SAND2018-1100C

IMAC XXXVI
Performance of Nonlinear Modal Model in Predicting Complex Bilinear Stiffness

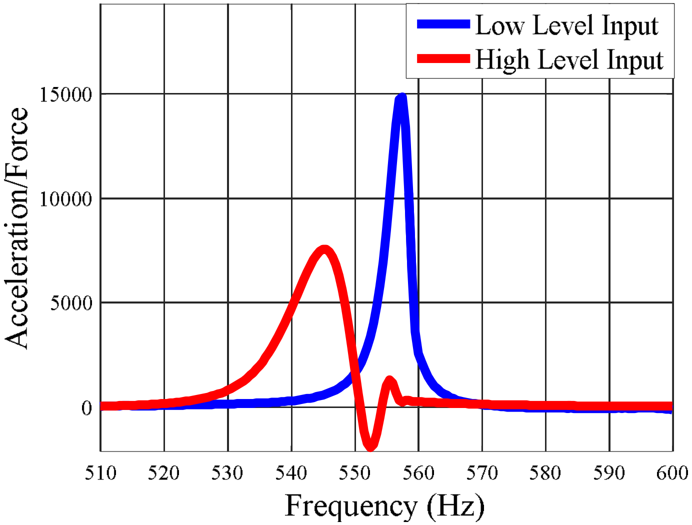
Benjamin Pacini, Wil Holzmann, Randall Mayes

February 12-15, 2018

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.

Motivation

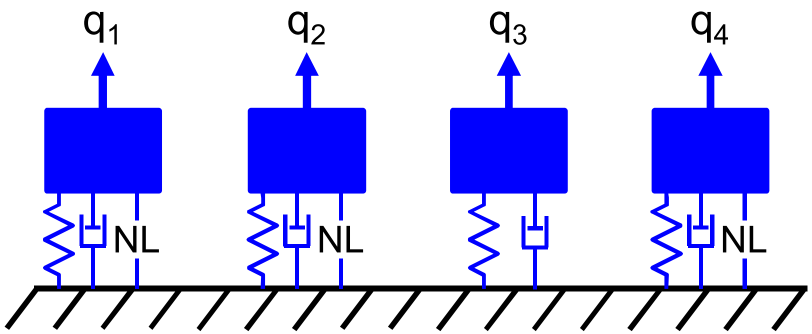
- The overall goal is to study the nonlinear characteristics of a joint of interest within a test article
- Two main methods to capture nonlinearities
  - **Local physical modeling**
  - **Pseudo-modal modeling**
- <u>**Local physical modeling**</u>
  - Physics based
  - Requires a description of each source of nonlinearity
  - Typically computationally expensive
  - Difficult to experimentally identify each model parameter associated with each nonlinear source for complex structures



- <u>**Pseudo-modal modeling**</u>
  - Captures nonlinearity on a mode-by-mode basis
  - Computationally inexpensive
  - Recent studies have shown successful implementation on extracting nonlinear models for complicated structures
- **This work demonstrates the performance of the pseudo-modal modeling approach in characterizing the joint of interest**

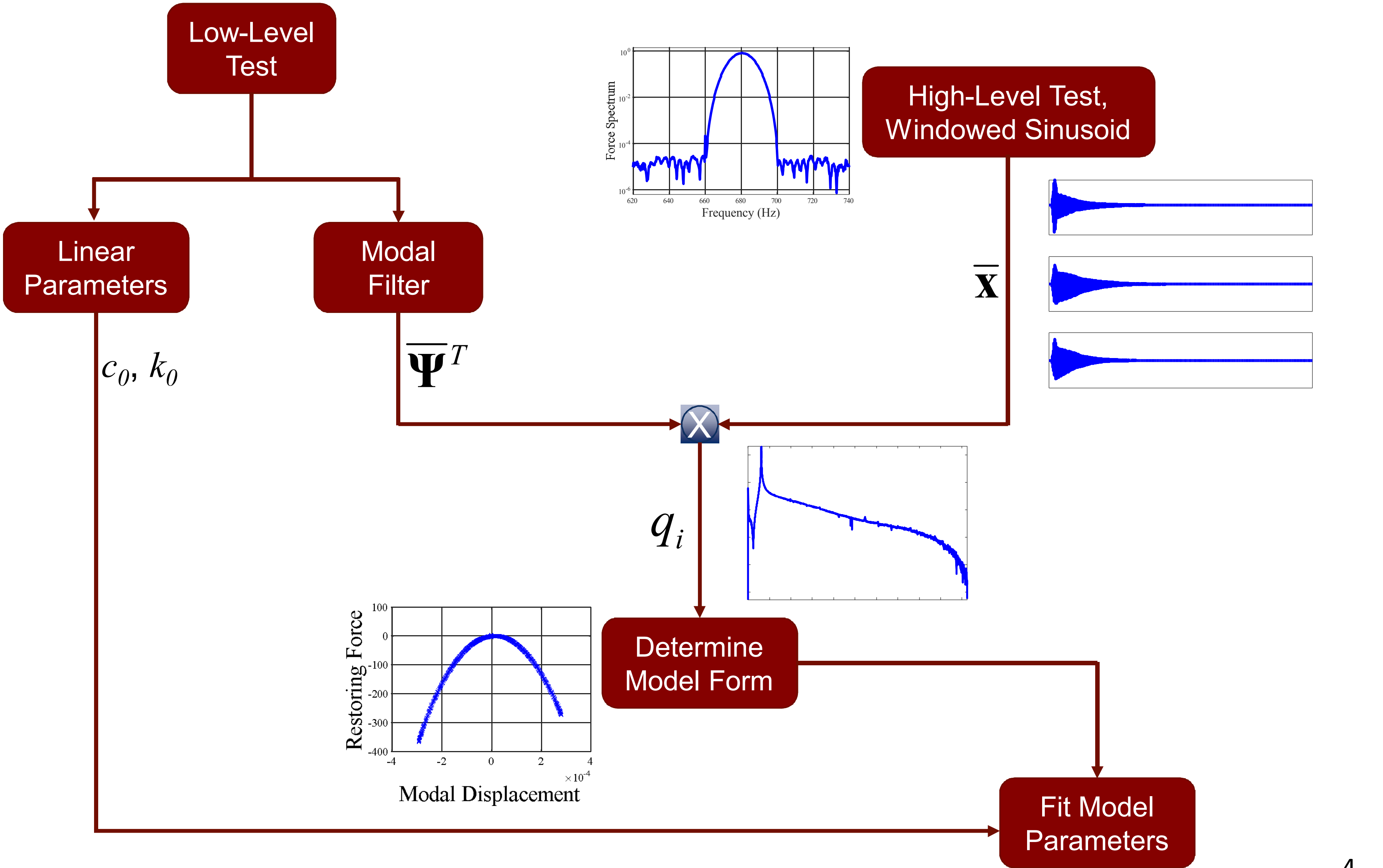
Pseudo-Modal Model

- Structure is decomposed into its modes and nonlinearity is captured on a mode-by-mode basis



- Assumptions
  - The **mode shapes** do not change with amplitude of response so $\bar{\mathbf{x}} = \mathbf{\Phi}\bar{\mathbf{q}}$
  - Nonlinear modes do not interact
  - Significant nonlinearity is captured by adding nonlinear elements supporting each modal mass

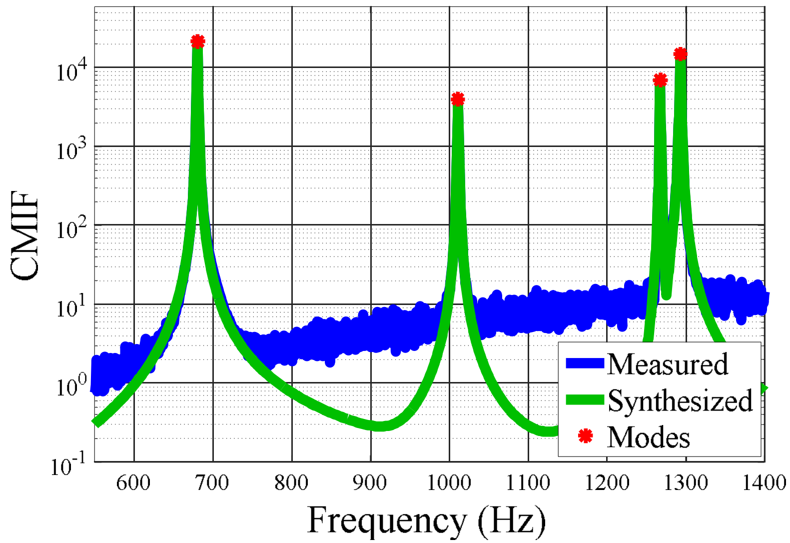
Nonlinear Identification Procedure

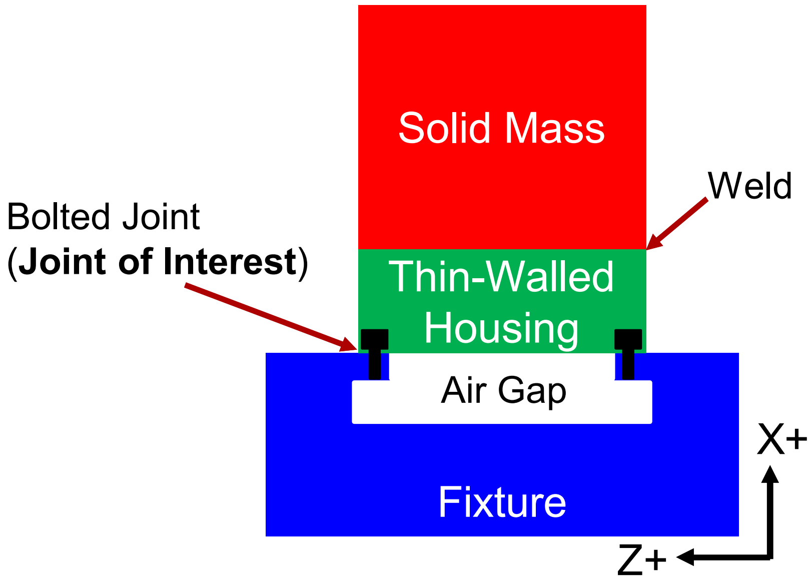


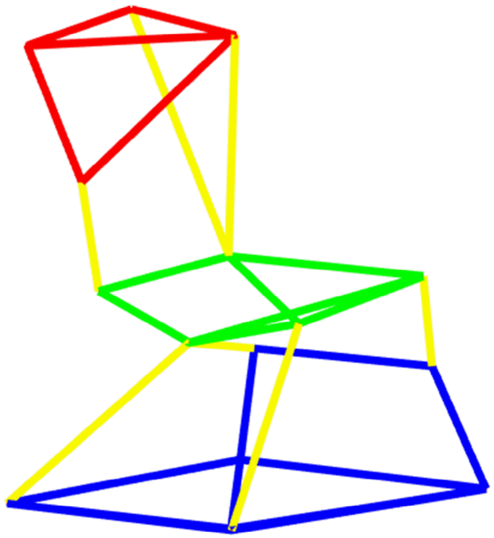
Linear Modal Analysis Results

Mode	Description	Frequency (Hz)	Damping (%)
7	1st bend of assembly in Z	680.7	0.088
8	1st bend of assembly in Y	1011.3	0.065
9	Inner wall bending	1267.8	0.039
10	Axial mode	1293.3	0.058

