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Approximating Two-Stage Chance-Constrained Programs with Classical Probability Bounds

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Chance Constraint Setting

Consider a linear Joint Chance Constraint:

$$P(x_t \leq y_t^\omega + w_t^\omega, \forall t \in T) \geq 1 - \varepsilon$$

Background:

- Two-stage stochastic program with recourse
- Possibly integer restrictions
- i.i.d. samples of uncertainty w_t^ω
- First stage decision, x_t , second-stage decision, y_t^ω

Singh, Morton, Santoso. "An Adaptive Model with Joint Chance Constraints for a Hybrid Wind-Conventional Generator System."
(Computational Management Science, 2018)

Challenges

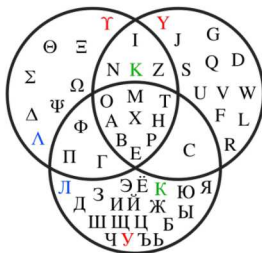
- CC models are computationally intractable
- A known NP-hard problem
- Existing algorithms not scalable to practical sized problems
- Feasible region is non-convex

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Approximations with classical probability bounds

Satisfying a JCC is an intersection of events. Failing a JCC is a union of events.



We can rewrite the JCC as follows:

$$\bigcup_{t \in T} F_t \leq \varepsilon$$

where F_t denotes the probability of “failure” at t ; i.e.,
 $F_t = P(x_t > y_t^\omega + w_t^\omega)$.

Approximations with Classical Probability Bounds

$$\bigcup_{t \in T} F_t \leq \varepsilon$$

Consider an optimization model with a JCC with a maximization objective.

- Lower Bound (LB): Approximate the LHS using a quantity **larger** than $\bigcup_{t \in T} F_t$. Feasible region is **restricted**.
- Upper Bound (UB): Approximate the LHS using a quantity **smaller** than $\bigcup_{t \in T} F_t$. Feasible region is **enlarged**.

Approximations with Classical Probability Bounds

$$S_k = \sum_{1 \leq i_1 < \dots < i_k \leq |T|} F_{i_1} \cap \dots \cap F_{i_k}.$$

Bonferroni bounds:

$$\bigcup_{t \in T} F_t \leq S_1 \leftarrow \text{LB} \quad (1a)$$

$$\bigcup_{t \in T} F_t \geq S_1 - S_2 \leftarrow \text{UB} \quad (1b)$$

Approximations with Classical Probability Bounds

$$S_k = \sum_{1 \leq i_1 < \dots < i_k \leq |T|} F_{i_1} \cap \dots \cap F_{i_k}.$$

Tighter bounds from Sathe et al. [1980]:

$$\bigcup_{t \in T} F_t \leq S_1 - \frac{2}{T} S_2 \leftarrow \text{LB} \quad (2a)$$

$$\bigcup_{t \in T} F_t \geq \frac{1}{T^2} (2S_2 + S_1) \leftarrow \text{UB} \quad (2b)$$

Approximations with Classical Probability Bounds

$$S_k = \sum_{1 \leq i_1 < \dots < i_k \leq |T|} F_{i_1} \cap \dots \cap F_{i_k}.$$

And more from Dawson and Sankoff [1967]:

$$\bigcup_{t \in T} F_t \geq \frac{S_1^2}{S_1 + 2S_2} \leftarrow \text{UB} \quad (3a)$$

can be linearized for $JCC \leq \varepsilon$:

$$2\varepsilon S_2 \geq \alpha_n S_1 + \beta_n, n = 0, 1, \dots, |N| - 1, \leftarrow \text{UB} \quad (3b)$$

Optimizing over JCCs

$u_t^\omega = 1$: failure at t in scenario ω

$v_{tt'}^\omega = 1$: failure at t and t' in scenario ω

$$x_t - y_t^\omega - w_t^\omega \leq M_t^\omega u_t^\omega, \forall t \in T, \omega \in \Omega$$

$$\text{McCormick envelope } \begin{cases} v_{t,t'}^\omega \leq u_t^\omega, (t, t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \leq u_{t'}^\omega, \forall (t, t') \in T, t < t', \omega \in \Omega \\ v_{t,t'}^\omega \geq u_t^\omega + u_{t'}^\omega - 1, \forall (t, t') \in T, t < t', \omega \in \Omega \end{cases}$$

$$u_t^\omega \in \{0, 1\}, \forall t \in T, v_{t,t'}^\omega \in \{0, 1\}, \forall (t, t') \in T, \omega \in \Omega$$

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$$\max_{x,y} \quad \sum_{t \in T} (R_t x_t - \mathbb{E}[B_t y_t^\omega]) \quad (4a)$$

