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A Method for Canceling Force Transducer Mass and Inertia Effects

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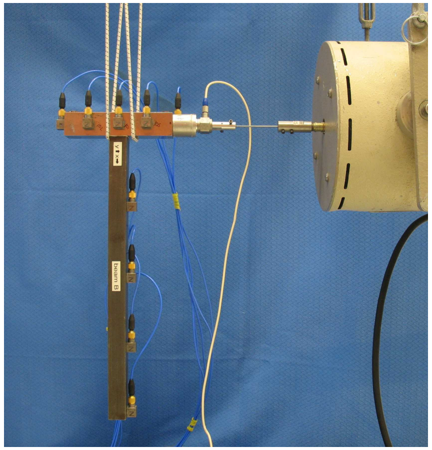

Shaker Test Setup

U.S. DEPARTMENT OF ENERGY
NNSA National Nuclear Security Administration

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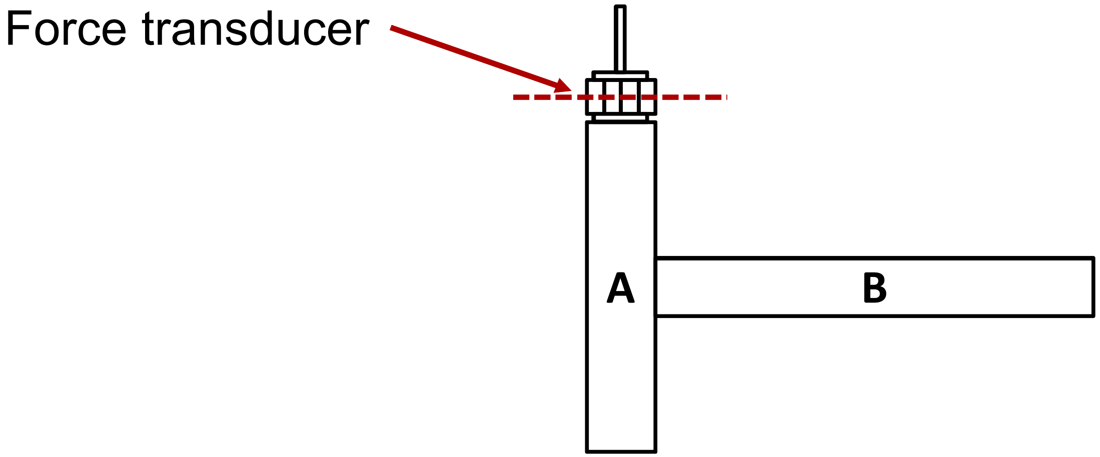
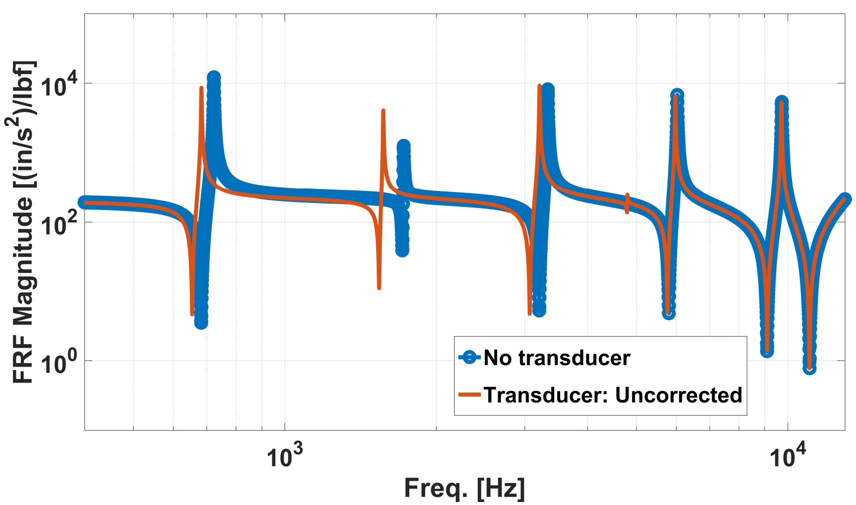
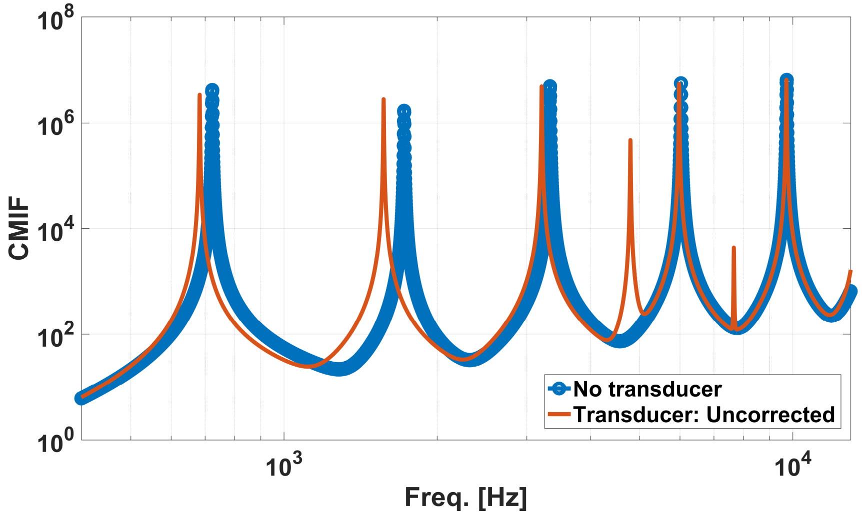
Force transducer may drastically alter system dynamics if not accounted for.