Rigid body modes not shown

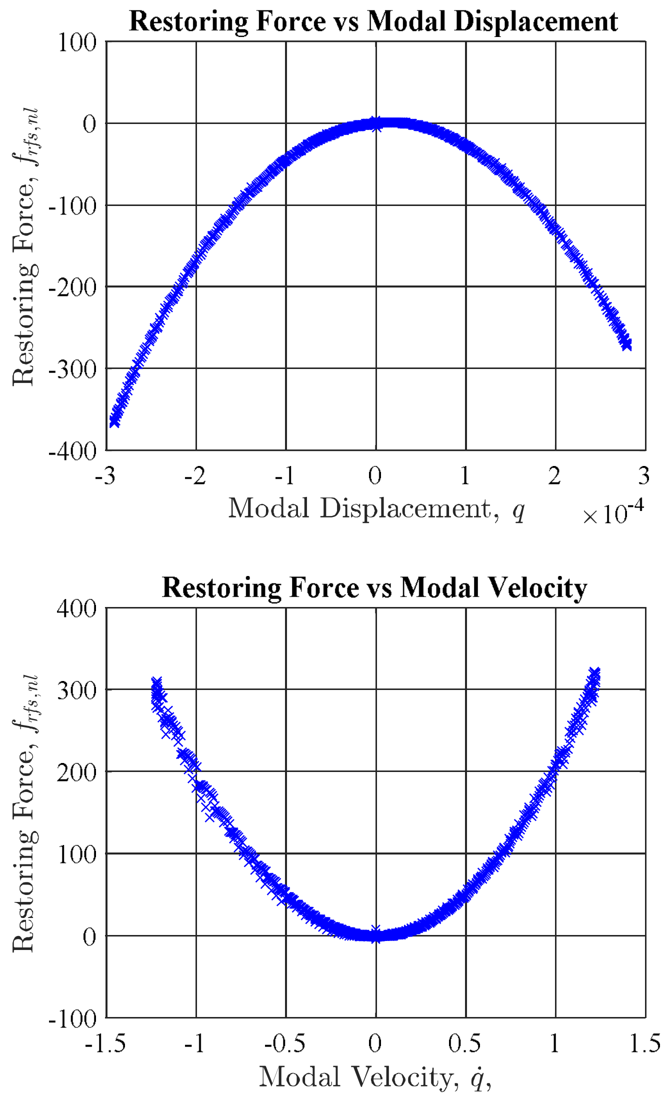






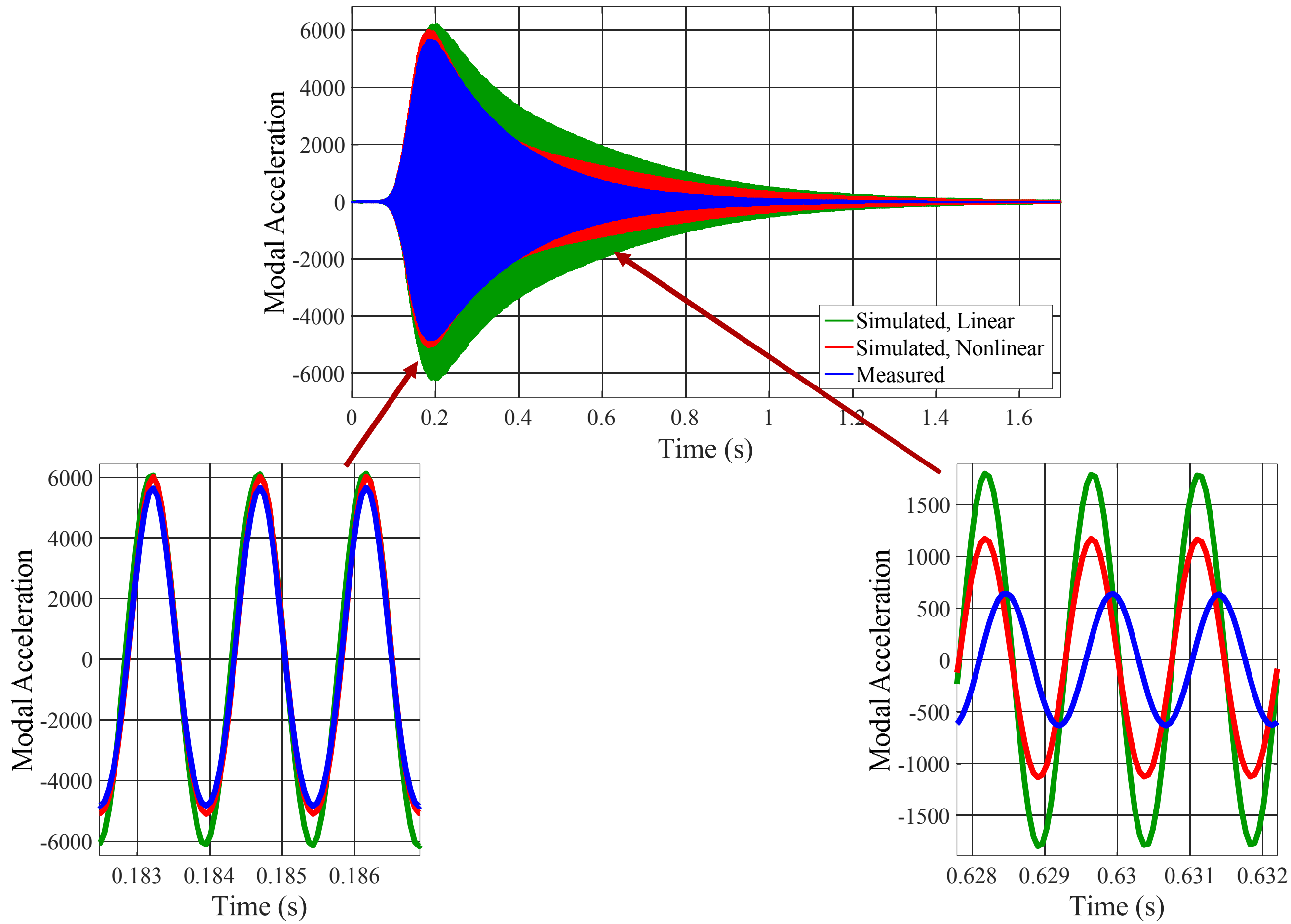
Nonlinear Model

- Nonlinear identification accomplished using the Restoring Force Surface (RFS) method
  - $\ddot{q}(t) + c_0\dot{q}(t) + k_0 q(t) + f_{rfs,nl}(q(t), \dot{q}(t)) = f(t)$
- To determine the model form, $f_{rfs,nl}$ is plotted versus $q(t)$ and $\dot{q}(t)$
  - RFS plots show large quadratic form
- Model form tuned through a series of simulations
  - $f_{rfs,nl}(q(t), \dot{q}(t)) = c_1\dot{q}^2(t) + c_2\dot{q}^3(t) + k_1 q^2(t)$
- The equation of motion can be reconfigured to
  - $[\dot{q}^2 \quad \dot{q}^3 \quad q^2]\begin{bmatrix} c_1 \\ c_2 \\ k_1 \end{bmatrix} = f - \ddot{q} - c_0\dot{q} - k_0 q$
- The nonlinear parameters are solved from this linear system of equations in frequency domain
  - c_1 = 206
  - c_2 = 5
  - k_1 = -3.83E9

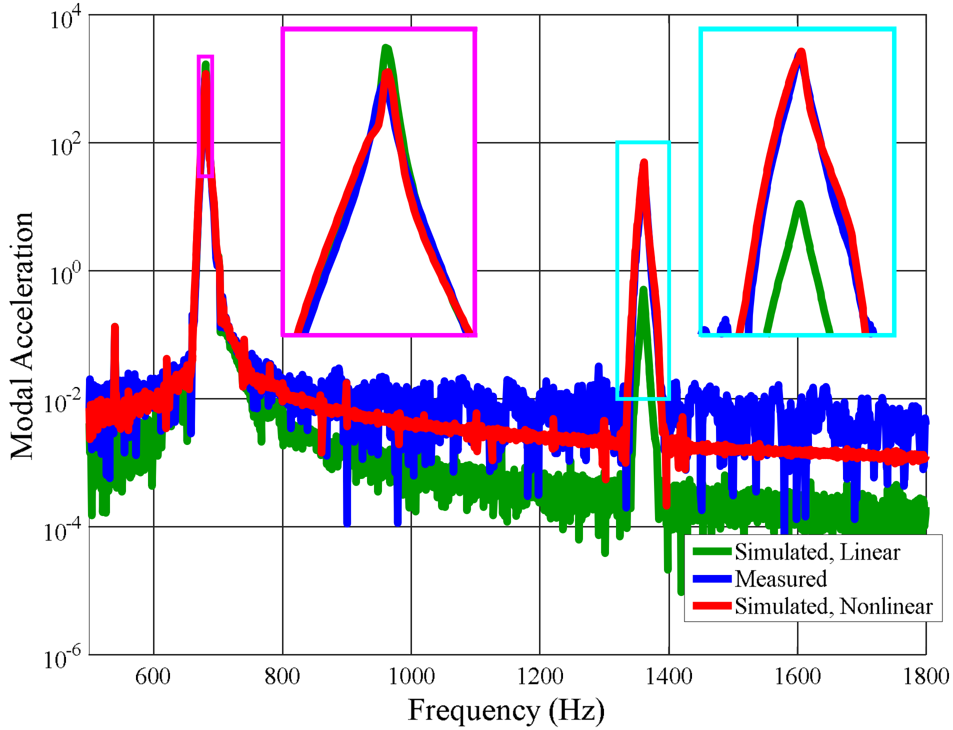


Simulation Results, Modal Domain

- The response of the nonlinear model for mode 7 was simulated using measured modal force and was compared to the corresponding measured modal response



