SAND2018-4275C

Some directions on algorithms for chance-constrained optimization models

Bismark Singh

Discrete Mathematics & Optimization
Sandia National Laboratories

bsingh@sandia.gov

April 24, 2018

Contents

What are Chance Constraints?

This is a linear Joint Chance Constraint:

$$P(x_t \leq y_t^\omega + w_t^\omega, \forall t \in T) \geq 1 - \varepsilon$$

Background:

- Two-stage stochastic program with recourse
- Possibly integer restrictions
- i.i.d. samples of uncertainty w_t^ω
- First stage decision, x_t, second-stage decision, y_t^ω

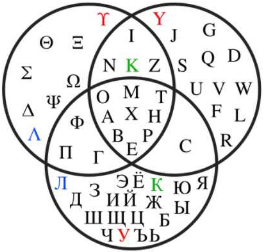
Challenges

- CC models are computationally intractable
- A known NP-hard problem
- Existing algorithms not scalable to practical sized problems
- Feasible region is non-convex

Contents

Approximations with classical probability bounds

Satisfying a JCC is an intersection of events. Failing a JCC is a union of events.



We can rewrite the JCC as follows:

$$\bigcup_{t \in T} F_t \leq \varepsilon$$

where F_t denotes the probability of "failure" at t; i.e., $F_t = P(x_t > y_t^\omega + w_t^\omega)$.

Approximations with Classical Probability Bounds

$$\bigcup_{t \in T} F_t \leq \varepsilon$$

Consider an optimization model with a JCC with a maximization objective.

- Lower Bound (LB): Approximate the LHS using a quantity **larger** than $\bigcup_{t \in T} F_t$. Feasible region is **restricted**.
- Upper Bound (UB): Approximate the LHS using a quantity **smaller** than $\bigcup_{t \in T} F_t$. Feasible region is **enlarged**.

Approximations with Classical Probability Bounds

$S_k = \sum_{1 \le i_1 < \cdots < i_k \le |T|} F_{i_1} \cap \cdots \cap F_{i_k}.$

Bonferroni bounds:

$$\bigcup_{t \in T} F_t \le \quad S_1 \leftarrow LB \tag{1a}$$

$$\bigcup_{t \in T} F_t \ge \quad S_1 - S_2 \leftarrow UB \tag{1b}$$

Approximations with Classical Probability Bounds

$S_k = \sum_{1 \leq i_1 < \cdots < i_k \leq |T|} F_{i_1} \cap \cdots \cap F_{i_k}$.

Tighter bounds from Sathe et al. [1980]:

$$\bigcup_{t \in T} F_t \leq \quad S_1 - \frac{2}{T} S_2 \leftarrow LB \tag{2a}$$

$$\bigcup_{t \in T} F_t \geq \quad \frac{1}{T^2}(2S_2 + S_1) \leftarrow UB \tag{2b}$$

Approximations with Classical Probability Bounds

$S_k = \sum_{1 \leq i_1 < \cdots < i_k \leq |T|} F_{i_1} \cap \cdots \cap F_{i_k}.$

And more from Dawson and Sankoff [1967]:

$$\bigcup_{t \in T} F_t \geq \frac{{S_1}^2}{S_1 + 2S_2} \leftarrow UB \tag{3a}$$

can be linearized for JCC $\leq \varepsilon$:

$$2\varepsilon S_2 \geq \alpha_k S_1 + \beta_k, k = 0, 1, \ldots |K| - 1, \leftarrow UB \tag{3b}$$

Optimizing over JCCs

$u_t^\omega = 1$: failure at t in scenario ω
$v_{tt'}^\omega = 1$: failure at t and t' in scenario ω

$$x_t - y_t^\omega - w_t^\omega \;\leq M_t^\omega u_t^\omega, \forall t \in T, \omega \in \Omega$$