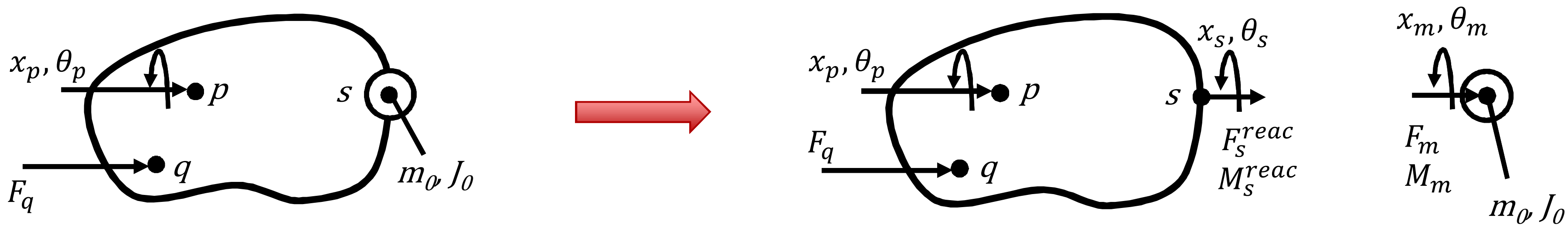






Both translational and rotational quantities can pollute the measured FRFs

Begin with a similar approach as Ashory (1998 IMAC) but include rotation terms



Frequency response at output *p:*

$$\begin{Bmatrix} x_p \\ \theta_p \end{Bmatrix} = \begin{Bmatrix} A_{pq} \\ C_{pq} \end{Bmatrix} F_q + \begin{bmatrix} A_{ps} & B_{ps} \\ C_{ps} & D_{ps} \end{bmatrix} \begin{Bmatrix} F_s^{\text{reac}} \\ M_s^{\text{reac}} \end{Bmatrix}$$

Frequency response of point mass *m:*

$$\begin{Bmatrix} x_m \\ \theta_m \end{Bmatrix} = \begin{bmatrix} -\frac{1}{\omega^2 m_0} & 0 \\ 0 & -\frac{1}{\omega^2 J_0} \end{bmatrix} \begin{Bmatrix} F_m \\ M_m \end{Bmatrix}$$

Subject to the constraints:

$$\begin{Bmatrix} x_s \\ \theta_s \end{Bmatrix} - \begin{Bmatrix} x_m \\ \theta_m \end{Bmatrix} = 0$$

$$\begin{Bmatrix} F_s^{\text{reac}} \\ M_s^{reac} \end{Bmatrix} + \begin{Bmatrix} F_m \\ M_m \end{Bmatrix} = 0$$

Considering the drive-point ($p = q = s$)

$$\begin{Bmatrix} x_p \\ \theta_p \end{Bmatrix} = \begin{bmatrix} 1 - \omega^2 m_0 A_{pp} & -\omega^2 J_0 B_{pp} \\ -\omega^2 m_0 C_{pp} & 1 - \omega^2 J_0 D_{pp} \end{bmatrix}^{-1} \begin{Bmatrix} A_{pp} \\ C_{pp} \end{Bmatrix} F_p$$

Both translational and rotational quantities modify the true FRF

- McConnell (1999 MSSP) shows a similar result

[1] Ashory, "Correction of Mass-Loading Effects of Transducers and Suspension Effects in Modal Testing," *IMAC,* 1998.
[2] McConnell and Cappa, "Transducer Inertia and Stinger Stiffness Effects on FRF Measurements," *MSSP,* 1999.

Presentation Outline

- Theoretical Development
- Numerical Results
- Experimental Results
- Conclusions

Presentation Outline

- **Theoretical Development**
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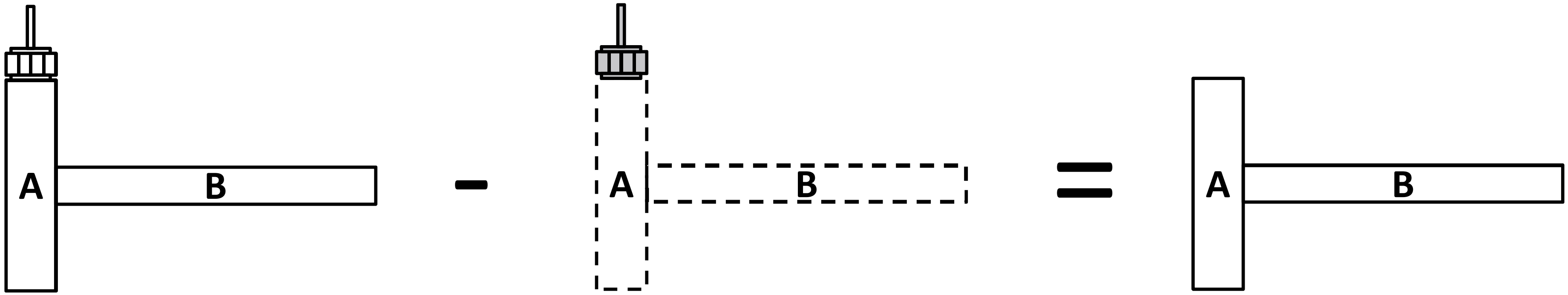
Substructuring enables removal of the force transducer from the system.



