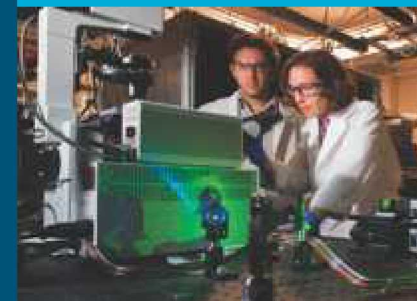
SAND2018-5605C

Modeling of Distortional Hardening via an Evolving Effective Stress Definition



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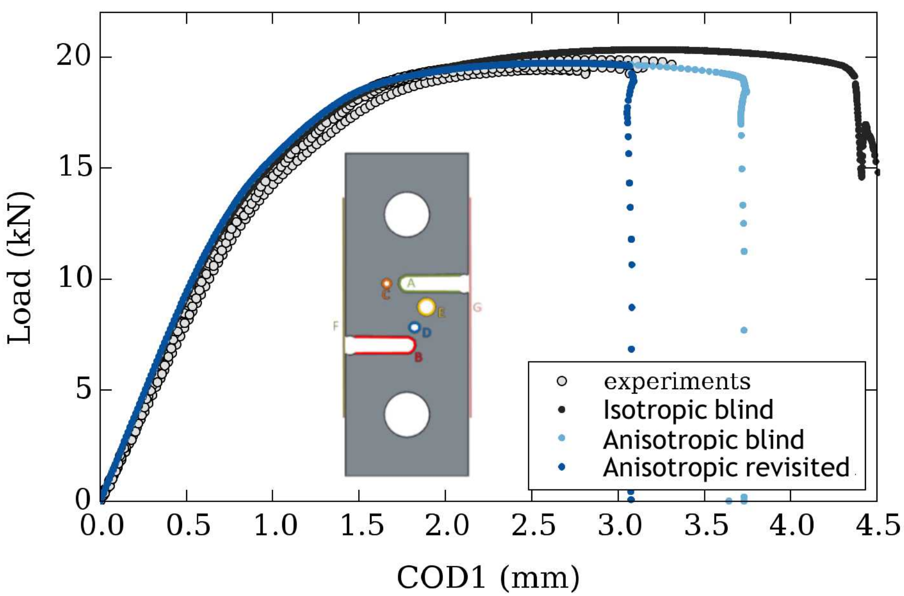
Anisotropic Plasticity

- Plastic anisotropy needed for modeling complex multiaxial loadings
  - Manufacturing processes (e.g. sheet metal forming)
  - Ductile Failure

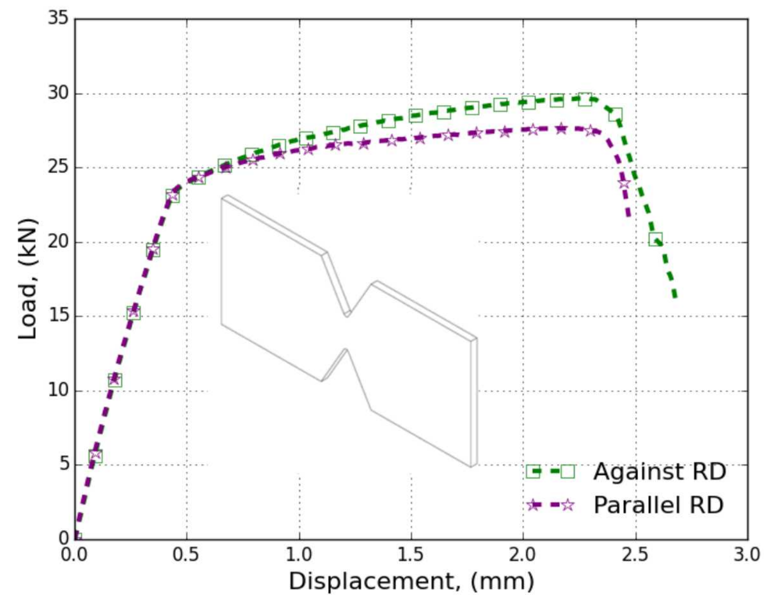
2nd Sandia Fracture Challenge (SFC2) (Ti-6Al-4V)