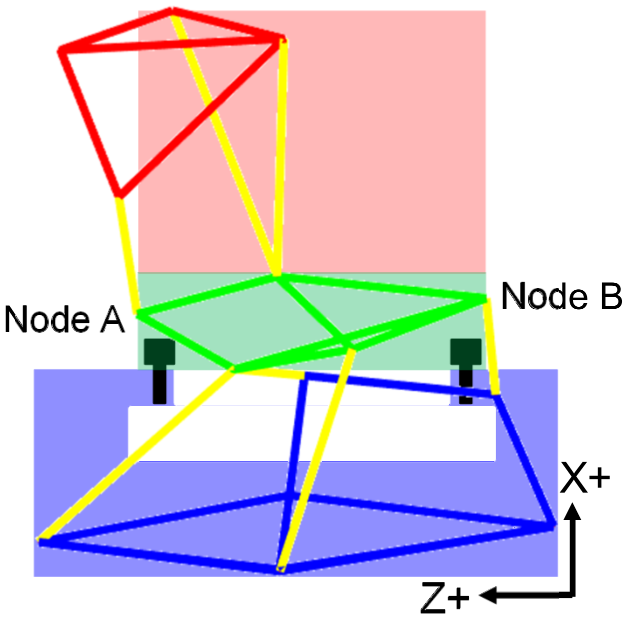
Simulation Results, Modal Domain

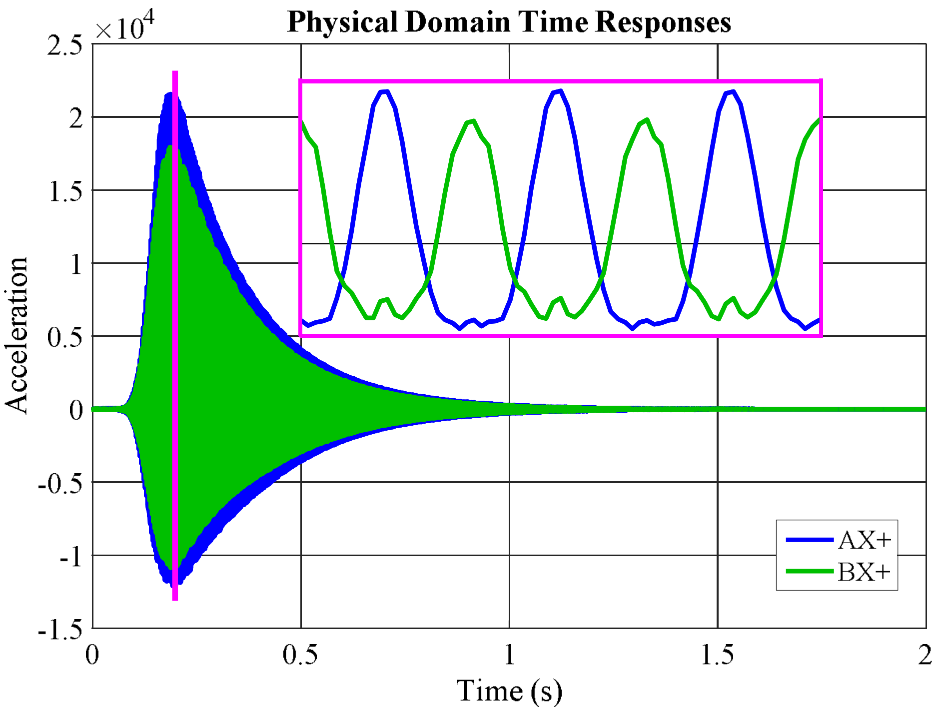


- Observations
  - Main peak (680 Hz) does not show a large frequency difference between linear and measured response (680.8 Hz vs 680.5 Hz)
  - There is a harmonic produced by the nonlinearity (1360 Hz)
- Typically harmonics are produced by stiffness nonlinearities, but the frequency of mode 7 is not appreciably changing from its linear value
- Why is there a relatively large harmonic response when there is negligible frequency shift?

Evidence of Bilinearity

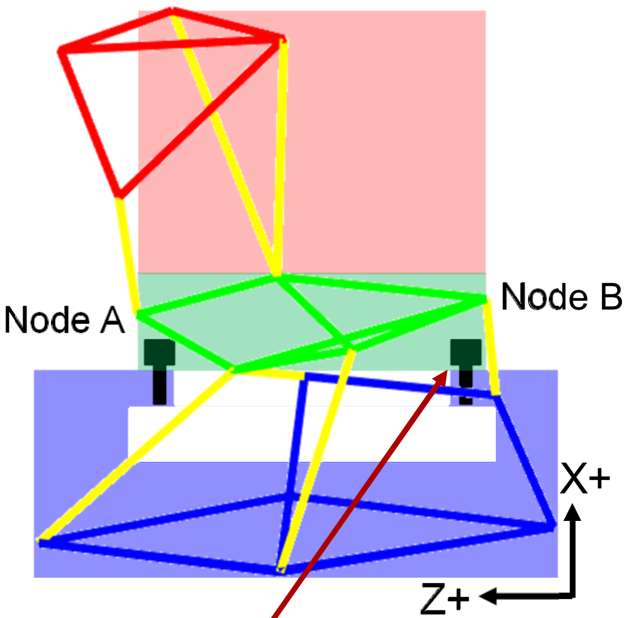
- Mode shape and physical domain data were studied to gain insight



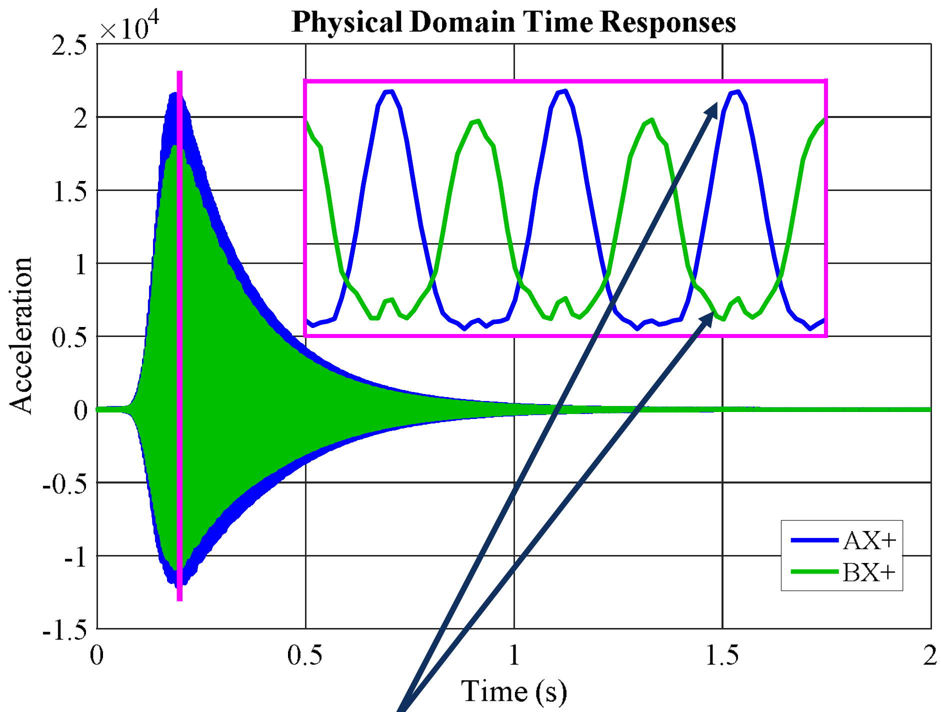


Evidence of Bilinearity

- Mode shape and physical domain data were studied to gain insight



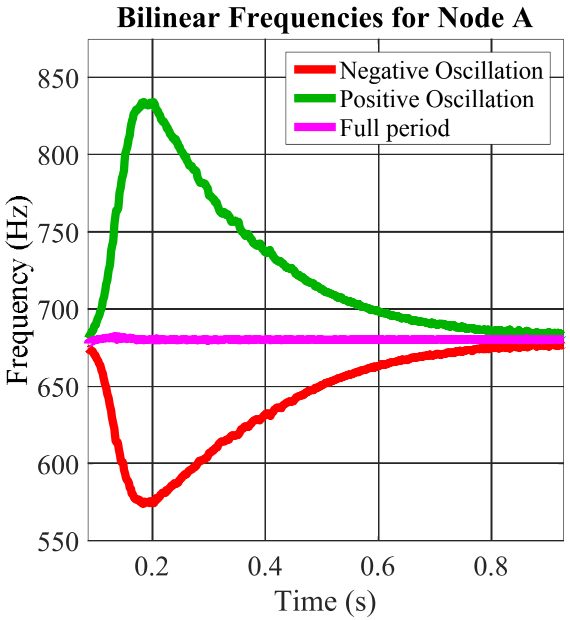
Housing is coming in and out of contact with fixture resulting in a bilinear response

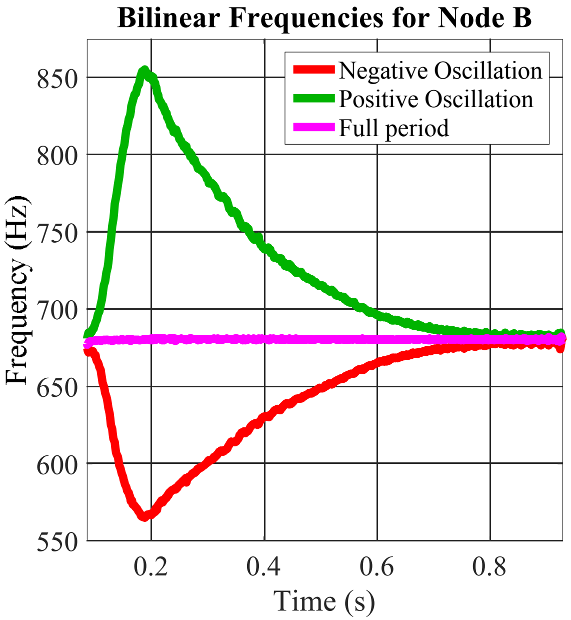


Phasing of bilinearity is complex: When A is in contact (stiff), B is pulling away (soft) and vice versa

Bilinear Frequencies

- Computed the bilinear frequency of each half-oscillation, $\frac{0.5}{T_{half}}$



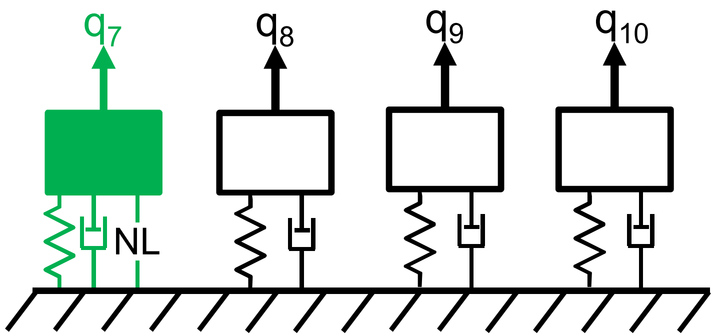


- The frequency of each half oscillation is changing dramatically but the frequency of the total period remains constant

The constant frequency of the full period results in a natural frequency that does not change with amplitude and the harmonics result from the asymmetry of the response about the origin

