

Toward Protecting Analog Simulations from Errors

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ENERGY



Sandia National Laboratories



Laboratory Directed
Research and Development


Apologia*



LIBERAL-ARTS MAJORS MAY BE ANNOYING SOMETIMES, BUT THERE'S NOTHING MORE OBNOXIOUS THAN A PHYSICIST FIRST ENCOUNTERING A NEW SUBJECT.

<https://xkcd.com/793>

*ap·o·lo·gi·a

/,apə'lōj(ē)ə/ 

noun

a formal written defense of one's opinions or conduct.
"an apologia for book banning"

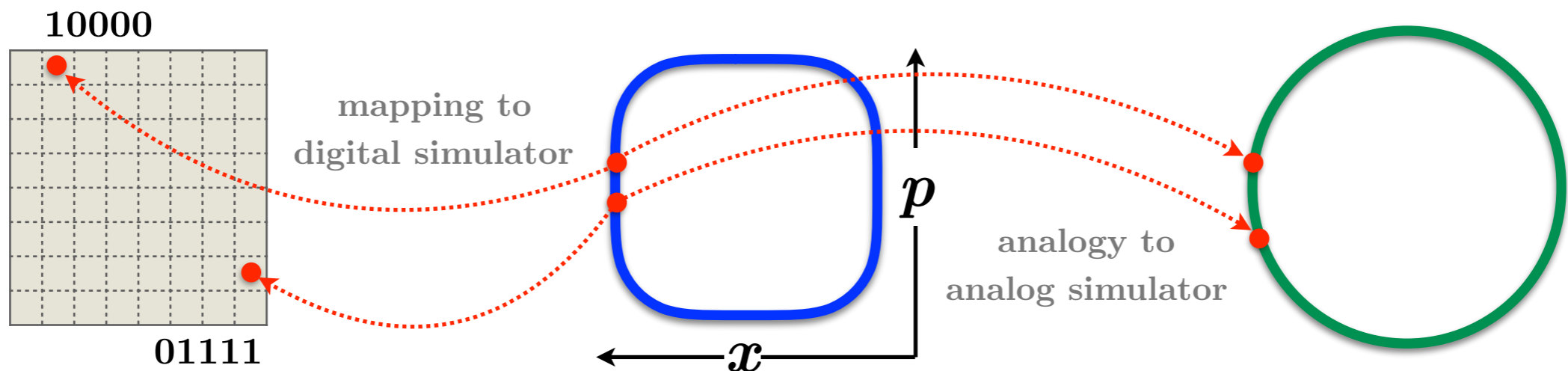
synonyms: [defense](#), [justification](#), [vindication](#), [explanation](#); [More](#)

Near-future qubits are noisy

- The digital quantum algorithms *known* to be useful require at least 200 qubits and $\gg 10^4$ consecutive operations.
 - \implies demand QEC to keep error rates low
 - \implies at least 5000 physical qubits, possibly many more.
 - \implies not “near future”
- **How can we find some quantum utility in the near term?**
- *One approach:* new digital algorithms requiring fewer gates.
- *Another approach:* Abandon the digital circuit framework and seek computational advantage from noise *analog* dynamics
- *This talk:* exploring how (and whether) analog quantum simulators (AQS) could provide useful computational advantages.

Analog and Analogue

- Analog = “*not digital*”, or “*continuous rather than discrete*”.
- Several QC paradigms are partly analog:
 - Adiabatic QC (time and Hamiltonian parameters are continuous)
 - Variational eigensolvers (parameters are continuous)
- But I’m going to focus on simulations of physical systems that are also *analogue* — the computer is **analogous** to that system.



- Topology is essential. Analogies preserve states’ adjacency w/r.t. noise and control... which is both good and bad.

Pros and Cons of Analog

Bad

- Probably incompatible with full QEC — QEC requires encoding that breaks topology even if it weren't "digital".
- No obvious way to encode or solve general computational problems — uniquely suited to simulating "real" systems.

Good

- Potentially error-resistant.
- Minimizes control overhead (states that are "close" in the system remain "close" in the computer).
- Physical errors in the simulator *should* correspond to "natural" noise rather than catastrophic errors.

