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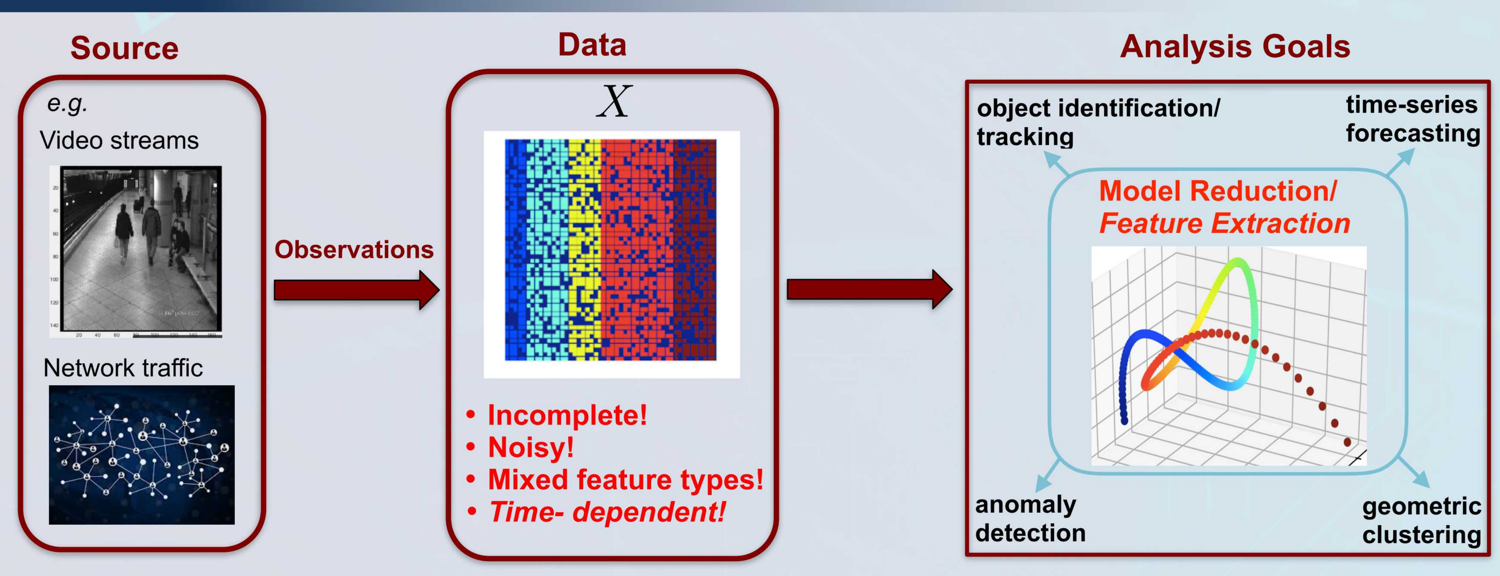
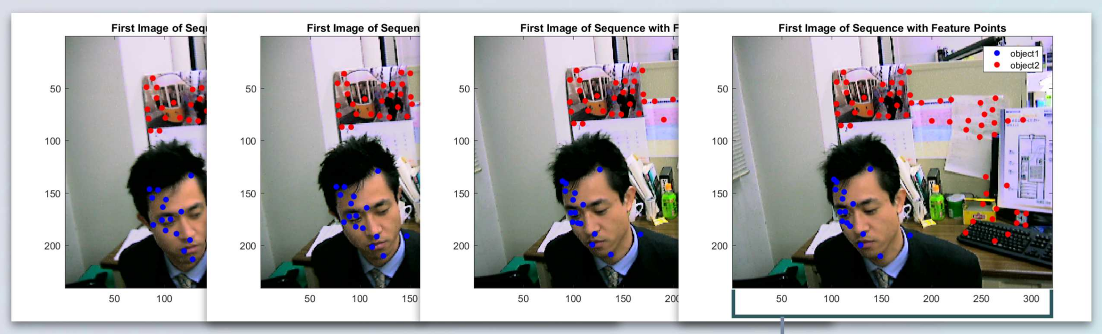
Dynamical Learning: Spectral methods for reducing large datasets with motion