Isotropic and Anisotropic Failure Predictions



Karlson et al., 2016, *Int. Jrnl. Frac.*, 198: 179-195

Notched Shear Calibration Data for SFC2

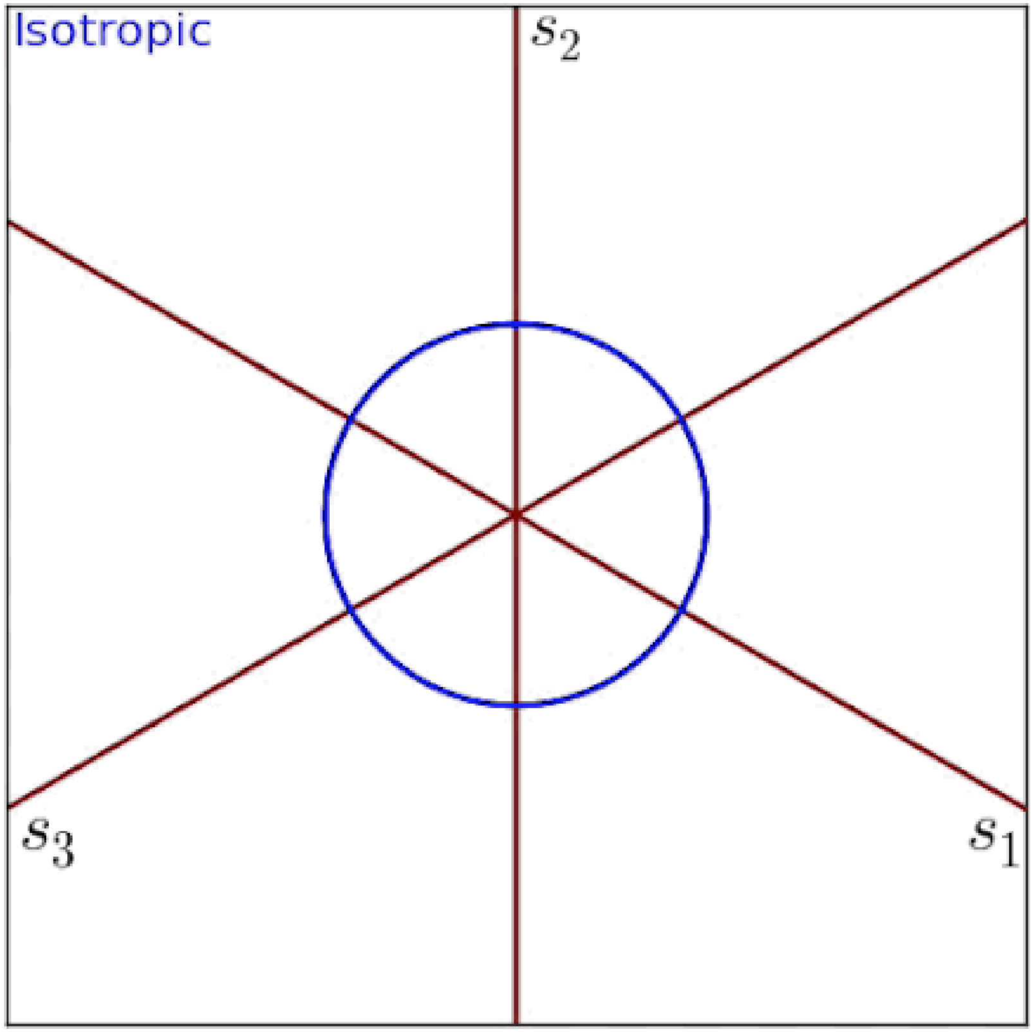


Boyce et al., 2016, *Int. Jrnl. Frac.*, 198: 5-100

(SFC2 Data Courtesy S. Kramer, B. Boyce, K. Karlson, J. T. Ostien et al., SNL)

Plastic Hardening

- Capturing multiaxial, history dependent response requires description of anisotropic yield and *hardening*



Objectives

- Ability to simulate forming/manufacturing processes requires accurate/efficient constitutive description
  - Existing implementations can be complicated, expensive to implement
  - Need flexible, efficient approach for use in such simulations

- Current work focuses on introduction/development of new, flexible approach via effective evolving stress approach
  - Thermodynamically consistent formulation through use of internal state variables (ISVs)
  - Simplified way of treating distortional hardening
  - Enables fully-implicit numerical implementation

Modeling

Free Energy

State Variables

Traditional: $\varepsilon_{ij}^{\mathrm{el}}, \quad \kappa$ New Distortional ISV: η

Free Energy

$$\psi = \hat{\psi}\left(\varepsilon_{ij}^{\mathrm{el}}, \kappa, \eta\right) = \psi^{\mathrm{el}}\left(\varepsilon_{ij}^{\mathrm{el}}\right) + \psi^{\mathrm{iso}}\left(\kappa\right) + \psi^{\mathrm{dis}}\left(\eta\right)$$

$$\psi^{\mathrm{el}}\left(\varepsilon_{ij}^{\mathrm{el}}\right) = \frac{1}{2\rho}\varepsilon_{ij}^{\mathrm{el}}\mathbb{C}_{ijkl}\varepsilon_{kl}^{\mathrm{el}} \quad \psi^{\mathrm{iso}}\left(\kappa\right) = \frac{1}{\rho}g\left(\kappa\right) \quad \psi^{\mathrm{dis}}\left(\eta\right) = \frac{1}{\rho}h\left(\eta\right)$$

Constitutive Behavior

$$\sigma_{ij} = \mathbb{C}_{ijkl}\varepsilon_{kl}^{\mathrm{el}} \qquad \varepsilon_{ij} = \varepsilon_{ij}^{\mathrm{el}} + \varepsilon_{ij}^{\mathrm{p}}$$

Dissipation Inequality

$$\mathcal{D} = \sigma_{ij}\dot{\varepsilon}_{ij}^{\mathrm{p}} - K\dot{\kappa} - N\dot{\eta} \geq 0$$

$$K := \rho\frac{\partial\psi}{\partial\kappa} = \frac{\partial g}{\partial\kappa} \qquad N := \rho\frac{\partial\psi}{\partial\eta} = \frac{\partial h}{\partial\eta}$$

- Traditional State Variables:
  - Elastic Strain Tensor, $\varepsilon_{ij}^{\mathrm{el}}$
  - Isotropic Hardening Variable (IHV), κ
- Introduce single scalar ISV for distortional hardening, η
  - Encapsulates all microstructural effects of distortional hardening
  - Likely multiple mechanisms
- Assume isotropic and distortional energetic effects are independent and separable

Yield Function Definition

- Introduce a new "Evolving Effective Stress" (EES)
- Weighted sum of different definitions for desired features

$$f = f\left(\sigma_{ij}, K, N\right) = \phi\left(\sigma_{ij}, N\right) - \sigma_y\left(K\right)$$

- "Evolving" Effective Stress (EES)

