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Coupling Acoustic-Structure Systems Using Dynamic Substructuring

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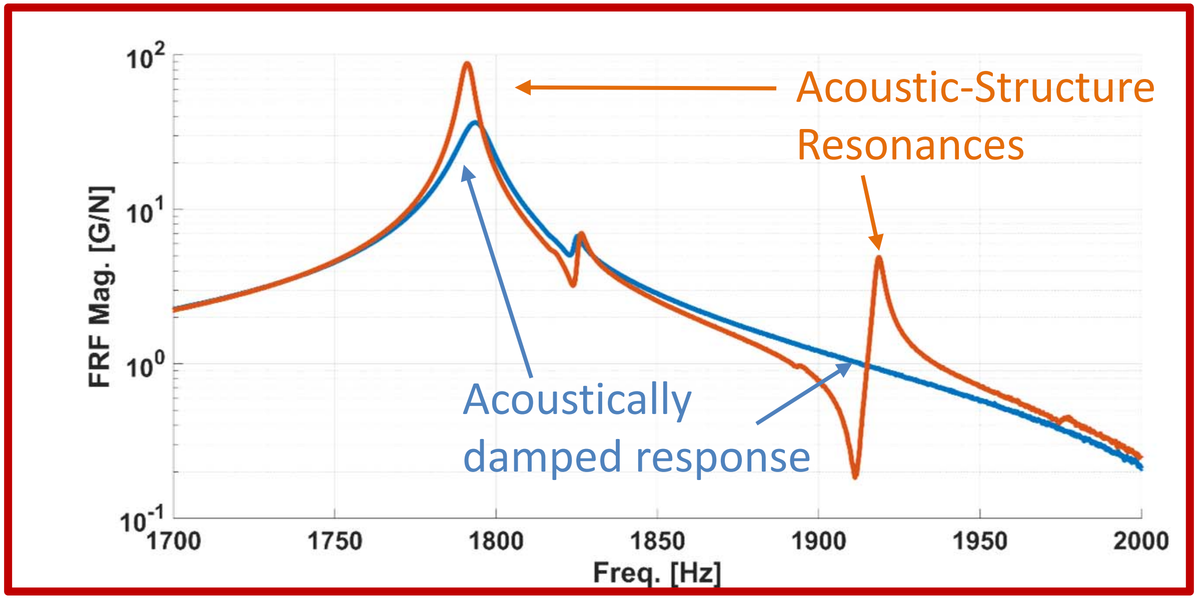
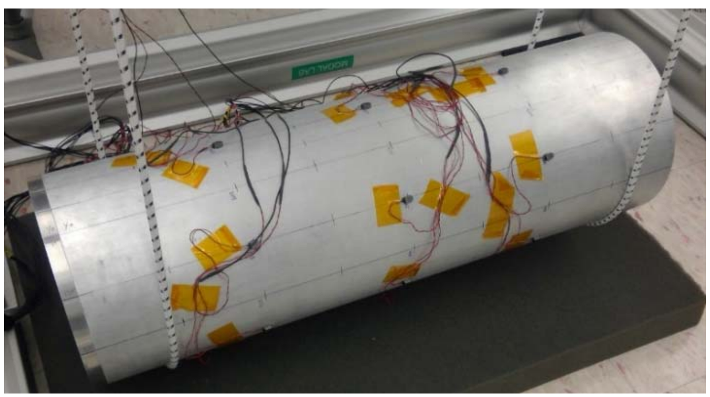
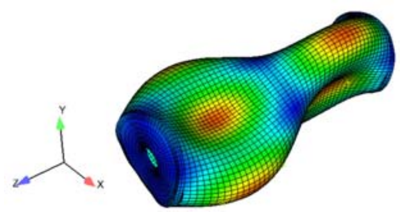
Motivation



