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## Finite-deformation diffusion of hydrogen in stainless steel alloys

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August 20, 2015

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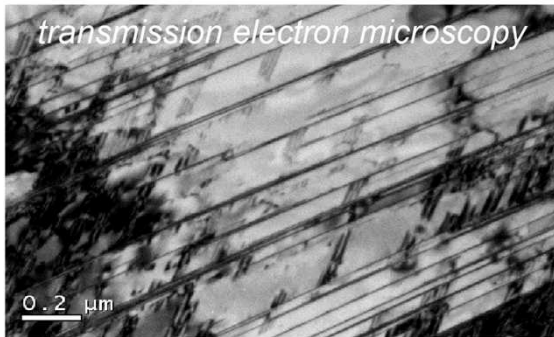
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# Introduction

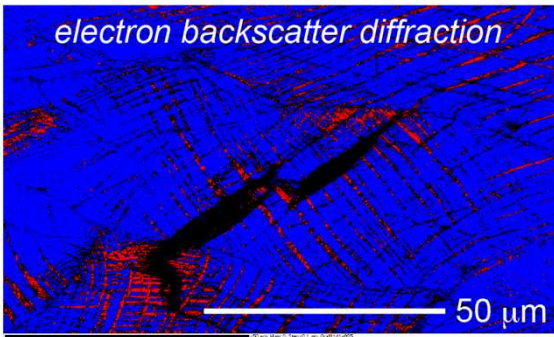
## Academic Background

- Undergraduate: UT Austin (Class of 2013)
- Graduate: UC Berkeley (M.S. and Ph.D.)
  - Anticipated Graduation: ~05/2018
  - Research Advisor: Dr. Shaofan Li
  - Co-authored Publications:
    - A Peridynamics-SPH Coupling Approach to Simulate Soil Fragmentation Induced by Shock Waves (Feb. 2015)
    - A Hybrid Peridynamics-SPH Simulation of Soil Fragmentation by Blast Loads of Buried Explosive (in Press)

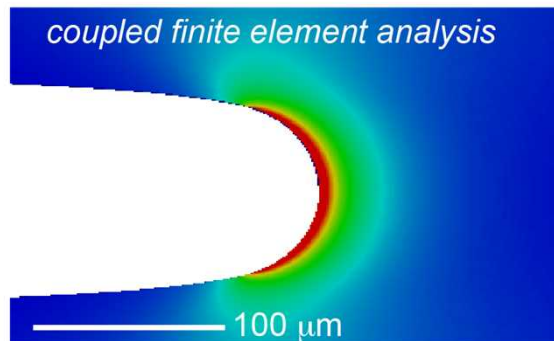
# Motivation



deformation twins



phase transformations, fcc, bcc



plastic zone at blunted crack tip

## Hydrogen Embrittlement

- Dissociated hydrogen molecules are introduced into a metal lattice through welding processes, environmental factors, etc..
- The embrittlement process can alter macroscopic material properties such as yield strength and ductility, which are driven by localized changes in the microstructure.
- Methods developed can be employed for phenomenological models of transport and trapping and continuum representations of microstructure.

# Theoretical Background

This path heavily leverages Sofronis/McMeeking (1989)\* and Krom (1998).  
Recent work by Leo and Anand (2013).

**chemical  
potential**

$$\mu_l = \mu_0 + RT \ln(\theta_l) - v_h \sigma_h$$

**model  
for flux**

$$\mathbf{j}_l = -\mathbf{m}_l c_l \nabla_x \mu_l$$

**equilibrium of  
lattice/trap sites**

$$\theta_t = \frac{1}{1 + \frac{1}{k_t \theta_l}}$$

**conservation  
of hydrogen**

$$\frac{d}{dt} \int_B c dv = - \int_{\partial B} \mathbf{j} \cdot \mathbf{n} da$$

