

Determining series resistance for equivalent circuit models of a PV module

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Abstract — Literature describes various methods for determining a series resistance for a photovoltaic device from measured IV curves. We investigate use of these techniques to estimate the series resistance parameter for a single diode equivalent circuit model. With simulated IV curves we demonstrate that the series resistance values obtained by these techniques differ systematically from the known series resistance parameter values used to generate the curves, indicating that these methods are not suitable for determining the series resistance parameter for the single diode model equation. We present an alternative method to determine the series resistance parameter jointly with the other parameters for the single diode model equation, and demonstrate the accuracy and reliability of this technique in the presence of measurement errors.

I. INTRODUCTION

Current-voltage (IV) characteristics of photovoltaic (PV) modules and systems are frequently modeled using an equivalent circuit comprising one or more diodes, a resistor in parallel with the diode(s), and a resistor in series with the load (Fig 1). An equivalent circuit model with a single diode gives rise to the ‘5 parameter’ equation (Eq. (1)) for the IV characteristic: the single diode equation parameters are photocurrent I_L (A), dark current I_o (A), diode or ideality factor n (unitless), shunt or parallel resistance R_{SH} (Ω) and series resistance R_s (Ω). The single diode equation is obtained by modeling the diode current I_d using Shockley’s ideal diode law and applying Kirchoff’s current law at the indicated junction.

$$I = I_L - I_o \left(\exp \left(\frac{V + IR_s}{nN_s V_{th}} \right) - 1 \right) - \frac{V + IR_s}{R_{SH}} \quad (1)$$

Eq. (1) describes a single IV characteristic, i.e., at a single irradiance level and cell temperature. To predict performance across a range of irradiance and temperature conditions, the diode equation is supplemented with auxiliary equations describing how the diode equation parameters vary with irradiance and/or temperature. Together, the diode equation and auxiliary equations comprise a PV performance model, most often applied to modules and PV systems rather than to single cells. Two popular single diode models for PV modules are PVsyst [1] and the CEC model [2], which is implemented in the System Advisor Model (SAM) software [3]. For both models, R_s is assumed to be constant for all irradiance and temperature conditions.

A fundamental task is to determine the parameters (i.e., the parameters in the auxiliary equations in Table 1) from measured IV curves. Determining the parameters for a single diode model in turn depends on determining values for the five parameters in Eq. (1) for each curve in a set of IV curves. Finding these five values for a single IV curve remains a challenge as evidenced by the multiplicity of published methods (see [4] for a recent survey).

When a single diode model fails to accurately predict measured IV curves, the fault could lie in incorrect model parameters, incorrect model equations, or both. Without methods to independently determine each parameter in the single diode equation, improvements to single diode models are difficult to verify. For example, a recent analysis [5] argues for including a temperature dependence of R_s to represent the dominant effect on series resistance of metals in PV module interconnects, and shows that model prediction error is reduced by including a temperature-dependent R_s . The analysis is persuasive but lacks verification by means of an explicit, empirical relationship between R_s and temperature for individual IV curves, relying instead on changes in the prediction error.

Considering these challenges, methods to determine individual parameters for the single diode equation are of interest. Validated methods may serve to confirm, or refute, model parameter sets and/or model equations. For example, a method to estimate R_s (the focus of this paper) for a single IV curve, when applied to a set of IV curves, would yield a set of R_s values that can expose dependencies on irradiance and temperature which should be represented in the model equations.

To this end, we analyze several published methods for determining a series resistance from measured IV curves. We demonstrate analytically that the values obtained differ systematically from the series resistance parameter needed for the single diode equation. We show numerically that this difference can be substantial and can be sensitive to measurement error in the IV curve. We propose an alternative: a novel procedure that determines R_s along with the other parameters for the single diode equation, and show the uncertainty in the resulting parameter estimates that arises from measurement error.

We use simulated rather than measured IV curves in our analysis of R_s estimation methods. Using simulated IV curves

ensures that a correct value of R_s is known a priori and that the single diode equation is the appropriate model for the IV curve. In contrast, using measured IV curves in this analysis would first require establishing the applicability of the single diode model to the measured device, and also defining a means to confirm the derived R_s values.