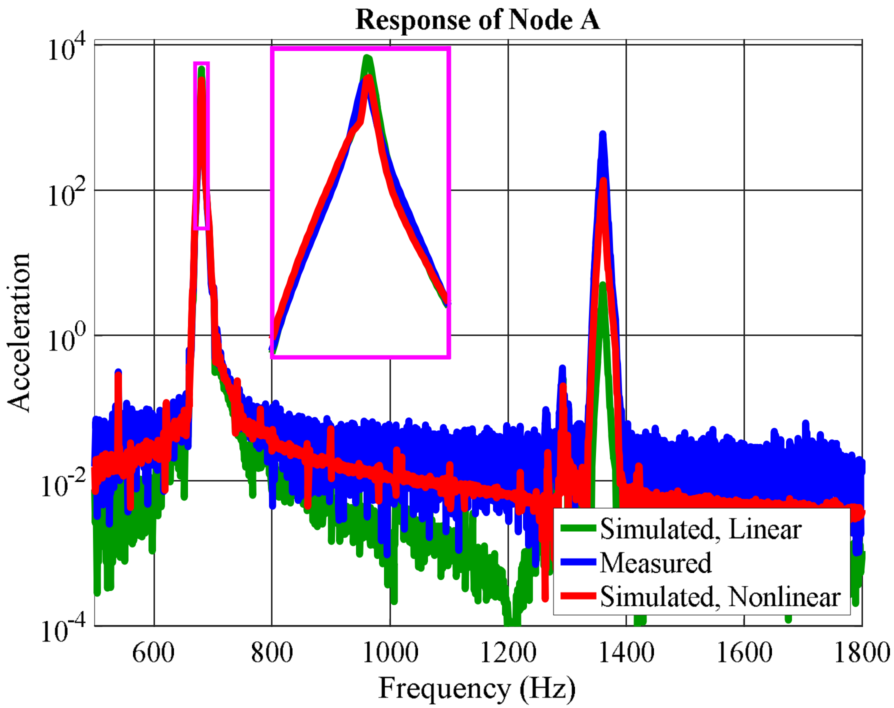
Simulation Results, Physical Domain

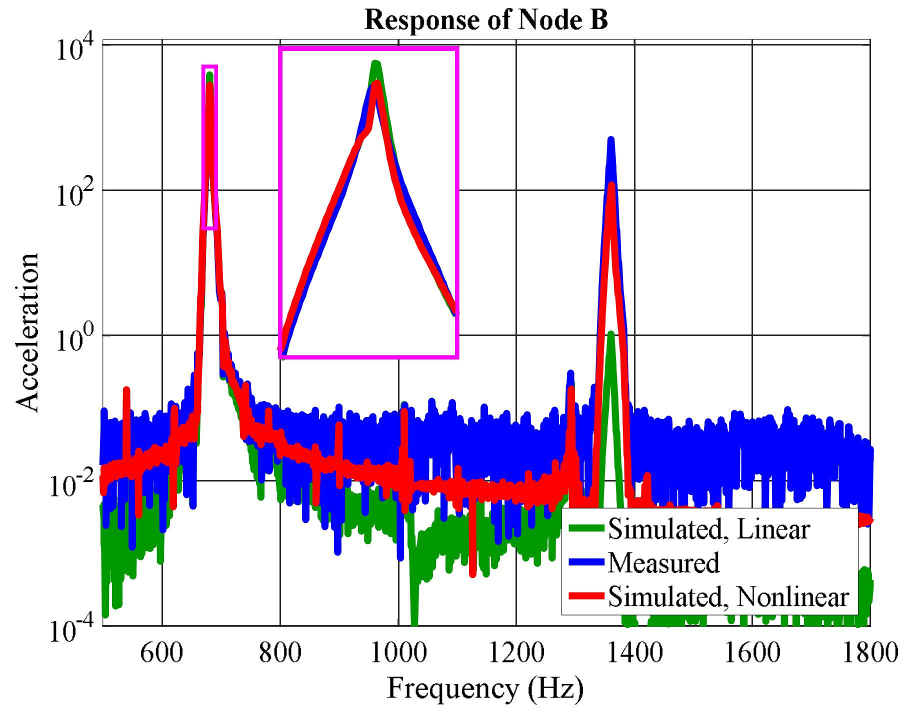
- A simulation was conducted to determine how well the pseudo-modal model replicates this complex bilinear behavior in the physical domain



- The pseudo-modal model comprised 10 modes (6 rigid, 3 linear, 1 nonlinear)
- The results were expanded back out to the physical domain via the linear mode shape matrix

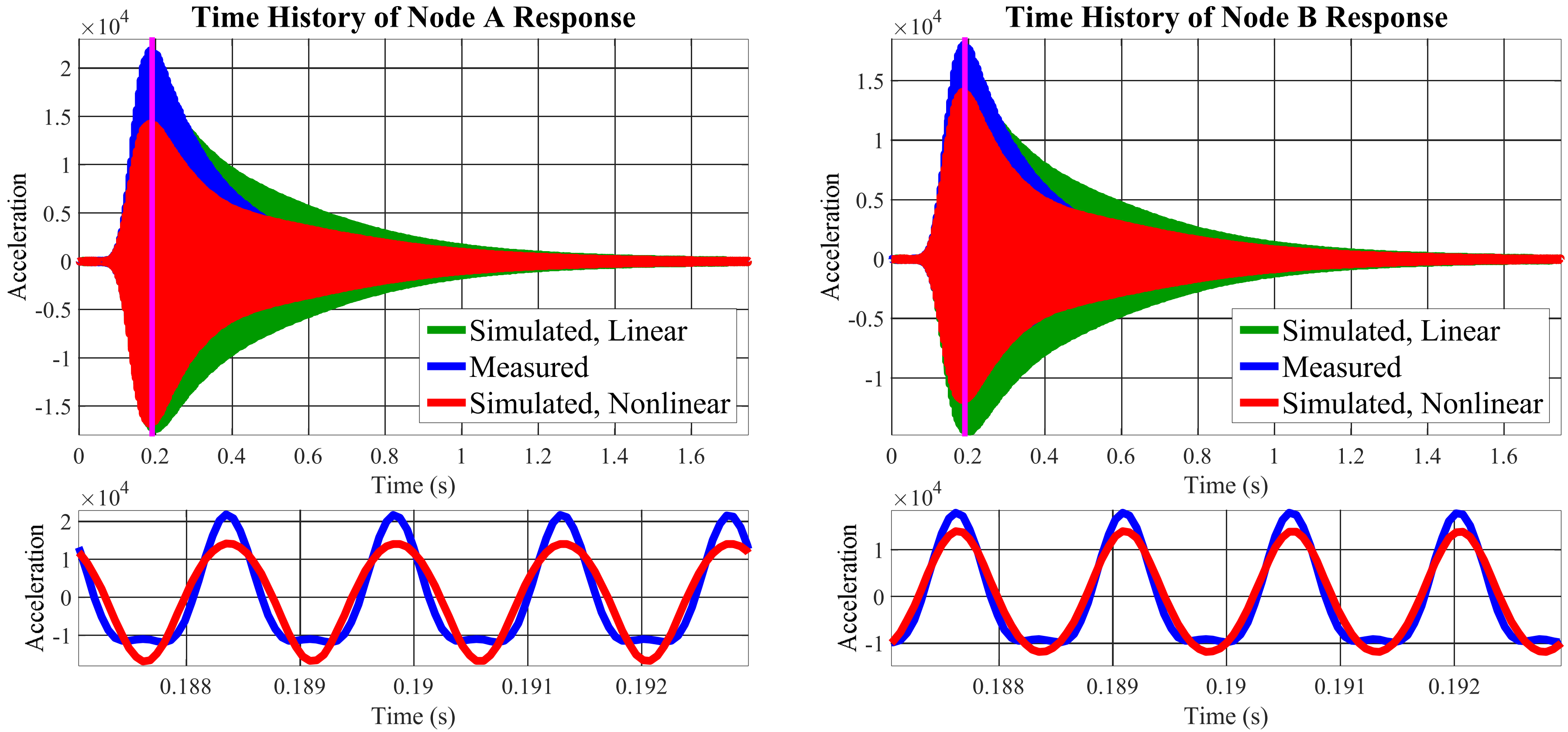
Simulation Results, Physical Domain





- Linear model slightly over predicts response amplitude of main peak
- Nonlinear model matches amplitude of the main peak
- Nonlinear model also shows harmonic response but not to the proper amplitude
- How does this translate into the time domain?

Simulation Results, Physical Domain



- Node A
  - Nonlinear model shows asymmetry but incorrect phasing of the stiff and soft oscillations
- Node B
  - Has very slight asymmetry (1.4e4 vs -1.2 e4 for positive and negative oscillations) but is correctly phased

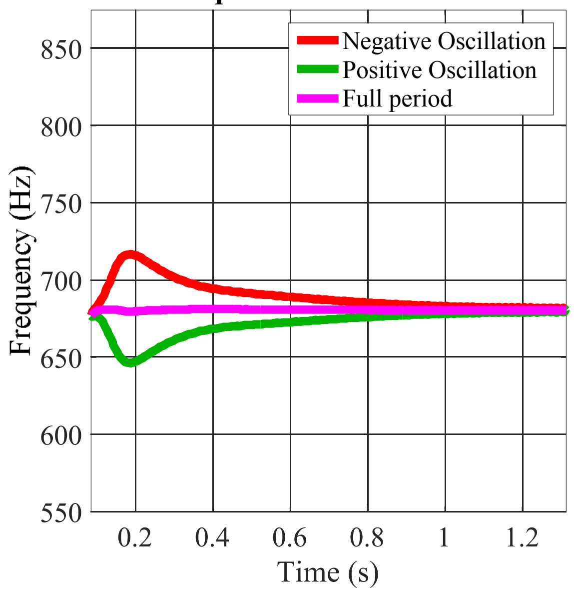
Conclusions

- Nonlinear modal model matched measured modal response
- Pseudo-modal model was not able to capture the detailed distortions and one-sided amplitudes in the physical domain
- Modal filter DID
  - Maintain the overall period of oscillation
  - Some asymmetry about the origin in peak response amplitude
- Modal filter did NOT
  - Capture the correct level of asymmetry
  - Maintain the phasing in the bilinearity in all nodes
- In the physical domain, the asymmetry is caused by a stiffness nonlinearity
- In contrast, the modal model accounts for the asymmetry with quadratic damping
- There are many types of nonlinearity, and one must consider whether an assumed nonlinear model form can meet the objectives of the analysis
  - Qualification acceleration random environment specifications vs capturing detailed physics of local response

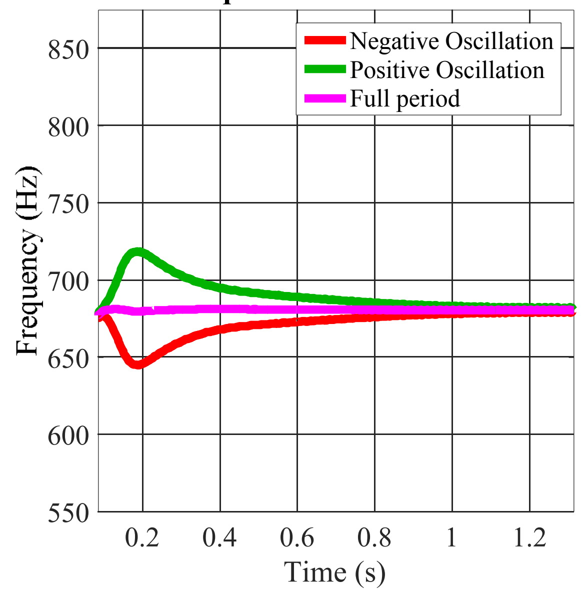
Backup Slides

Bilinear Frequencies

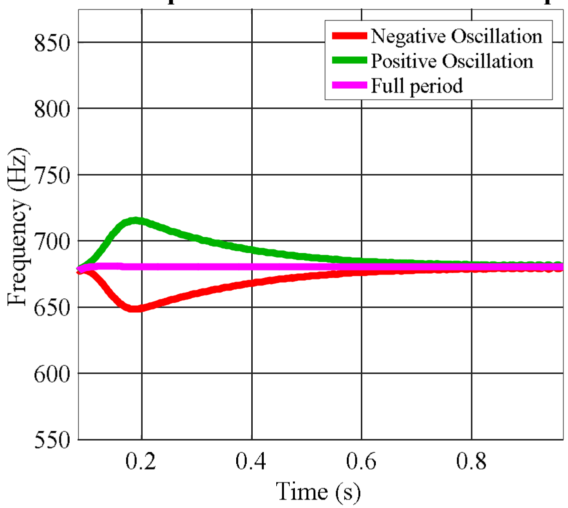
Bilinear Frequencies for Simulated Node A



