3600 s). The reported gap of 0.59% was achieved in 423 s.

Defining the Reference Bus

- The reference bus is used to specify the voltage angle at one bus.

- The solution to the ACOPF problem is not unique without a reference bus, defined as:

Assume Voltage Phasor: $\mathrm{V}\angle\theta = V\cos\theta + \mathbf{j}V\sin\theta = v^r + \mathbf{j}v^j$

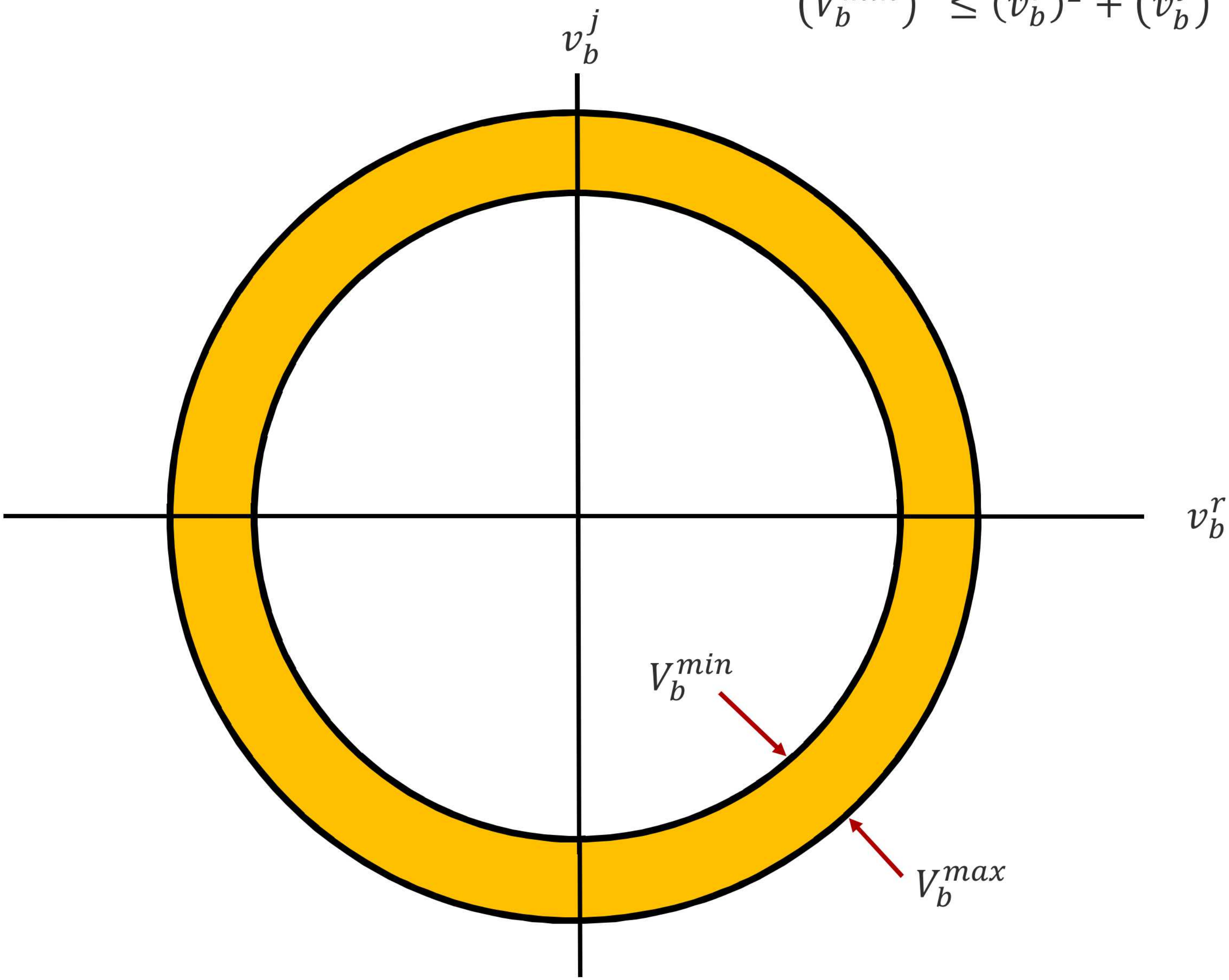
The reference bus (ref) with $\theta_{ref} = 0 \rightarrow$

$$v_b^r = \mathrm{V_b}$$
$$V_b^{min} \leq v_b^r \leq V_b^{max}$$
$$v_b^j = 0$$

- Can we tighten the SOCP?