Sandia National Laboratories
A. Kumar*, I. Mezić†, R. Mohr†, C. Brown†
* Sandia National Laboratories, California 94551
† Department of Mechanical Engineering, UCSB, California 93106

Problem





$N \times N$ pixels $\implies X \subset \mathbb{R}^M$ $\quad M \geq N^2$ evolving variables with complex dynamics

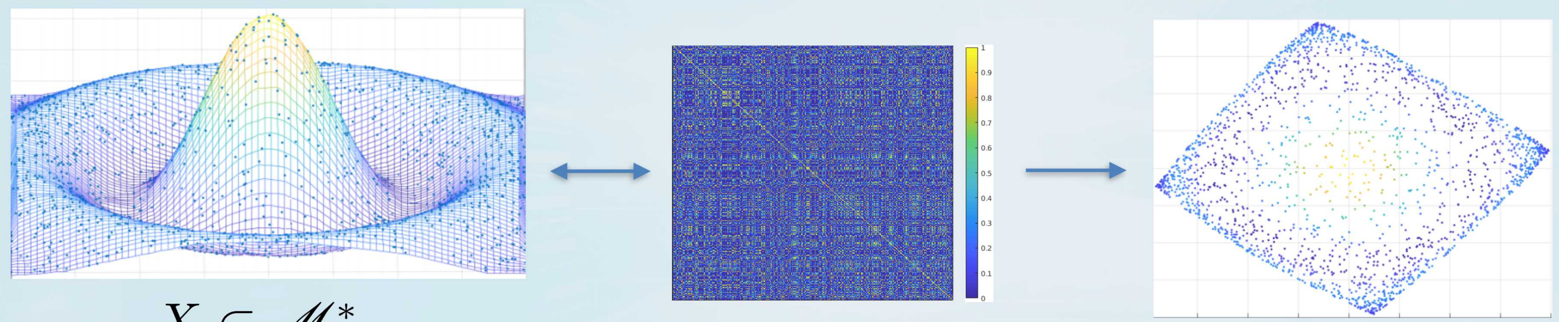
*Data has internal structure (**spatial**) that evolves according to its **dynamical** characteristics*

Goals: Develop *efficient, data-driven* representations of time-dependent datasets with high dimensionality and presence of noise/chaos:
- *Spatial* dimensionality reduction, with
- Dynamical (temporal) separation & robust evolution

Approach

Hypothesis: *Data with structure* has low *intrinsic* dimensionality and small number of primary governing dynamical features

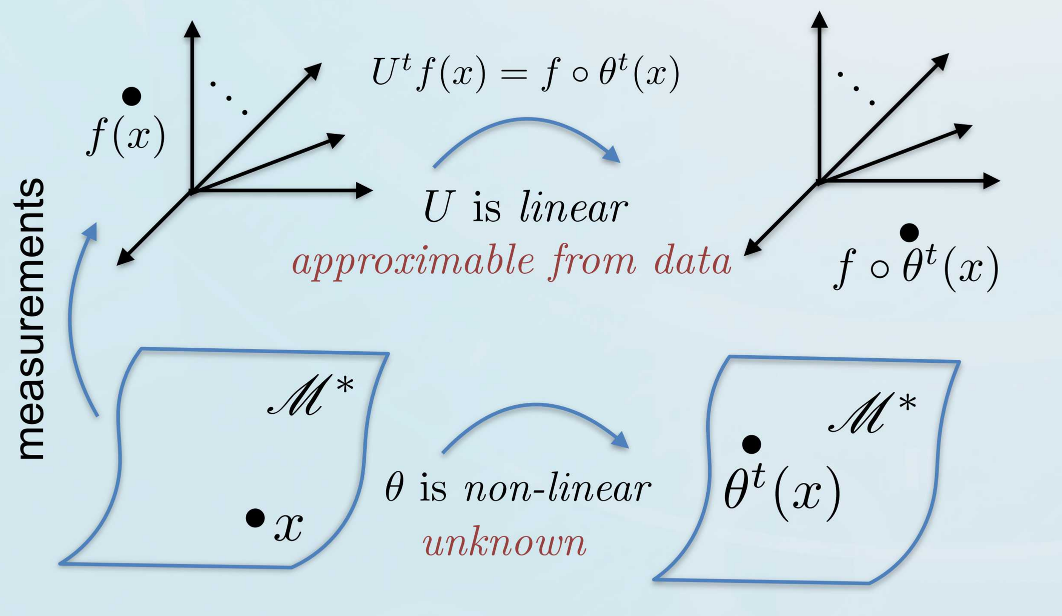
Spatial organization & reduction: Manifold learning



