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Final Report on Subcontract B629176: Algebraic Multigrid with Optimal Interpolation

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May 15, 2019

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This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

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August 1, 2018 – May 31, 2019

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Progress has been made in several of the proposed areas of research, as summarized below.

Compatible Relaxation and Optimal Interpolation

The project team has continued with work on developing new AMG techniques for solving symmetric and non-symmetric PDE systems. The focus of this work is to derive more reliable measures of the quality of the coarse variable set than the convergence rate of the standard F-relaxation form of CR and a more robust form of interpolation than the so-called ideal form. We have successfully derived a sharp variant of CR that gives the precise convergence rate of the two-grid method using this optimal interpolation and, in addition, we derived a new Generalized Bootstrap AMG setup algorithm that uses as its main tool a multilevel eigensolver for the generalized eigenvalue problem involving the system matrix and the symmetrized smoother. In addition, the approach allows for general block smoothers with overlap. We have applied the method to scalar diffusion problems, linear elasticity, Maxwell's equations and Stokes equations and the method shows marked improvements over existing AMG methods for these problems.

Bootstrap Algebraic Multigrid and non-Galerkin AMG for the Dirac equation.

The team also continued to develop Bootstrap AMG (BAMG) solvers for non-symmetric problems, and in particular for the non-Hermitian Wilson Dirac system and Overlap system. The BAMG approach employs a bootstrap setup algorithm to compute interpolation, based on the notion of least squares fitting of test vectors (left and right singular vectors that are computed adaptively). This is an alternative, and generally far less expensive approach to the adaptive aggregation-based setup algorithm we originally developed for the Wilson system. In 2D, the BAMG approach has been shown to yield a solver whose performance is better than that of the aggregation based scheme with less work required in the adaptive setup and with lower overall complexities in the solve phase. Currently, we are working with members of CASC to extend the BAMG approach to the full 4D system. The aim here is to implement the solver in the parallel solver library *hypre*.

In addition, we continued to investigate the use of non-Galerkin coarsening in this same context. In particular, we considered non-Galerkin algebraic multigrid solvers for lattice discretizations of the Dirac system with the aim of reducing the number of nonzero entries in the Galerkin coarse-level operators constructed in the Bootstrap AMG setup. The approaches we considered are: the parallel semi-coarsening multigrid (PFMG) strategy proposed that is now being implemented in *hypre* for the Wilson-Dirac system; and a stencil collapsing approach recently developed by Falgout and Schroder.

Our preliminary findings are promising and suggest that these techniques should lead to efficient and robust solvers for various discretizations of the Dirac PDE.