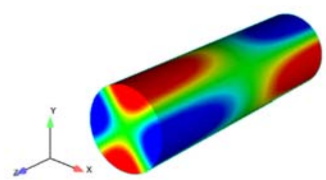
Experimental FRF



Cylindrical test article



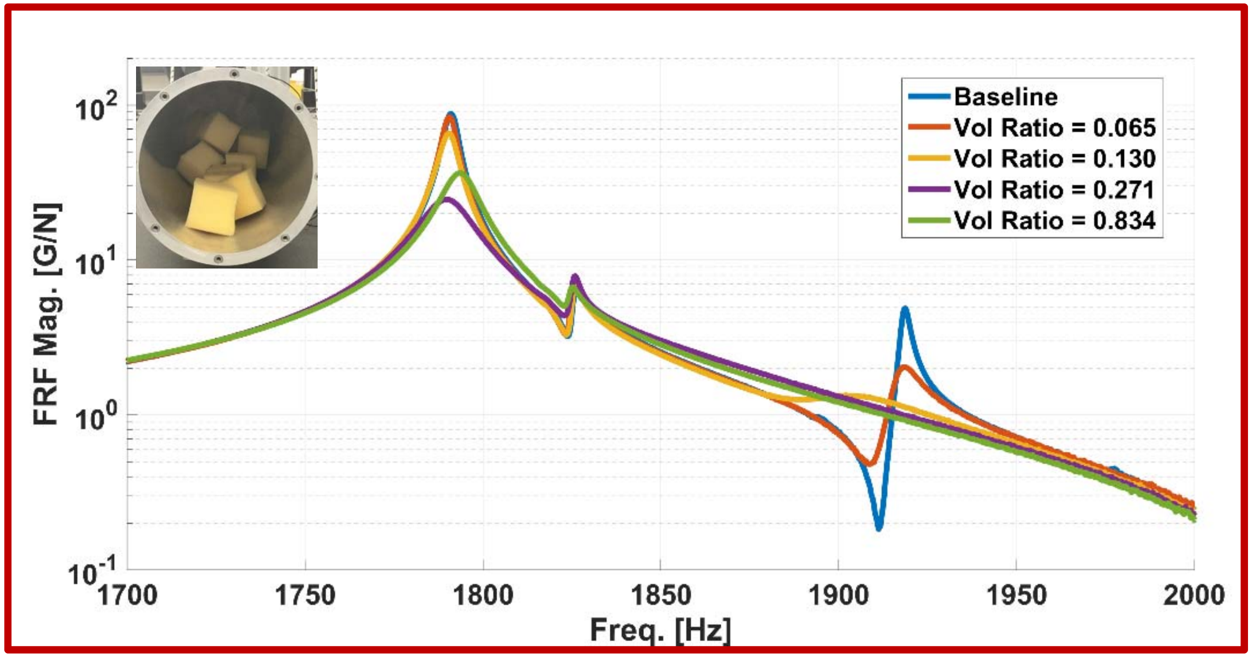
Structural component mode

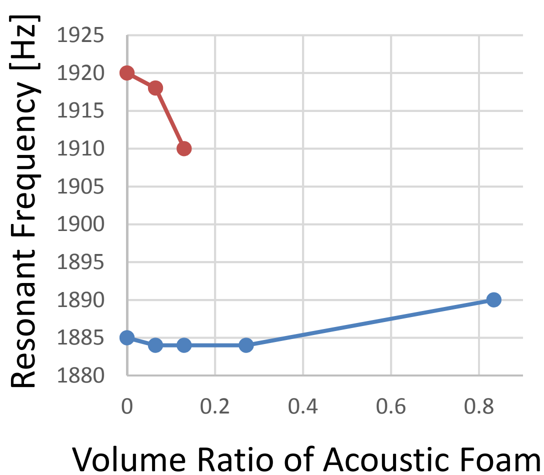


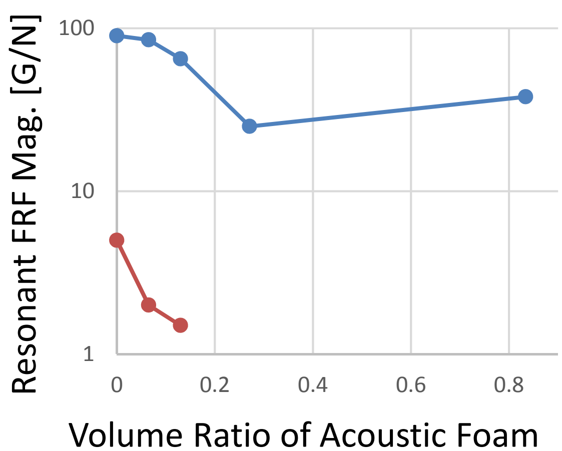
Acoustic component mode

- Coupling with acoustic modes of internal cavities can lead to acoustic-structure resonances in experimental FRFs
- Since FEMs typically assume the structure is *in vacuo*, this coupling can confound test-analysis correlation
- Simply adding acoustic damping to the cavity does not recover the *in vacuo* response

Motivation







- In fact, the resonant behavior with increasing acoustic damping is not straightforward
- At some level of acoustic damping, second resonance peak disappears
- As more damping is added, remaining peak goes up in frequency and amplitude

Goal

- Ultimately, we want to take measured acoustic-structural coupled responses and remove the acoustic part
  - Inspired by transmission simulator method[1,2]
  - Decoupling approach (paper 334) will be presented tomorrow at 2:30pm by Ryan Schultz

- But first, can we couple acoustic and structural subsystems using a generalized coordinate assembly component mode synthesis method (GCA-CMS)[3]?
  - This will pave the way for the decoupling technique

- The GCA-CMS formulation also provides a convenient framework for looking at the resonant behavior of acoustoelastic systems with varying levels of acoustic subsystem damping

[1]D. R. Roettgen, "Experimental Dynamic Substructuring Using Nonlinear Modal Joint Models," Ph.D. Dissertation, University of Wisconsin, Madison, 2016.
[2]R. L. Mayes and M. Arviso, "Design studies for the Transmission Simulator Method of Experimental Dynamic Substructuring," in *International Seminar on Modal Analysis (ISMA2010)*, Lueven, Belgium, 2010.
