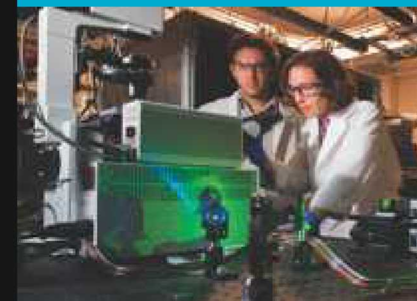
SAND2018-5572C

Radial Inertia Effect on Dynamic Compressive Response of Polymeric Foam Materials



2018 SEM Annual Conference and Exposition on Experimental and Applied Mechanics, Greenville, SC, June 4-7, 2018

PRESENTED BY

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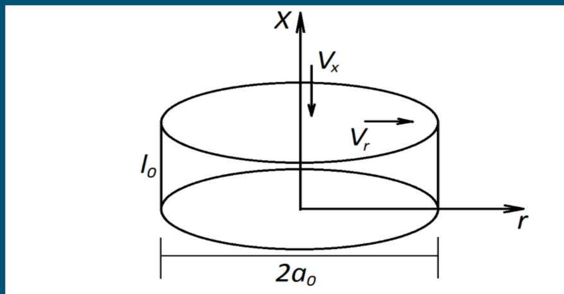
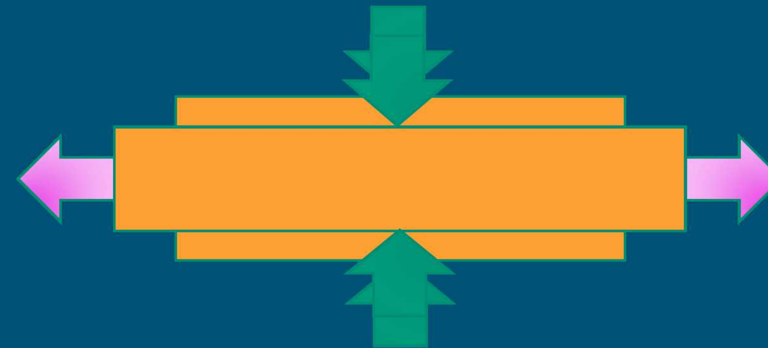
Radial Inertia during Dynamic Loading





$$V_r(t) = \frac{r}{2l} V_x(t)$$

Dharan and Hauser (1970)

In a compression test with a constant axial velocity:

- ❖ At a specific location, particle velocity along lateral (radial) direction increases with time (or increasing strain/decreasing specimen thickness)

- ❖ At a specific time, particle at outer diameter moves faster along lateral (radial) direction than the particle at inner diameter

Radial Confinement

Consequence of Radial Inertia

- ❖ Abnormal stress history

Soft Materials

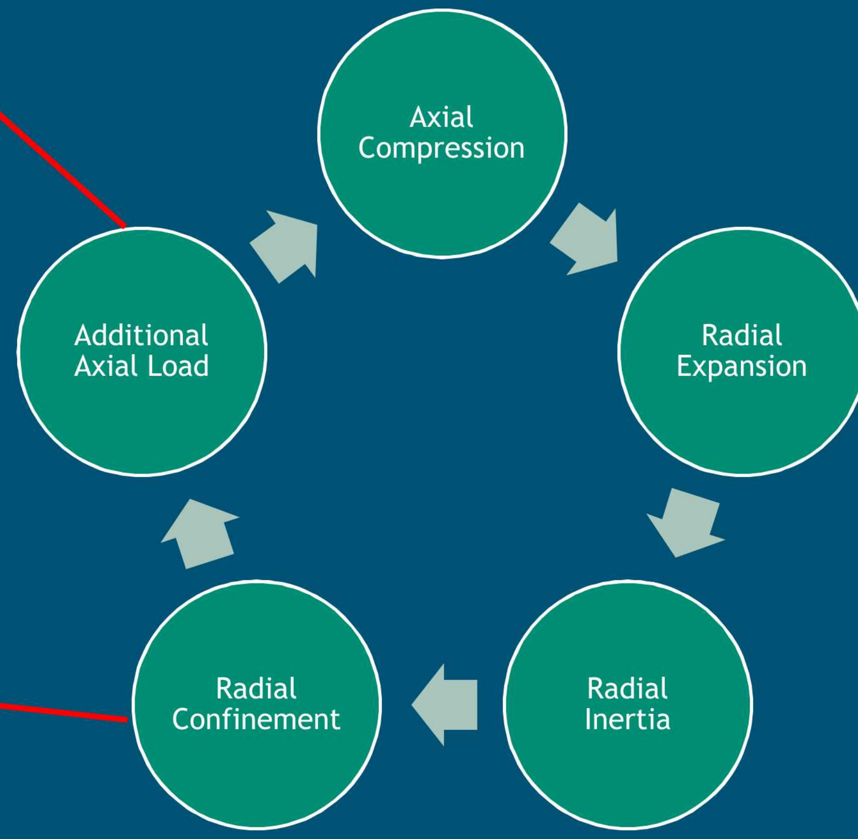
Intrinsic material stress-strain response may be overcome.