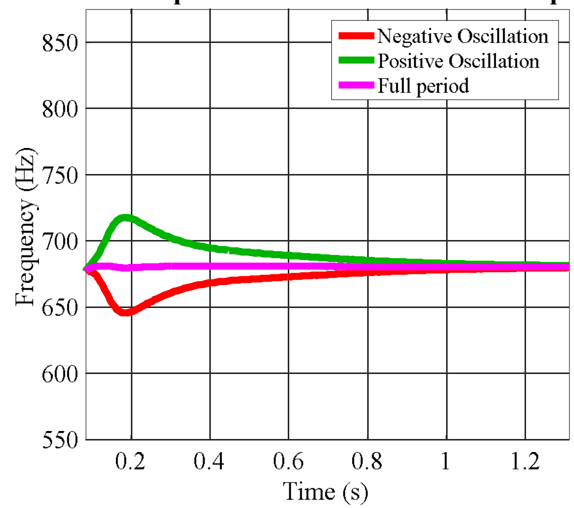
Bilinear Frequencies for Simulated Node B



Bilinear Frequencies for Measured Mode 7 Response



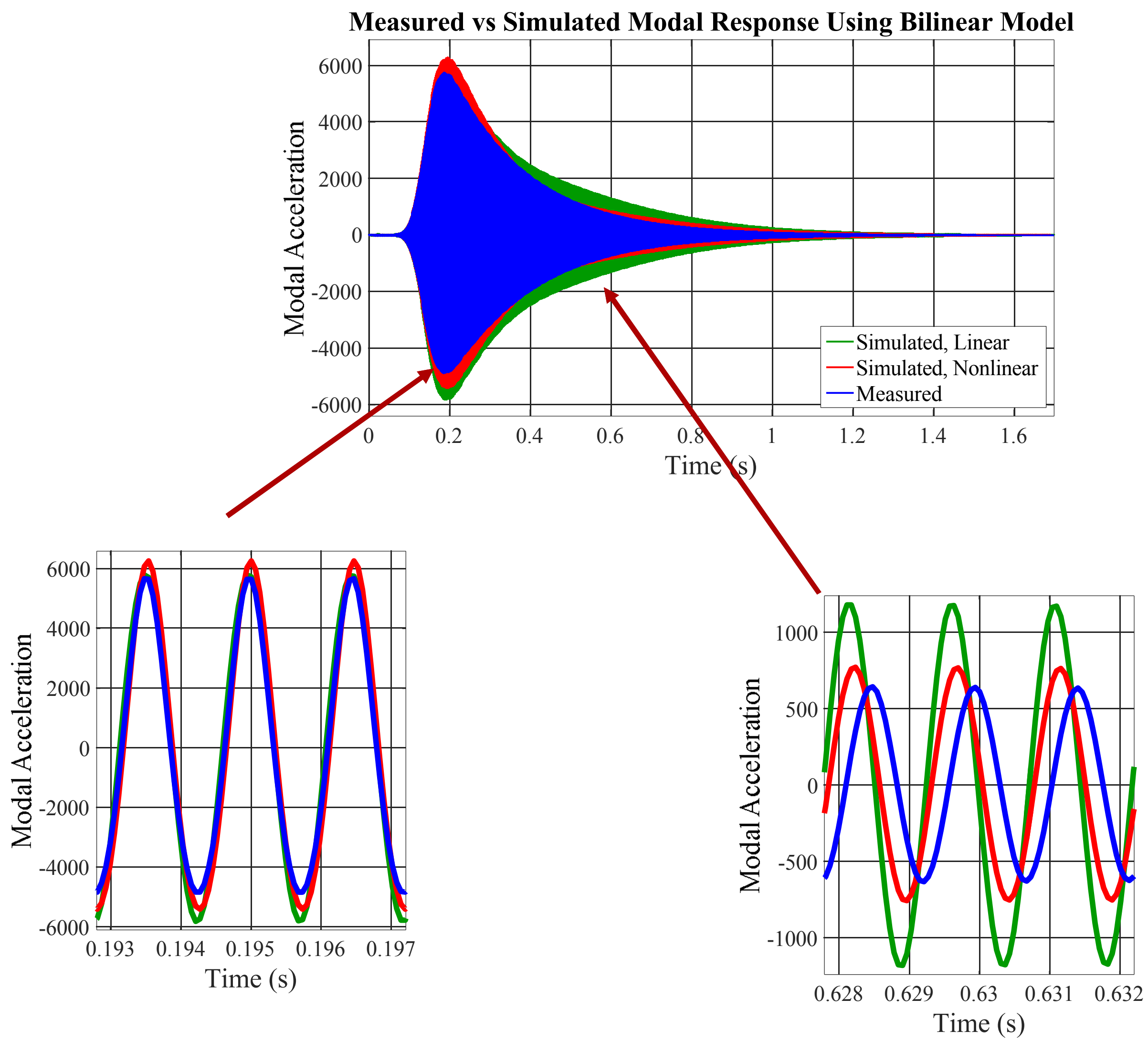
Bilinear Frequencies for Simulated Mode 7 Response



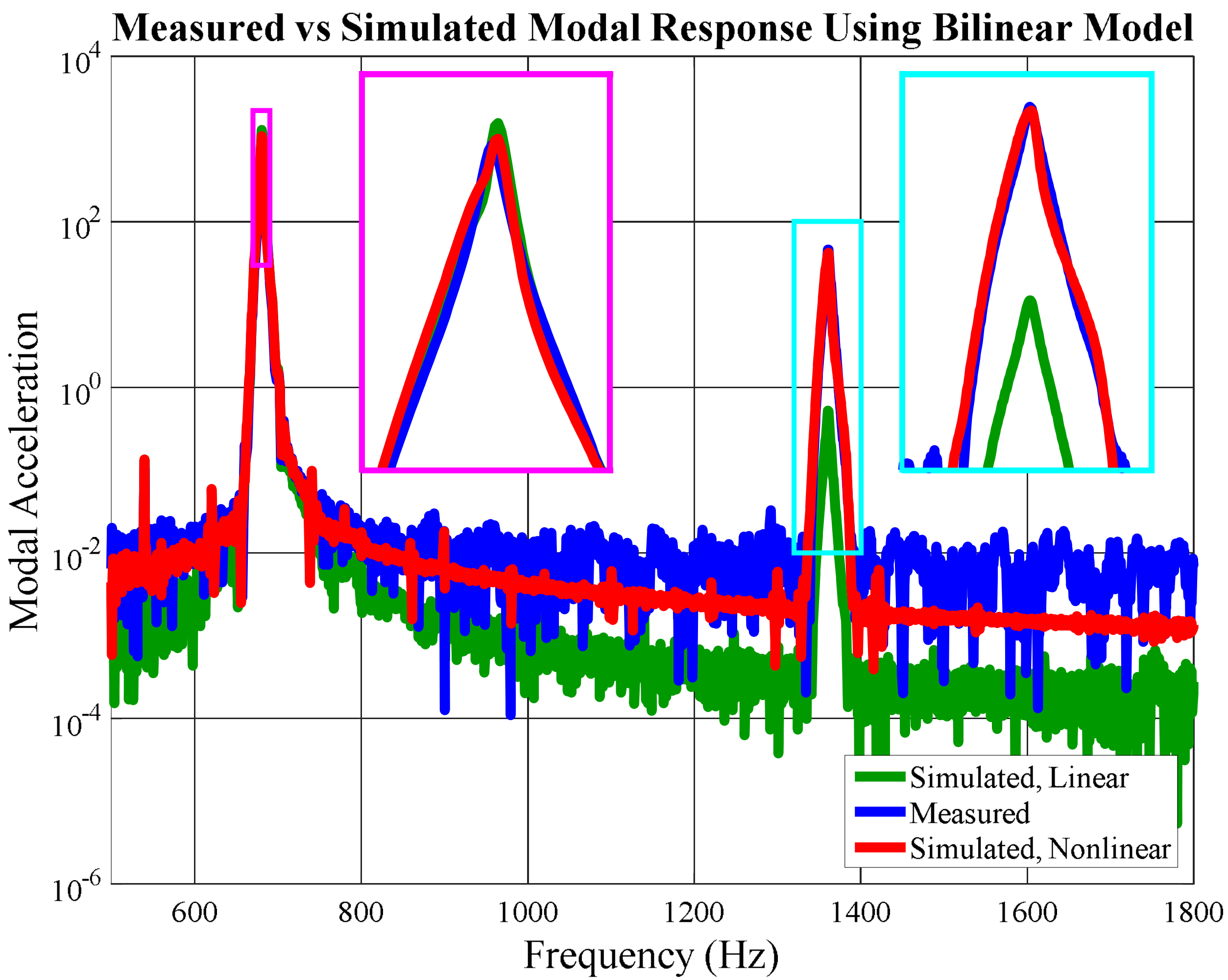
Bilinear Model Results, Modal Domain

- $\omega_{n,nl}(q) = \begin{cases} \omega_n + a_{1p}q + a_{2p}q^2, q \geq 0 \\ \omega_n + a_{1n}q + a_{2n}q^2, q < 0 \end{cases}$
- $f_s = \omega_{n,nl}(q)^2 q$
- $f_d = 2\zeta\omega_{n,nl}(q)$
- Modified ζ to be 35% larger than the value extracted in linear modal testing