$X \subset \mathscr{M}^*$, $\mathscr{M}^*$ is a manifold (induced by K) $\quad$ $K :=$ affinity kernel from X $\quad$ reduced embedding $\varphi(X) \subset \mathscr{M}$

Computation of φ amounts to a low-order eigen-decomposition
Robust to noise & sampling density; typically, $\dim(\mathscr{M}) \ll \dim(X)$

Data-driven dynamical recovery: Koopman theory



$$f_t(x) = \sum_{k=1}^{\infty} e^{\lambda_k t}\phi_k(x) f_k$$

temporal $\quad$ spatial

$\phi_k :=$ eigenfunctions of $U : L^2(\mathscr{M}^*) \to L^2(\mathscr{M}^*)$

Approach: Combine these *spectral methods* to achieve reduced representations of complex, high-dimensional datasets

Results

Techniques reveal structure in video datasets with several motions, allowing decomposition into simpler primitives & lower-dimensional representations

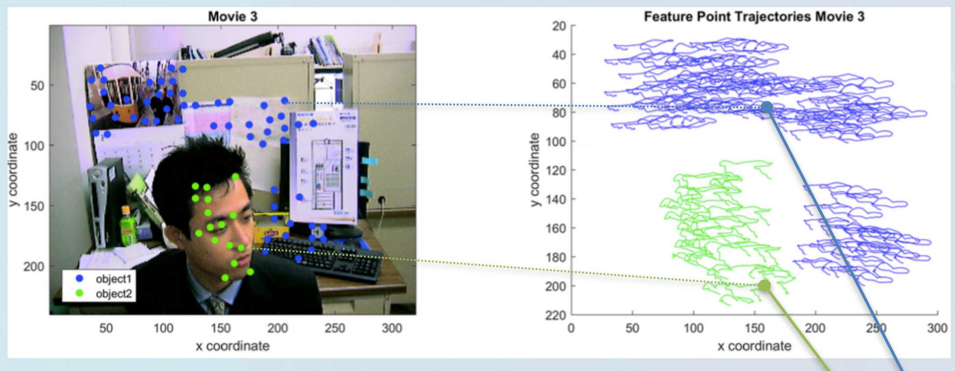
Method:

$$(X,\theta) \to ((f^1,\dots,f^m) : X \to \mathbb{R}^m, \theta)$$
$$\downarrow$$
$$((\lambda_1,\dots,\lambda_n), \vec{f}_k)$$
$$\downarrow$$