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$$

$$J = \det[\mathbf{F}]$$

$$\theta_l = c_l / n_l$$

$$\theta_t = c_t / n_t$$

$$n_l = N_L / J$$

$$n_t = N_T(\epsilon_p) / J$$

$$v_h = V_H J$$

$$\tau_h = J \sigma_h$$

$$\mathbf{d}_l = RT \mathbf{m}_l$$

$$\dot{c} = \dot{c}_l + \frac{\partial c_t}{\partial c_l} \dot{c}_l + \frac{\partial c_t}{\partial n_t} \frac{\partial n_t}{\partial \epsilon_p} \dot{\epsilon}_p + \frac{\partial c_t}{\partial n_t} \frac{\partial n_t}{\partial J} \dot{J}$$

$$c_t = c_t(c_l, \epsilon_p, J)$$

$$C^* = c_l + \frac{n_t}{1 + \frac{n_t}{k_t c_l}} \quad D^* = 1 + \frac{\partial c_t}{\partial c_l}$$

\*P. Sofronis and R.M. McMeeking, J. Mech. Phys. Solids 37 (1989) 317

# FEM Implementation

**conservation equation (reference configuration)**

$$\frac{d}{dt} \int_{B_0} C dV = - \int_{\partial B_0} J F^{-1} \mathbf{j} \cdot \mathbf{N} dA$$

$$\Rightarrow \int_{B_0} \left( \dot{C} - \nabla_{\mathbf{X}} \cdot d_l \mathbf{C}^{-1} \nabla_{\mathbf{X}} C_L + \nabla_{\mathbf{X}} \cdot \frac{d_l V_H}{RT} \mathbf{C}^{-1} \nabla_{\mathbf{X}} \tau_h C_L \right) dV = 0$$

**Multi-field variational formulation (Q1/P0 elements)**

*Variations in Displacement ( $\eta$ ):*

$$\int_{B_0} \mathbf{P} \cdot \nabla_{\mathbf{X}} \eta + \mathbf{B} \cdot \eta dV - \int_{\partial \Gamma_0} \mathbf{t} \cdot \eta dA = 0$$

*Variations in Concentration ( $v$ ):*

$$\int_{B_0} \left[ \left( D^* \dot{C}_L + \theta_T \frac{dN_T}{d\epsilon_p} \dot{\epsilon}_p \right) v + d_l \mathbf{C}^{-1} \nabla_{\mathbf{X}} C_L \cdot \nabla_{\mathbf{X}} v - \frac{d_l C_L V_H}{RT} \mathbf{C}^{-1} \nabla_{\mathbf{X}} \hat{\tau}_h \cdot \nabla_{\mathbf{X}} v \right] dV + \int_{\partial \Gamma_0} J F^{-1} \mathbf{j}_{app} \cdot \mathbf{N} v dA = 0$$

*Variations in Pressure ( $\psi$ ):*

$$\int_{B} \left( \tau_h - \hat{\tau}_h \right) \psi dV = 0$$

# Example: 2D Block

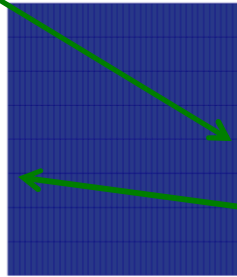
**Uniaxial extension of 2D block  
With transient hydrogen diffusion:**

Imposed Displacement 
$$u = \begin{bmatrix} AtX_1 \\ BtX_2 \\ 0 \end{bmatrix}$$

Neo-hookean hyperelastic model 
$$\sigma = \frac{1}{2}\kappa \left[ J_e - J_e^{-1} \right] \mathbf{I} + J_e^{-5/3} \mu dev[\mathbf{b}_e]$$

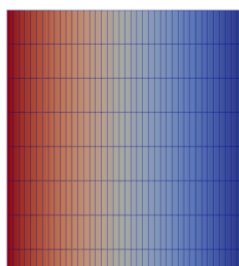
Time-dependent Displacement  
Fixed Concentration