[3]D. de Klerk, D. J. Rixen and S. N. Voormeeren, "General Framework for Dynamic Substructuring: History, Review, and Classification of Techniques," *AIAA Journal,* vol 46(5), pp 1169-1180, 2008.

GCA-CMS for Acoustic-Structure Systems

First assemble disjoint system in generalized coordinates:

$$\begin{bmatrix} [I_A] & [0] \\ [0] & [I_S] \end{bmatrix} \begin{Bmatrix} \ddot{q}_A \\ \ddot{q}_S \end{Bmatrix} + \begin{bmatrix} [\omega_A^2] & [0] \\ [0] & [\omega_S^2] \end{bmatrix} \begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The transformation between physical and generalized coordinates is given by:

$$\begin{Bmatrix} u_A \\ u_S \end{Bmatrix} = \begin{bmatrix} [\nabla\Phi_A] & [0] \\ [0] & [\Phi_S] \end{bmatrix} \begin{Bmatrix} q_A \\ q_S \end{Bmatrix}$$

$[\nabla\Phi_A]$ Normalized gradient of pressure-release acoustic modes. Modes come from FEM. Calculation of gradient and normalization performed outside of FEM. Modes normalized according to: $\nabla\Phi_j = \frac{\nabla\phi_j}{\sqrt{m_j}}$ where $m_j = \rho_f \int_V \nabla\phi_j \cdot \nabla\phi_j dV$

$[\Phi_S]$ Mass normalized structural modes (comes directly from FEM)

Gradient of pressure mode shape is used here because we want to enforce displacement compatibility and the relationship between pressure and fluid particle displacement for harmonic responses is

$$u_A = \frac{\nabla p}{\rho_f \omega^2}$$

So the gradient of the pressure mode shape is proportional to the displacement mode shape

GCA-CMS for Acoustic-Structure Systems

Constraints will be enforced in terms of physical displacements and will take the form

$$[a]\begin{Bmatrix} u_A \\ u_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

where $[a]$ is the Boolean constraint matrix. In terms of generalized coordinates:

Equals 0 for rigid wall modes, which leads to a 0=0 constraint equation. This is why rigid wall acoustic modes can't be used here.