Experimental system with transducer, *CT*

Analytical model of transducer, *T*

Experimental system, *C*

Equations of motion:

$$\begin{bmatrix} \mathbf{M}_{CT} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M}_T \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_{CT} \\ \ddot{\mathbf{u}}_T \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{CT} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_{CT} \\ \dot{\mathbf{u}}_T \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{CT} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{CT} \\ \mathbf{u}_T \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_{CT} \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} \mathbf{g}_{CT} \\ \mathbf{g}_T \end{Bmatrix}$$

Subject to:

Compatibility at interface: $\mathbf{u}_{CT,T} - \mathbf{u}_T = \mathbf{0}$ → $\mathbf{B} \begin{Bmatrix} \mathbf{u}_{CT} \\ \mathbf{u}_T \end{Bmatrix} = \mathbf{0}$

Interface force equilibrium: $\mathbf{g}_{CT} + \mathbf{g}_T = \mathbf{0}$ → $\mathbf{L}^T \begin{Bmatrix} \mathbf{g}_{CT} \\ \mathbf{g}_T \end{Bmatrix} = \mathbf{0}$

System analytical model is unknown; instead, use modal equations derived from tests.

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Transformation to the modal coordinate system retaining R_{CT} measured system modes and R_T transducer rigid body modes :

$$\begin{Bmatrix} \mathbf{u}_{CT} \\ \mathbf{u}_T \end{Bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{CT} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_T \end{bmatrix} \begin{Bmatrix} \mathbf{q}_{CT} \\ \mathbf{q}_T \end{Bmatrix}$$

Modal equations of motion:

$$\begin{bmatrix} \mathbf{I}_{CT} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_T \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_{CT} \\ \ddot{\mathbf{q}}_T \end{Bmatrix} + \begin{bmatrix} [\backslash 2\zeta_r\omega_r \backslash]_{CT} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}_{CT} \\ \dot{\mathbf{q}}_T \end{Bmatrix} + \begin{bmatrix} [\backslash \omega_r^2 \backslash]_{CT} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_{CT} \\ \mathbf{q}_T \end{Bmatrix}$$

$$= \begin{Bmatrix} \mathbf{\Phi}_{CT}{}^T \mathbf{f}_{CT} \\ \mathbf{0} \end{Bmatrix} + \begin{bmatrix} \mathbf{\Phi}_{CT}{}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_T{}^T \end{bmatrix} \begin{Bmatrix} \mathbf{g}_{CT} \\ \mathbf{g}_T \end{Bmatrix}$$

Subject to:

Compatibility at interface:

$$\mathbf{B} \begin{bmatrix} \mathbf{\Phi}_{CT} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_T \end{bmatrix} \begin{Bmatrix} \mathbf{q}_{CT} \\ \mathbf{q}_T \end{Bmatrix} = \mathbf{B}_p \begin{Bmatrix} \mathbf{q}_{CT} \\ \mathbf{q}_T \end{Bmatrix} = \mathbf{0}$$

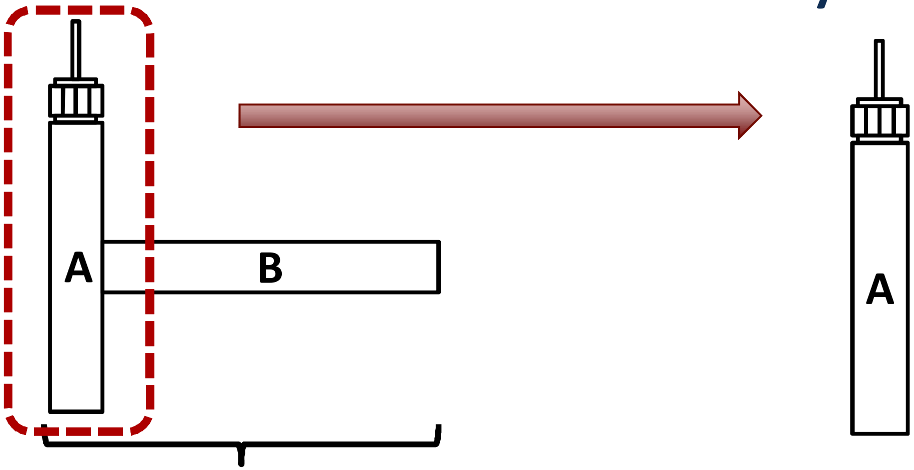
Interface force equilibrium:

$$\mathbf{L}_p{}^T \begin{bmatrix} \mathbf{\Phi}_{CT} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_T{}^T \end{bmatrix}^T \begin{Bmatrix} \mathbf{g}_{CT} \\ \mathbf{g}_T \end{Bmatrix} = \mathbf{L}_p^T \begin{Bmatrix} \mathbf{g}_{p,CT} \\ \mathbf{g}_{p,T} \end{Bmatrix} = \mathbf{0}$$

Requires rotational mode shapes at connection point which are unavailable

Need to find a different set of B_p and B_q using measurable quantities!

Can estimate connection point motion using an analytical model of the local system.



Measured system CT **Analytical model of AT**

Constrain translation measurements on system **CT** and local system **AT:**

$$\mathbf{u}_{AT,m} = \mathbf{u}_{CT,m}$$

Transform into the modal coordinate system:

$$\mathbf{\Phi}_{AT,m}\mathbf{q}_{AT} = \mathbf{\Phi}_{CT,m}\mathbf{q}_{CT}$$

Take pseudoinverse of analytical mode shape matrix

$$\mathbf{q}_{AT} = \mathbf{\Phi}_{AT,m}{}^{\dagger}\mathbf{\Phi}_{CT,m}\mathbf{q}_{CT}$$

Expand response to estimate full motion at transducer connection point:

$$\mathbf{u}_{AT,T} = \mathbf{\Phi}_{AT,T}\mathbf{q}_{AT}$$

$$\mathbf{u}_{AT,T} = \underbrace{\left[\mathbf{\Phi}_{AT,T}\mathbf{\Phi}_{AT,m}{}^{\dagger}\mathbf{\Phi}_{CT,m}\right]}_{\hat{\mathbf{\Phi}}_{CT,T}}\mathbf{q}_{CT} = \hat{\mathbf{u}}_{CT,T}$$

Estimated mode shape matrix now contains rotational information!