II. METHODS FOR DETERMINING SERIES RESISTANCE

We consider five published methods for determining a series resistance from IV curves which we term: Swanson, suns-Voc, Bowden and Rohatgi, IEC 60891-1 and IEC 60891-2. Pysch [6] analyzes methods for determining the series resistance of solar cells, of which we consider two methods, Swanson and suns-Voc; other methods considered in [6] include two that rely on dark IV curves, and another method which fits a single diode equation that omits the shunt resistance term. Bowden and Rohatgi [7] describe a technique to extract a series resistance value from a pair of IV curves at different irradiance levels. IEC 60891 [8] describes two procedures for translating IV curves to desired temperature and irradiance conditions which we term IEC60891-1 and IEC60891-2; each procedure involves estimating a series resistance quantity.

The Swanson technique (described in [6]) applies to two IV curves at different irradiances and a constant cell temperature. A fixed difference in current ΔI is selected such that $V_1(I_{SC1} - \Delta I)$ is slightly greater than V_{MP1} for the IV curve for which R_s is desired. The points $(V_1, I_1 = I_{SC1} - \Delta I)$ and $(V_2, I_2 = I_{SC2} - \Delta I)$ are found on the respective IV curves, and a resistance \hat{R}_s is estimated by Eq. (2):

$$\hat{R}_s = (V_2 - V_1) / (I_1 - I_2) \quad (2)$$

Pysch [6] recommends an extension of the Swanson technique by considering three or more IV curves, with irradiance ranging from just below to just above one sun, and estimating \hat{R}_s as the slope of a line fit to the collection of (V_i, I_i) pairs, i.e., if done by ordinary least squares,

$$\hat{R}_s = \frac{\sum_{i \neq j} (V_j - V_i)(I_i - I_j)}{\sum_{i \neq j} (I_i - I_j)^2} \quad (3)$$

The suns-Voc technique [6], [10] compares (V_{MP}, I_{MP}) from a target IV curve with a corresponding point on the suns-Voc curve. The suns-Voc curve $(E, V_{OC}(E))$ is translated to a voltage-current curve $(V_{OC}(E), I'(E) = (1 - E/E_0)I_{SC0})$ where $E_0 = 1000 \text{ W/m}^2$ and I_{SC0} is the short circuit current at E_0 . Setting $E = (1 - I_{MP}/I_{SC0})E_0$, \hat{R}_s is estimated as

$$\hat{R}_s = (V_{OC}(E) - V_{MP}) / I_{MP} \quad (4)$$

The Bowden and Rohatgi method [7] applies to two IV curves, one at high irradiance denoted by \sim_H and a second at low irradiance (\sim_L) and a constant cell temperature. The method anticipates that $I_{SC,H} - I_{SC,L} \approx I_{MP,H}$. Obtaining $V_A = V(I_{SC,F} - I_{SC,L})$, \hat{R}_s is estimated as

$$\hat{R}_s = (V_{OC,L} - V_A) / (I_{SC,H} - I_{SC,L}) \quad (5)$$

IEC60891-1. Correction procedure 1 in [8] translates a point (V_1, I_1) on an IV curve measured at irradiance G_1 and cell temperature T_1 to the corresponding point (V_2, I_2) on an unobserved IV curve at irradiance G_2 and cell temperature T_2 . The translation of V_1 to V_2 reduces V_1 by a voltage drop $\hat{R}_s(I_2 - I_1)$ across the 'internal series resistance' of the module ([IEC 60891], Eq. 2). \hat{R}_s is found by an iterative search over a set of IV curves measured at different irradiance levels and constant cell temperature: first, current for each IV curve is translated linearly to a common irradiance, then \hat{R}_s is found by minimizing the variance of P_{MP} of the translated IV curves.

IEC60891-2. Correction procedure 2 in [8] also translates a point (V_1, I_1) on one IV curve to the corresponding point (V_2, I_2) on a different IV curve. A series resistance \hat{R}_s operates in the translation of voltage in the same manner as in method IEC60891-1; however, additional terms are involved to accommodate IV curves with greater disparity in irradiance. Determination of \hat{R}_s involves a translation to a common irradiance as well as a second IV curve translation to minimize variance at V_{OC} .