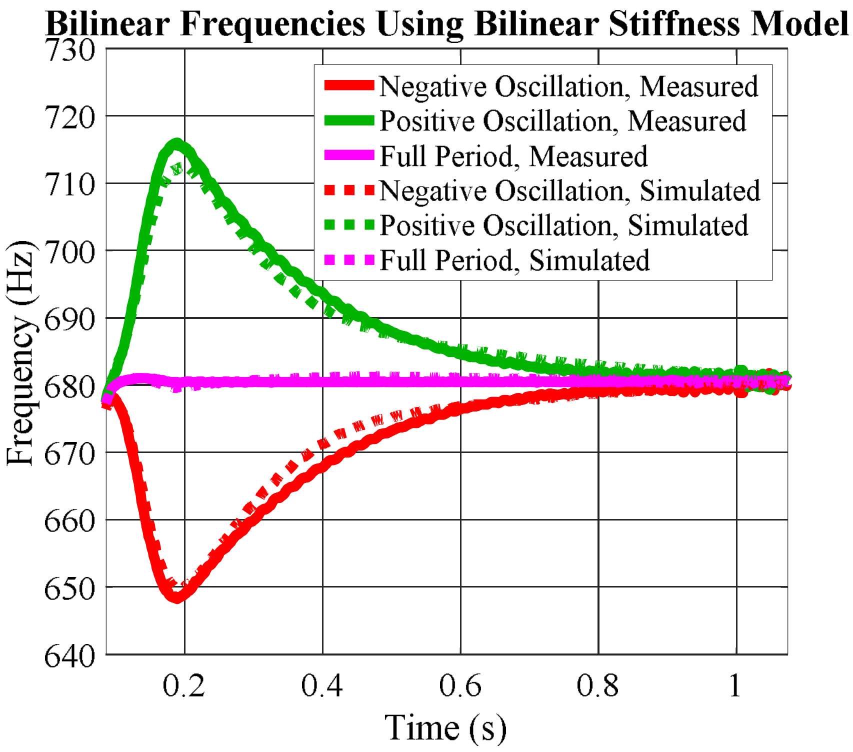
Bilinear Model Results, Modal Domain



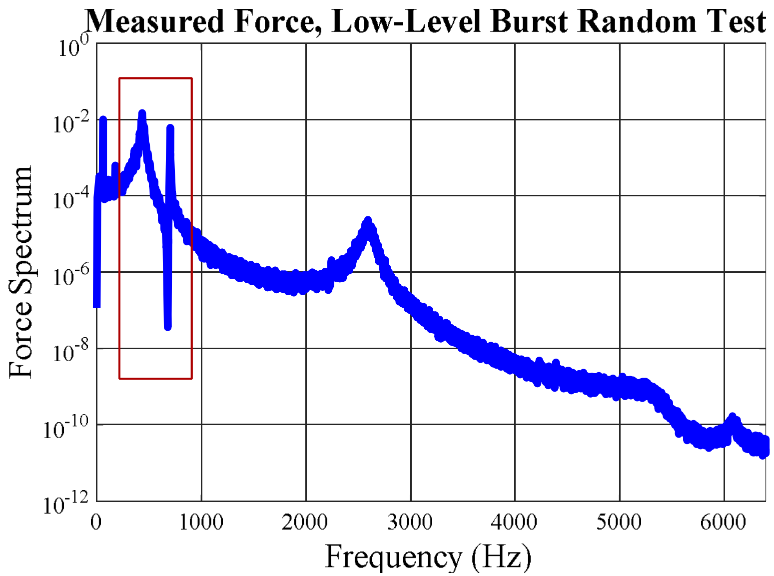
Bilinear Model Results, Modal Domain

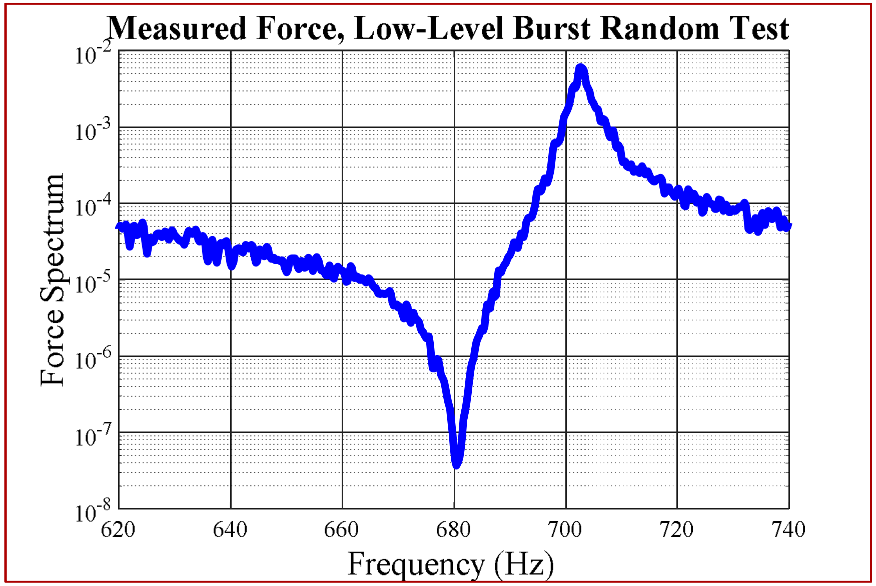


Bilinear Model Results, Modal Domain

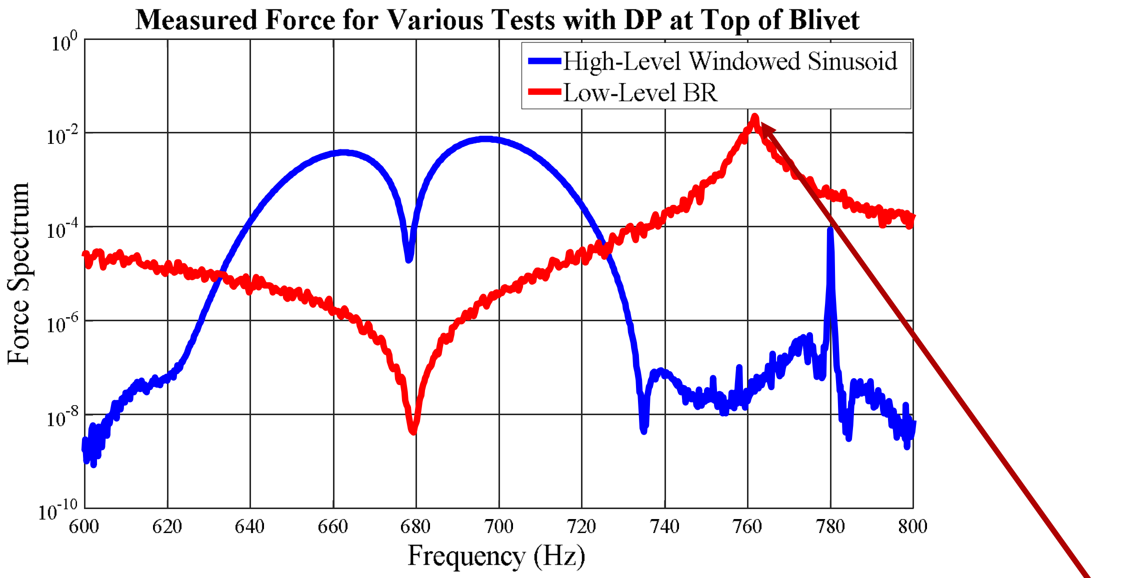


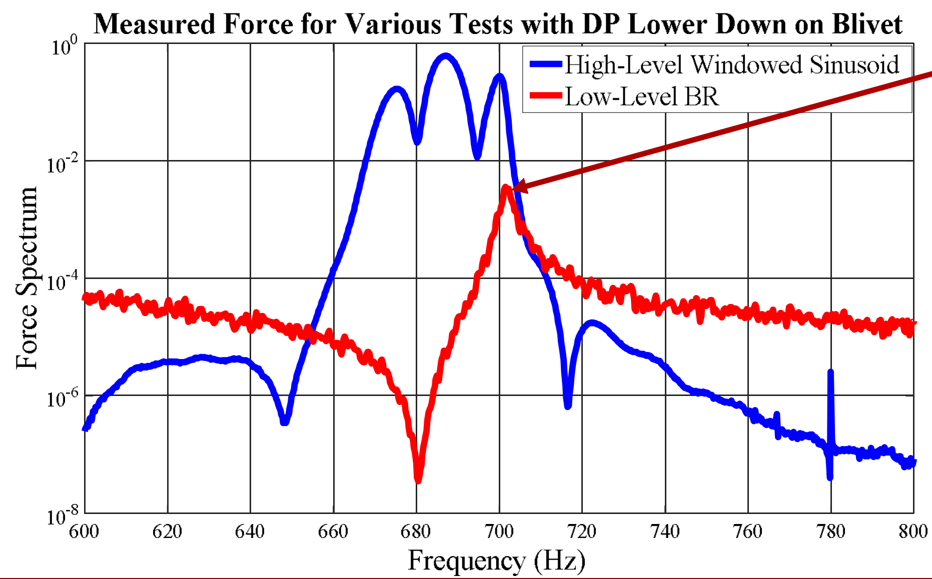
700 Hz Peak in Force





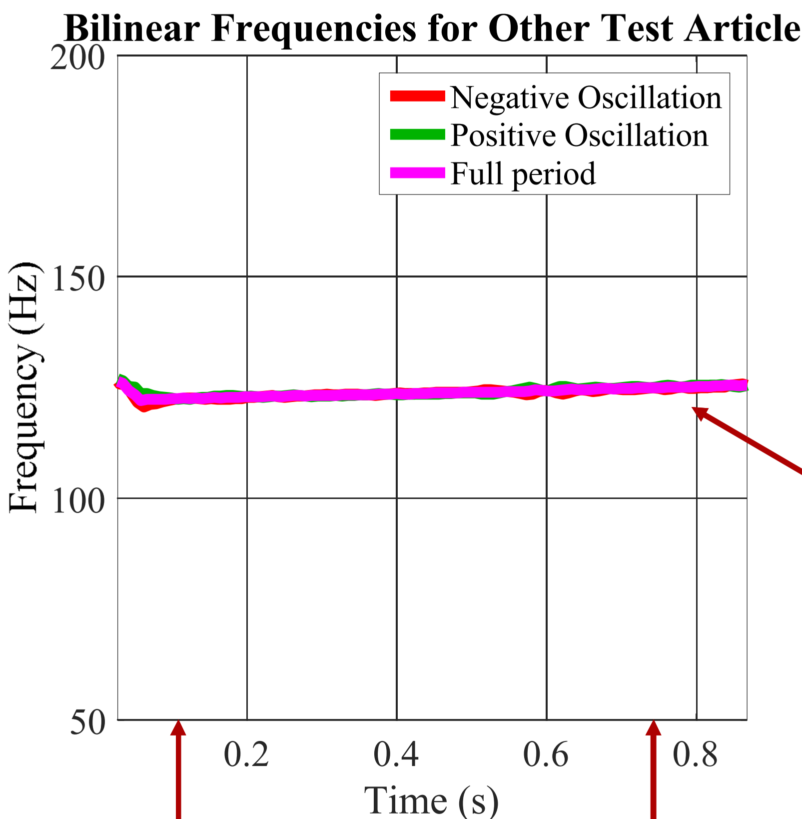