Initial Layout

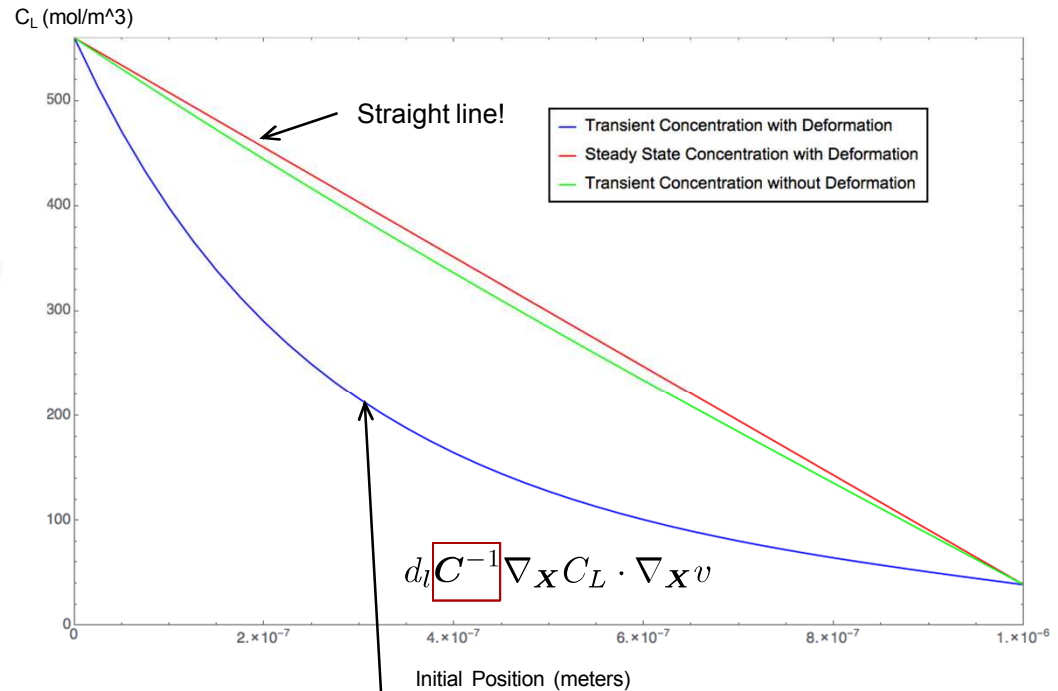


Fixed Displacement  
Time-dependent Concentration

Undeformed



Deformed

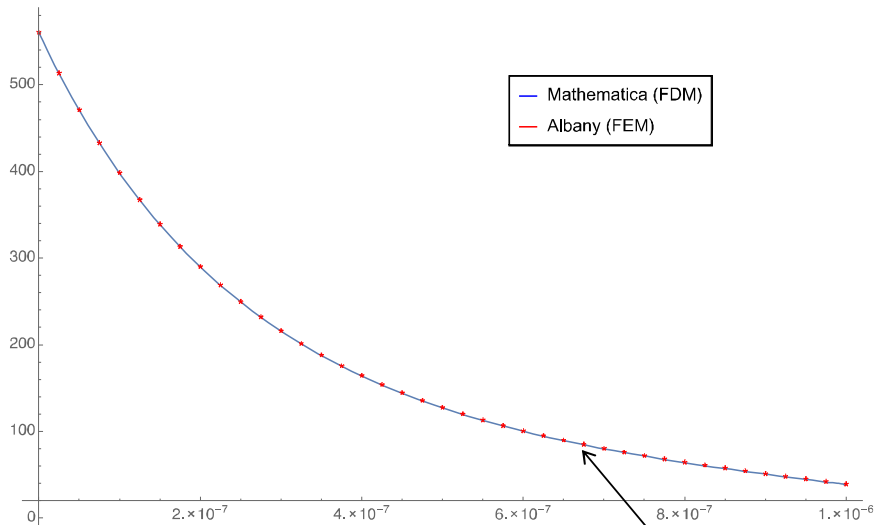


Pushback operation has distinct effect on the "Laplacian term" and hence, the concentration profile