Song, B., Chen, W.W., Ge, Y., Weerasooriya, T., (2007) Radial inertia effects in Kolsky bar testing of extra-soft materials. Experimental Mechanics, 47:659-670.

- ❖ Change in specimen stress state from uniaxial to triaxial stress

Brittle Materials

Failure strength and associated strain rate effect may be overestimated.



Li, Q.M., and Meng, H. (2003) About the dynamic strength enhancement of concrete-like materials in a split Hopkinson pressure bar test. International Journal of Solids and Structures. 40:343-360.

Currently Existing Analysis on Radial Inertia Induced Axial Stress

- Compressible Solid
- Small deformation

- Incompressible Solid
- Large deformation

➢ Kolsky (1949)

$$\sigma_z = \frac{\nu^2 a_0^2 \rho_0}{2} \cdot \ddot{\varepsilon}$$

❖ Dharan and Hauser (1970)

$$\bar{\sigma}_z = \frac{\rho_0 a_0^{\,2}}{4 l_0 \left(1 - e_x\right)^2} \left[\frac{3 V_x^2}{2 l_0 \left(1 - e_x\right)} + \frac{d V_x}{dt} \right]$$

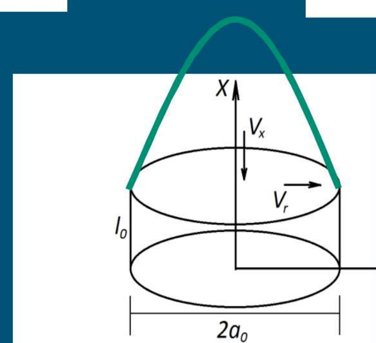
$$\bar{\sigma}_z = \frac{1}{\pi a^2} \int_0^a \int_0^{2\pi} \sigma_z(r)\, r\, dr\, d\theta$$

➢ Forrestal et al. (2007)

$$\sigma_z = \frac{\nu^2 (3 - 2\nu)}{4(1 - \nu)} \left[a_0^2 - \frac{2 r^2}{(3 - 2\nu)} \right] \rho_0 \cdot \ddot{\varepsilon}$$

➢ Warren and Forrestal (2010)

$$\sigma_z = \frac{\rho}{4 \left(1 - e_x\right)^2} \left[\frac{3 \dot{e}_x^2}{2 \left(1 - e_x\right)} + \ddot{e}_x \right] \left(a_0^2 - r^2\right)$$



Key Factors to Radial Inertia

- Compressible Solid
- Small deformation

$$\sigma_z = \frac{\nu^2 a_0^2 \rho_0}{2} \cdot \&$$

- Incompressible Solid
- Large deformation

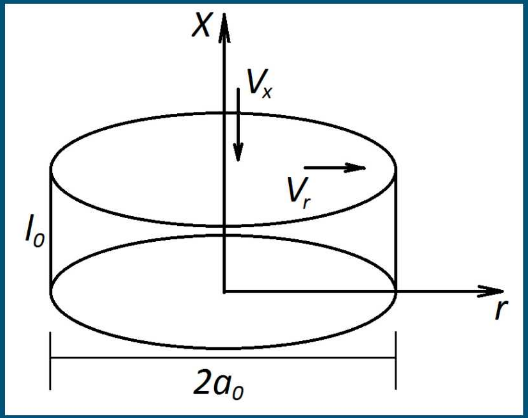
$$\bar{\sigma}_z = \frac{\rho_0 a_0^{\,2}}{4 l_0 \left(1 - e_x\right)^2} \left[\frac{3 V_x^2}{2 l_0 \left(1 - e_x\right)} + \frac{d V_x}{dt} \right]$$

- Specimen Geometry/Dimensions
  - Specimen diameter
  - Specimen thickness
- Loading conditions
  - Impact velocity
  - Strain rate history
  - Specimen strain
- Specimen Material Parameters
  - Density
  - **Poisson's ratio**

All current analyses are based on constant Poisson's ratio.