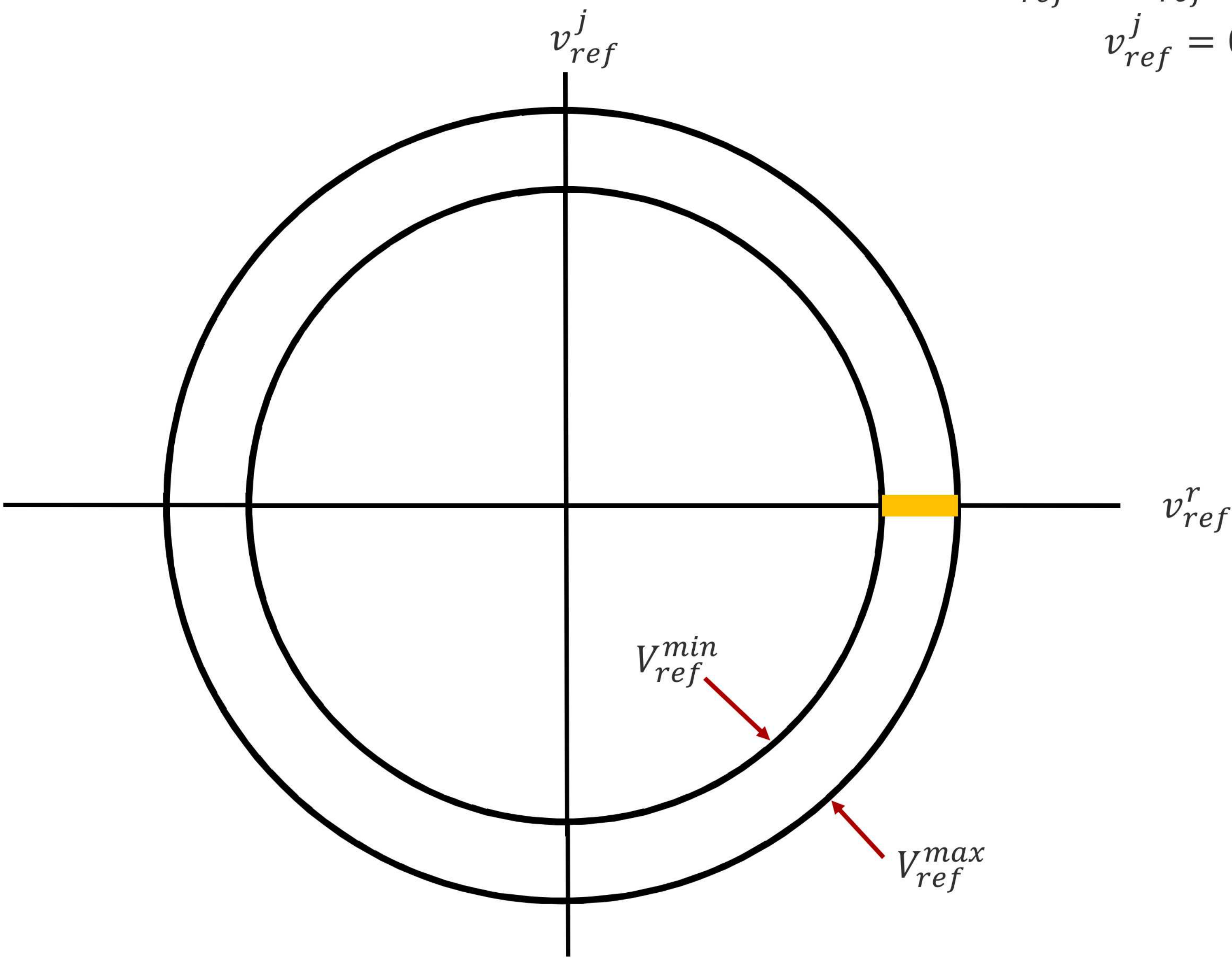
Feasible region of bus voltage constraint

$$\left(V_b^{min}\right)^2 \leq (v_b^r)^2 + \left(v_b^j\right)^2 \leq (V_b^{max})^2$$



Feasible region of reference bus voltage constraints

$$V_{ref}^{min} \le v_{ref}^{r} \le V_{ref}^{max}$$
$$v_{ref}^{j} = 0$$



Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \le \widehat{v_b^r} \le (V_b^{r,\max} + V_b^{r,\min}) v_b^r - V_b^{r,\max} V_b^{r,\min}$$

$$(v_b^j)^2 \le \widehat{v_b^j} \le (V_b^{j,\max} + V_b^{j,\min}) v_b^j - V_b^{j,\max} V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \le v_b^r, v_b^j \le V_b^{\max}$$

$$V_{ref}^{\min} \le v_{ref}^r \le V_{ref}^{\max}$$

$$v_{ref}^j = 0$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \le \widehat{v_b^r} \le (V_b^{r,\max} + V_b^{r,\min}) v_b^r - V_b^{r,\max} V_b^{r,\min}$$

$$(v_b^j)^2 \le \widehat{v_b^j} \le (V_b^{j,\max} + V_b^{j,\min}) v_b^j - V_b^{j,\max} V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \le v_b^r, v_b^j \le V_b^{\max}$$

$$V_{ref}^{\min} \le v_{ref}^r \le V_{ref}^{\max}$$

$$v_{ref}^j = 0$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \le \widehat{v_b^r} \le (V_b^{r,\max} + V_b^{r,\min})v_b^r - V_b^{r,\max}V_b^{r,\min}$$

$$(v_b^j)^2 \le \widehat{v_b^j} \le (V_b^{j,\max} + V_b^{j,\min})v_b^j - V_b^{j,\max}V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \le v_b^r, v_b^j \le V_b^{\max}$$

$$V_{ref}^{\min} \le v_{ref}^r \le V_{ref}^{\max}$$

$$v_{ref}^j = 0$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \leq \widehat{v_b^r} \leq (V_b^{r,\max} + V_b^{r,\min})v_b^r - V_b^{r,\max}V_b^{r,\min}$$

$$(v_b^j)^2 \leq \widehat{v_b^j} \leq (V_b^{j,\max} + V_b^{j,\min})v_b^j - V_b^{j,\max}V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \leq v_b^r, v_b^j \leq V_b^{\max}$$

