The Aurora

Electron Transport in the Upper Atmosphere

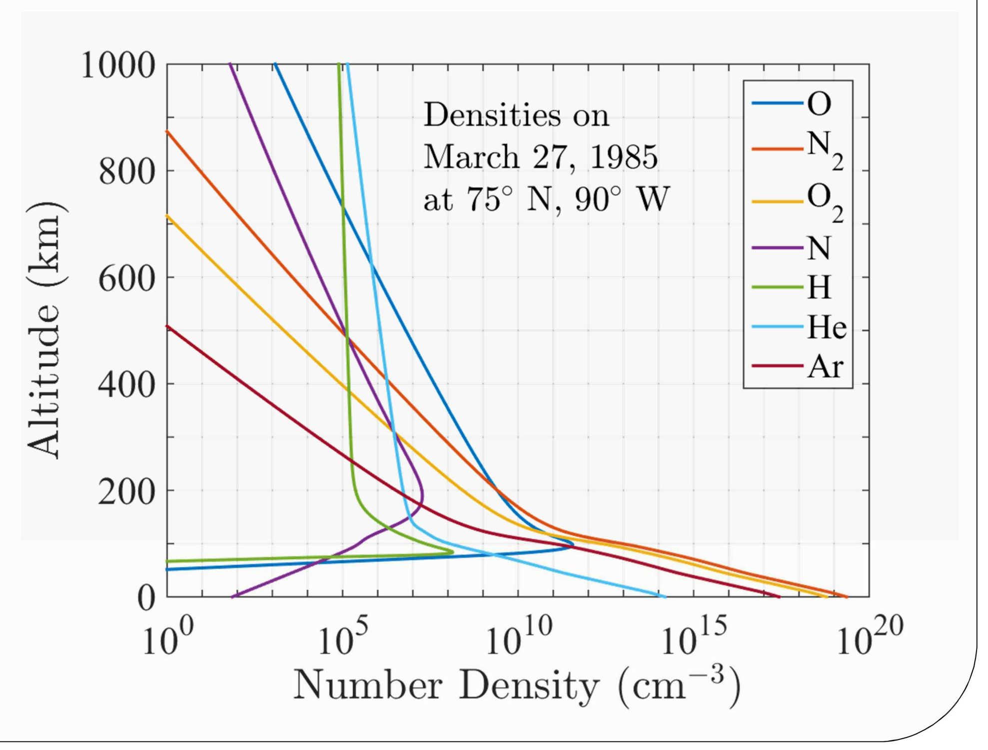
Mark Woods†‡ William Sailor† Mark Holmes‡
†Sandia National Laboratories ‡Rensselaer Polytechnic Institute

Abstract

A new approach for solving the electron transport equation in the upper atmosphere is given. The problem is a very stiff boundary value problem, and it is solved by explicitly decoupling the fast and slow modes. A simplified problem is solved and compared to the solution obtained using a boundary element method. Good agreement is found, demonstrating the effectiveness of the decoupling method. We then solve the full problem and calculate excitation and ionization rates that contribute to visible auroral light and the energy deposition rate.

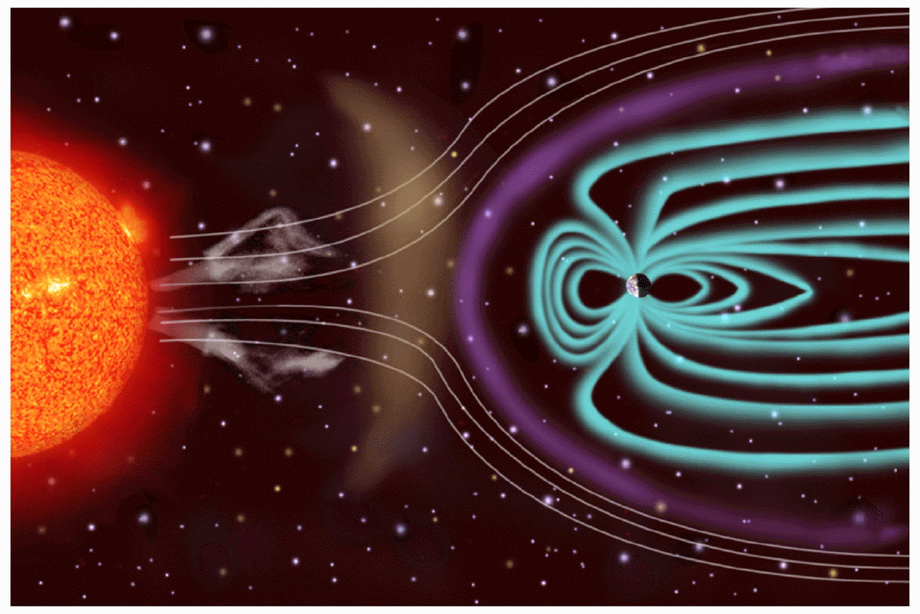
What is the Upper Atmosphere?