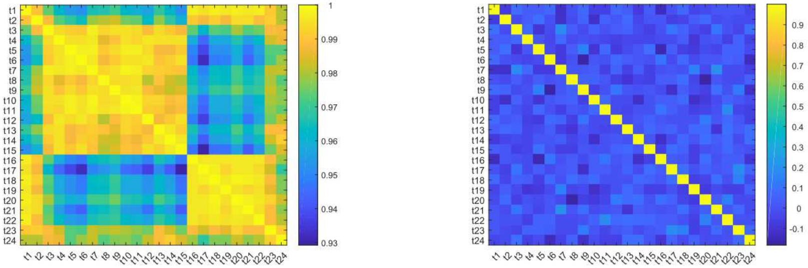
$$\text{McCormick envelope} \begin{cases} v_{t,t'}^\omega \leq u_t^\omega, (t, t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \leq u_{t'}^\omega, \forall (t, t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \geq u_t^\omega + u_{t'}^\omega - 1, \forall (t, t') \in T, t < t', \omega \in \Omega \end{cases}$$

$$u_t^\omega = \{0, 1\}, \forall t \in T, v_{t,t'}^\omega = \{0, 1\}, \forall (t, t') \in, \omega \in \Omega$$

Contents

Computational results

We compare two sampling procedures: (a) ARMA(2,2) process, and (b) normal random variables. Both samples have the same hourly means and variances.



(a) (b)

Figure: Correlation structure

Computational results: ARMA (large correlation)

ε	Bounding constraint	Optimal objective value: Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(1a)	**8,351.3**	8,351.3	0%	4	3.3%
	(1b)	**21,282.8**	21,282.8	0%	18	59.4%
	(2a)	**8,339.3**	8,410.9	0.85%	2100	3.4%
	(2b)	8,339.3	**10,682.2**	21%	2100	19.2%
	(3a)	8,339.3	**39,905**	79.1%	2100	78.4%
	(3b)	8,688.9	**8,708.9**	0.20%	2100	0.9%
0.03	(1a)	**8,374.6**	8,374.6	0%	9	8.5%
	(1b)	**22,353.2**	22,353.2	0%	95	59.0%
	(2a)	**8,339.6**	8,787.3	5.0%	2100	8.9%
	(2b)	8,339.3	**13,300.4**	37.3%	2100	31.2%
	(3a)	8,339.3	**40,975**	79.6%	2100	77.7 %
	(3b)	8610.0	**9,559.2**	9.9%	2100	4.2%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 8,351.3 and 8,708.9; true optimal value is 8,634.1
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 8,374.6 and 9,559.2; true optimal value is 9,154.9

Computational results: Gaussian (weak correlation)

ε	Bounding constraint	Optimal objective value Lower bound	Upper bound	MIP gap	Time (seconds)	Gap from optimal
0.01	(1a)	**9,100.8**	9,100.8	0%	1	2.7%
	(1b)	**21,606.6**	21,606.6	0%	22	56.7%
	(2a)	**9,092.3**	9,106.9	0.16%	2100	2.8%
	(2b)	9092.3	**11,266.8**	19%	2100	17.0%
	(3a)	9,092.3	**40,054.2**	77.3%	2100	76.7%
	(3b)	9,428.2	**9,449.9**	0.23%	2100	1.0%
0.03	(1a)	**9,124.3**	91,24.3	0%	2	7.7%
	(1b)	**22,762.1**	22,762.1	0%	28	56.6%
	(2a)	**9,092.3**	9,174.9	0.9%	2100	8.0%
	(2b)	9,092.3	**13,981.7**	34.9%	2100	29.3%
	(3a)	9,092.3	**41,366.2**	78.0%	2100	76.1%
	(3b)	9,485.1	**9,994.8**	5.1%	2100	1.1%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 9,100.8 and 9,449.9; true optimal value is 9,353.2
Tightest lower and upper bounds for $\varepsilon = 0.03$ are 9,124.3 and 9,994.8; true optimal value is 9,884.0

Computational results

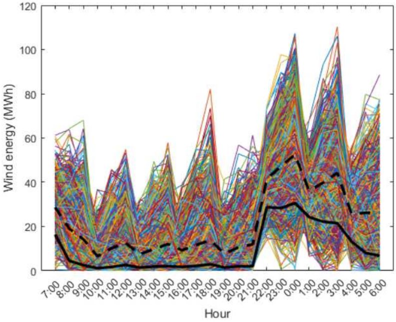
- Bonferroni lower bound and Dawson & Sankoff upper bound consistently perform better than others
- Weaker correlation in uncertainty leads to easier-to-solve models
- MIQCP formulation of Dawson & Sankoff bound is challenging

Contents

A regularization algorithm

Motivation: A hybrid wind-diesel generator supplying reliably in the day-ahead market. Real-time dispatch decisions. Example: natural gas plant.