$$V_{ref}^{\min} \leq v_{ref}^r \leq V_{ref}^{\max}$$

$$v_{ref}^j = 0$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \le \widehat{v_b^r} \le (V_b^{r,\max} + V_b^{r,\min})v_b^r - V_b^{r,\max}V_b^{r,\min}$$

$$(v_b^j)^2 \le \widehat{v_b^j} \le (V_b^{j,\max} + V_b^{j,\min})v_b^j - V_b^{j,\max}V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \le v_b^r, v_b^j \le V_b^{\max}$$

$$V_{ref}^{\min} \le v_{ref}^r \le V_{ref}^{\max}$$

$$v_{ref}^j = 0$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \le \widehat{v_b^r} \le (V_b^{r,\max} + V_b^{r,\min})v_b^r - V_b^{r,\max} V_b^{r,\min}$$

$$(v_b^j)^2 \le \widehat{v_b^j} \le (V_b^{j,\max} + V_b^{j,\min})v_b^j - V_b^{j,\max} V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

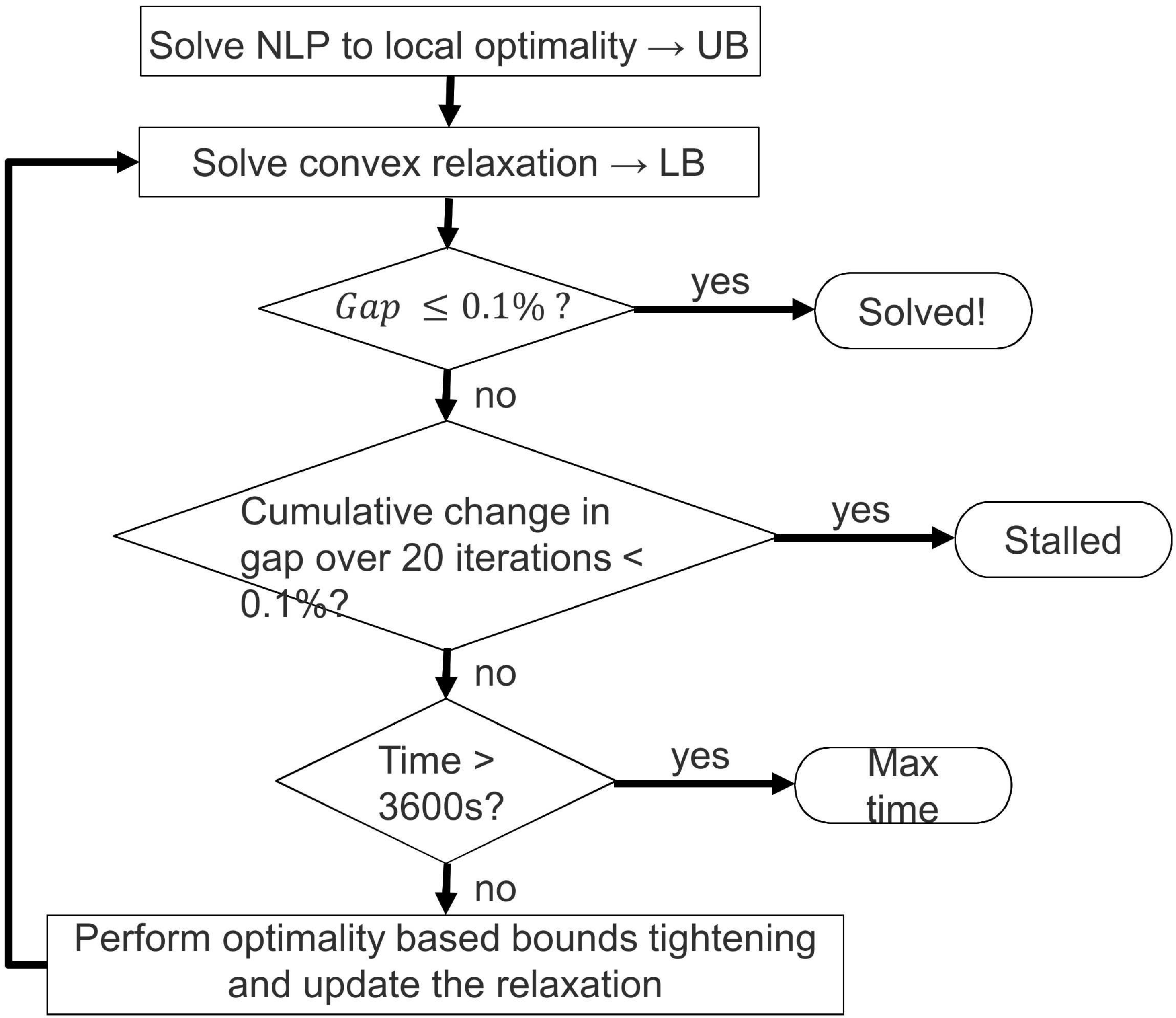
$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \le v_b^r, v_b^j \le V_b^{\max}$$

$$V_{ref}^{\min} \le v_{ref}^r \le V_{ref}^{\max}$$

$$v_{ref}^j = 0$$

Computational Tests



Optimality Based Bounds Tightening (OBBT)

$$\min / \max \quad [variable]$$

s.t.