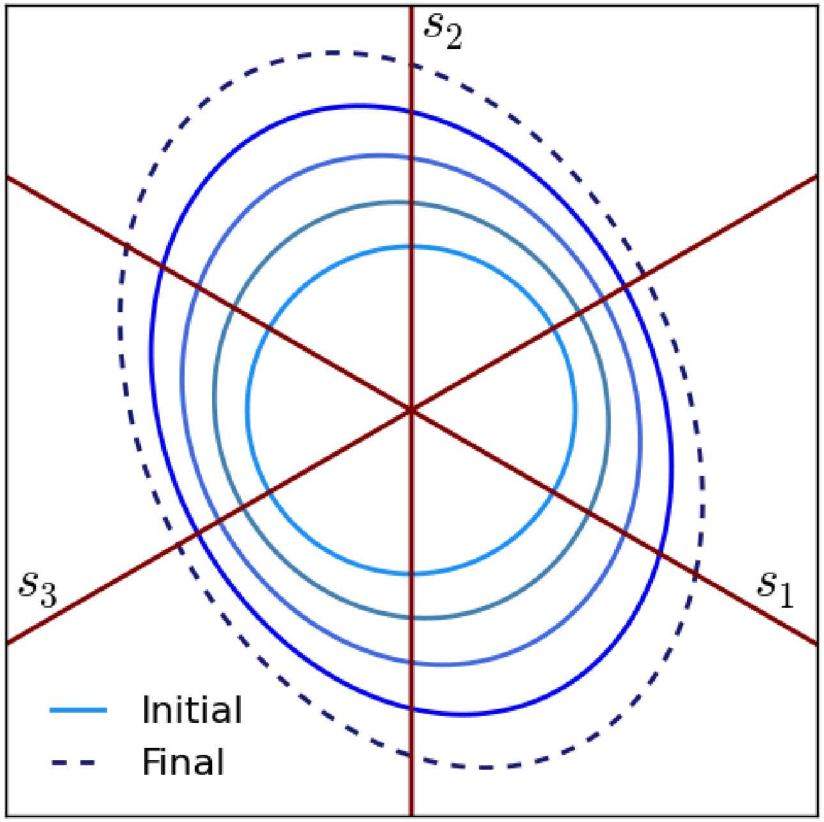
$$\phi\left(\sigma_{ij}, N\right) = \sum_{k=1}^{n_{\text{es}}} \zeta^{(k)}\left(N\right) \phi^{(k)}\left(\sigma_{ij}\right)$$

- Weighting Function Constraints

$$\sum_{k=1}^{n_{\text{es}}} \zeta^{(k)} = 1 \;;\; \zeta^{(k)} \geq 0$$

- Flow Stress

$$\sigma_y\left(K\right) = \sigma_y^0 + K$$



Evolution Equations

- Evolution equations found by trying to maximize dissipation
- Flow rules correspond to Karush-Kuhn-Tucker conditions

$$\dot{\kappa} = \lambda$$

$$\dot{\varepsilon}^{\mathrm{p}}_{ij} = \lambda \frac{\partial \phi}{\partial \sigma_{ij}} \qquad \lambda f\left(\sigma_{ij}, K, N\right) = 0$$

$$\dot{\eta} = -\lambda \frac{\partial \phi}{\partial N}$$

- Leads to rate of dissipation density of

$$\mathcal{D} = \left(\sigma^0_y + N \frac{\partial \phi}{\partial N}\right) \dot{\kappa}$$

Can be positive or negative

Weighting Function Definition

- For current cases consider a two effective stress definition

$$\phi\left(\sigma_{ij}, N\right) = \zeta\left(N\right)\phi^{(1)}\left(\sigma_{ij}\right) + \left(1 - \zeta\left(N\right)\right)\phi^{(2)}\left(\sigma_{ij}\right)$$

$$\frac{\partial \phi}{\partial N} = \frac{\partial \zeta}{\partial N}\left(\phi^{(1)} - \phi^{(2)}\right)$$

- For weighting functions want:
  - Non-zero initial derivatives
  - Satisfy positivity constraints
  - Eventually saturate
  - Continuous

$$\zeta = \exp\left(-kN\right) \qquad N\left(\eta\right) = \frac{1}{2}P^{\mathrm{mod}}\eta^2$$

$k,\ P^{\mathrm{mod}}$ Fitting constants

Numerical Implementation

- Use Line-Search Augmented Newton-Raphson (LS-NR) approach

Minimize $$\psi = \frac{1}{2}\left[\left(\frac{E}{\sigma_y^0}\right)^2 r_{ij}^{\varepsilon} r_{ij}^{\varepsilon} + \left(\frac{r^f}{\sigma_y^0}\right)^2 + \left(\frac{P^{\mathrm{mod}} r^{\eta}}{\sigma_y^0}\right)^2\right]$$

Residuals

$$-r^{f(k)} = f(\sigma_{ij}, \kappa, \eta)$$ Consistency

$$-r_{ij}^{\varepsilon(k)} = -\mathscr{L}_{ijkl}^{p-1} + d\kappa \frac{\partial \phi}{\partial \sigma_{ij}}$$ Plastic Strain Flow Rule

$$-r^{\eta(k)} = -\eta + d\kappa \frac{\partial \phi}{\partial N}$$ DHV Flow Rule