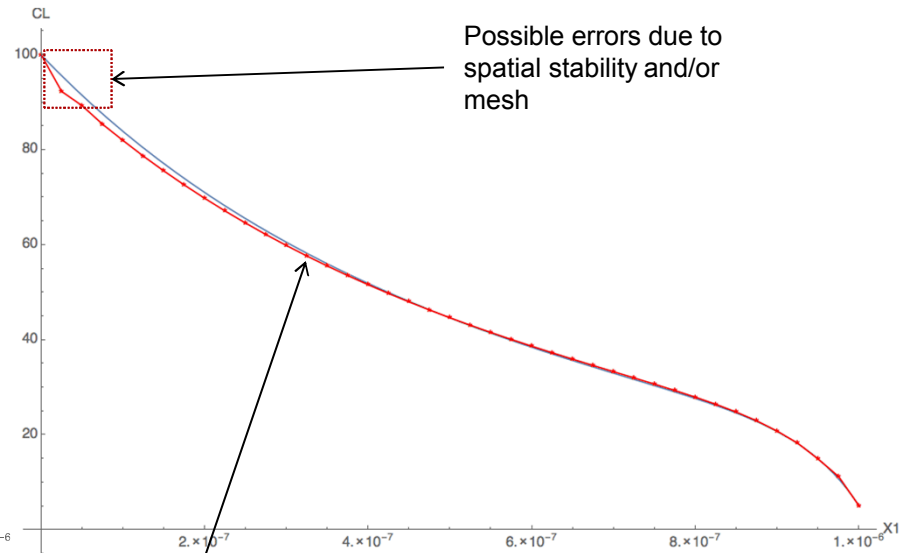
# Example: 2D Block

**Verification of solution: Goal is to implement tests to run in Albany**

**2D block diffusion without pressure gradient:**



**2D block diffusion with pressure gradient:**



Solutions  
match pretty  
well

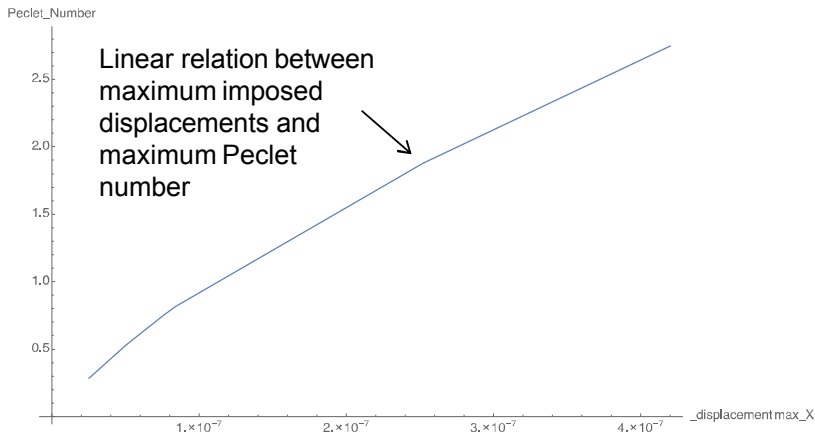
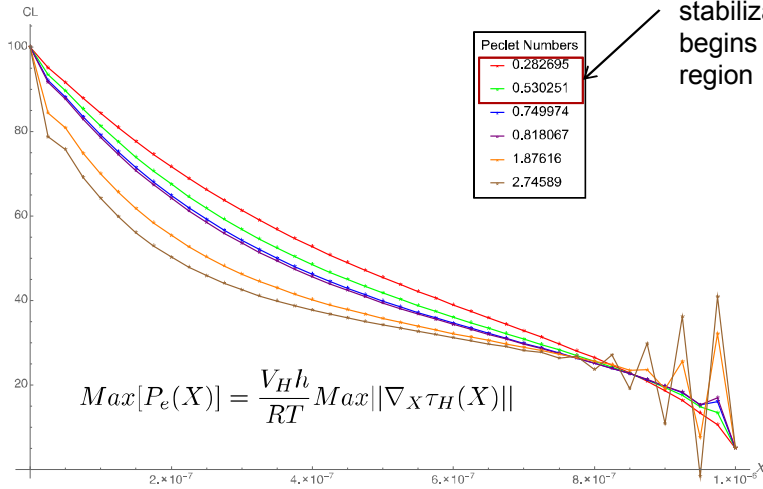
Possible errors due to  
spatial stability and/or  
mesh