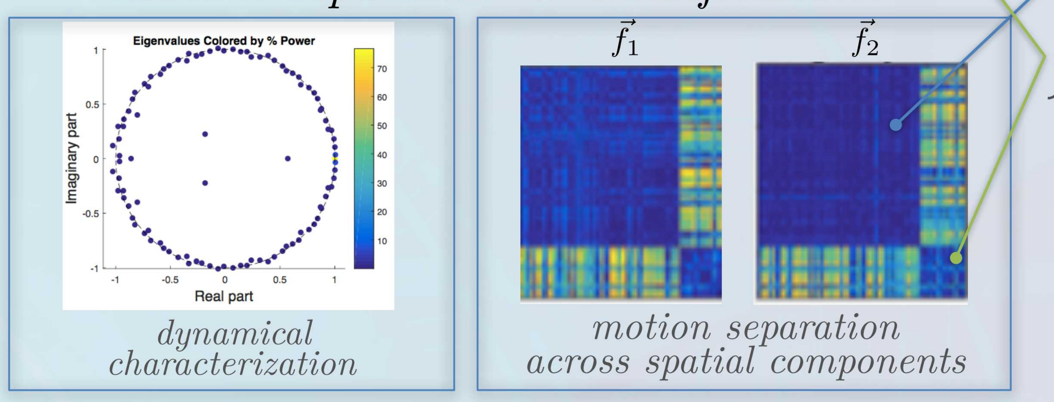
Dynamical classification & spatial reduction through manifold learning

Data-driven dynamical feature extraction with application-specific observables f^j

Application:



Dynamical clustering: Motion separation & classification



dynamical characterization $\quad$ motion separation across spatial components

Modal structures separate objects by geometry & dynamics

Structural recovery & low-dimensional representation enable efficient identification & tracking of multiple motions

Significance

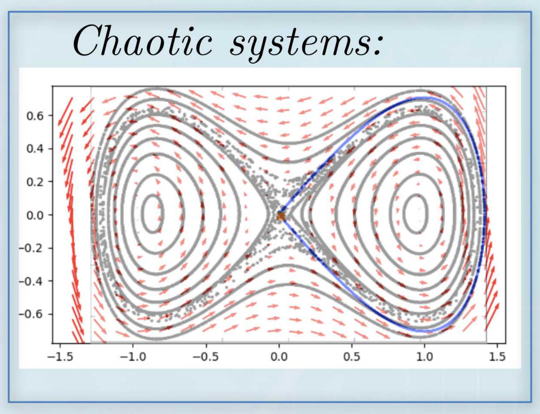
Algorithms grounded firmly in theory: framework and theory are kept general so they are effective across analysis on all types of time-dependent datasets; *Application-dependence* is separated into choice of observables & embedding parameters

Results reveal structure in *natural* data, extendible to a *dictionary* between structure in reduced representations & dynamical characteristics

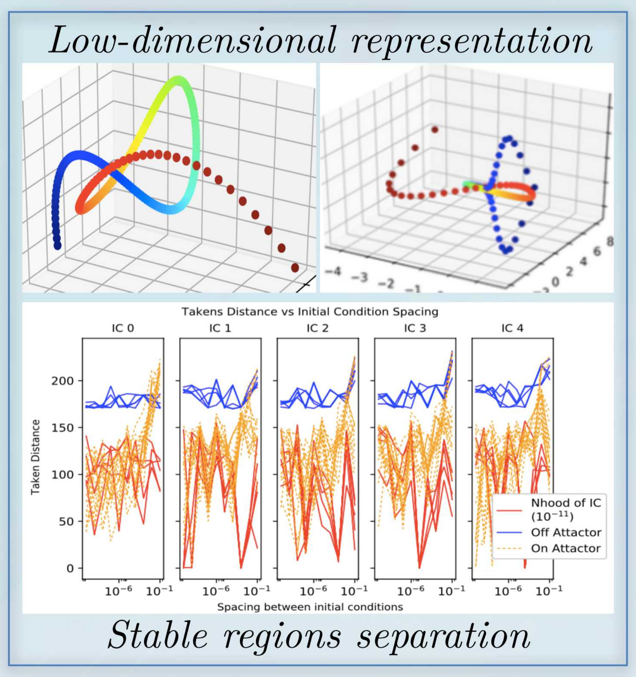
Wide application space across mission areas: Fluid dynamics, anomaly detection, pattern classification & tracking, quantum dynamics

Example:

Chaotic systems:



Low-dimensional representation



Stable regions separation

Future work: Extend to datasets with *implicit* time-dependence & complex *small-scale* dynamics through *local-to-global principles*

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