[3] Allen, Mayes, and Bergman, "Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connects," *JSV*, 2010.

Previously unknown constraint matrices are found using measurable quantities.

Constrain modal motion of transducer **T** and measured system **CT**

$$\hat{\mathbf{\Phi}}_{CT,T}\mathbf{q}_{CT} - \mathbf{\Phi}_T\mathbf{q}_T = \mathbf{0}$$

$$\underbrace{\begin{bmatrix}\hat{\mathbf{\Phi}}_{CT,T} & -\mathbf{\Phi}_T\end{bmatrix}}_{\mathbf{B}_p}\begin{Bmatrix}\mathbf{q}_{CT}\\ \mathbf{q}_T\end{Bmatrix} = \mathbf{0}$$

B_p enforces as many constraints as there are transducer rigid body modes

Have R_T constraints and R_{CT} + R_T modal equations of motion – can transform to a set unconstrained equations by simply choosing q_{CT} as the unconstrained modes:

$$\begin{Bmatrix}\mathbf{q}_{CT}\\ \mathbf{q}_T\end{Bmatrix} = \underbrace{\begin{bmatrix}\mathbf{I}_{CT}\\ {\mathbf{\Phi}_T}^{-1}\hat{\mathbf{\Phi}}_{CT,T}\end{bmatrix}}_{\mathbf{L}_p}\mathbf{q}_{CT}$$

Force transducer can now be immediately removed from the experimental system.

Apply the transformation to the modal EOM

$$\left\{ \begin{array}{c} \mathbf{q}_{CT} \\ \mathbf{q}_T \end{array} \right\} = \mathbf{L}_p \mathbf{q}_{CT}$$

Unconstrained equations of motion

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}}_{CT} + \tilde{\mathbf{C}}\dot{\mathbf{q}}_{CT} + \tilde{\mathbf{K}}\mathbf{q}_{CT} = \tilde{\mathbf{f}} + \tilde{\mathbf{g}}$$

$$\tilde{\mathbf{M}} = \mathbf{L}_p{}^T \begin{bmatrix} \mathbf{I}_C & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_T \end{bmatrix} \mathbf{L}_p = \mathbf{I}_{CT} - \hat{\mathbf{\Phi}}_{CT,T}^T \left[\mathbf{\Phi}_T \mathbf{\Phi}_T{}^T\right]^{-1} \hat{\mathbf{\Phi}}_{CT,T}$$

$$\tilde{\mathbf{M}} = \mathbf{I}_{CT} - \mathbf{\Phi}_{CT,m}^T \underbrace{\left[\left(\mathbf{\Phi}_{AT,T} \mathbf{\Phi}_{AT,m}{}^{\dagger}\right)^T \left(\mathbf{\Phi}_T \mathbf{\Phi}_T{}^T\right)^{-1} \mathbf{\Phi}_{AT,T} \mathbf{\Phi}_{AT,m}{}^{\dagger} \right]}_{M_{T,m}} \mathbf{\Phi}_{CT,m}$$

This process creates equivalent transducer mass with zero inertia at measurement locations allowing for immediate removal!

Presentation Outline

- Theoretical Development
- **Numerical Results**
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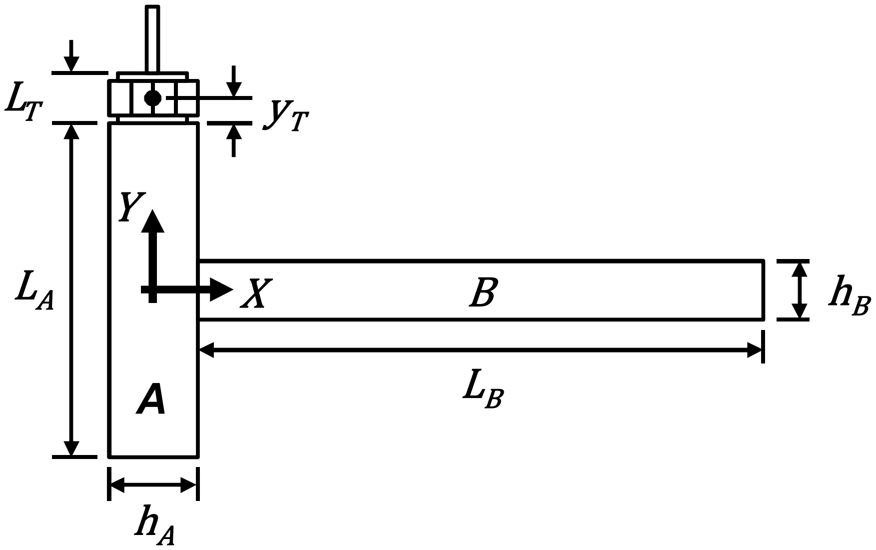
Overview of numerical model

Beam system modeled using FEM

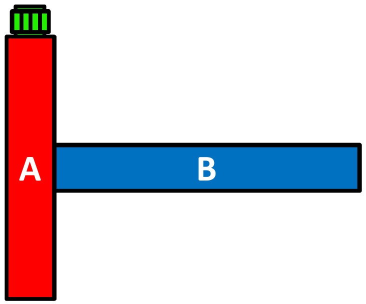


	Beam A	Beam B
Material	Steel	Steel
Length [mm]	114	305
Width [mm]	25.4	25.4
Thickness [mm]	25.4	19.1