# Example: 1D Stabilization Studies

**Spatial Stabilization and Spurious oscillation studies:**

Imposed Displacements:  
 $u(X) = C_0 + C_1 X + C_2 X^2 + C_3 X^3$

**Based on FEM (Albany):**



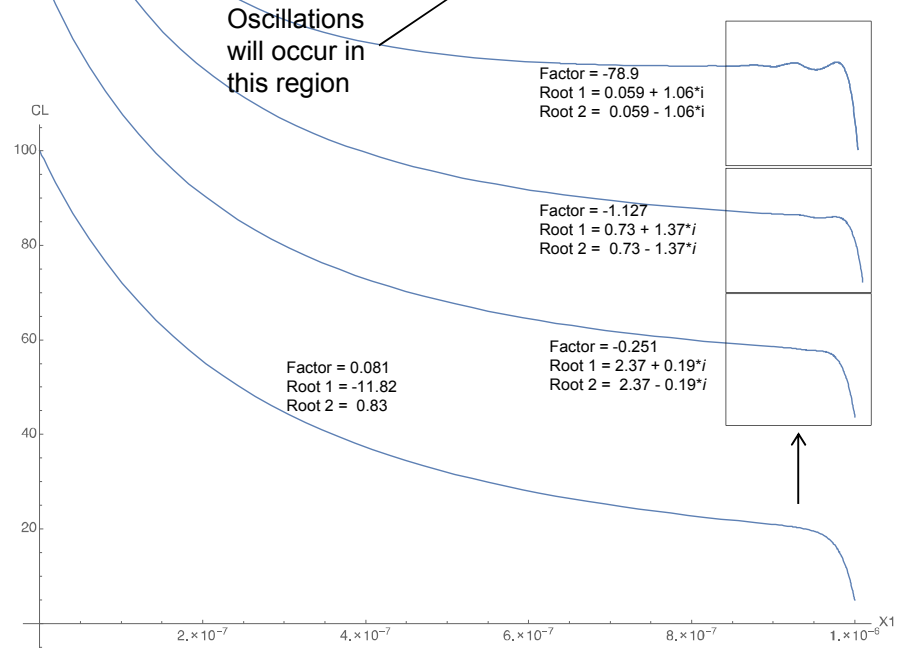
**Based on FDM (Mathematica):**

$$-\nabla_X \cdot d_l C^{-1} \nabla_X C_L + \nabla_X \cdot \frac{d_l V_H}{RT} C^{-1} \nabla_X \tau_h C_L = 0$$

$$\implies \alpha C_{L,xx} + \beta C_{L,x} + \gamma C_L = 0$$

Plug FDM stencil into ODE:

$$\frac{1 \pm \sqrt{1 + 4 \frac{\alpha^2 - \beta^2}{(\gamma + 2\alpha)^2}}}{\frac{2(\beta - \alpha)}{\gamma + 2\alpha}} = 0 \implies \frac{\alpha^2 - \beta^2}{(\gamma + 2\alpha)^2} \leq \frac{1}{4}$$



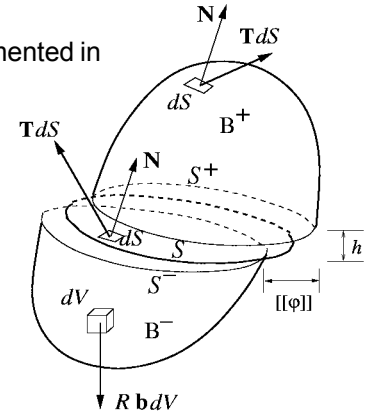
# Example: Simple channel fast pathway

## Implementation of Fast pathway (no deformation):

Cohesive Surface Element Formulation

$$C(\mathbf{X}) = \bar{C}(\phi[\xi^1, \xi^2]) + \frac{[[C]](\phi[\xi^1, \xi^2])}{h} \xi^3$$