III. ANALYSIS OF RS ESTIMATORS

For the Swanson, suns-Voc and Bowden and Rohatgi methods, from the single diode equation we derive an expression for the difference between the series resistance estimated by the method, denoted as \hat{R}_s , and the series resistance parameter in the single diode model, denoted as R_s . For these three methods, we demonstrate the difference between estimated and known R_s both analytically and numerically. The methods in IEC 60891 find \hat{R}_s by applying an optimization procedure to several IV curves; for these methods, we show the differences numerically.

Our primary tool is the analytic solution $V = V(I)$ to Eq. (1) from [9] which uses Lambert's W function, i.e., $W(x) = y \Leftrightarrow x = y \exp(y)$:

$$V = (I_L + I_O - I)R_{SH} - IR_s - aW(\psi(I)) \quad (6)$$

$$\psi(I) = \frac{I_O R_{SH}}{a} \exp\left(\frac{(I_L + I_O - I) R_{SH}}{a}\right) \quad (7)$$

where we write $a = nN_s V_{th}$ for convenience. In Eq. (6) as V increases the terms $(I_L + I_O - I) R_{SH}$ and $aW(\psi(I))$ become large but approximately equal in magnitude. An approximation of Eq. (6) accurate to within 1% in V (see Appendix) is

$$V \approx -IR_S - a \ln\left(\frac{I_O R_{SH}}{a}\right) + a \ln \ln \xi(I) \times \left(1 - \frac{1.04}{\ln(\xi(I))}\right) \quad (8)$$

$$\xi(I) = \frac{I_O R_{SH}}{a} \exp\left(\frac{(I_{SC} - I) R_{SH}}{a}\right) \quad (9)$$

Applying Eq. (8) to both V_2 and V_1 in Eq. (2), for any two points (V_1, I_1) and (V_2, I_2) we obtain

$$\frac{V_2 - V_1}{I_1 - I_2} = R_S + \frac{a}{I_1 - I_2} \left(\begin{array}{l} \ln \frac{R_{SH1}}{R_{SH2}} + \ln \frac{\ln \xi_2}{\ln \xi_1} \\ + 1.04 \ln \left(\frac{\ln \ln \xi_1}{\ln \xi_1} - \frac{\ln \ln \xi_2}{\ln \xi_2} \right) \end{array} \right) \quad (10)$$

where $\xi_1 = \xi(I_1; I_{SC1}, I_{O1}, R_{SH1}, a_1)$ and the subscript ~ 1 indicates the first IV curve. Eq. (10) applies to points on the same IV curve or two different IV curves with constant cell temperature.

Eq. (10) provides an expression for the error in \hat{R}_S when a value is obtained from Eq. (2)

$$\begin{aligned} \hat{R}_S - R_S &\approx \frac{a}{I_1 - I_2} \left(\begin{array}{l} \ln \frac{R_{SH1}}{R_{SH2}} + \ln \frac{\ln \xi_2}{\ln \xi_1} \\ + 1.04 \ln \left(\frac{\ln \ln \xi_1}{\ln \xi_1} - \frac{\ln \ln \xi_2}{\ln \xi_2} \right) \end{array} \right) \\ &= \frac{a}{I_1 - I_2} \left(\begin{array}{l} \ln \frac{I_{SC2} - I_2 + (a/R_{SH2}) \ln(I_O R_{SH2}/a)}{I_{SC1} - I_1 + (a/R_{SH1}) \ln(I_O R_{SH1}/a)} \\ + 1.04 \ln \left(\frac{\ln \ln \xi_1}{\ln \xi_1} - \frac{\ln \ln \xi_2}{\ln \xi_2} \right) \end{array} \right) \end{aligned} \quad (11)$$

Eq. (11) shows that \hat{R}_S and R_S differ systematically, and that the difference scales linearly with the diode factor, is inversely proportional to the difference in current $I_1 - I_2$ at the selected points, and scales in a complex manner depending on the values of I_{SC} , I , I_O and R_{SH} . Unless $I_{SC2} - I_2$ and $I_{SC1} - I_1$ differ substantially the left term inside the parenthesis in Eq. (11) is nearly zero and thus the lower order term $1.04(*)$ in Eq. (11) must be retained.