$$[a]\begin{bmatrix} [\nabla\Phi_A] & [0] \\ [0] & [\Phi_S] \end{bmatrix}\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = [\hat{a}]\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Using Lagrange multipliers, the complete set of EOMs for the assembled system is then

$$\begin{bmatrix} [I_A] & [0] \\ [0] & [I_S] \end{bmatrix}\begin{Bmatrix} \ddot{q}_A \\ \ddot{q}_S \end{Bmatrix} + \begin{bmatrix} [\omega_A^2] & [0] \\ [0] & [\omega_S^2] \end{bmatrix}\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = [\hat{a}]^T\{\lambda\};$$

$$[\hat{a}]\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The first of these equations has N unknowns equal to the total number of retained modes from the two subsystems and has K unknown Lagrange multipliers. The second set of equations represents the K constraint equations. So we have N+K equations with N+K unknowns

GCA-CMS for Acoustic-Structure Systems

Only *N*-*K* DOFs are independent because of the constraints. Need to find a set of *N*-*K* unconstrained DOFs. This set is related to the constrained set by the coordinate transformation

$$\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = [B]\{\xi\}$$

From the constraint equations and this transformation it is clear that

$$[\hat{a}][B]\{\xi\} = \{0\}$$

So $[B]$ is the null space of $[\hat{a}]$ and can be easily computed. Multiply both sides of the EOM by $[B]^T$

$$[B]^T \begin{bmatrix} [I_A] & [0] \\ [0] & [I_S] \end{bmatrix} [B]\{\xi\} + [B]^T \begin{bmatrix} [\omega_A^2] & [0] \\ [0] & [\omega_S^2] \end{bmatrix} [B]\{\xi\} = \cancelto{0}{[B]^T [\hat{a}]^T}\{\lambda\}$$

$$[\hat{a}][B]\{\xi\} = \{0\}$$

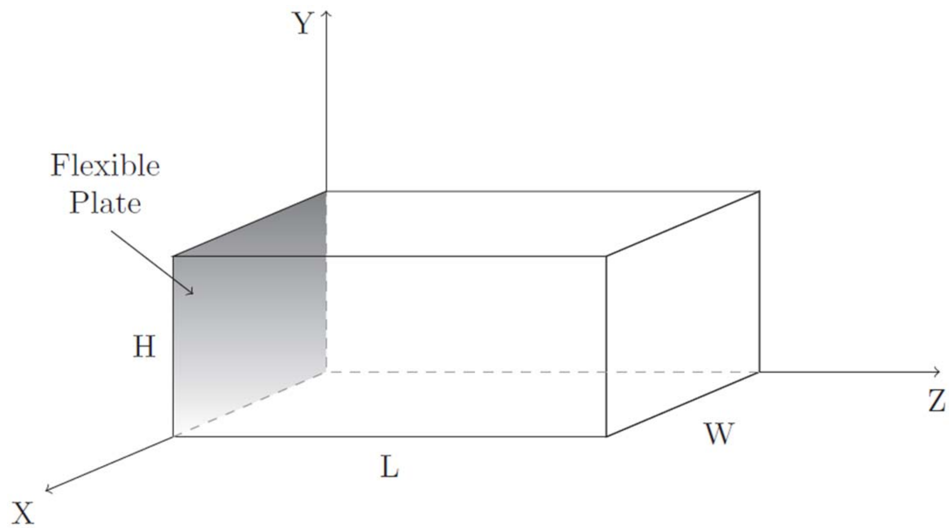
The coupled natural frequencies and modes can then be calculated from

$$\left(\left[\hat{K}\right] - \omega^2\left[\hat{M}\right]\right)\{\phi\} = \{0\}$$

where

$$\left[\hat{M}\right] = [B]^T \begin{bmatrix} [I_A] & [0] \\ [0] & [I_S] \end{bmatrix} [B] \qquad \left[\hat{K}\right] = [B]^T \begin{bmatrix} [\omega_A] & [0] \\ [0] & [\omega_S] \end{bmatrix} [B]$$