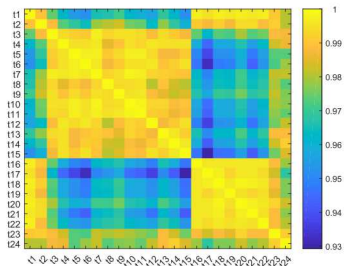
$$\text{s.t.} \quad \mathbb{P}(y_t^\omega + w_t^\omega \geq x_t, \forall t \in T) \geq 1 - \varepsilon \quad (4b)$$

$$0 \leq y_t^\omega \leq \Delta, \forall t \in T, \omega \in \Omega \quad (4c)$$

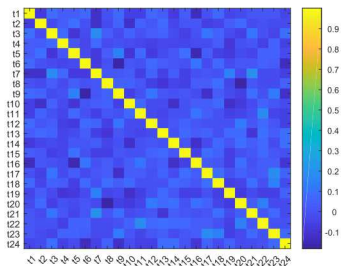
$$x_t \geq 0, \forall t \in T. \quad (4d)$$

Computational results

We compare two sampling procedures: (a) ARMA(2,2) process, and (b) normal random variables. Both samples have the same hourly means and variances.



(a)



(b)

Figure: Correlation structure of w_t

Computational results: ARMA (large correlation)

ε	Bounding constraint	Optimal objective value			Time (seconds)	Gap from optimal
		Lower bound	Upper bound	MIP gap		
0.01	(1a)	8,351.3	8,351.3	0%	4	3.3%
	(1b)	21,282.8	21,282.8	0%	18	59.4%
	(2a)	8,339.3	8,410.9	0.85%	2100	3.4%
	(2b)	8,339.3	10,682.2	21%	2100	19.2%
	(3a)	8,339.3	39,905	79.1%	2100	78.4%
	(3b)	8,688.9	8,708.9	0.20%	2100	0.9%
0.03	(1a)	8,374.6	8,374.6	0%	9	8.5%
	(1b)	22,353.2	22,353.2	0%	95	59.0%
	(2a)	8,339.6	8,787.3	5.0%	2100	8.9%
	(2b)	8,339.3	13,300.4	37.3%	2100	31.2%
	(3a)	8,339.3	40,975	79.6%	2100	77.7 %
	(3b)	8610.0	9,559.2	9.9%	2100	4.2%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 8,351.3 and 8,708.9; true optimal value is 8,634.1

Tightest lower and upper bounds for $\varepsilon = 0.03$ are 8,374.6 and 9,559.2; true optimal value is 9,154.9

Computational results: Gaussian (weak correlation)

ε	Bounding constraint	Optimal objective value			Time (seconds)	Gap from optimal
		Lower bound	Upper bound	MIP gap		
0.01	(1a)	9,100.8	9,100.8	0%	1	2.7%
	(1b)	21,606.6	21,606.6	0%	22	56.7%
	(2a)	9,092.3	9,106.9	0.16%	2100	2.8%
	(2b)	9092.3	11,266.8	19%	2100	17.0%
	(3a)	9,092.3	40,054.2	77.3%	2100	76.7%
	(3b)	9,428.2	9,449.9	0.23%	2100	1.0%
0.03	(1a)	9,124.3	91,24.3	0%	2	7.7%
	(1b)	22,762.1	22,762.1	0%	28	56.6%
	(2a)	9,092.3	9,174.9	0.9%	2100	8.0%
	(2b)	9,092.3	13,981.7	34.9%	2100	29.3%
	(3a)	9,092.3	41,366.2	78.0%	2100	76.1%
	(3b)	9,485.1	9,994.8	5.1%	2100	1.1%

Table: Tightest lower and upper bounds for $\varepsilon = 0.01$ are 9,100.8 and 9,449.9; true optimal value is 9,353.2

Tightest lower and upper bounds for $\varepsilon = 0.03$ are 9,124.3 and 9,994.8; true optimal value is 9,884.0

- Bonferroni lower bound and Dawson & Sankoff upper bound consistently perform better than others
- Weaker correlation in uncertainty leads to easier-to-solve models
- MIQCP formulation of Dawson & Sankoff bound is challenging

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Summary

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- Modeling interdiction with AC optimal power flow
- Investigating other applications of chance-constrained models, such as public health

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- Working on algorithm development for approximating chance-constrained optimization models
- Modeling interdiction with AC optimal power flow
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We seek collaborations with faculty and students!

Won a \$80,000 Laboratory Directed Research and Development grant on “Chance-Constrained Optimization for Critical Infrastructure Protection”.

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Possible reasons for long computation time of naive solve

- No extended variable formulation above
- Big M
- Less reliable regime, more combinations to choose from

Acknowledgements



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