IV. ACCURACY OF RS ESTIMATORS

The accuracy of each method is evaluated by simulating IV curves with assumed values for the CEC model (Table I), estimating \hat{R}_S with each method, and comparing \hat{R}_S with the known R_S values (Table II). We consider two sets of parameters representative of a 240W, 60 cell cSi module (i.e., high I_{sc} and moderate V_{oc}), and a 72W, 114 cell CdTe module (low I_{sc} and high V_{oc}). To judge the effect of measurement error, the IV curve simulation is repeated 100 times with each current value in each IV curve multiplied by a factor independently sampled from normal distribution with mean 1 and standard deviation of 0.15%, resulting in 100 estimates of \hat{R}_S (Table II). We note that this application of measurement error does not alter the calculated V_{oc} .

TABLE I. CEC MODEL PARAMETERS FOR THE SIMULATED MODULES

Parameter	cSi 240W	CdTe 72W
I_{L0}	8.0 A	1.15 A
α_{Isc}	0.8 mA/K	0.35 mA/K
<i>Adjust</i>	0.0	0.0
I_{O0}	0.5 nA	0.3 nA
E_{g0}	1.121 eV	1.475 eV
R_{S0}	0.2 Ω	0.5 Ω
R_{SH0}	1000 Ω	800 Ω
n_0	1.05	1.40
N_s (cells in series)	60	114

The results in Table II show that some methods (notably Bowden and IEC60891-2) obtain reasonable values for the cSi module and that values remain within 10% in the presence of simulated measurement noise. However, none of the methods considered here accurately recovers the R_S value for the CdTe module from noise-free IV curves. Using the Pysch extension of the Swanson technique (results not shown) did not improve the estimates of \hat{R}_S .

Conceptually, estimates of R_S can be improved by using the additional terms appearing in Eq. (10) to adjust the value for \hat{R}_S obtained from Eq. (2). However, doing so requires values for a , R_{SH} and I_O for the IV curves of interest. We explored whether this approach leads to accurate recovery of R_S from simulated IV curves, by positing techniques to estimate the required values (we set $I_L = I_{SC}$ and estimate a , R_{SH} and I_O as described in Section V without the iteration in Step 5) and using these values to calculate $R_S = \hat{R}_S$ - correction using Eq. (11). With the correction the Bowden estimate is closer to the known value, but estimates by other methods remain poor (Table II, 'w/correction' column). Moreover, no method (even with correction) performs acceptably when simulated noise is applied to the IV curve. We conclude that for the single diode

TABLE II. ESTIMATED R_S VALUES FOR SIMULATED MODULES AT STC

Rs method	cSi: true R_S value 0.2000 Ω				CdTe: true R_S value 0.5000 Ω				
	No noise	With noise (100 replicates)			No noise	No noise w/ correction	With noise (100 replicates)		
		5th	Mean	95th			5th	Mean	95th
Bowden	0.2051	0.2031	0.2054	0.2085	1.4808	0.4882	1.4710	1.5186	1.5580
Swanson	0.2694	-0.2054	0.2244	0.5187	7.3866	0.8576	-0.1040	7.1869	12.6157
Pysch	0.2584	0.0881	0.2352	0.4076	6.8445	0.8538	3.1044	6.4761	9.7333
suns-Voc	0.2144	0.2031	0.2129	0.2230	4.5713	1.7291	4.1225	4.2122	4.2642
IEC60891-1	0.2129	0.1992	0.2147	0.2273	6.1033	-	5.4196	5.6214	5.8245
IEC60891-2	0.1989	0.1798	0.1985	0.2159	1.0352	-	0.3968	0.9513	1.3454

model R_S cannot reliably be estimated by the five methods considered here. Instead, it is our view that R_S must be estimated in conjunction with the other parameters in Eq. (1). Indeed, a procedure to calculate R_S using Eq. (11) essentially amounts to a procedure to calculate all five parameters for Eq. (1) for each IV curve, since the correction term requires values for the other four parameters (as written Eq. (11) approximates $I_L \approx I_{SC}$).

V. ESTIMATING R_S JOINTLY WITH OTHER PARAMETERS

We present a method to determine R_S jointly with the other four parameters for an IV curve, and demonstrate the accuracy values recovered from simulated IV curves with measurement noise. This procedure is a simplification of that published in [12]. The method first uses a set of IV curves at different irradiances but constant cell temperature to find a value for a . Next, for each IV curve, a value for R_{SH} found, and then values for I_O , R_S , and I_L are found with a few iterations.