$$[convex\ relaxation]$$

$$\sum_{g \in \mathcal{G}} [A_g^2 (p_g^G)^2 + A_g^1 p_g^G + A_g^0] \leq UB$$

Computational Tests

- Comparison of Convex Relaxations of the ACOPF
  - SOCR
  - SOCR + Rectangular McCormick (R-McC)
  - SOCR + QCR
  - Variants:
    - With and Without Reference Bus in R-McC
    - With and Without Upper Bound (UB) in OBBT
- OBBT parallelized: 24 nodes, 16 cores per node (2.6 GHz Intel SB) – 12 tasks per node
- NESTA Archive (opf, sad, api) of cases up to 300 buses [Coffrin et al. 2014]

Pyomo Tools

- Python-based mathematical programming environment
- Packages in development:
  - minlp-tools
    - Parallel bounds tightening
    - Parallel branch & bound
    - Outer approximations, relaxations, and piecewise refinement management
    - MIP solver callbacks
  - electric grid research toolkit (egret)
    - Modular power systems model building
    - OPF and its approximations and relaxations
    - Unit commitment
    - Line switching
    - Policy/stakeholder-driven operations and planning
    - Integration in production cost model and stochastic programs

Optimality Gap (%)

<=0.1	0.1 - 5	>5

case_name	SOC
3_lmbd	1.32
4_gs	0.00
5_pjm	14.54
6_c	0.30
6_ww	0.63
9_wscc	0.00
14_ieee	0.11
24_ieee_rts	0.01
29_edin	0.12
30_as	0.06
30_fsr	0.39
30_ieee	15.88
39_epri	0.05
57_ieee	0.06
73_ieee_rts	0.03
118_ieee	1.83
162_ieee_dtc	4.03
189_edin	0.21
300_ieee	1.18
3_lmbd__api	3.30
4_gs__api	0.65
5_pjm__api	0.28
6_c__api	0.35
9_wscc__api	0.00
14_ieee__api	1.34
24_ieee_rts__api	20.75
29_edin__api	0.42
30_as__api	4.76
30_fsr__api	45.97

case_name	SOC
30_ieee__api	1.01
39_epri__api	2.99
57_ieee__api	0.21
73_ieee_rts__api	14.39
89_pegase__api	20.43
118_ieee__api	43.91
162_ieee_dtc__api	1.34
189_edin__api	5.67
300_ieee__api	0.71
3_lmbd__sad	4.28
4_gs__sad	4.90
5_pjm__sad	3.61
6_c__sad	1.36
6_ww__sad	0.80
9_wscc__sad	1.50
14_ieee__sad	0.06
24_ieee_rts__sad	11.42
29_edin__sad	34.68
30_as__sad	9.16
30_fsr__sad	0.62
30_ieee__sad	5.84
39_epri__sad	0.11
57_ieee__sad	0.11
73_ieee_rts__sad	8.37
89_pegase__sad	0.28
118_ieee__sad	12.77
162_ieee_dtc__sad	7.08
189_edin__sad	2.25
300_ieee__sad	1.26

Optimality Gap (%)

<=0.1	0.1 - 5	>5

case_name	SOC	SOC R-McC (no RB) (with UB)
3_lmbd	1.32	1.32
4_gs	0.00	0.00
5_pjm	14.54	14.54
6_c	0.30	0.30
6_ww	0.63	0.63
9_wscc	0.00	0.00
14_ieee	0.11	0.11
24_ieee_rts	0.01	0.01
29_edin	0.12	0.12
30_as	0.06	0.06
30_fsr	0.39	0.39
30_ieee	15.88	15.88
39_epri	0.05	0.05
57_ieee	0.06	0.06
73_ieee_rts	0.03	0.03
118_ieee	1.83	1.83
162_ieee_dtc	4.03	4.03
189_edin	0.21	0.21
300_ieee	1.18	1.18
3_lmbd__api	3.30	3.30
4_gs__api	0.65	0.65
5_pjm__api	0.28	0.28
6_c__api	0.35	0.35
9_wscc__api	0.00	0.00
14_ieee__api	1.34	1.34
24_ieee_rts__api	20.75	20.75
29_edin__api	0.42	0.42
30_as__api	4.76	4.76
30_fsr__api	45.97	45.97

case_name	SOC	SOC R-McC (no RB) (with UB)
30_ieee__api	1.01	1.01
39_epri__api	2.99	2.99
57_ieee__api	0.21	0.21
73_ieee_rts__api	14.39	14.39
89_pegase__api	20.43	20.43
118_ieee__api	43.91	43.91
162_ieee_dtc__api	1.34	1.34
189_edin__api	5.67	5.67
300_ieee__api	0.71	0.71
3_lmbd__sad	4.28	4.28
4_gs__sad	4.90	4.90
5_pjm__sad	3.61	3.61
6_c__sad	1.36	1.36
6_ww__sad	0.80	0.80
9_wscc__sad	1.50	1.50
14_ieee__sad	0.06	0.06
24_ieee_rts__sad	11.42	11.42
29_edin__sad	34.68	34.68
30_as__sad	9.16	9.16
30_fsr__sad	0.62	0.62
30_ieee__sad	5.84	5.84
39_epri__sad	0.11	0.11
57_ieee__sad	0.11	0.11
73_ieee_rts__sad	8.37	8.37
89_pegase__sad	0.28	0.28
118_ieee__sad	12.77	12.77
162_ieee_dtc__sad	7.08	7.08
189_edin__sad	2.25	2.25
300_ieee__sad	1.26	1.26