What will happen if Poisson's ratio is no longer a constant?

Comprehensive Radial Inertia Analysis



Mass conservation:

$$\frac{1}{\rho(t)}\cdot\frac{d\rho(t)}{dt}=-\left(\frac{V_x(t)}{l(t)}+2\frac{V_r(r,t)}{r(t)}\right)$$

Momentum conservation:

$$\frac{\partial\sigma_r(r,t)}{\partial r}=-\rho(t)\left(\frac{\partial V_r(r,t)}{\partial t}+V_r(r,t)\frac{\partial V_r(r,t)}{\partial r}\right)$$

$$\frac{\partial\sigma_r(r,t)}{\partial r}=-\rho(t)\cdot\frac{r}{1-e_x(t)}\cdot\left[\nu(t)\ddot{e}_x(t)+\nu(t)\cdot\left(\nu(t)+1\right)\cdot\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{e}_x^2(t)\right]$$

$$\sigma_r(r,t)=\rho(t)\cdot\frac{a^2(t)-r^2(t)}{2\left(1-e_x(t)\right)}\cdot\left[\nu(t)\ddot{e}_x(t)+\nu(t)\cdot\left(\nu(t)+1\right)\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{e}_x^2(t)\right]$$

Comprehensive Radial Inertia Analysis

$$\sigma_r(r,t)=\rho(t)\cdot\frac{a^2(t)-r^2(t)}{2\left(1-e_x(t)\right)}\cdot\left[\nu(t)\ddot{e}_x(t)+\nu(t)\cdot\left(\nu(t)+1\right)\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{e}_x^2(t)\right]$$

0

Radial inertia in a solid with a constant Poisson's ratio $\frac{d\nu}{de_x}=0$

$$\sigma_r(r_0,t)=\rho_0\cdot\frac{a_0^2-r_0^2(t)}{2\left(1-e_x(t)\right)^2}\cdot\left[\nu\ddot{e}_x(t)+\nu\cdot(\nu+1)\frac{\dot{e}_x^2(t)}{1-e_x(t)}\right]$$

for incompressible solid $\nu=\frac{1}{2}$

$$\sigma_r(r_0,t)=\rho_0\cdot\frac{a_0^2-r_0^2}{4\left(1-e_x(t)\right)^2}\cdot\left[\ddot{e}_x(t)+\frac{3}{2}\cdot\frac{\dot{e}_x^2(t)}{1-e_x(t)}\right]$$

Exactly same as that given by Warren and Forrestal (2010) for an incompressible solid

Understanding Radial Inertia

Specimen geometry

Strain acceleration

Strain rate

$$\sigma_r\left(r,t\right)=\rho\left(t\right)\cdot\frac{a^2\left(t\right)-r^2\left(t\right)}{2\left(1-e_x\left(t\right)\right)}\cdot\left[\nu\left(t\right)\ddot{e}_x\left(t\right)+\nu\left(t\right)\cdot\left(\nu\left(t\right)+1\right)\frac{\dot{e}_x^2\left(t\right)}{1-e_x\left(t\right)}+\frac{d\nu\left(e_x\right)}{de_x}\dot{e}_x^2\left(t\right)\right]$$

Specimen density

Poisson's ratio

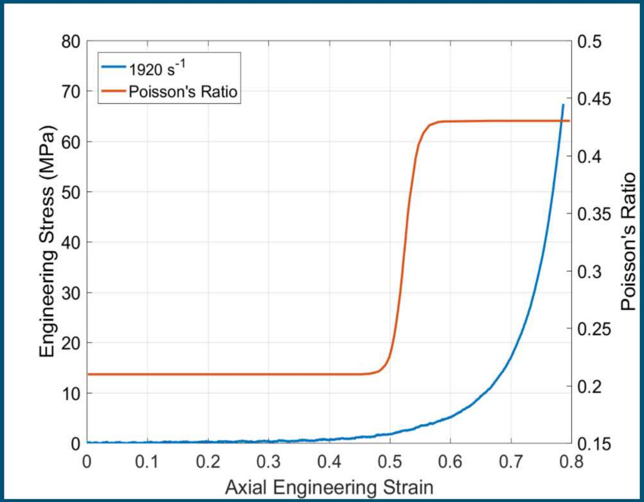
Poisson's ratio change

Specimen strain (deformation)