700 Hz Peak in Force





Peak in force spectrum after mode altered by moving location of drive point

Bilinear Frequencies, Other Test Article

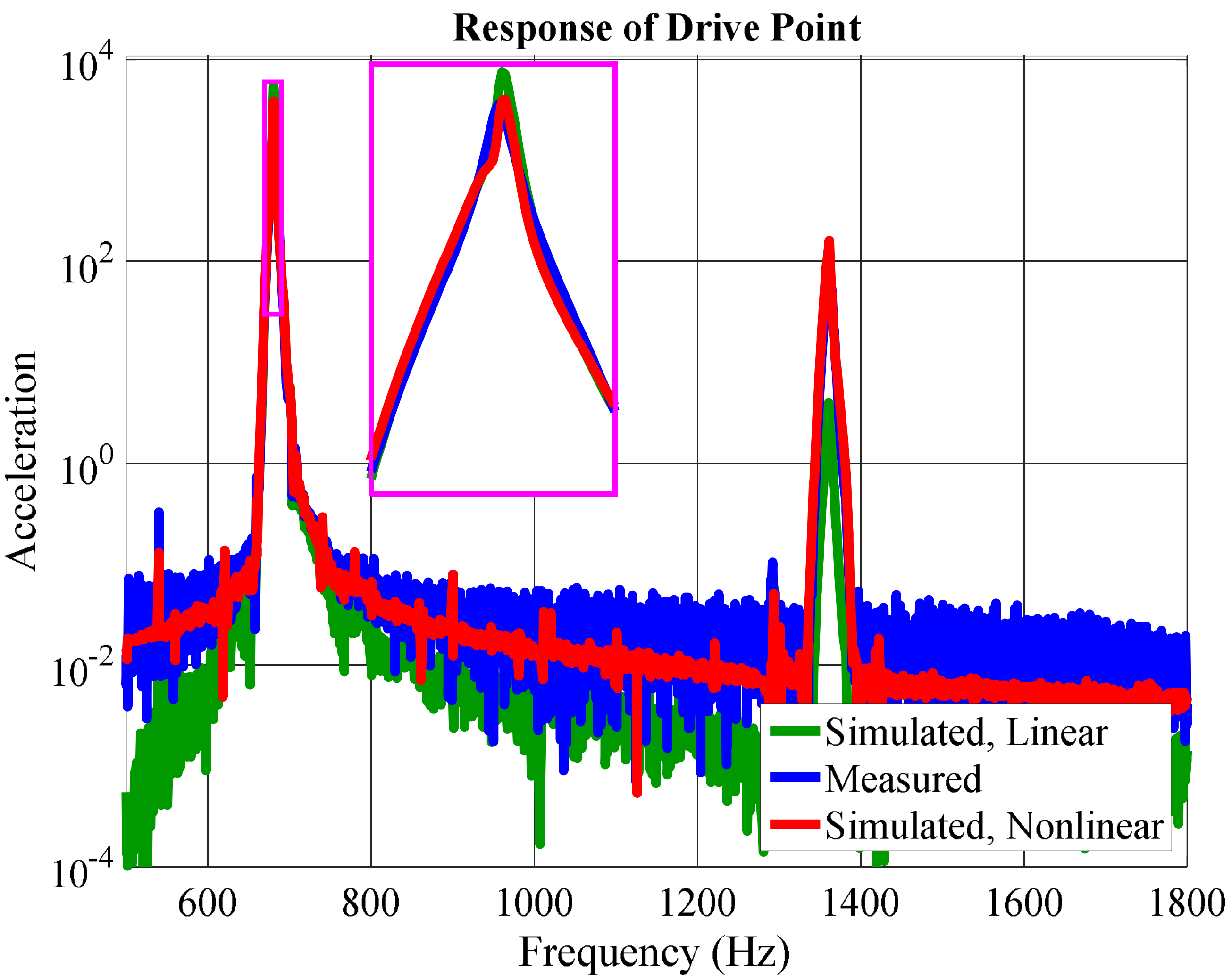


High response Amplitude

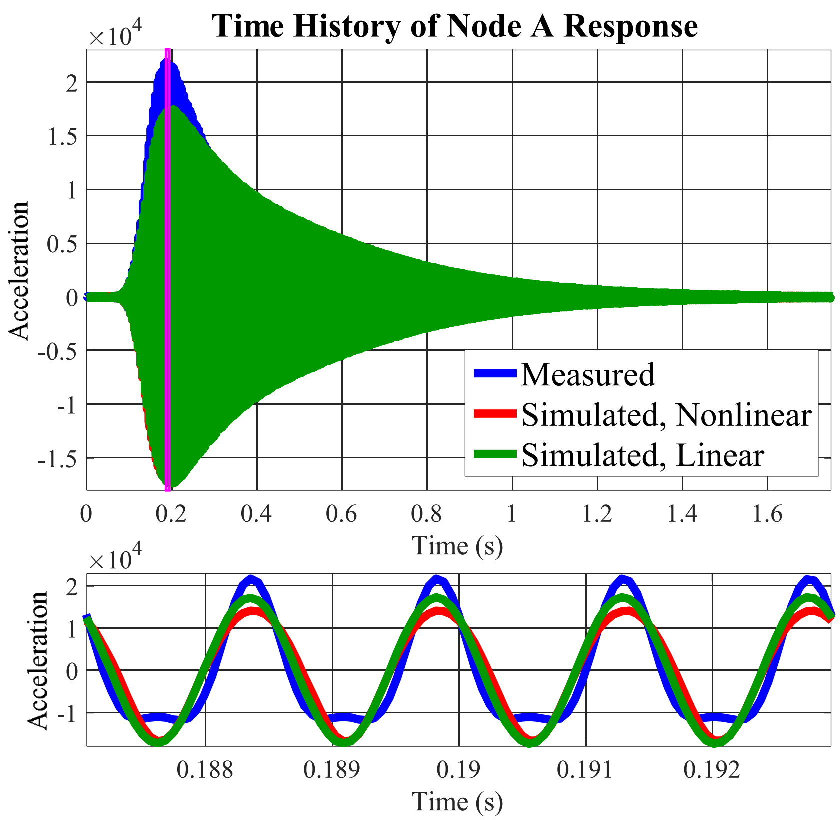
Low response Amplitude

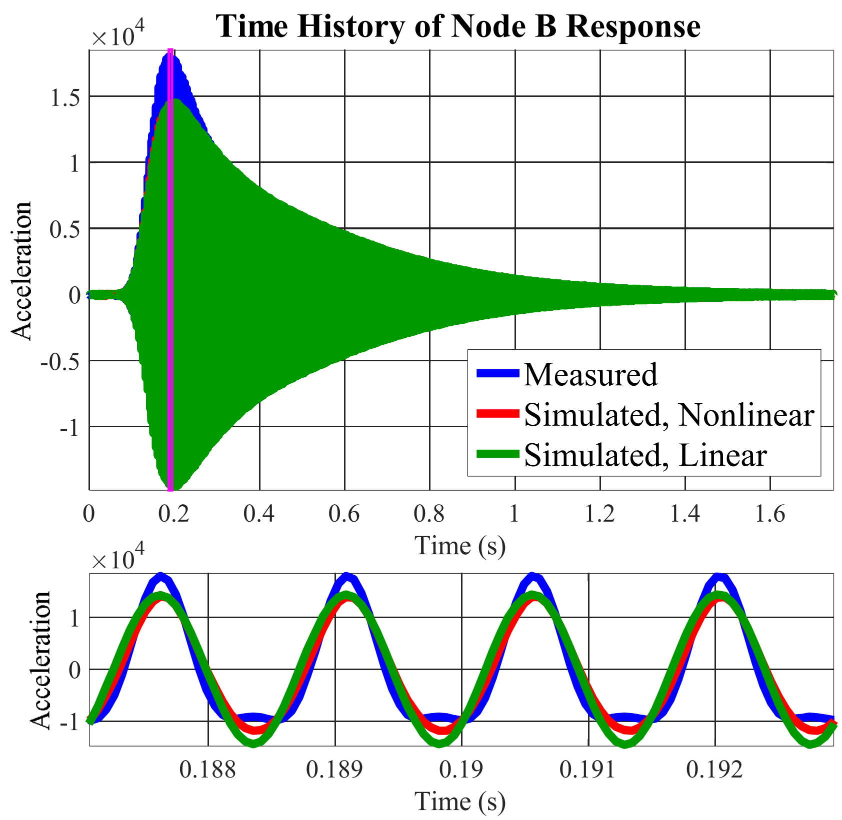
Can see the softening nonlinearity in this test article