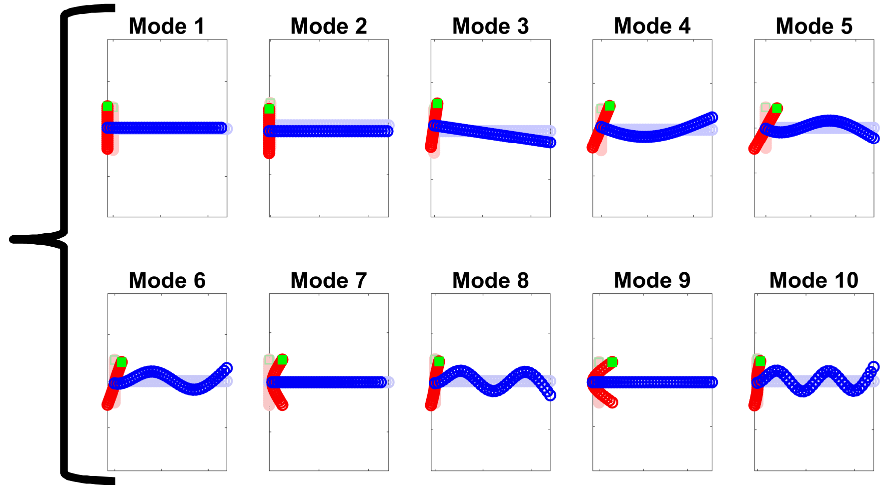
Transducer modeled as rigid body

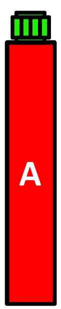
$$\mathbf{M}_T = \begin{bmatrix} m_T & 0 & -m_T y_T \\ 0 & m_T/2 & 0 \\ -m_T y_T & 0 & I_{zz} \end{bmatrix}$$