Radial Inertia in a Silicone Foam

Brett Sanborn's earlier presentation

$$\nu(e_x) = \frac{\nu_1 - \nu_2}{1 + \exp\left(\dfrac{e_x - e_{x0}}{\delta}\right)} + \nu_2$$



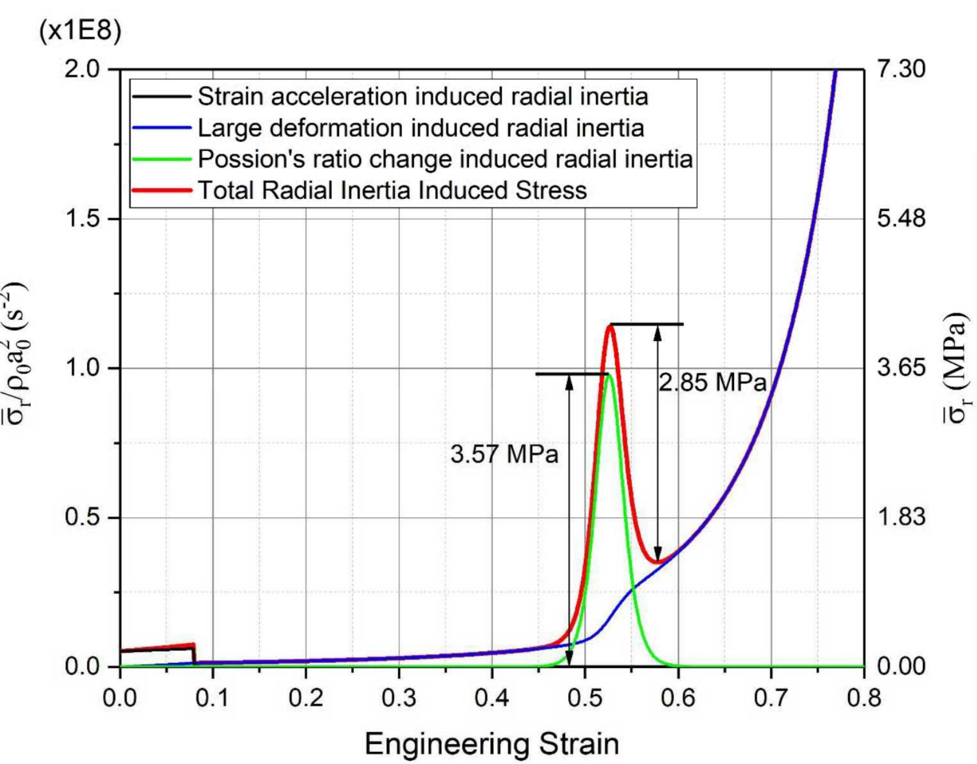
$$\sigma_r(r_0,t) = \rho_0 \cdot \frac{a_0^2 - r_0^2}{2\left(1 - e_x(t)\right)^2} \cdot \left[\begin{array}{l} \ddot{e}_x(t) \cdot \left(\nu_2 + \dfrac{\nu_1 - \nu_2}{1 + \exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)} \right) \\ + \left(\nu_2 + \dfrac{\nu_1 - \nu_2}{1 + \exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)} \right) \cdot \left(1 + \nu_2 + \dfrac{\nu_1 - \nu_2}{1 + \exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)} \right) \cdot \dfrac{\dot{e}_x^2(t)}{1 - e_x(t)} \\ + \dot{e}_x^2(t) \cdot \dfrac{\nu_2 - \nu_1}{\delta} \cdot \dfrac{\exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)}{\left[1 + \exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)\right]^2} \end{array} \right]$$

Radial Inertia in a Silicone Foam

Average additional stress

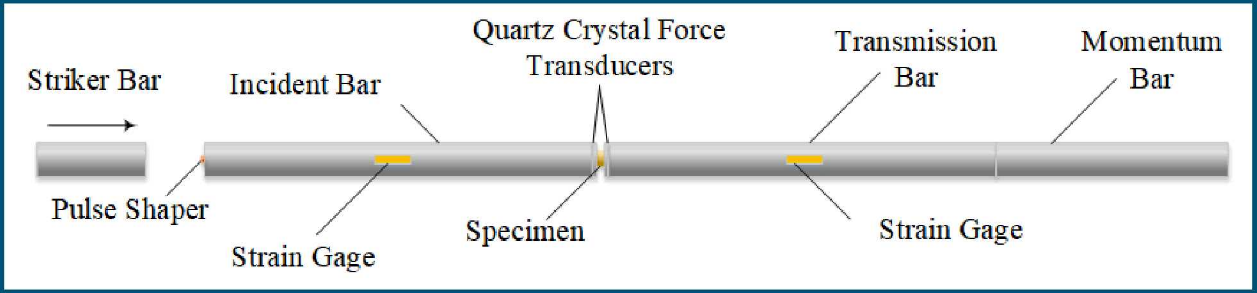
$$\bar{\sigma}_z = \frac{1}{\pi a^2}\int_0^a\int_0^{2\pi}\sigma_z(r)\,r\,dr\,d\theta$$