- Thermosphere and parts of the ionosphere
- Altitudes strictly below 1000 km
- O, N_2, and O_2 contribute most of the auroral color
- Densities are taken from the MSIS-E-90 atmospheric model



What Causes the Aurora?

- Sun releases charged particles via a coronal mass ejection
- Charged particles scatter off atmospheric particles
- Scattering imparts energy to atmospheric particles
- Atmospheric particles release energy via fluorescence



Electron Transport Assumptions and Equation

- Electron intensity ($cm^{-2}\ s^{-1}\ eV^{-1}\ sr^{-1}$) is a quantity that allows us to calculate excitation rates, ionization rates, and the energy deposition rate
- An equation for electron intensity can be derived from the continuity equation
- Assumptions include
  1. Steady state
  2. Atmosphere is horizontally stratified
  3. Earth's magnetic field is uniform along the local vertical
  4. Collective effects are negligible

$$\mu\frac{\partial I}{\partial z} = \sum_{\substack{\text{species}\\ \xi}} \sum_{\substack{\text{channel}\\ \eta}} n_\xi(z)\left(\int_0^\infty \int_{-1}^1 S_\xi^\eta(E,E',\mu,\mu')I(z,E',\mu')\,d\mu' dE' - \sigma_\xi^\eta(E)I(z,E,\mu)\right)$$

$$I(1000,E,\mu<0) = I_{top}(E,\mu<0) \qquad I(50,E,\mu>0)=0$$

$$50 \le z \le 1000 \text{ km} \qquad E \le 10^5 \text{ eV} \qquad -1 \le \mu \le 1$$

- $I(z,E,\mu)$ is the electron intensity as a function of altitude, energy, and pitch angle cosine
- $n_\xi(z)$ is the number density of species ξ
- $S_\xi^\eta(E,E',\mu,\mu')$ is a function closely related to the differential cross section, which governs how electrons are redistributed in energy and pitch angle upon scattering with species ξ through channel η
- $\sigma_\xi^\eta(E)$ is the cross section for species ξ and channel η

Solution Method

- Discretization yields a linear boundary value problem (BVP)

$$\frac{d\mathbf{I}}{dz} = \mathbf{A}(z)\mathbf{I} + \mathbf{q}(z)$$

- Eigenvalues grow exponentially as electrons approach lower altitudes
- Method finds a smooth, invertible matrix function $\mathbf{V}(z)$ such that we can explicitly decouple the system with a similarity transform

$$\mathbf{A}(z) = \mathbf{V}(z)\mathbf{\Lambda}(z)\mathbf{V}^{-1}(z), \qquad \mathbf{\Lambda}(z) = \begin{bmatrix} \mathbf{\Lambda}_-(z) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_0(z) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Lambda}_+(z) \end{bmatrix}$$

Submatrix	Eigenvalues	Difference Method
$\mathbf{\Lambda}_-(z)$	large negative	backward Euler form left to right
$\mathbf{\Lambda}_0(z)$	small	trapezoid rule
$\mathbf{\Lambda}_+(z)$	large positive	backward Euler from right to left

Verification: Boundary Element Method

- Let atmosphere be entirely atomic oxygen
- Transform equation to be in terms of scattering depth τ (analogous to optical depth) instead of z
- A fundamental solution $F(\tau,\mu;\tau_0,\mu_0)$ can be found such that