Optimality Gap (%)

<=0.1	0.1 - 5	>5

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)
3_lmbd	1.32	1.32	0.49
4_gs	0.00	0.00	0.00
5_pjm	14.54	14.54	5.00
6_c	0.30	0.30	0.10
6_ww	0.63	0.63	0.09
9_wscc	0.00	0.00	0.00
14_ieee	0.11	0.11	0.06
24_ieee_rts	0.01	0.01	0.01
29_edin	0.12	0.12	0.10
30_as	0.06	0.06	0.06
30_fsr	0.39	0.39	0.39
30_ieee	15.88	15.88	0.08
39_epri	0.05	0.05	0.05
57_ieee	0.06	0.06	0.06
73_ieee_rts	0.03	0.03	0.03
118_ieee	1.83	1.83	1.59
162_ieee_dtc	4.03	4.03	4.03
189_edin	0.21	0.21	0.21
300_ieee	1.18	1.18	1.18
3_lmbd__api	3.30	3.30	0.07
4_gs__api	0.65	0.65	0.07
5_pjm__api	0.28	0.28	0.06
6_c__api	0.35	0.35	0.05
9_wscc__api	0.00	0.00	0.00
14_ieee__api	1.34	1.34	0.22
24_ieee_rts__api	20.75	20.75	2.03
29_edin__api	0.42	0.42	0.42
30_as__api	4.76	4.76	0.28
30_fsr__api	45.97	45.97	41.63

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)
30_ieee__api	1.01	1.01	0.09
39_epri__api	2.99	2.99	0.29
57_ieee__api	0.21	0.21	0.08
73_ieee_rts__api	14.39	14.39	14.39
89_pegase__api	20.43	20.43	20.20
118_ieee__api	43.91	43.91	26.87
162_ieee_dtc__api	1.34	1.34	0.98
189_edin__api	5.67	5.67	5.45
300_ieee__api	0.71	0.71	0.71
3_lmbd__sad	4.28	4.28	0.05
4_gs__sad	4.90	4.90	0.01
5_pjm__sad	3.61	3.61	0.03
6_c__sad	1.36	1.36	0.01
6_ww__sad	0.80	0.80	0.05
9_wscc__sad	1.50	1.50	0.01
14_ieee__sad	0.06	0.06	0.06
24_ieee_rts__sad	11.42	11.42	0.08
29_edin__sad	34.68	34.68	2.39
30_as__sad	9.16	9.16	0.24
30_fsr__sad	0.62	0.62	0.23
30_ieee__sad	5.84	5.84	0.08
39_epri__sad	0.11	0.11	0.04
57_ieee__sad	0.11	0.11	0.10
73_ieee_rts__sad	8.37	8.37	2.59
89_pegase__sad	0.28	0.28	0.27
118_ieee__sad	12.77	12.77	5.13
162_ieee_dtc__sad	7.08	7.08	7.08
189_edin__sad	2.25	2.25	2.17
300_ieee__sad	1.26	1.26	1.26

Optimality Gap (%)

<=0.1 | 0.1 - 5 | >5

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)
3_lmbd	1.32	1.32	0.49	0.03
4_gs	0.00	0.00	0.00	0.00
5_pjm	14.54	14.54	5.00	0.09
6_c	0.30	0.30	0.10	0.01
6_ww	0.63	0.63	0.09	0.05
9_wscc	0.00	0.00	0.00	0.00
14_ieee	0.11	0.11	0.06	0.06
24_ieee_rts	0.01	0.01	0.01	0.01
29_edin	0.12	0.12	0.10	0.08
30_as	0.06	0.06	0.06	0.06
30_fsr	0.39	0.39	0.39	0.07
30_ieee	15.88	15.88	0.08	0.03
39_epri	0.05	0.05	0.05	0.05
57_ieee	0.06	0.06	0.06	0.06
73_ieee_rts	0.03	0.03	0.03	0.03
118_ieee	1.83	1.83	1.59	0.66
162_ieee_dtc	4.03	4.03	4.03	4.03
189_edin	0.21	0.21	0.21	0.21
300_ieee	1.18	1.18	1.18	1.18
3_lmbd__api	3.30	3.30	0.07	0.09
4_gs__api	0.65	0.65	0.07	0.01
5_pjm__api	0.28	0.28	0.06	0.02
6_c__api	0.35	0.35	0.05	0.01
9_wscc__api	0.00	0.00	0.00	0.00
14_ieee__api	1.34	1.34	0.22	0.04
24_ieee_rts__api	20.75	20.75	2.03	0.56
29_edin__api	0.42	0.42	0.42	0.42
30_as__api	4.76	4.76	0.28	0.07
30_fsr__api	45.97	45.97	41.63	41.20