1. Determine a . Fit $V_{OC} = \beta_0 + a \log(G/1000)$ using a set of IV curves where G is in-plane irradiance (W/m^2) ranging over $400 \text{ W/m}^2 \leq G \leq 1100 \text{ W/m}^2$ and constant cell temperature ([12], Appendix D).
2. For each IV curve, determine R_{SH} . For $V < V_{OC}/3$ (or other suitable restriction on V) fit a line $I = \beta_0 + bV$ and set $R_{SH} = -1/b$ (e.g., [13], but see caution in [12] Appendix E).
3. For each IV curve, initially set $I_L = I_{SC}$.
4. For each IV curve, perform in order:
 - a. $I_O = (I_L - V_{OC}/R_{SH}) / (\exp(V_{OC}/a) - 1)$ [12]
 - b. $R_S = \frac{1}{I^*} \left[(I_L + I_O - I^*) R_{SH} - V^* - aW(\psi(I^*)) \right]$ [13]
where $V^* \approx (V_{MP} + V_{OC})/2$.
 - c. $I_L = I_{SC} (1 + R_S/R_{SH})$ [14]
5. Iterate step 4 a few (e.g., 5) times.

Eq (12) is obtained from Eq. (1) at V_{OC} , Eq. (13) from Eq. (6), and Eq. (14) approximates Eq. (1) at $I = I_{SC}$.

To demonstrate the method's accuracy, we use the CEC model [14], [15] to calculate a set of IV curves for the representative cSi and CdTe modules described by the model parameters given in Table I, at combinations of irradiance ranging from 400 W/m^2 to 1100 W/m^2 in steps of 50 W/m^2 and cell temperature of 25, 35 or 45°C , for a total of 45 combinations. As a reminder the CEC model uses the following equations to compute the five values for Eq. (1) at specific irradiance and temperature conditions.

$$I_L = E \left[I_{L0} + \alpha'_{sc} (T_C - T_0) \right] \quad (15)$$

$$\alpha'_{sc} = \alpha_{sc} (1 - \text{Adjust} / 100) \quad (16)$$

$$I_O = I_{O0} \left(\frac{T_C}{T_0} \right)^3 \exp \left(\frac{1}{k} \left(\frac{E_{g0}}{T_0} - \frac{E_g}{T_C} \right) \right) \quad (17)$$

$$E_g = E_{g0} (1 - 0.0002677(T_C - T_0)) \quad (18)$$

$$R_{SH} = R_{SH0} \frac{E_0}{E}, \quad R_S = R_{S0}, \quad a = N_S n_0 V_{th} \quad (19)$$

In Eq. (15) to (19), E is the in-plane irradiance producing photocurrent (i.e., broadband irradiance adjusted for surface losses and solar spectrum), $E_0 = 1000 \text{ W/m}^2$, $T_0 = 25^\circ\text{C}$, V_{th} is the usual thermal voltage per cell and $k = 8.6173 \times 10^{-5} \text{ eV}$.

From simulated IV curves without noise applied, recovered values for the diode factor are 1.0498 for the cSi module (true value 1.05) and 1.393 for the CdTe module (true value 1.4). Figure 1 displays error in each of the other four recovered parameters for both modules. Parameters are recovered with at least 3 digits of accuracy for the cSi module, but less accurately for the CdTe module. For R_S , the joint determination method outlined above outperforms each of the five R_S methods considered in our analysis, for either module.

We repeated parameter estimation for 100 sets of 45 IV curves, with simulated noise applied to each IV curve as described earlier. Because the applied noise does not alter V_{OC} , the diode factor estimate is the same for each set of 45 IV curves: 1.0498 for the cSi module, and 1.393 for the CdTe

module. We obtain a total of 4500 estimates for each of the other four parameters, one estimate for each IV curve. Figure 2 displays error in each recovered parameter for both modules. For the cSi module, error in all parameters is zero-centered and error in each of R_S , I_L and I_O is relatively small. Error in R_{SH} is greater; most values are within 25% of the true value (corresponding to error in $\log_{10} R_{SH} \approx 10\%$) but some estimates differ by up to a factor of 10. These large errors can result when the random error applied to a few points on the IV curve substantially alters the linear fit to the data, because the R_{SH} value is determined from the slope of the fitted line. However, even large error in R_{SH} has only a minor effect on calculated IV curves, causing visually slight alterations in the slope of the IV curves near I_{SC} .