Jump is implemented in formulation



Diffusivity is a function of position when fast pathway is utilized

CSE thickness is needed to properly integrate the residual

CSE residual

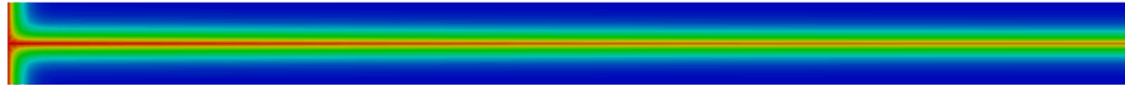
$$\int_{BG} \nabla_{\mathbf{X}} v D_L(\mathbf{X}) \nabla_{\mathbf{X}} C dV \approx \frac{h}{\sum_{j=1}^{N_{int}^M} \bar{W}(\xi_j^1, \xi_j^2)} [v_a^+(\xi_j^1, \xi_j^2) \quad v_a^-(\xi_j^1, \xi_j^2)] \begin{bmatrix} \vec{B}^+(\xi_j^1, \xi_j^2) \\ \vec{B}^-(\xi_j^1, \xi_j^2) \end{bmatrix} D_L(\mathbf{X})$$

$$\begin{bmatrix} \vec{B}^+(\xi_j^1, \xi_j^2) & \vec{B}^-(\xi_j^1, \xi_j^2) \end{bmatrix} \begin{bmatrix} C_a^+ \\ C_a^- \end{bmatrix}$$

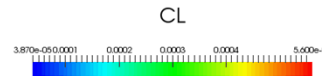
( )

Increasing these terms will increase diffusion of the fast pathway

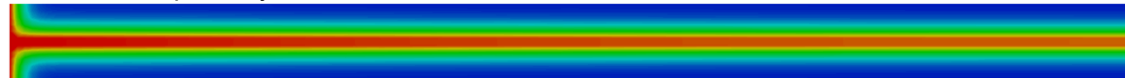
CSE fast pathway



Comparison between channel and block



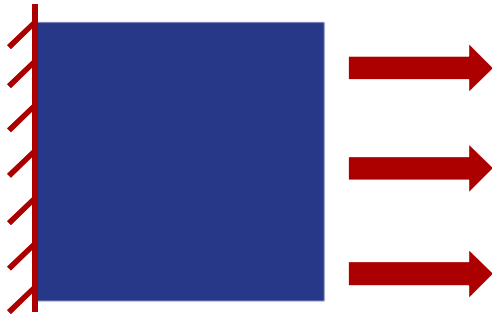
Channel fast pathway



Both cases exhibit similar trends

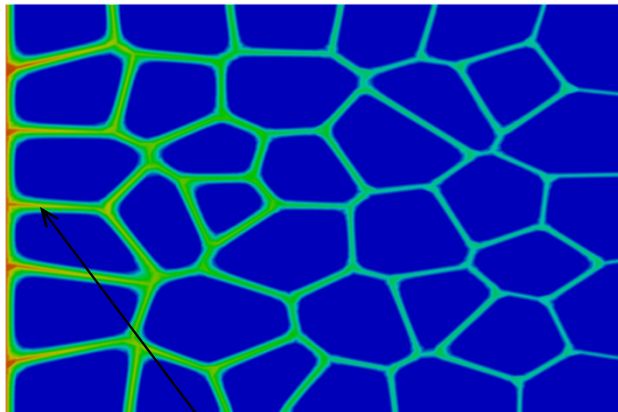
# Example: Polycrystals with fast pathway

*Grain interface diffusion coupled with mechanical loading:*



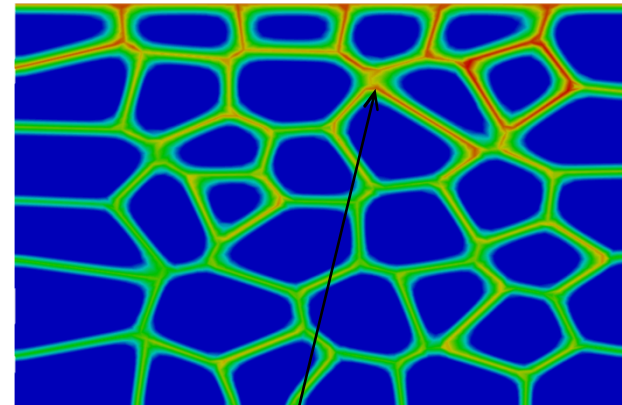
- Stretched in horizontal direction to a displacement of 2
- Fast pathway =  $10^5 \cdot D_0$