Simulation Results, Physical Domain



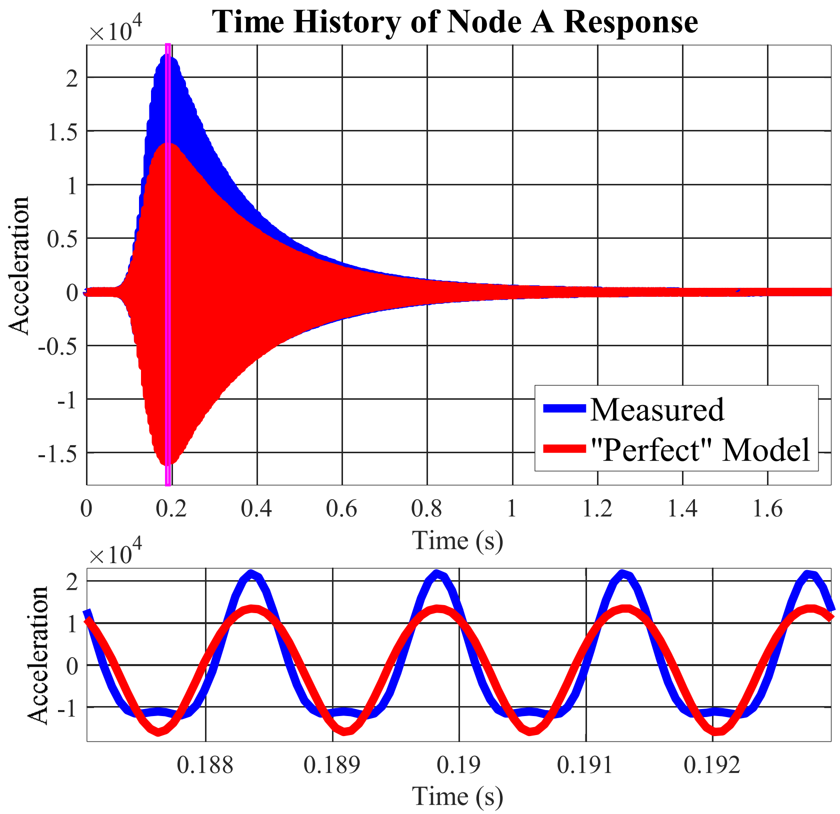
Simulation Results, Physical Domain

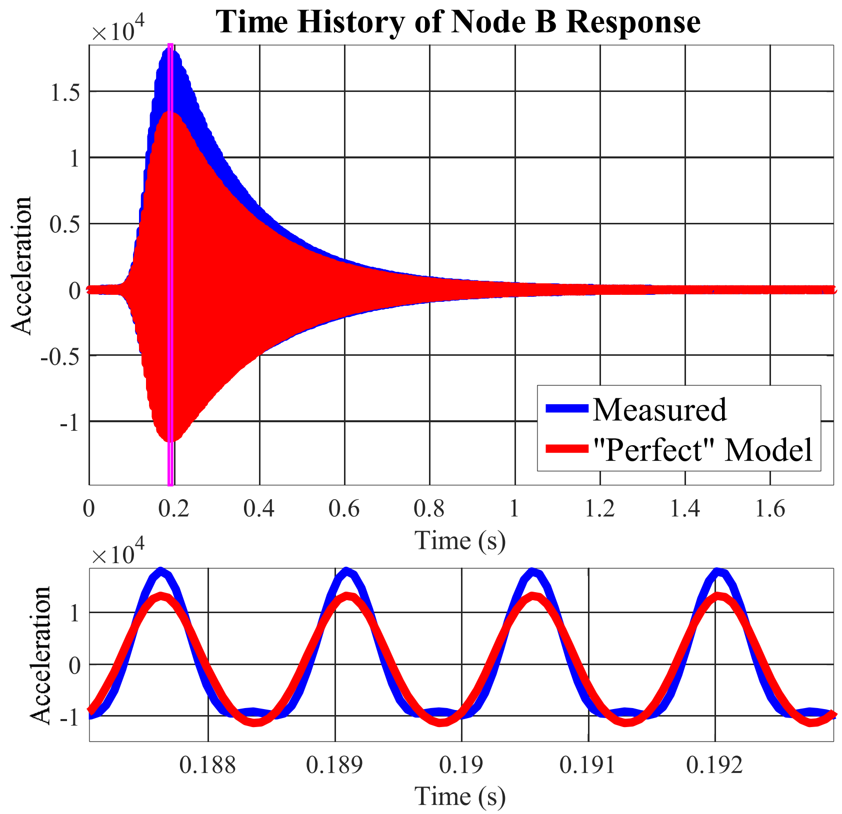




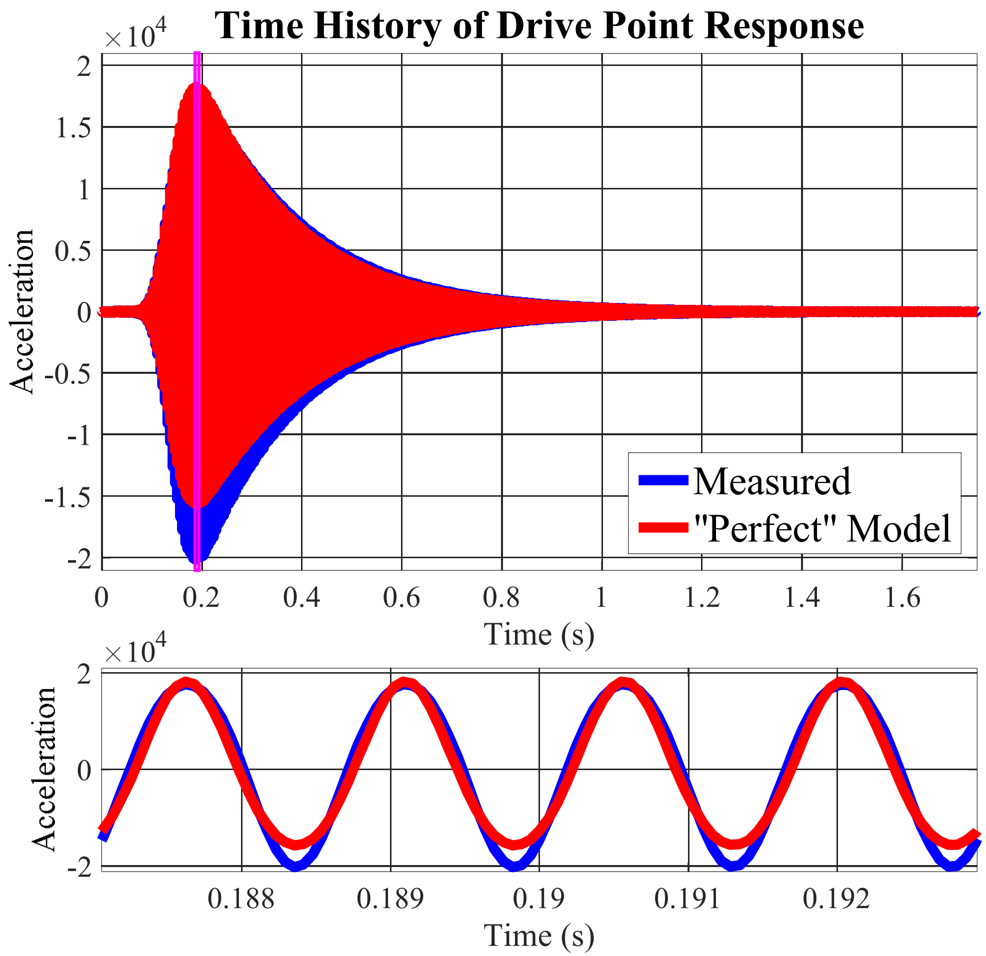
Simulation Results, Physical Domain

Starting with modal responses of all 10 modes, replacing mode 7 nonlinear model response with "measured" modal response and expanding back to physical domain (i.e. if the nonlinear model of mode 7 was perfect, how good does it replicate the response in the physical domain?)

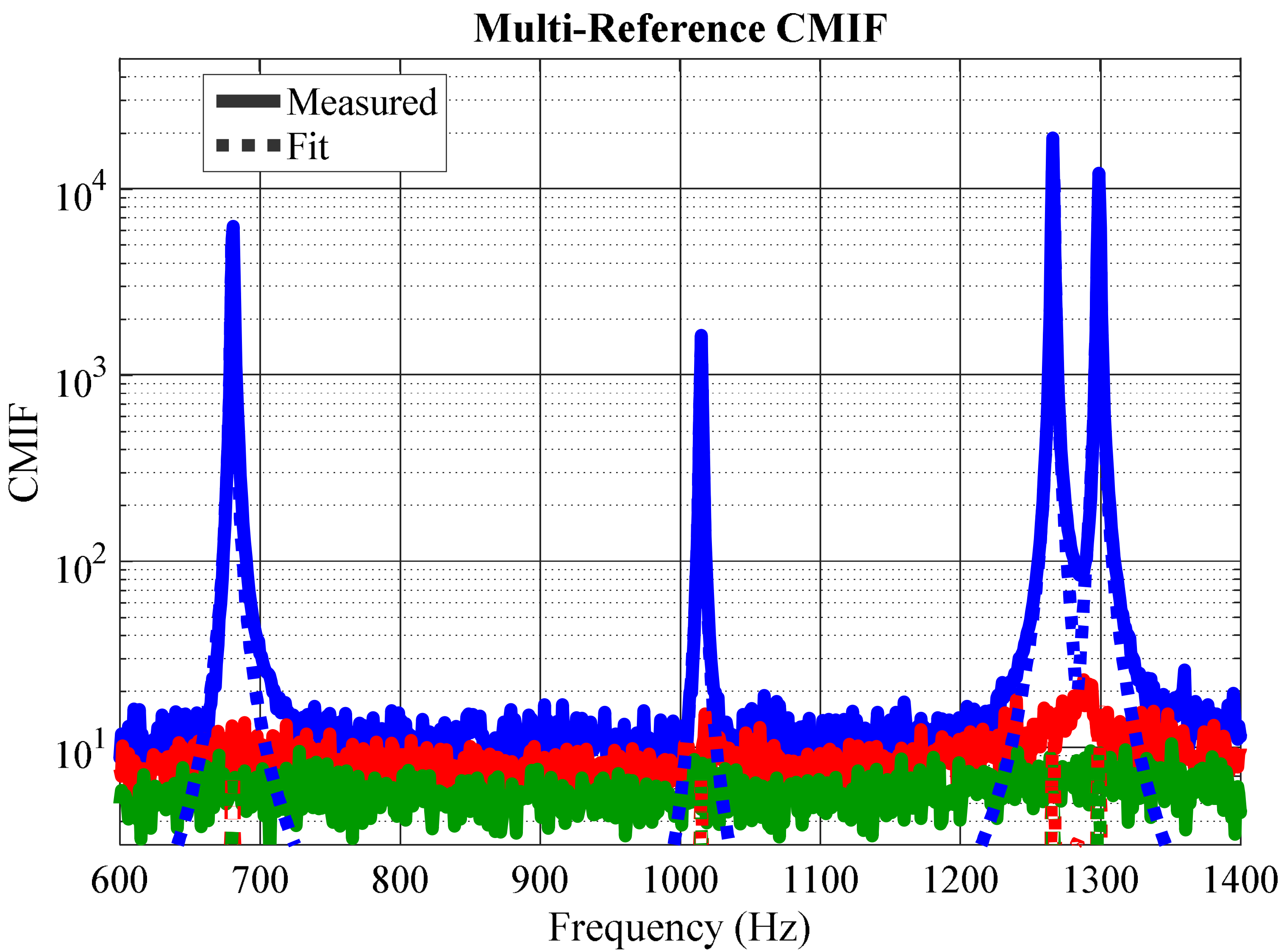




Simulation Results, Physical Domain



Multi-Reference CMIF

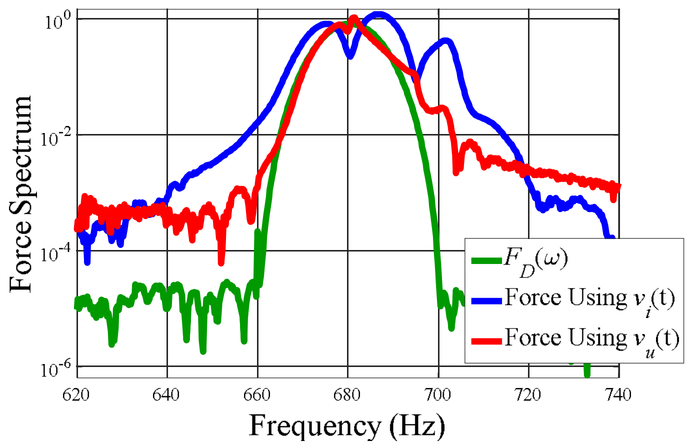


High-Level Testing

- Windowed sinusoidal modal shaker inputs used to maximize the excitation of mode 7
  - 680 Hz sine wave windowed by a Blackman-Harris window
  - Force dropout at resonance required updating shaker voltage signal

$$G_{vf}(\omega) = \frac{F(\omega)}{V_i(\omega)}$$

$$V_u(\omega) = \frac{F_D(\omega)}{G_{vf}(\omega)}$$



v_i(t), $V_i(\omega)$ = initial voltage input to shaker
$F(\omega)$ = measured force using v_i(t)
$F_D(\omega)$ = desired force profile
v_u(t), $V_u(\omega)$ = updated voltage input to shaker

Modal Filter

- Necessary to isolate the nonlinear effects for a specific mode
- Data from all accelerometers is weighted and summed with a modal filter to obtain a single mode response

$$\overline{\boldsymbol{\Psi}}^T \overline{\mathbf{x}} = q_i$$

- Modal filter calculated using the Synthesize Modes And Correlate (SMAC) parameter estimation algorithm
  - Generally shown to eliminate non-targeted modes better than some other methods

$$\overline{\boldsymbol{\Psi}}^T \overline{\mathbf{H}}_{\mathbf{x}} = \mathrm{H}_{\mathrm{q}}$$