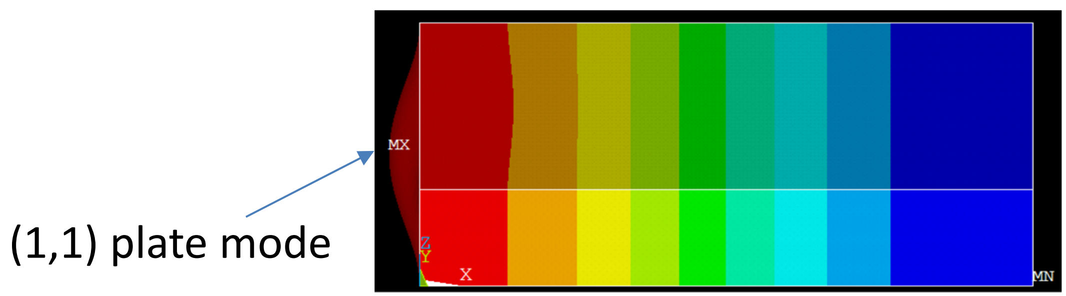
Example Problem



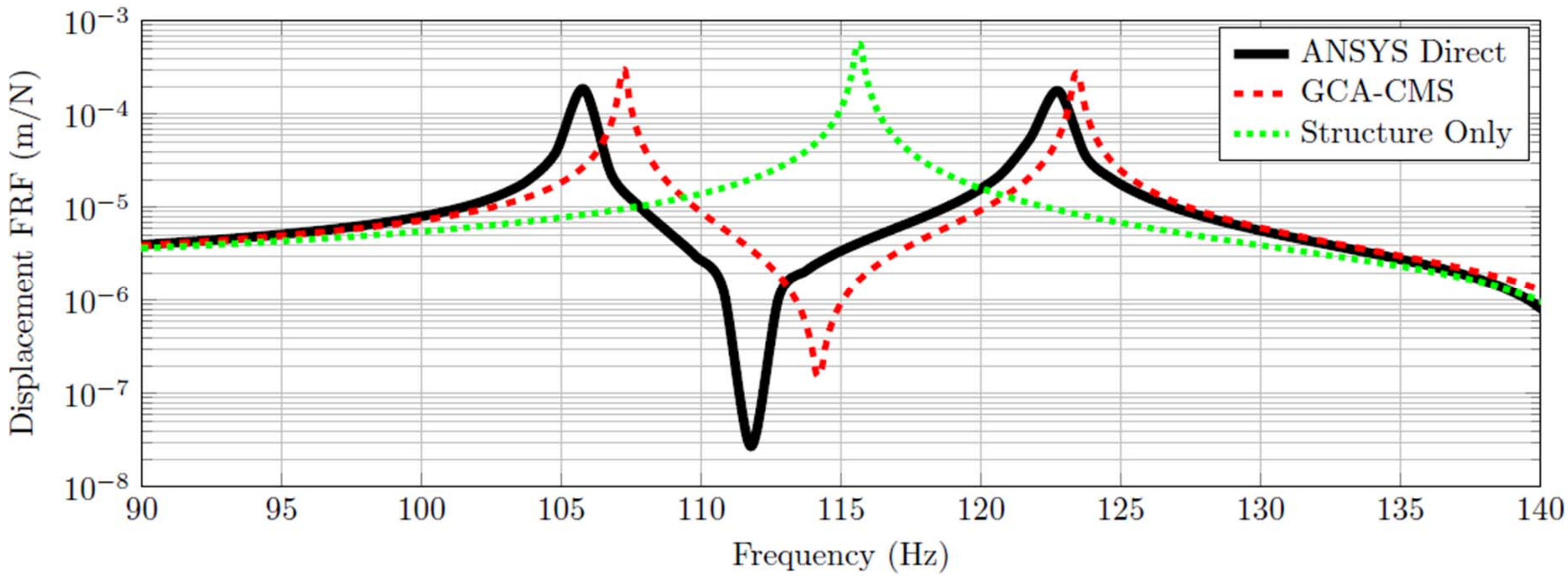
Length	1.524	[m]
Width	0.9144	[m]
Height	0.3048	[m]
Plate thickness	4	[mm]