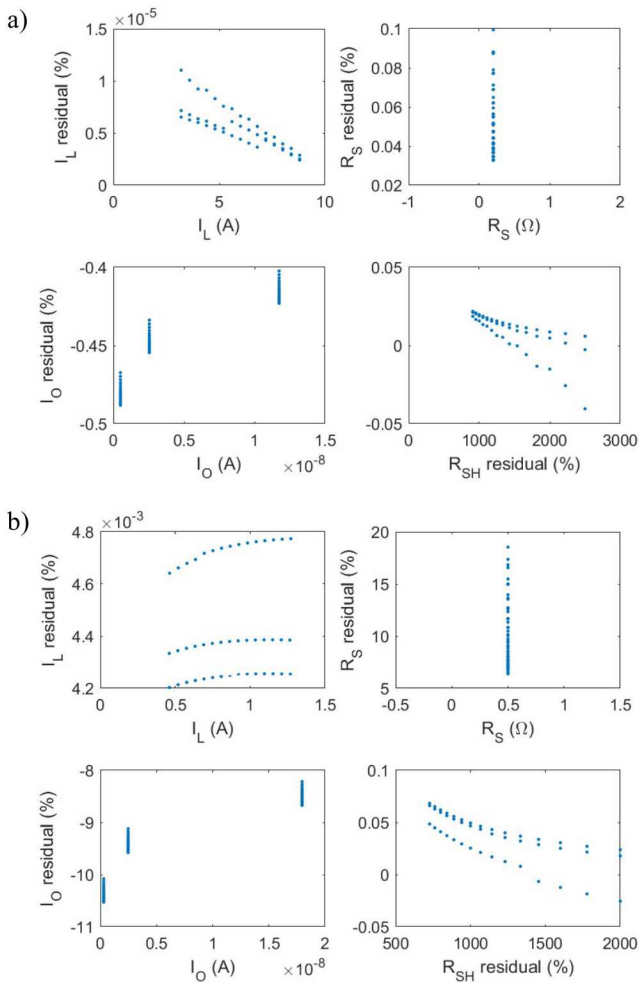


Figure 1. Error (%) in recovered parameters for (a) cSi and (b) CdTe module.

In contrast, for the CdTe module, error in R_S is biased toward overestimated values with compensating underestimates of I_O . The median errors are corrected to zero

if the exact value of the diode factor (1.4) is imposed in place of the estimated value (1.393) (Figure 3). This demonstrated sensitivity of estimation error in R_S (and I_O) is disheartening considering the superior skill of the method outlined in Sect. V to obtain R_S as compared to the five empirical methods described in Sect. II.

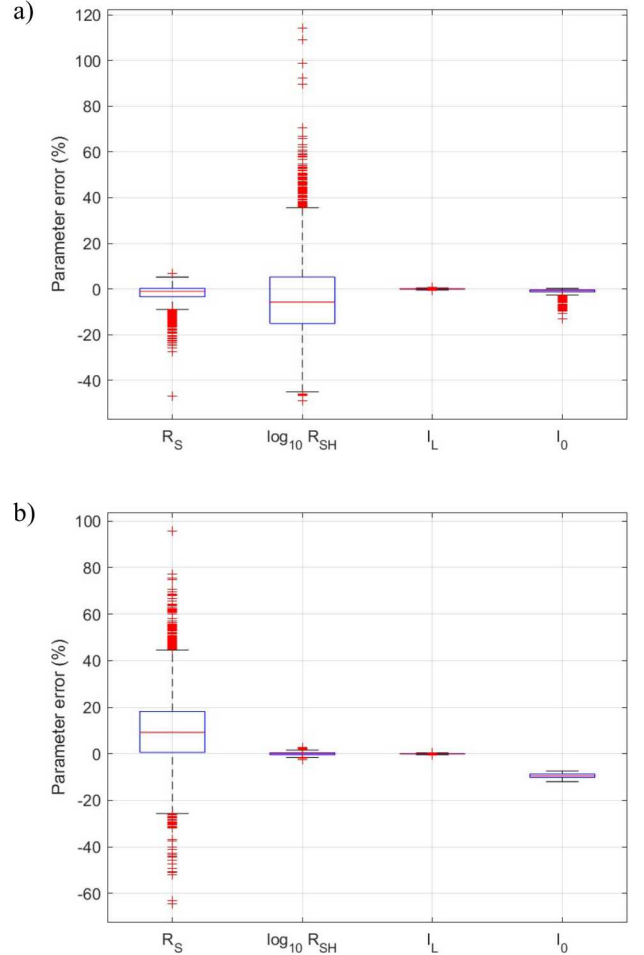


Figure 2. Error (%) in recovered parameters for the (a) cSi and (b) CdTe module.