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)
30_ieee__api	1.01	1.01	0.09	0.09
39_epri__api	2.99	2.99	0.29	0.10
57_ieee__api	0.21	0.21	0.08	0.06
73_ieee_rts__api	14.39	14.39	14.39	14.39
89_pegase__api	20.43	20.43	20.20	20.16
118_ieee__api	43.91	43.91	26.87	26.45
162_ieee_dtc__api	1.34	1.34	0.98	0.94
189_edin__api	5.67	5.67	5.45	2.90
300_ieee__api	0.71	0.71	0.71	0.71
3_lmbd__sad	4.28	4.28	0.05	0.00
4_gs__sad	4.90	4.90	0.01	0.00
5_pjm__sad	3.61	3.61	0.03	0.01
6_c__sad	1.36	1.36	0.01	0.04
6_ww__sad	0.80	0.80	0.05	0.05
9_wscc__sad	1.50	1.50	0.01	0.01
14_ieee__sad	0.06	0.06	0.06	0.06
24_ieee_rts__sad	11.42	11.42	0.08	0.04
29_edin__sad	34.68	34.68	2.39	0.55
30_as__sad	9.16	9.16	0.24	0.05
30_fsr__sad	0.62	0.62	0.23	0.04
30_ieee__sad	5.84	5.84	0.08	0.09
39_epri__sad	0.11	0.11	0.04	0.03
57_ieee__sad	0.11	0.11	0.10	0.10
73_ieee_rts__sad	8.37	8.37	2.59	1.96
89_pegase__sad	0.28	0.28	0.27	0.08
118_ieee__sad	12.77	12.77	5.13	1.47
162_ieee_dtc__sad	7.08	7.08	7.08	7.08
189_edin__sad	2.25	2.25	2.17	1.84
300_ieee__sad	1.26	1.26	1.26	1.26

Optimality Gap (%)

<=0.1	0.1 - 5	>5

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)	SOC QC (no UB)
3_lmbd	1.32	1.32	0.49	0.03	0.19
4_gs	0.00	0.00	0.00	0.00	0.00
5_pjm	14.54	14.54	5.00	0.09	9.29
6_c	0.30	0.30	0.10	0.01	0.10
6_ww	0.63	0.63	0.09	0.05	0.01
9_wscc	0.00	0.00	0.00	0.00	0.00
14_ieee	0.11	0.11	0.06	0.06	0.02
24_ieee_rts	0.01	0.01	0.01	0.01	0.01
29_edin	0.12	0.12	0.10	0.08	0.10
30_as	0.06	0.06	0.06	0.06	0.06
30_fsr	0.39	0.39	0.39	0.07	0.10
30_ieee	15.88	15.88	0.08	0.03	0.05
39_epri	0.05	0.05	0.05	0.05	0.05
57_ieee	0.06	0.06	0.06	0.06	0.06
73_ieee_rts	0.03	0.03	0.03	0.03	0.03
118_ieee	1.83	1.83	1.59	0.66	0.47
162_ieee_dtc	4.03	4.03	4.03	4.03	0.66
189_edin	0.21	0.21	0.21	0.21	0.21
300_ieee	1.18	1.18	1.18	1.18	0.18
3_lmbd__api	3.30	3.30	0.07	0.09	0.08
4_gs__api	0.65	0.65	0.07	0.01	0.03
5_pjm__api	0.28	0.28	0.06	0.02	0.01
6_c__api	0.35	0.35	0.05	0.01	0.08
9_wscc__api	0.00	0.00	0.00	0.00	0.00
14_ieee__api	1.34	1.34	0.22	0.04	0.30
24_ieee_rts__api	20.75	20.75	2.03	0.56	0.30
29_edin__api	0.42	0.42	0.42	0.42	0.09
30_as__api	4.76	4.76	0.28	0.07	0.04
30_fsr__api	45.97	45.97	41.63	41.20	2.41

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)	SOC QC (no UB)
30_ieee__api	1.01	1.01	0.09	0.09	0.07
39_epri__api	2.99	2.99	0.29	0.10	0.05
57_ieee__api	0.21	0.21	0.08	0.06	0.02
73_ieee_rts__api	14.39	14.39	14.39	14.39	0.19
89_pegase__api	20.43	20.43	20.20	20.16	18.88
118_ieee__api	43.91	43.91	26.87	26.45	9.26
162_ieee_dtc__api	1.34	1.34	0.98	0.94	0.10
189_edin__api	5.67	5.67	5.45	2.90	0.33
300_ieee__api	0.71	0.71	0.71	0.71	0.13
3_lmbd__sad	4.28	4.28	0.05	0.00	0.04
4_gs__sad	4.90	4.90	0.01	0.00	0.01
5_pjm__sad	3.61	3.61	0.03	0.01	0.05
6_c__sad	1.36	1.36	0.01	0.04	0.03
6_ww__sad	0.80	0.80	0.05	0.05	0.00
9_wscc__sad	1.50	1.50	0.01	0.01	0.00
14_ieee__sad	0.06	0.06	0.06	0.06	0.06
24_ieee_rts__sad	11.42	11.42	0.08	0.04	0.07
29_edin__sad	34.68	34.68	2.39	0.55	1.16
30_as__sad	9.16	9.16	0.24	0.05	0.06
30_fsr__sad	0.62	0.62	0.23	0.04	0.08
30_ieee__sad	5.84	5.84	0.08	0.09	0.02
39_epri__sad	0.11	0.11	0.04	0.03	0.04
57_ieee__sad	0.11	0.11	0.10	0.10	0.10
73_ieee_rts__sad	8.37	8.37	2.59	1.96	0.07
89_pegase__sad	0.28	0.28	0.27	0.08	0.08
118_ieee__sad	12.77	12.77	5.13	1.47	1.35
162_ieee_dtc__sad	7.08	7.08	7.08	7.08	0.46
189_edin__sad	2.25	2.25	2.17	1.84	0.97
300_ieee__sad	1.26	1.26	1.26	1.26	0.21