- A rectangular acoustic enclosure with a simply-supported plate on one end is a commonly studied ASI system.
- Dimensions chosen to create a coupling between the (1,1) plate mode and the (0,0,1) acoustic mode

(0,0,1) cavity mode



Example FRF



	f_{c_1} (Hz)	f_{c_2} (Hz)
FEM	105.6	122.5
GCA-CMS	107.2	123.4

- Coupled resonances appear on either side of uncoupled resonance and are lower in amplitude
- Coupled resonance prediction agrees with direct FEM by 1.5% for first resonance and 0.7% for second
- Modal damping applied at system level
  - GCA-CMS lends itself to classical modal analysis as long as system model damping is used

Resonant Behavior with Varying Acoustic Damping

- A numerical experiment is devised:
  - Vary viscous damping ratio of the acoustic subsystem
  - Track system eigenvalues, resonant frequencies, and amplitudes

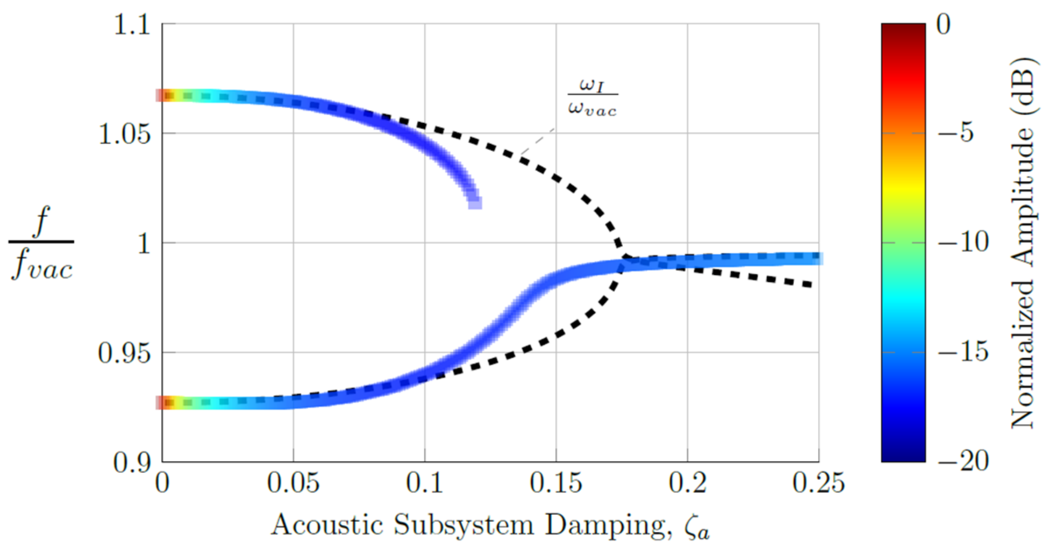
- Acoustic damping is more accurately modeled by invoking complex-valued frequency dependent impedance boundary conditions
  - But modeling the fluid dissipation effects as a single viscous damping ratio is a convenient way to qualitatively assess the system behavior

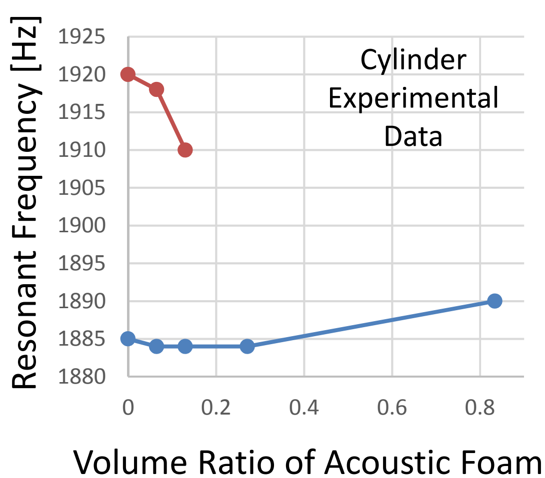
- With the introduction of subsystem damping, the coupled system frequencies and modes become complex-valued so a complex modal analysis procedure is used[4]