Correction approach requires a sufficient number of analytical modes

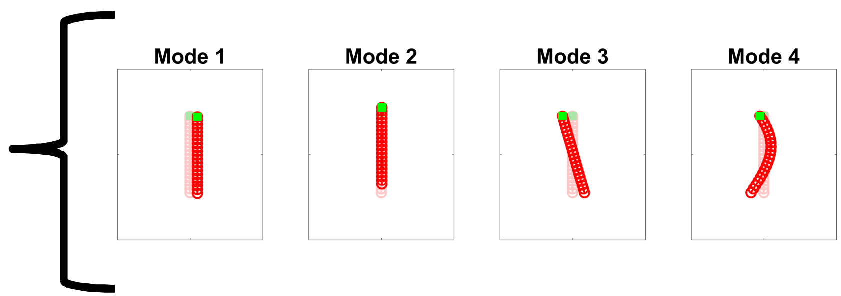


Simulated experimental system with transducer, *CT*



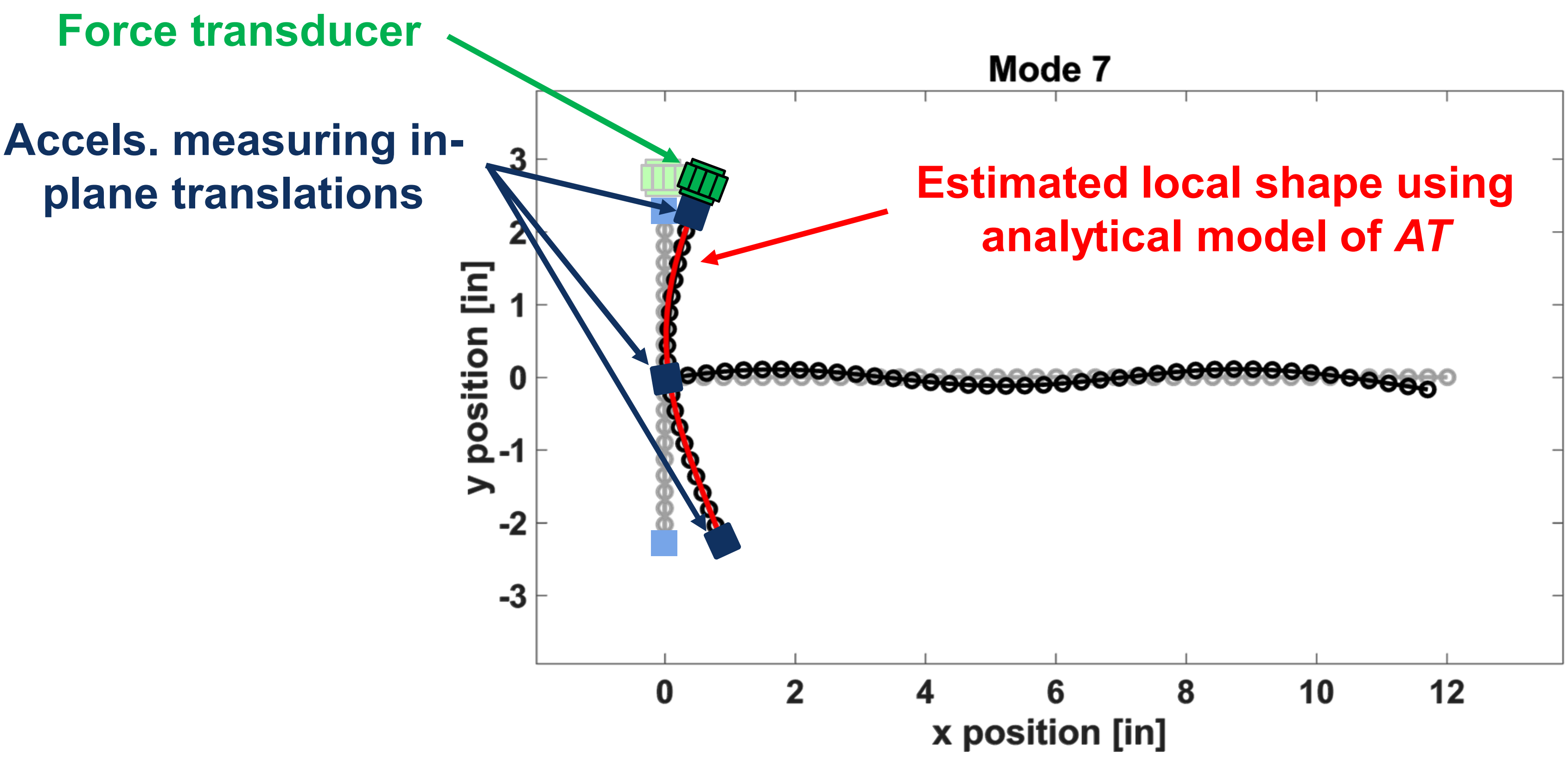


Analytical model of local system with transducer, *AT*

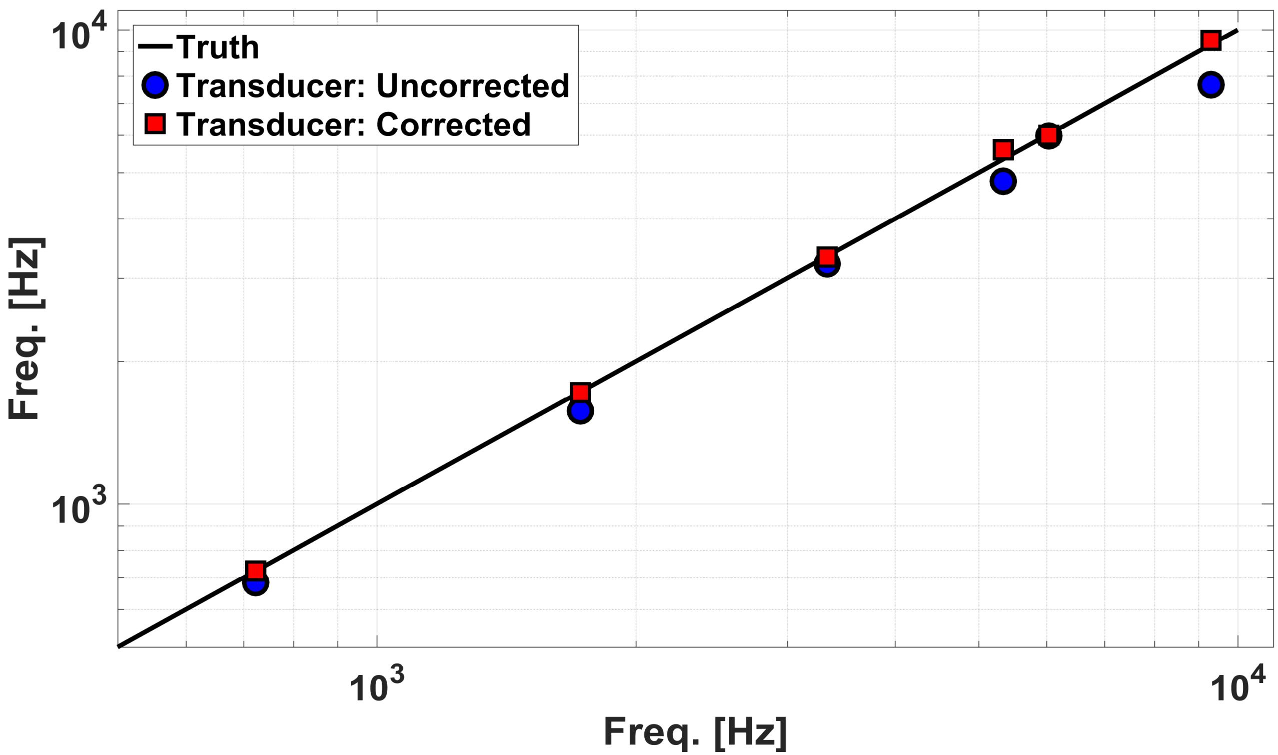


First 4 analytical modes (3 rigid body, 1 bending) sufficiently capture the full system modal motion

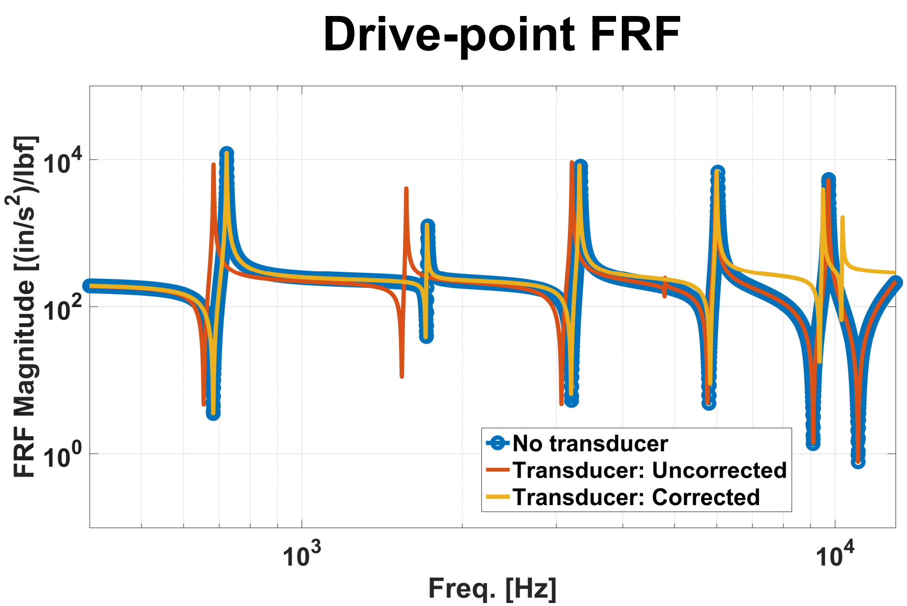
The analytical model of beam AT estimates full motion at transducer connection point.