Optimality Gap (%)

<=0.1 | 0.1 - 5 | >5

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)	SOC QC (no UB)	SOC QC (with UB)
3_lmbd	1.32	1.32	0.49	0.03	0.19	0.01
4_gs	0.00	0.00	0.00	0.00	0.00	0.00
5_pjm	14.54	14.54	5.00	0.09	9.29	5.68
6_c	0.30	0.30	0.10	0.01	0.10	0.08
6_ww	0.63	0.63	0.09	0.05	0.01	0.01
9_wscc	0.00	0.00	0.00	0.00	0.00	0.00
14_ieee	0.11	0.11	0.06	0.06	0.02	0.01
24_ieee_rts	0.01	0.01	0.01	0.01	0.01	0.01
29_edin	0.12	0.12	0.10	0.08	0.10	0.10
30_as	0.06	0.06	0.06	0.06	0.06	0.06
30_fsr	0.39	0.39	0.39	0.07	0.10	0.08
30_ieee	15.88	15.88	0.08	0.03	0.05	0.05
39_epri	0.05	0.05	0.05	0.05	0.05	0.05
57_ieee	0.06	0.06	0.06	0.06	0.06	0.06
73_ieee_rts	0.03	0.03	0.03	0.03	0.03	0.03
118_ieee	1.83	1.83	1.59	0.66	0.47	0.08
162_ieee_dtc	4.03	4.03	4.03	4.03	0.66	0.08
189_edin	0.21	0.21	0.21	0.21	0.21	0.09
300_ieee	1.18	1.18	1.18	1.18	0.18	0.04
3_lmbd__api	3.30	3.30	0.07	0.09	0.08	0.02
4_gs__api	0.65	0.65	0.07	0.01	0.03	0.06
5_pjm__api	0.28	0.28	0.06	0.02	0.01	0.01
6_c__api	0.35	0.35	0.05	0.01	0.08	0.04
9_wscc__api	0.00	0.00	0.00	0.00	0.00	0.00
14_ieee__api	1.34	1.34	0.22	0.04	0.30	0.08
24_ieee_rts__api	20.75	20.75	2.03	0.56	0.30	0.09
29_edin__api	0.42	0.42	0.42	0.42	0.09	0.05
30_as__api	4.76	4.76	0.28	0.07	0.04	0.02
30_fsr__api	45.97	45.97	41.63	41.20	2.41	0.06

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)	SOC QC (no UB)	SOC QC (with UB)
30_ieee__api	1.01	1.01	0.09	0.09	0.07	0.01
39_epri__api	2.99	2.99	0.29	0.10	0.05	0.04
57_ieee__api	0.21	0.21	0.08	0.06	0.02	0.06
73_ieee_rts__api	14.39	14.39	14.39	14.39	0.19	0.03
89_pegase__api	20.43	20.43	20.20	20.16	18.88	9.09
118_ieee__api	43.91	43.91	26.87	26.45	9.26	8.72
162_ieee_dtc__api	1.34	1.34	0.98	0.94	0.10	0.07
189_edin__api	5.67	5.67	5.45	2.90	0.33	0.13
300_ieee__api	0.71	0.71	0.71	0.71	0.13	0.04
3_lmbd__sad	4.28	4.28	0.05	0.00	0.04	0.03
4_gs__sad	4.90	4.90	0.01	0.00	0.01	0.00
5_pjm__sad	3.61	3.61	0.03	0.01	0.05	0.03
6_c__sad	1.36	1.36	0.01	0.04	0.03	0.03
6_ww__sad	0.80	0.80	0.05	0.05	0.00	0.00
9_wscc__sad	1.50	1.50	0.01	0.01	0.00	0.00
14_ieee__sad	0.06	0.06	0.06	0.06	0.06	0.06
24_ieee_rts__sad	11.42	11.42	0.08	0.04	0.07	0.03
29_edin__sad	34.68	34.68	2.39	0.55	1.16	0.49
30_as__sad	9.16	9.16	0.24	0.05	0.06	0.03
30_fsr__sad	0.62	0.62	0.23	0.04	0.08	0.02
30_ieee__sad	5.84	5.84	0.08	0.09	0.02	0.01
39_epri__sad	0.11	0.11	0.04	0.03	0.04	0.04
57_ieee__sad	0.11	0.11	0.10	0.10	0.10	0.10
73_ieee_rts__sad	8.37	8.37	2.59	1.96	0.07	0.01
89_pegase__sad	0.28	0.28	0.27	0.08	0.08	0.08
118_ieee__sad	12.77	12.77	5.13	1.47	1.35	0.10
162_ieee_dtc__sad	7.08	7.08	7.08	7.08	0.46	0.04
189_edin__sad	2.25	2.25	2.17	1.84	0.97	0.82
300_ieee__sad	1.26	1.26	1.26	1.26	0.21	0.01