- In the numerical experiment, the structural subsystem damping is held fixed at ζ_s=0.001 while the acoustic damping is varied from ζ_a= 0 to ζ_a= 0.25

[4] *Davis, R. B., Techniques to assess acoustic-structure interaction in liquid rocket engines, Ph.D. thesis, Duke University, 2008.*

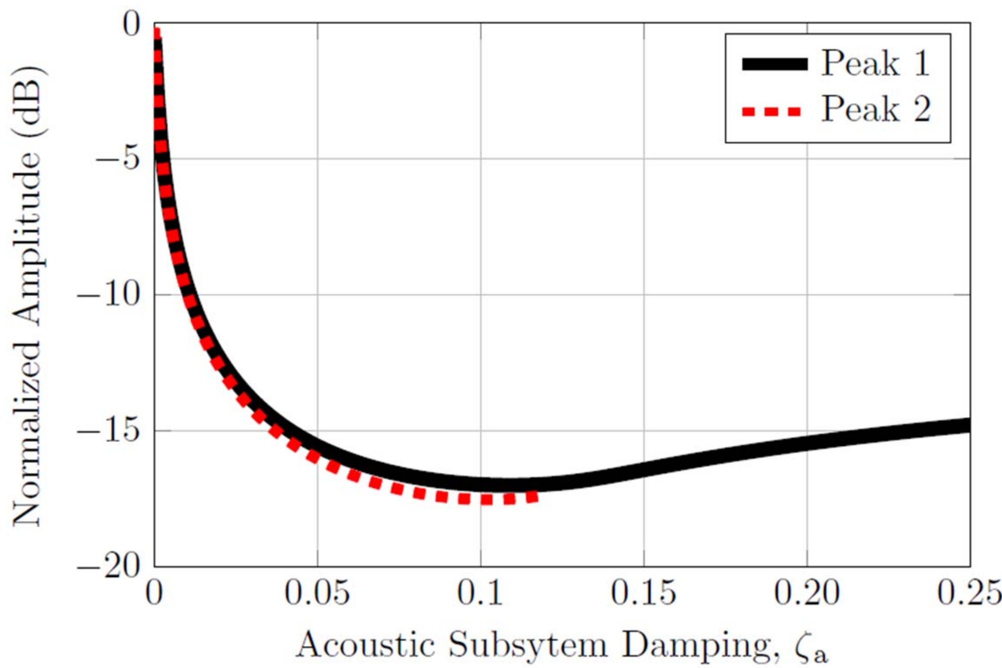
Resonant Behavior with Varying Acoustic Damping

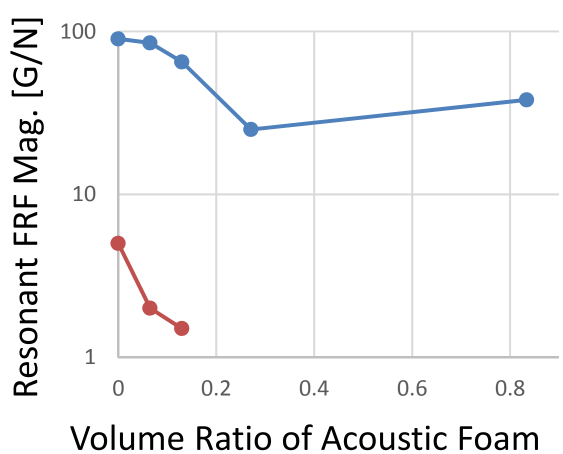




- For low ζ_a, the resonant frequencies closely follow the curves representing the imaginary portion of the eigenvalues
- Near ζ_a= 0.10, the two resonant frequencies begin to deviate from the imaginary portion of the eigenvalues until the second resonance abruptly disappears near ζ_a = 0.12.
- The first resonance persists at higher values of ζ_a and eventually follows the higher of the two eigenvalue curves
- Resonant frequency behavior is very qualitatively similar to that observed by filling cylindrical test article with foam