The corrected frequencies show much better agreement with the truth case.

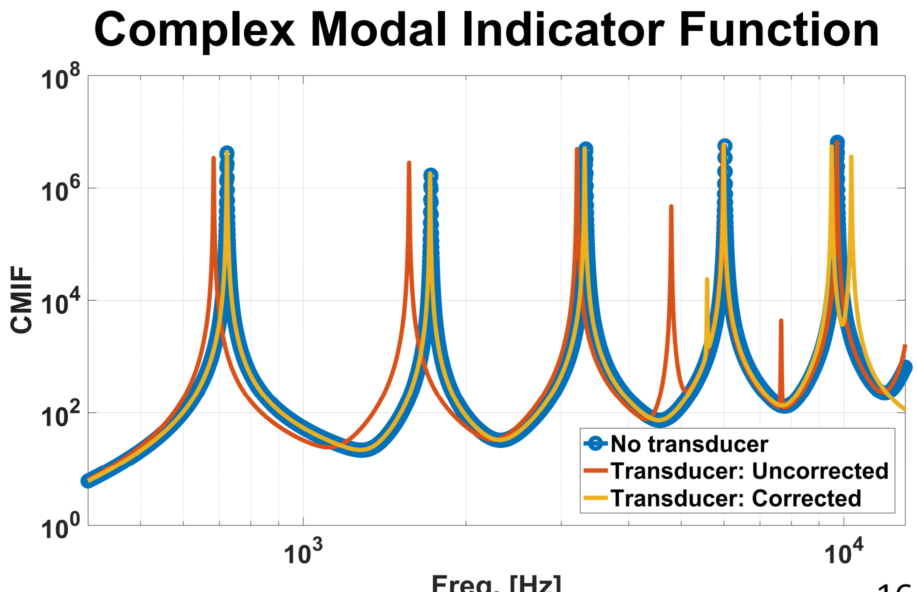


The corrected FRF and CMIF show much better agreement to the truth case.



Only modes with frequencies < 10 kHz were "measured" -- Leads to the deviations in the FRF starting around 8 kHz

The spurious peaks in the CMIF were mostly removed – increasing measurement points showed further improvements



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Hammer test with no attached transducer used for the truth baseline.

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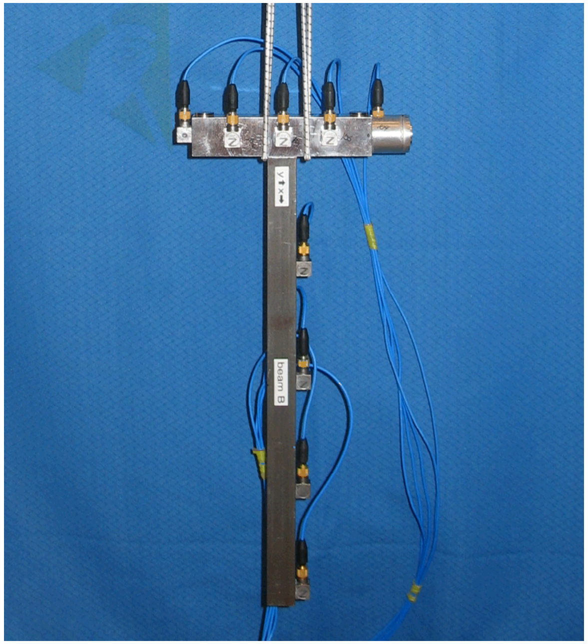
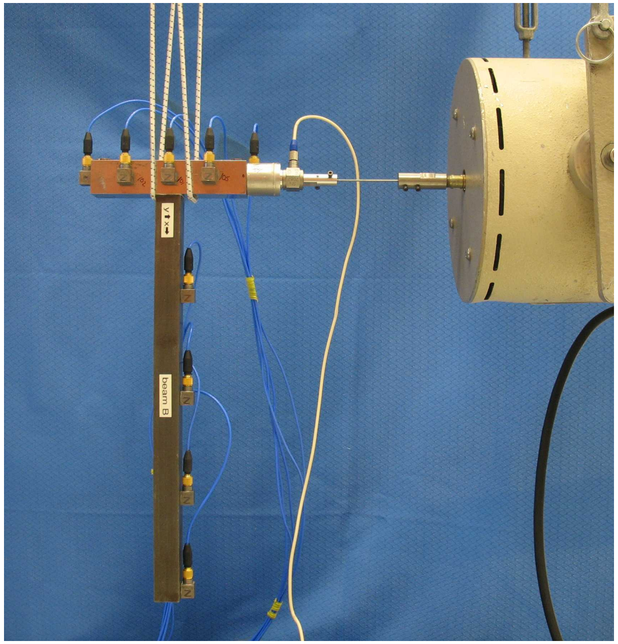