System of Linearized Residuals

$$-r^f = \frac{\partial \phi}{\partial \sigma_{ij}} \Delta\sigma_{ij} - \frac{\partial \sigma_y}{\partial \kappa} \Delta\kappa + \frac{\partial \phi}{\partial \eta} \Delta\eta$$

$$-r_{ij}^{\varepsilon} = \mathscr{L}_{ijkl} \Delta\sigma_{kl} + \frac{\partial \phi}{\partial \sigma_{ij}} \Delta\kappa + d\kappa \frac{\partial^2 \phi}{\partial \sigma_{ij}\partial \eta} \Delta\eta$$

$$-r^{\eta} = d\kappa \frac{\partial^2 \phi}{\partial N \partial \sigma_{ij}} \Delta\sigma_{ij} + \frac{\partial \phi}{\partial N} \Delta\kappa + \left(1 + d\kappa \frac{\partial^2 \phi}{\partial N \partial \eta}\right) \Delta\eta$$

$$\Delta\kappa = \frac{-\frac{\partial \phi}{\partial \sigma_{ij}} \mathscr{L}_{ijkl} r_{kl}^{\varepsilon} + r^f}{\frac{\partial \phi}{\partial \sigma_{ij}} \mathscr{L}_{ijkl} \frac{\partial \phi}{\partial \sigma_{kl}} + \frac{\partial \sigma_y}{\partial \kappa}}$$

"Classical" solution for isotropic hardening plasticity

$$\Delta\kappa = \frac{-\frac{\partial \phi}{\partial \sigma_{ij}} \mathscr{L}_{ijkl} r_{kl}^{\varepsilon} + r^f + \frac{1}{\omega}\left(\frac{\partial \phi}{\partial \eta} - d\kappa \frac{\partial \phi}{\partial \sigma_{ij}} \mathscr{L}_{ijkl} \frac{\partial^2 \phi}{\partial \sigma_{kl}\partial \eta}\right)\left(r^{\eta} - d\kappa \frac{\partial^2 \phi}{\partial N \partial \sigma_{ij}} \mathscr{L}_{ijkl} r_{kl}^{\varepsilon}\right)}{\frac{\partial \phi}{\partial \sigma_{ij}} \mathscr{L}_{ijkl} \frac{\partial \phi}{\partial \sigma_{kl}} + \frac{\partial \sigma_y}{\partial \kappa} + \frac{1}{\omega}\left(\frac{\partial \phi}{\partial \eta} - d\kappa \frac{\partial \phi}{\partial \sigma_{ij}} \mathscr{L}_{ijkl} \frac{\partial^2 \phi}{\partial \sigma_{kl}\partial \eta}\right)\left(\frac{\partial \phi}{\partial N} - d\kappa \frac{\partial \phi}{\partial \sigma_{ij}} \mathscr{L}_{ijkl} \frac{\partial^2 \phi}{\partial \sigma_{kl}\partial N}\right)}$$

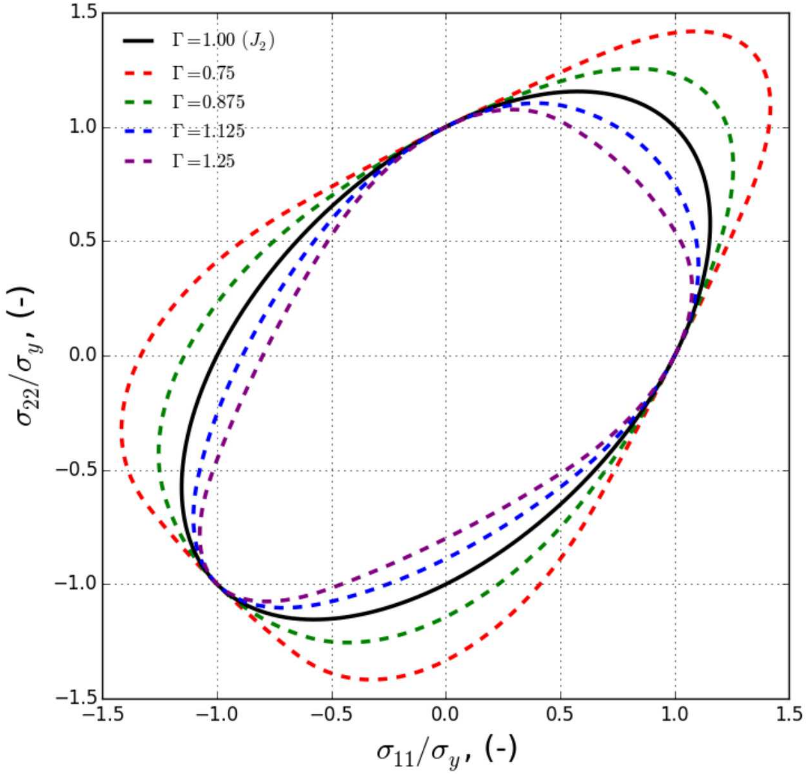
(Scherzinger, 2017, CMAME, 317, pp.526-553; Lester and Scherzinger, 2017, IJNME, 112, pp. 257-282)

Results

Strength-Differential Evolution

- Want to look at effect of developing a strength-differential effect
  - Consider isotropic form of Cazacu *et al.* effective stress

$$\phi^{(C)} = \{[|s_1| - k_c s_1]^a + [|s_2| - k_c s_2]^a + [|s_3| - k_c s_3]^a\}^{1/a}$$