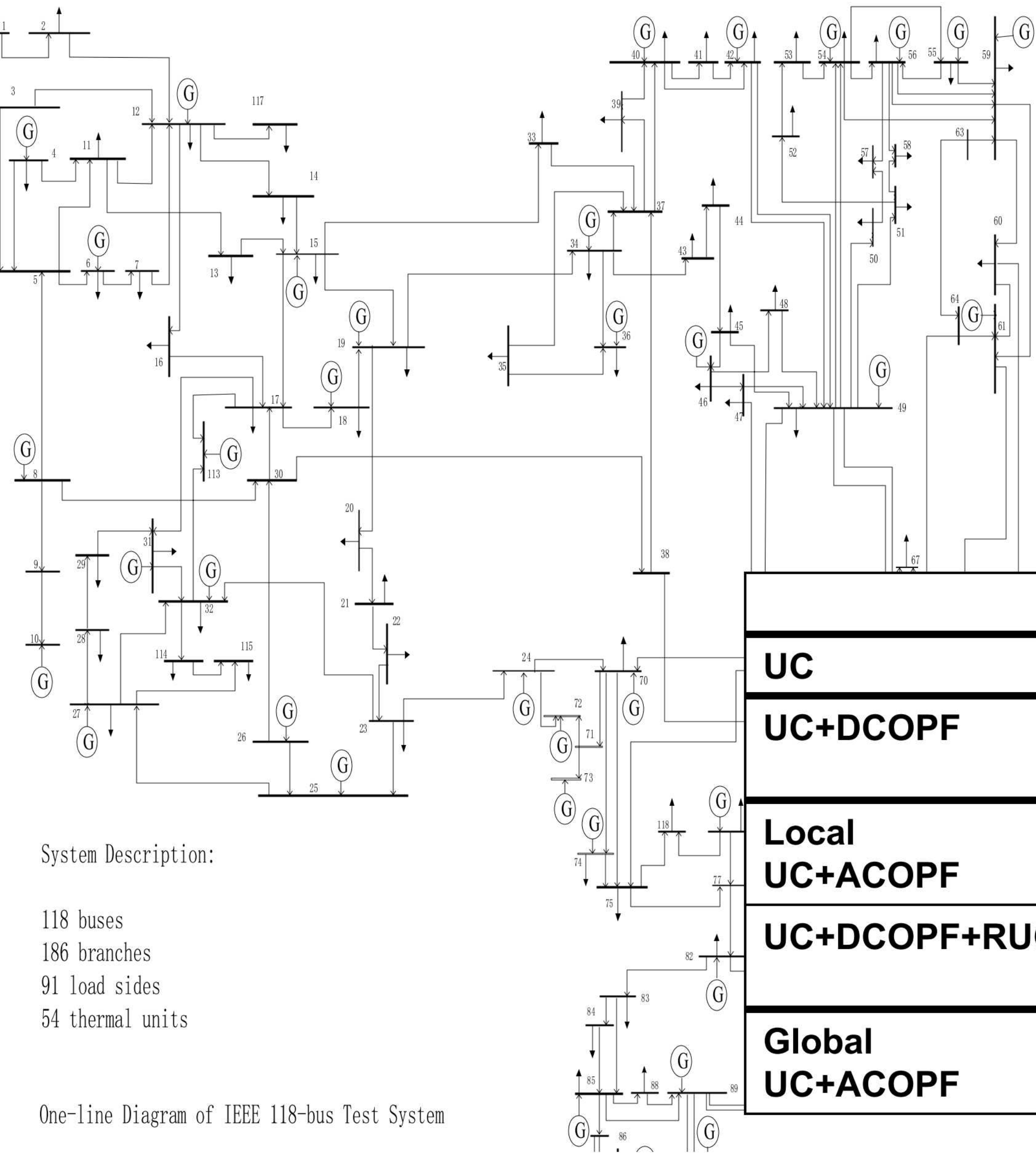
Summary Remarks

- The SOC and QC base relaxations are already effective for most cases, but not all
- Incorporating the reference bus and UB in OBBT improve the performance of pre-existing convex relaxations, e.g.,
  - Between SOC+QC w/ UB and SOC+R-McC, only 5 cases had more than a 0.1% Optimality Gap
  - Computational performance improves
- We present results achieved without branching (only bounds tightening) … straightforward extension ...

ONGOING CHALLENGES

IEEE-118 §

Sandia National Laboratories



118 nodes
54 generators
91 loads
186 network elements/lines
24-hour hourly commitment

	Cost ($)	AC Feasible?
UC	811,658 (base)	NO
UC+DCOPF	814,715 (+0.4%)	NO
Local UC+ACOPF	843,591 (+3.9%)	YES
UC+DCOPF+RUC	844,922 (+4.1%)	YES
Global UC+ACOPF	835,926 (+3.0%)	YES

- Key Takeaway: Results indicate considerable divergence between the market settlements and stability/reliability requirements

§ Data from Fu et al. (2006)

Local v. Global UC+ACOPF Method

Case	Problem Formulation	Upper Bound	Lower Bound	Relative Gap (%)	CPU Time (s)
6-Bus	Global Local	101,763 101,763	101,655 -	0.11% *0.11%*	3.6 0.95
RTS-79	Global Local	895,096 895,281	893,967 -	0.13% *0.15%*	266.4 89.46
IEEE-118	Global Local	835,926 843,591	833,057 -	0.34% *1.25%*	8480 115.23

- **Note**: Thermal limits different in global solution method (apparent power thermal limit) and local solution method (current thermal limit) so a direct comparison (above) is *inexact*
- On the largest test case, the approximation method is over 70x faster, at the cost of 0.91% in relative optimality gap change

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