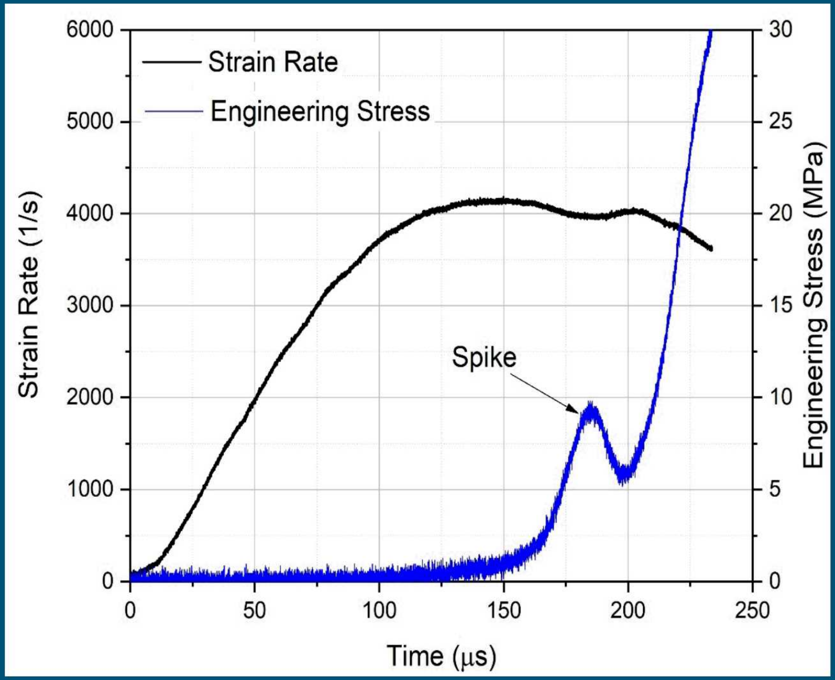
$$\bar{\sigma}_r(t) = \frac{\rho_0\cdot a_0^2}{4\left(1-e_x(t)\right)^2}\cdot\left[\begin{array}{l}\ddot{e}_x(t)\cdot\left(\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\\ +\left(\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\left(1+\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\dfrac{\dot{e}_x^2(t)}{1-e_x(t)}\\ +\dot{e}_x^2(t)\cdot\dfrac{\nu_2-\nu_1}{\delta}\cdot\dfrac{\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}{\left[1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)\right]^2}\end{array}\right]$$