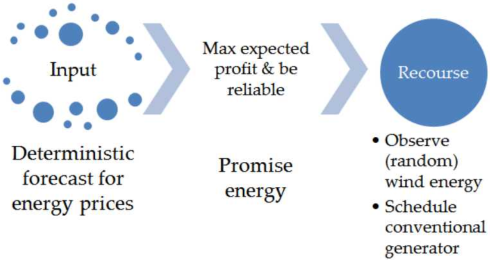
`Singh, Morton, Santoso. "An Adaptive Model with Joint Chance Constraints for a Hybrid Wind-Conventional Generator System." (Accepted, Computational Management Science, 2018)`



1500 hourly scenarios for wind energy generated using Monte Carlo sampling

A regularization algorithm

Decisions for each hour: (i) how much energy to promise, and (ii) how much energy to schedule from conventional generator.



This problem is intractable if solved naively!!

A regularization algorithm

A computationally efficient regularization algorithm:

Require: m scenarios, δ, ρ, *time*, 1500 i.i.d. scenarios of w.
Ensure: $\hat{z}$: objective function value of original model with 1500 scenarios.

1: Generate m i.i.d. realizations of w, and solve the SAA of original model to obtain x_m^*. Let $\hat{x} \leftarrow x_m^*$.
2: **while** time $\leq$ *time* **do**
3:     Let $m \leftarrow \lceil m(1+\delta) \rceil$. (Increase number of scenarios)
4:     Generate m i.i.d. realizations of w, and solve the SAA of regularized model to obtain x_m^*. Let $\hat{x} \leftarrow x_m^*$.
5:     Solve original model with 1500 scenarios with x fixed to $\hat{x}$, and let $\hat{z}$ denote the objective function value.
6:     Update time to the cumulative wall-clock time consumed so far.
7: **end while**

A regularization algorithm

A computationally efficient regularization algorithm:

Require: m scenarios, δ, ρ, *time*, 1500 i.i.d. scenarios of w.
Ensure: $\hat{z}$: objective function value of original model with 1500 scenarios.

1: Generate m i.i.d. realizations of w, and solve the SAA of original model to obtain x_m^*. Let $\hat{x} \leftarrow x_m^*$.
2: **while** time $\leq$ *time* **do**
3: Let $m \leftarrow \lceil m(1+\delta) \rceil$. (Increase number of scenarios)
4: Generate m i.i.d. realizations of w, and solve the SAA of regularized model to obtain x_m^*. Let $\hat{x} \leftarrow x_m^*$.
5: Solve original model with 1500 scenarios with x fixed to $\hat{x}$, and let $\hat{z}$ denote the objective function value.
6: Update time to the cumulative wall-clock time consumed so far.
7: **end while**

Scenarios	Reliability	Solution method	Profit	Gap	Time
1500	95%	Naive (CPLEX)	3422.3	67.4%	2100
1500	95%	Regularization	6043.8	n/a	700

Contents

Future directions

Summary

- Working on algorithm development for approximating chance-constrained optimization models
- Modeling interdiction with AC optimal power flow
- Investigating other applications of chance-constrained models, such as public health

Future directions

Summary

- Working on algorithm development for approximating chance-constrained optimization models
- Modeling interdiction with AC optimal power flow
- Investigating other applications of chance-constrained models, such as public health

We seek collaborations with faculty and students!
We recently won a Laboratory Directed Research and Development grant on "Chance-Constrained Optimization for Critical Infrastructure Protection".

Contents

Possible reasons for long computation time of naive solve

- No extended variable formulation above
- Big M
- Less reliable regime, more combinations to choose from

Acknowledgements

Sandia National Laboratories

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.