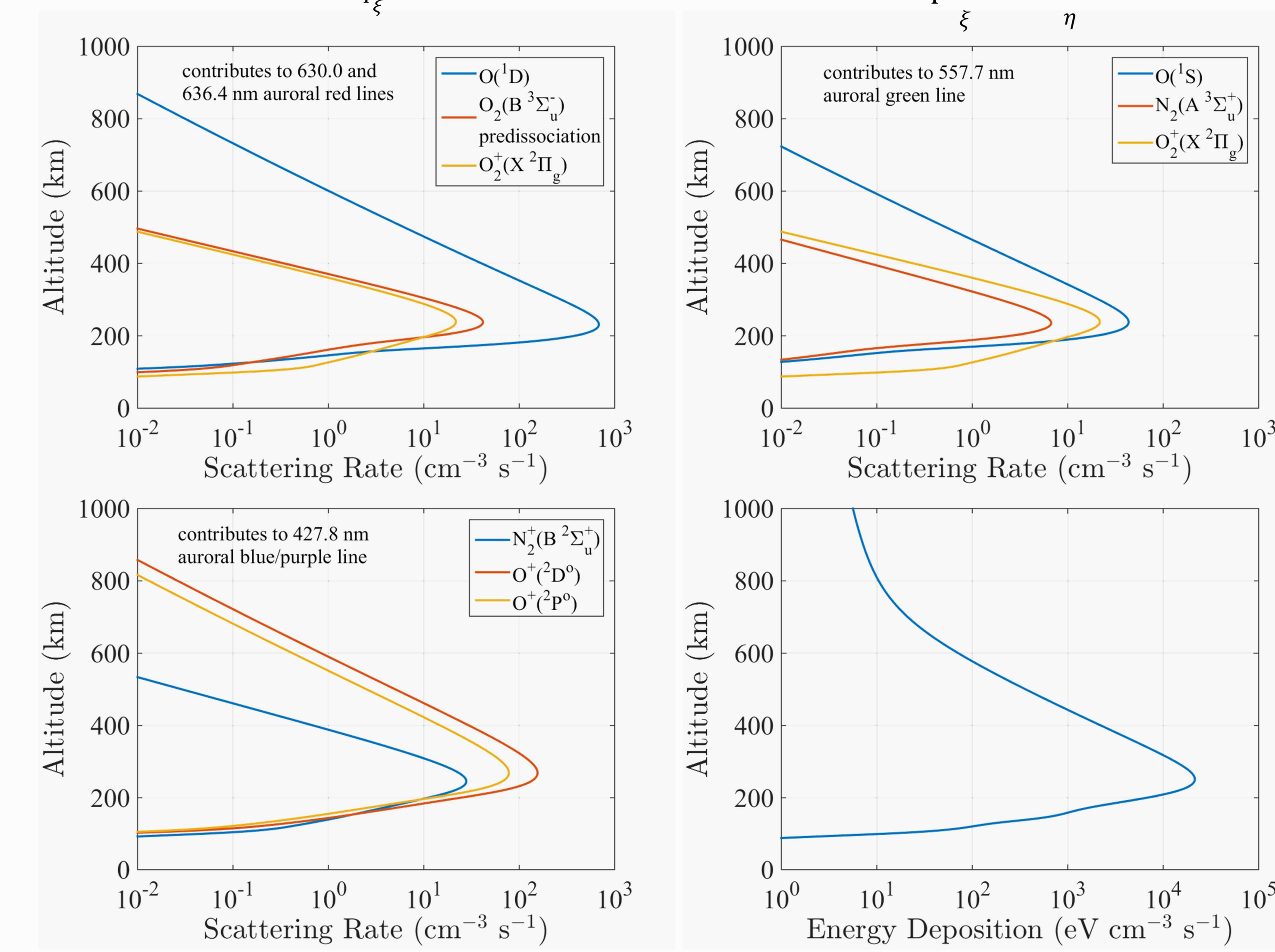
$$I(\tau,\mu) = \int_0^{\tau_{max}}\int_{-1}^1 F(\tau,\mu;\tau_0,\mu_0)q(\tau,\mu)\,d\mu_0\,d\tau_0 - \int_{-1}^1 \mu_0 F(\tau,\mu;\tau_0,\mu_0)I(\tau_0,\mu_0)\Big|_{\tau_0=0}^{\tau_{max}} d\mu_0$$

- Only approximation is the numerical evaluation of the integrals
- Finite difference solution agrees with boundary element solution everywhere to at least three decimal places

Full Numerical Solution

- Incorporate seven neutral species and their cross sections into the model
- Use the electron intensity to calculate excitation and ionization rates that contribute to visible auroral light and the energy deposition rate

$$r_\xi^\eta(z) = n_\xi(z)\int_0^{2\pi}\int_{-1}^1\int_{T_\xi^\eta}^\infty \sigma_\xi^\eta(E)I(z,E,\mu)\,dE\,d\mu\,d\phi, \qquad E_{dep} = \sum_{\substack{\text{species}\\ \xi}}\sum_{\substack{\text{channel}\\ \eta}} T_\xi^\eta r_\xi^\eta(z)$$



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The author gratefully acknowledges support from the Sandia National Laboratories Campus Executive Program, which is funded by the Laboratory Directed Research and Development (LDRD) Program. Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.
SAND No. XXXX

Contact Information
mcwood@sandia.gov woodsm4@rpi.edu

Rensselaer