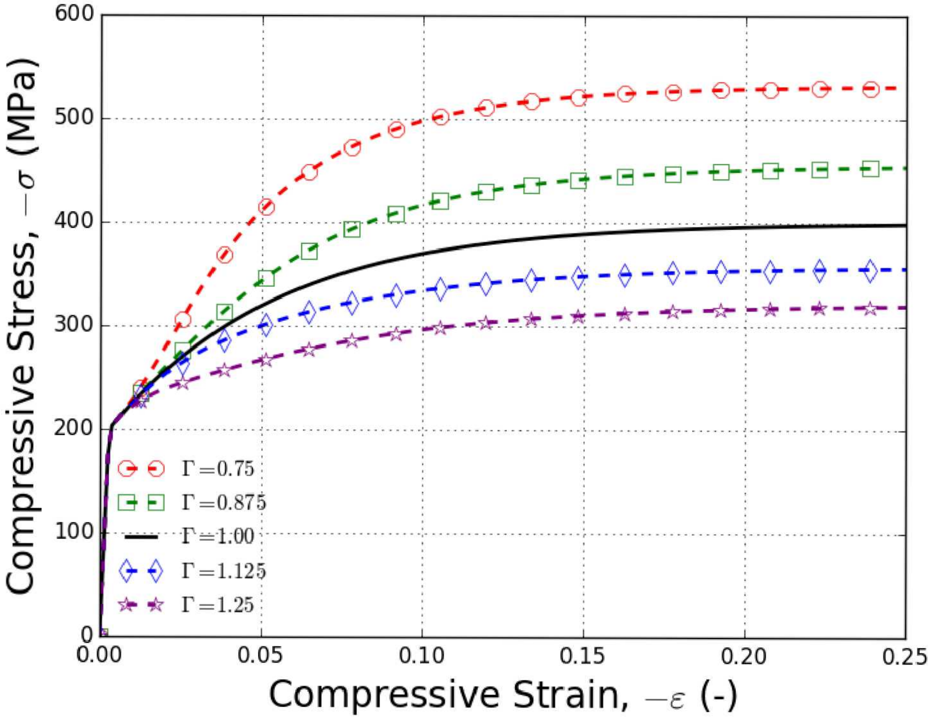
$$\Gamma = \frac{\sigma_y^{0(t)}}{\sigma_y^{0(c)}}$$

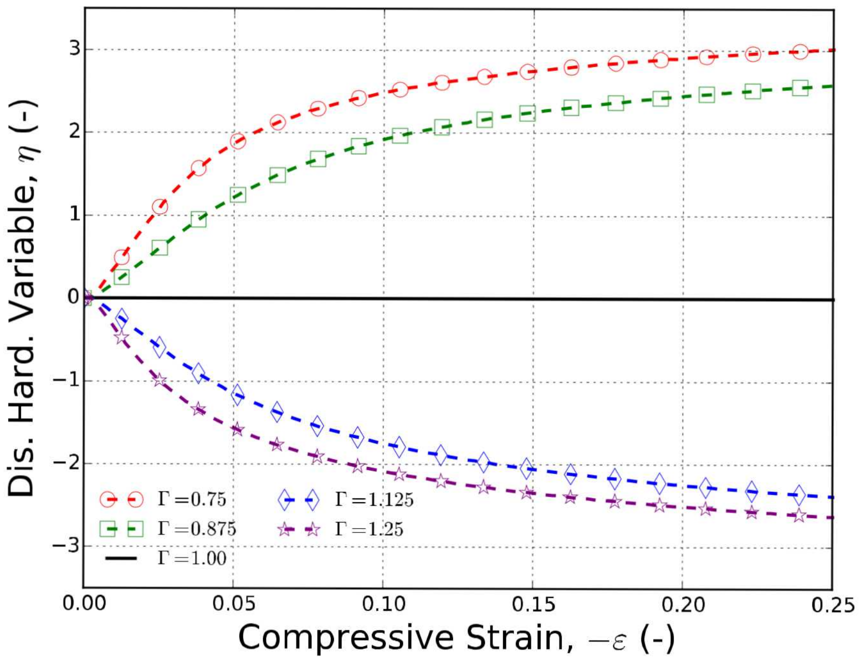
$$k_c = \frac{1 - h(\Gamma)}{1 + h(\Gamma)}$$

$$h(\Gamma) = \left[\frac{2^a - 2\Gamma^a}{(2\Gamma)^a - 2}\right]^{\frac{1}{a}}$$

(Cazacu *et al.*, 2006, IJP, 22, pp.1171-1194)