Horizontal diffusion (parallel to displacement)



Diffusion slows down.  
Poisson effect will compress  
lattice along vertical direction.

Vertical diffusion (perpendicular to displacement)

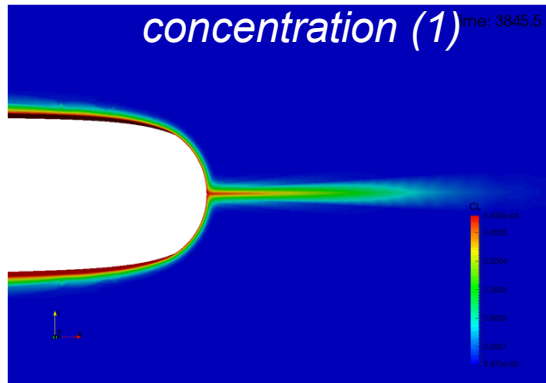


Diffusion speeds up as a result  
of the stretching of the lattice

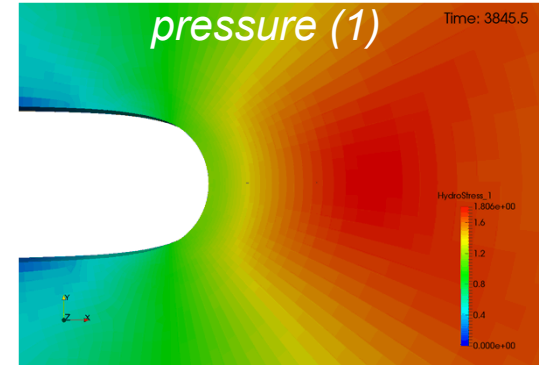
# Example: Kfield simulation with embedded crack