VI. CONCLUSIONS

Our purpose is to evaluate methods to determine the series resistance parameter for an equivalent circuit model of a photovoltaic device comprising a diode with parallel and series resistances. We summarize five methods from literature that determine series resistance from an IV curve and apply these methods to recover series resistance values from IV curves simulated using a single diode model. We found none of these methods to be reliable at recovering the known values for both representative c-Si and CdTe PV modules, although some methods showed acceptable results for the c-Si module in the absence of measurement noise. All methods

approximate the series resistance as a ratio of a voltage difference to a current difference. We show analytically that this ratio is a low order approximation of an expression for the series resistance parameter for the single diode equation, and that higher accuracy requires knowledge of other equation parameters such as the diode factor and dark current. We conclude that the series resistance parameter for the single diode model cannot be determined accurately in isolation; estimation of its value must also consider estimation of the other model parameters.

Eq. (8) results from combining Eq. (20) with three approximations:

$$W(x) \approx \log x - \log \log x + \frac{1}{2} \left(\frac{1}{2} + \frac{e}{e-1} \right) \frac{\log \log x}{\log x} \quad (21)$$

$$I_L + I_o \approx I_{sc}, \text{ and } 1.04 \cong \frac{1}{2} \left(\frac{1}{2} + \frac{e}{e-1} \right).$$

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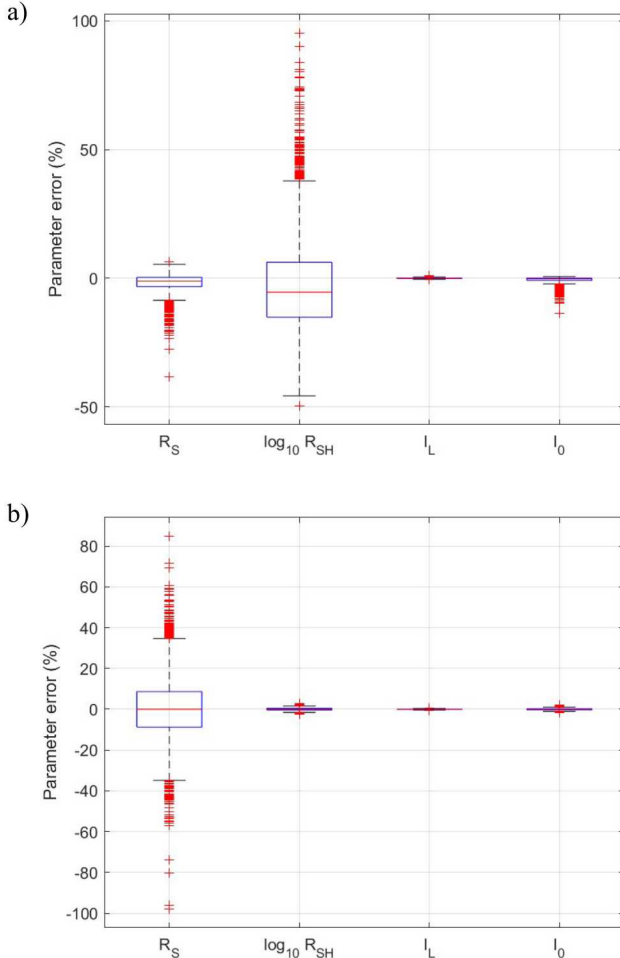


Figure 3. Error (%) in recovered parameters for the (a) cSi and (b) CdTe module with the exact value for the diode factor imposed.

IV. APPENDIX

Sharp bounds on $W(x)$ valid for $x \geq e$ are given by Hoorfar and Hassani [11] (Theorem 2.7):

$$\frac{1}{2} \frac{\log \log x}{\log x} \leq W(x) - \log x + \log \log x \leq \frac{e}{e-1} \frac{\log \log x}{\log x} \quad (20)$$