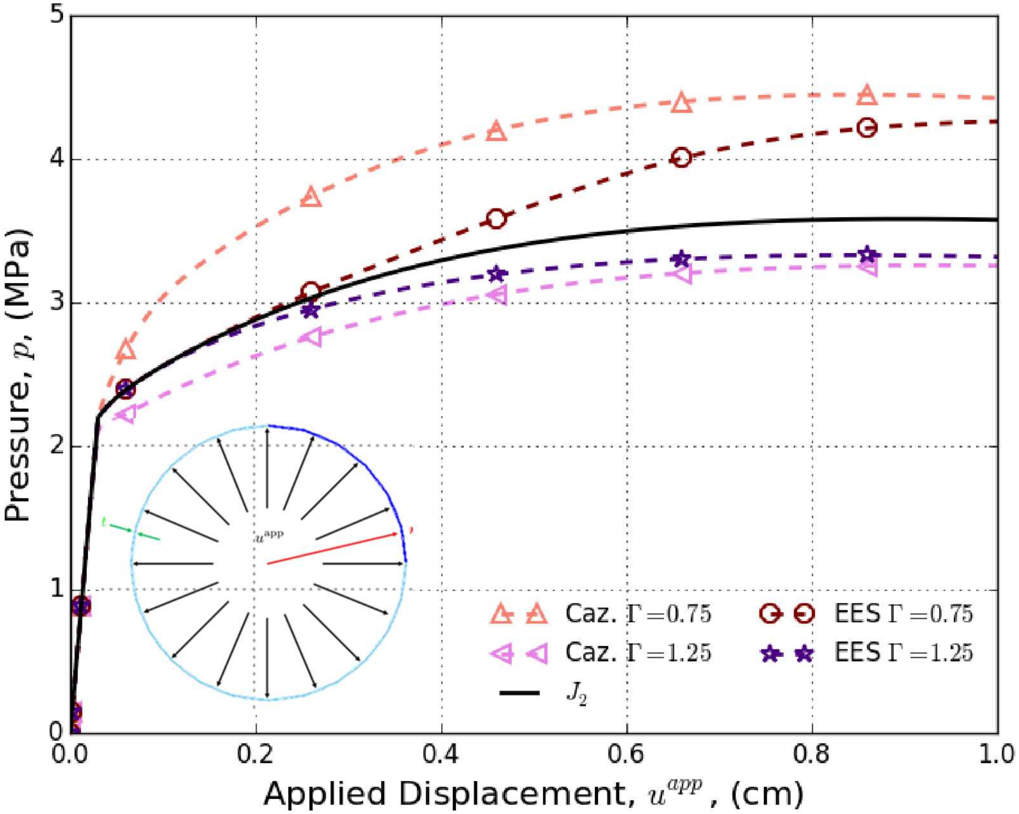
Constitutive Behavior

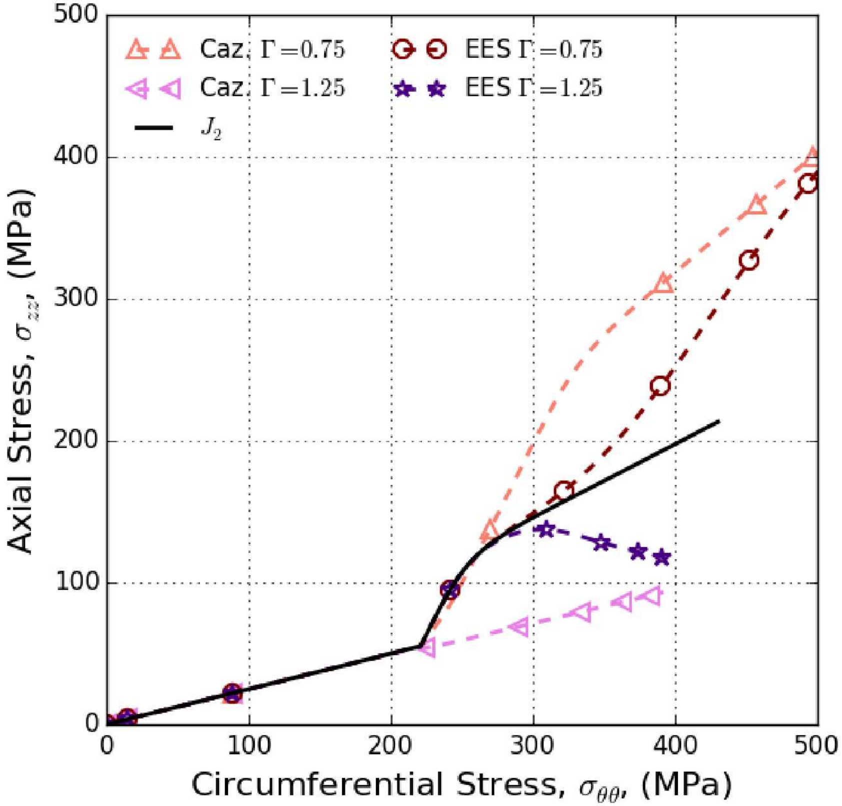




- EES approach enables the description of developing tension-compression asymmetry