*Examining crack-tip diffusion:*

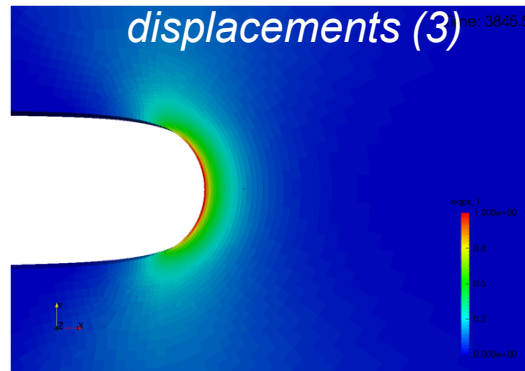
$$D^* \dot{C}_L$$



- Path:  $10^5 D_0$
- $K_{app}$ : 85 MPa m<sup>1/2</sup>
- Time: 3850 s



$$\nabla_{\mathbf{X}} \cdot d_l \mathbf{C}^{-1} \nabla_{\mathbf{X}} C_L$$

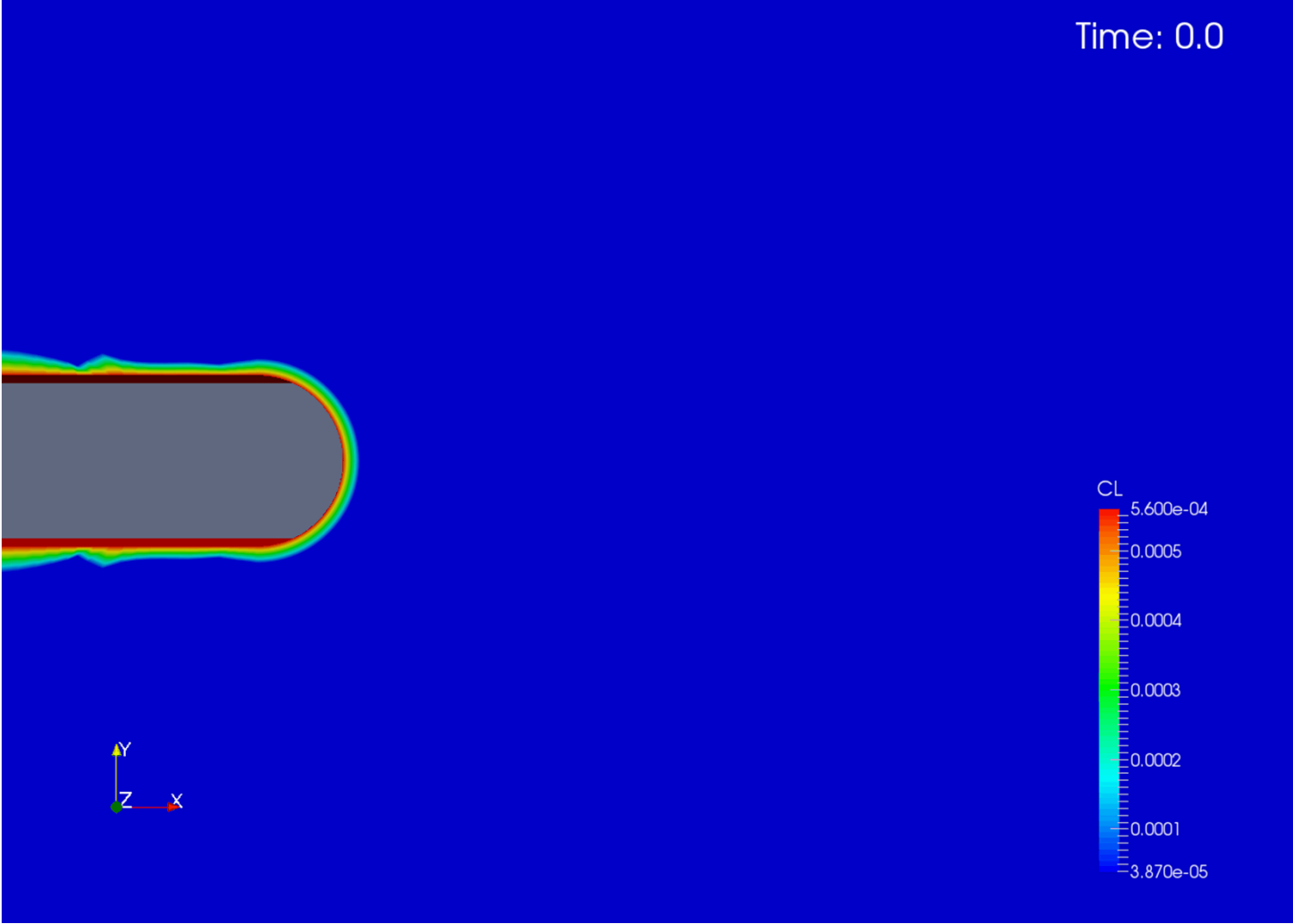


$$\nabla_{\mathbf{X}} \cdot \frac{d_l V_H}{RT} \mathbf{C}^{-1} \nabla_{\mathbf{X}} \tau_h C_L$$

NOTE: 85 MPa m<sup>1/2</sup> is well below  $J_Q$  for 21Cr-6Ni-9Mn (220 MPa m<sup>1/2</sup>)

$$+ \theta_T \frac{dN_T}{d\epsilon_p} \dot{\epsilon}_p$$

# Example: Kfield simulation with embedded crack (movie)



# Summary

- Pushback operation significantly alters concentration profile (as seen in 2D block and polycrystal examples).
- The code can accommodate finite deformations and finite rotations when computing the concentrations.
- Spatial stabilization and the peclet number play a key role on the concentration profiles when using a coarse mesh, low diffusivities, or high pressure gradients.
- CSE provides efficient and accurate representation (relative to bulk elements) of a fast pathway.