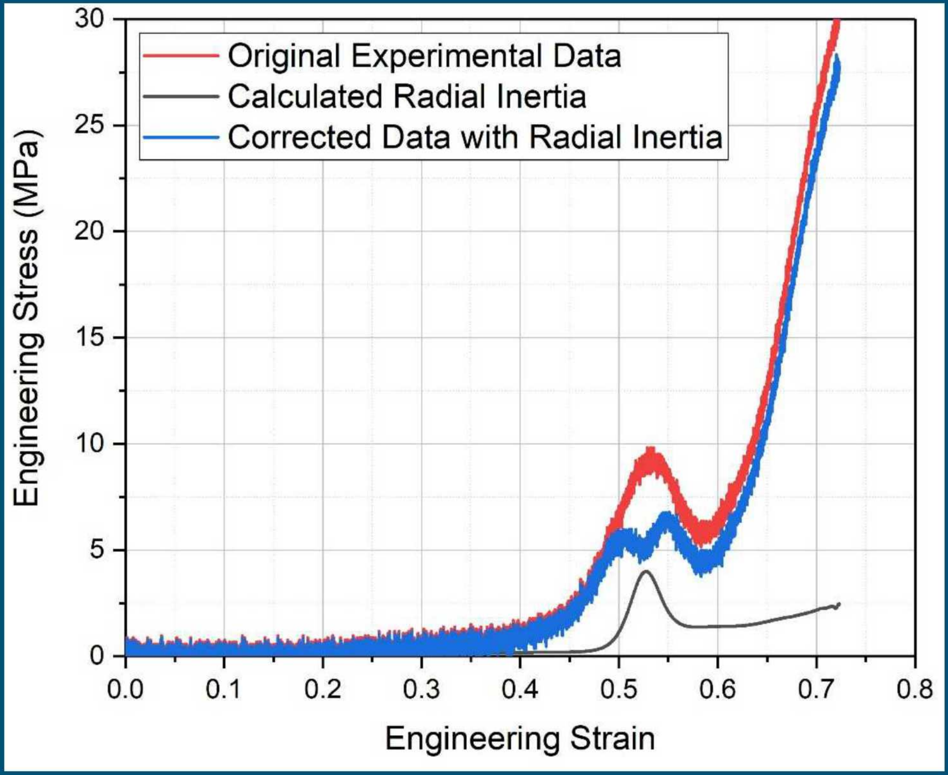


- **Strain acceleration**
- **Large deformation**
- **Poisson's ratio Change**

Experimental Verification of Radial Inertia in a Silicone Foam







Radial Inertia in Foam Materials

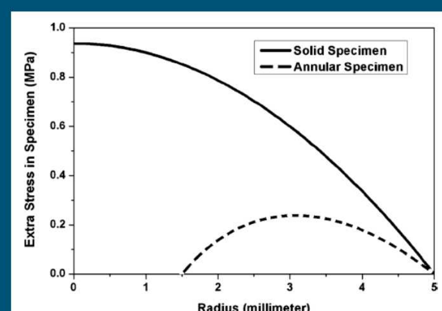
$$\sigma_r\left(r,t\right)=\rho\left(t\right)\cdot\frac{a^2\left(t\right)-r^2\left(t\right)}{2\left(1-e_x\left(t\right)\right)}\cdot\left[\quad\cdots\quad\frac{d\nu\left(e_x\right)}{de_x}\dot{e}_x^2\left(t\right)\right]$$

$$\sigma_r\left(r,t\right)=\rho\left(t\right)\cdot\frac{a^2\left(t\right)-r^2\left(t\right)}{2\left(1-e_x\left(t\right)\right)}\cdot\left[\frac{d\nu\left(e_x\right)}{de_x}\dot{e}_x^2\left(t\right)\right]$$

Poisson's Ratio Change Dominated During Densification

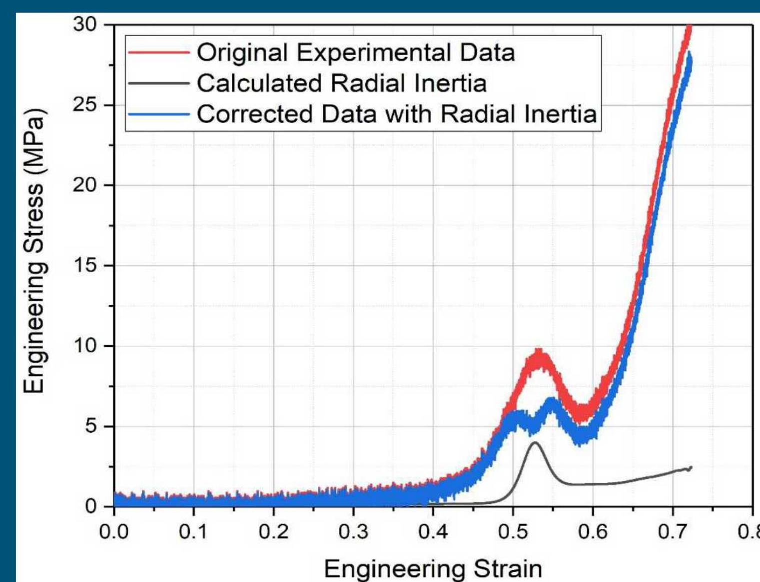
Experimental Solutions

- ✓ Constant strain rate
- ✓ Annular specimen?



Numerical Correction

- ✓ Challenge: It is highly sensitive to strain-dependent Poisson's ratio (derivation)



Conclusions

- Unique Poisson's ratio of unique materials, i.e., polymeric foams, results in unique radial inertia during dynamic tests
- Radial inertia has been comprehensively analyzed, particularly accounting for the effect of Poisson's ratio change
  - Key factors dominating different regions
    - Strain acceleration
      – ***before strain rate achieves a constant***
    - Large deformation
      – ***after specimen is densified at large deformation***
    - Poisson's ratio change
      – ***during specimen densification***
    - More significant at higher strain rates
- Suggested solutions
  - Numerical correction
  - Experimental solution