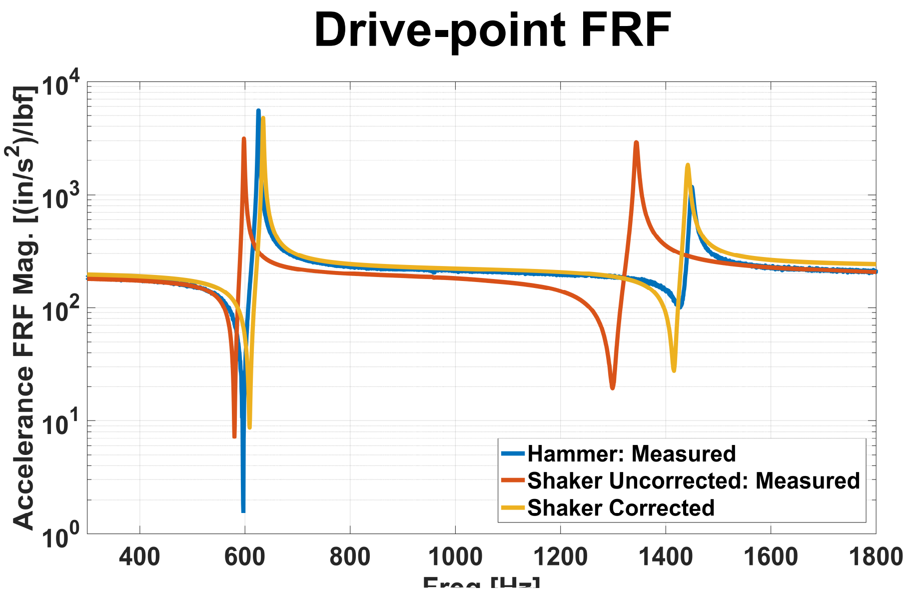
Hammer Test Setup



Shaker Test Setup

Excitation up to ~3 kHz provided information for the first 2 flexible modes

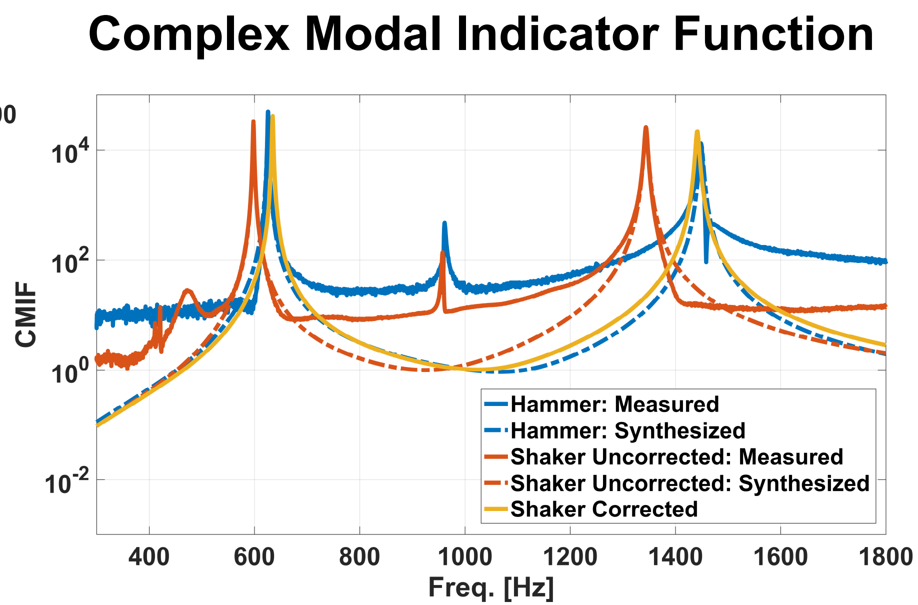
The corrected FRFs shows much better agreement to the truth case.



Correction procedure improves modal frequency estimates:

Uncorrected freq. error:
-4.44%, -7.25%

Corrected freq. error:
1.39%, -0.51%



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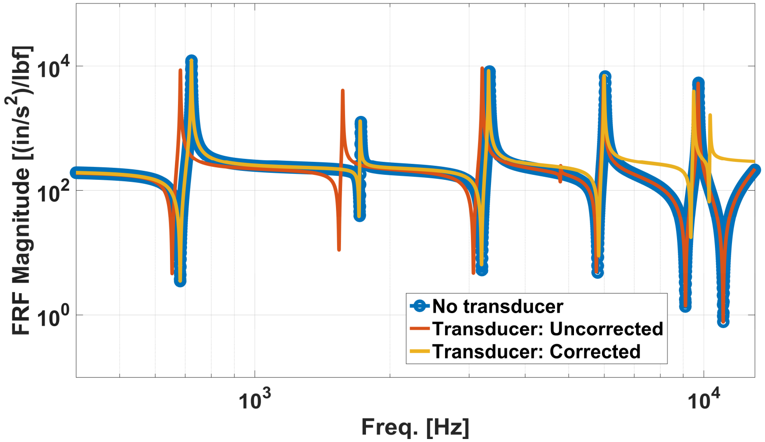
Successfully developed a method to remove force transducer effects during shaker testing.

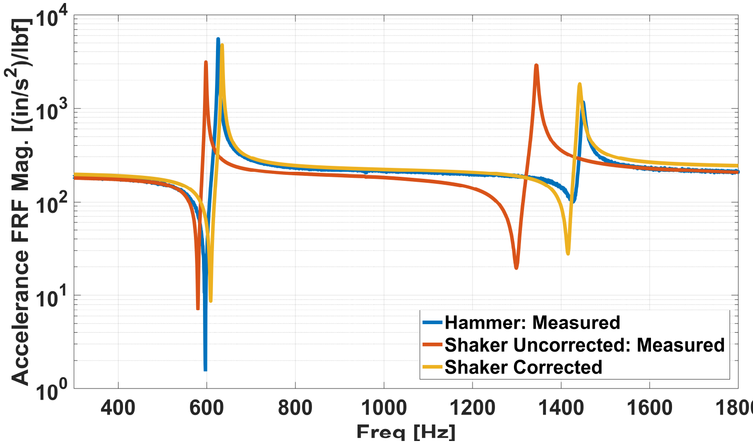
- **A local analytical model of experimental system facilitated estimation of connection point motion**
  - Removed requirement for measurements of rotation to correct for mass moment of inertia

- **Numerical results show the correction method provides excellent agreement with the case of no attached transducer.**

- **Experimental tests provide validation for the method, especially for correcting the freqs.**



Numerical Results



Experimental Results