Pressurized Cylinder



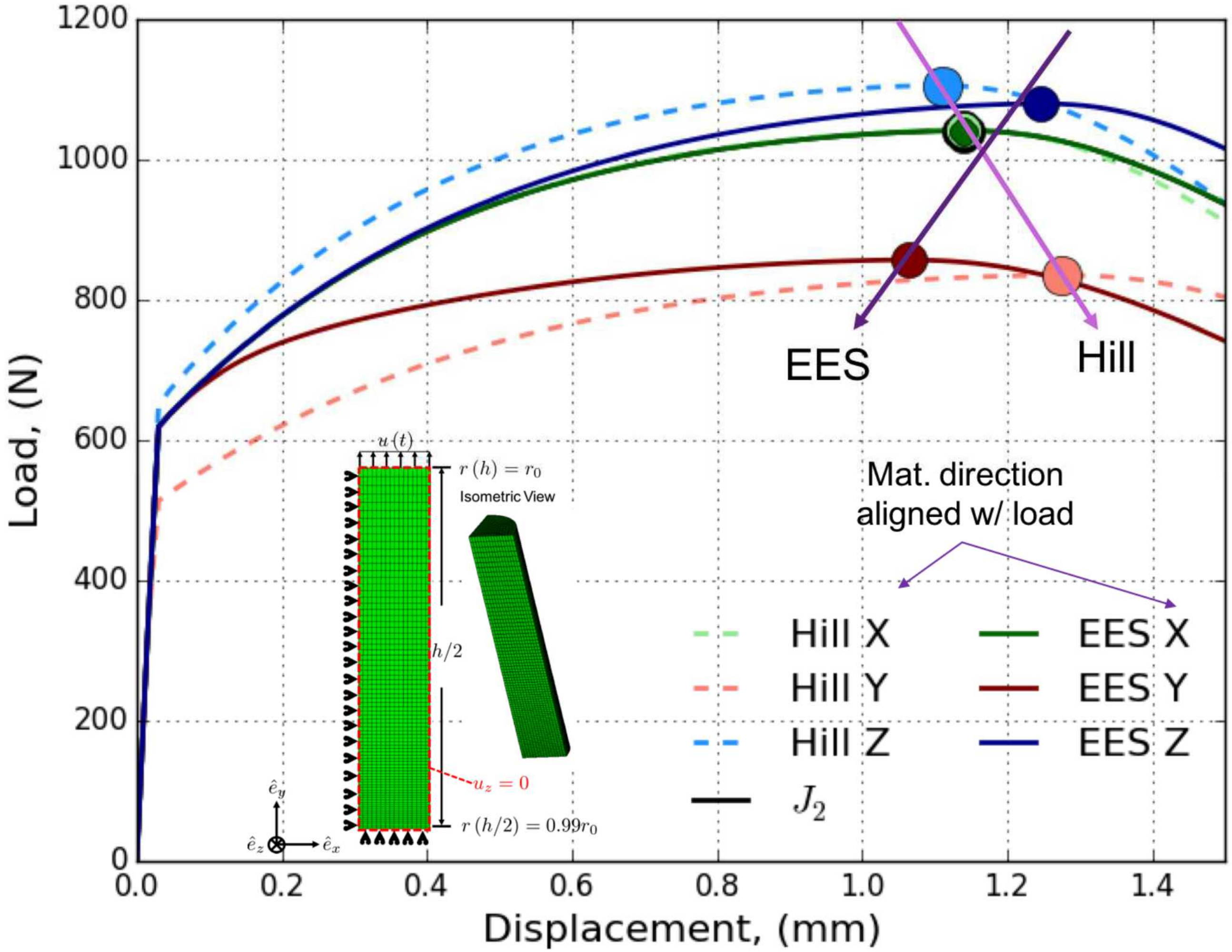


$$p = \frac{F}{(r + \bar{u}_r)\, h}$$

- Implementation is robust under complex, non-proportional, multiaxial loading paths

Tensile Cylinder

- Consider loading of a uniaxial tensile bar with the classic Hill'48 yield surface



Conclusion

- Developed theory and numerical implementation for evolving effective stress (EES) distortional hardening model
  - Introduce additional scalar internal state variable associated specifically with distortional hardening
  - Evolution equations derived in a thermodynamically consistent fashion producing associative flow rules
  - Numerical implementation via fully implicit, closest point projection line-search augmented Newton-Raphson return mapping algorithm
  - Demonstrated capability to solve structural problems

- Future work
  - Extension with kinematic hardening and experimental validation
  - Convexity – Open issue with distortional hardening
  - Dissipation

- Lester and Scherzinger, 2018, "An evolving effective stress approach to anisotropic distortional hardening", IJSS, *In Press*

Acknowledgements

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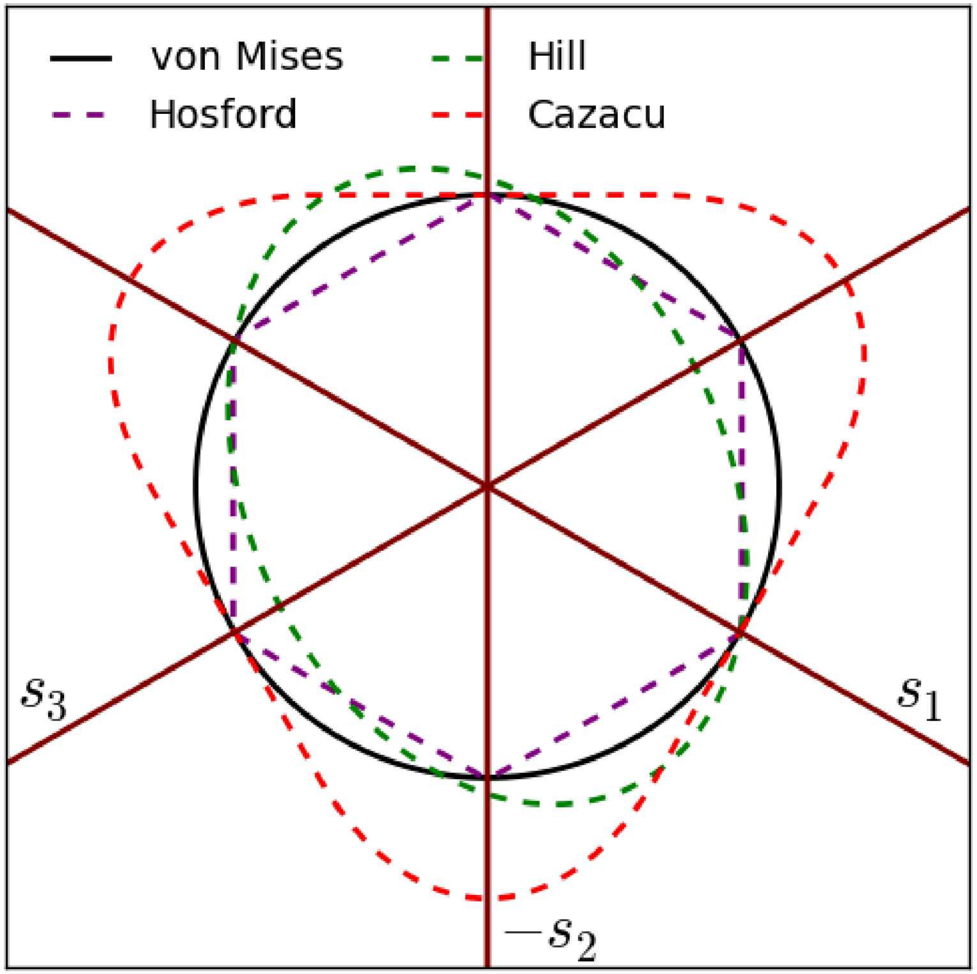
Questions?