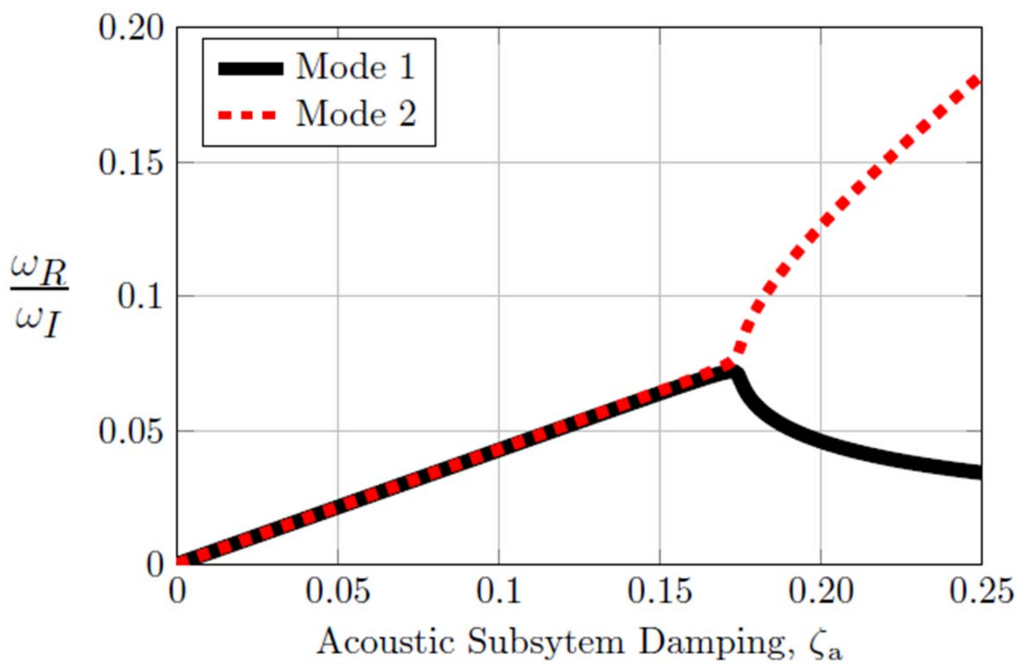
Resonance Amplitude vs. Acoustic Subsystem Damping





- For low ζ_a, two amplitudes are roughly equivalent
- Near $\zeta_a = 0.05$, the amplitude of the second resonance drops relative to the first
- Near $\zeta_a = 0.10$, both resonances reach a minimum. Beyond this point, increasing acoustic damping somewhat increases both resonant amplitudes
- The abrupt disappearance of the second resonance can be seen near $\zeta_a = 0.12$,
- Beyond $\zeta_a = 0.12$, there is a single resonance that increases in amplitude as acoustic damping is increased
- Resonant amplitude behavior is qualitatively similar to that observed by filling cylindrical test article with foam

Acoustic Damping Study



- How does the acoustic damping distribute to the two coupled modes?
- The ratio of the real and imaginary parts of the eigenvalues is equivalent to the modal viscous damping ratio of the coupled system
- The results indicate that for $\zeta_a < 0.17$, the total damping ($\zeta_a + \zeta_s$) is split between the two modes
- Near $\zeta_a = 0.17$, the real parts of the system eigenvalues begin to diverge
- Beyond this point, the total system damping is noticeably less than the total subsystem damping

Conclusions

GCA-CMS Method:

- GCA-CMS formulation works well in the simple example case
- This formulation has some appealing features:
    - Only required inputs are subsystem natural frequencies and modes
    - No calculation of coupling coefficients between component mode pairs
        - Computing these with FEM modes can be non-trivial for axi-symmetric geometries where the component modes set up at arbitrary azimuthal angles and need to be "clocked"
        - Lends itself to classical modal analysis as long as system level modal damping is assumed

Damping Study:

- It is observed that at some critical value of subsystem damping, the imaginary parts of the coupled system eigenvalues cross while the real parts diverge
    - Explains experimental observations where d two coupled resonances become a single resonance at sufficient levels of acoustic damping
- Interestingly, the response analysis indicates that this transition from two resonances to one occurs abruptly at levels of damping that are well below this the damping levels predicted by the eigenvalues alone