Traditional Yield Function

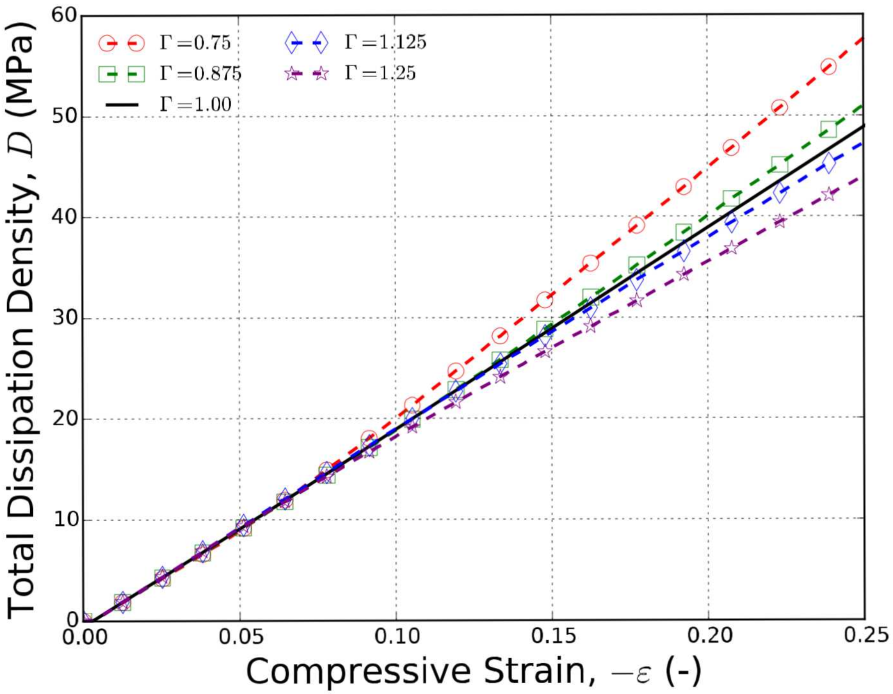
$$f = f\left(\sigma_{ij}, K\right) = \phi\left(\sigma_{ij}\right) - \sigma_y\left(K\right)$$

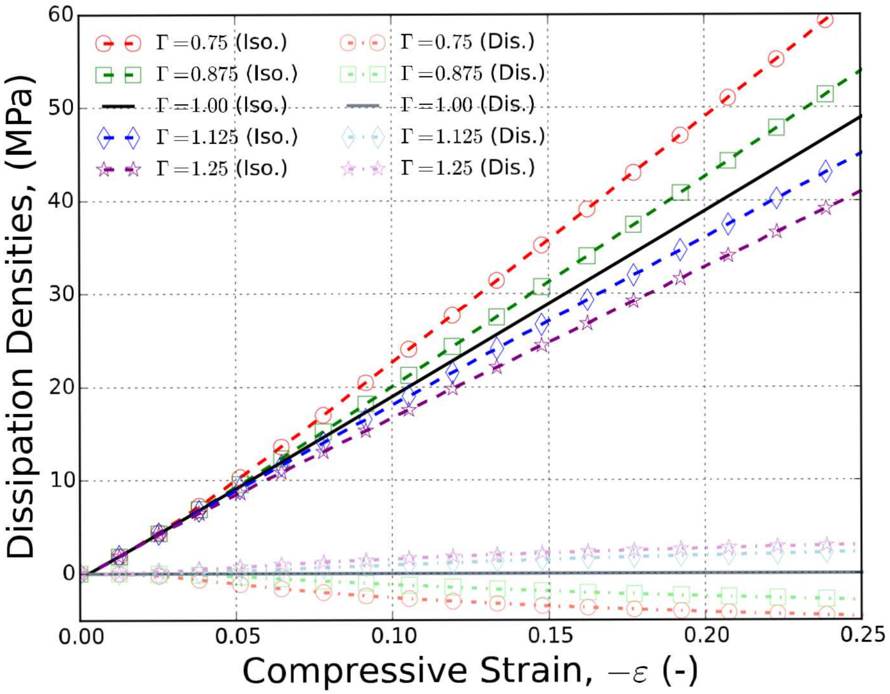
$\phi\left(\sigma_{ij}\right)$ Effective Stress $\qquad \sigma_y\left(K\right) = \sigma_y^0 + K$ Flow Stress



- Many existing effective stress definitions:
  - Non-quadratic
  - Strength-differential
  - Anisotropic
- Can they be leveraged for distortional capabilities?

Constitutive Dissipation





Convexity

- To maximize dissipation, minimize constrained Lagrangian

$$\mathcal{L}\left(\sigma_{ij}, K, N, \lambda\right) = -\mathcal{D}\left(\sigma_{ij}, K, N\right) + \lambda f\left(\sigma_{ij}, K, N\right)$$

$$\mathcal{D} = \sigma_{ij}\dot{\varepsilon}^{\mathrm{p}}_{ij} - K\dot{\kappa} - N\dot{\eta} \geq 0$$

- Second-order necessary and sufficient conditions for relative minimum satisfied if

$$y \cdot \nabla^2 \mathcal{L} y = \lambda y \cdot \nabla^2 f y \geq= 0 \qquad \forall y \qquad \text{s.t.} \qquad y \cdot \nabla f = 0$$

$$\hat{\sigma}_{ij}\frac{\partial^2 \phi^*}{\partial \sigma_{ij}\partial \sigma_{kl}}\hat{\sigma}_{kl} + 2\hat{N}\frac{\partial^2 \phi^*}{\partial \sigma_{ij}\partial N}\hat{\sigma}_{ij} + \hat{N}^2\frac{\partial^2 \phi^*}{\partial N^2}$$

- Some issues need to be addressed for general convexity of distortional hardening

(Luenberger and Ye, 2008, "Linear and Non-linear Programming")