- The natural problem for analog simulators is "What's the behavior of real(ish) quantum physical systems under real(ish) conditions?"

Can/will AQS be useful?

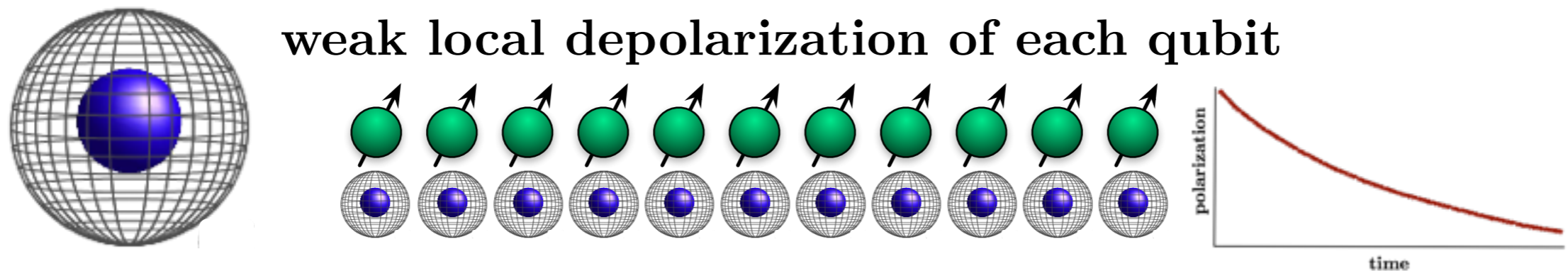
- **Goal:** figure out whether AQS has enough potential to invest in.
First step: identify potential showstoppers & evaluate workarounds!
 - ☞ What questions (about physical systems) do we want answered by an AQS?
 - ☞ Are any of those questions actually hard for classical algorithms?
 - ☞ If so, what makes a problem “classically hard”?
 - ☞ Which of those questions *can* AQS answer (reliably and efficiently)?
 - ☞ Which questions/problems do AQS fail on?
 - ☞ What is a/the computational model for AQS as a computation?
 - ☞ What should an AQS “simulate”? Ground states, thermal states, excited states, unitary dynamics, thermalization, quasistatic equilibrium dynamics, ...?
 - ☞ Spins, bosons, fermions, or what?
 - ☞ Lattice models, quantum chemistry, or something else?
 - ☞ Real-world models, variations on real-world models, or crazy Hamiltonians?
 - ☞ How do we characterize, verify, validate, and benchmark an AQS?
 - ☞ How do imperfections (noise, errors, control variations, etc) impact performance?
 - ☞ What control/architecture techniques can enhance performance of imperfect AQS?

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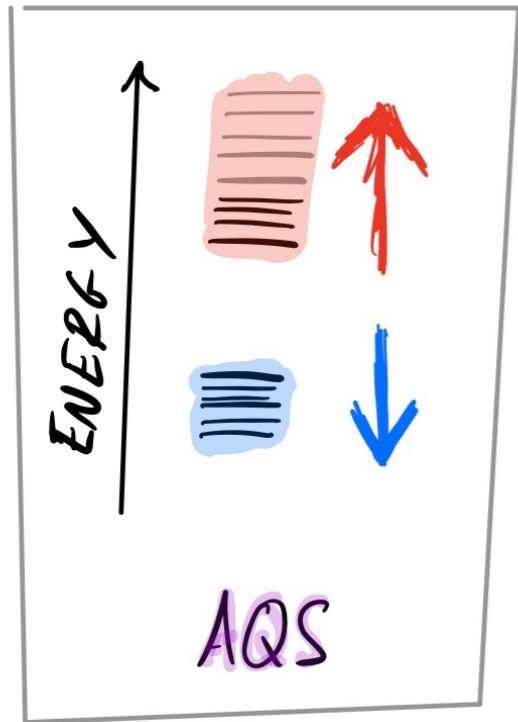
A simple, nasty model of noise

- For now, we are focusing (among all these questions) on *noise*.
- Neither the cause nor the effect of noise in AQS is obvious.
- So let's start with the same model used for digital QCs:

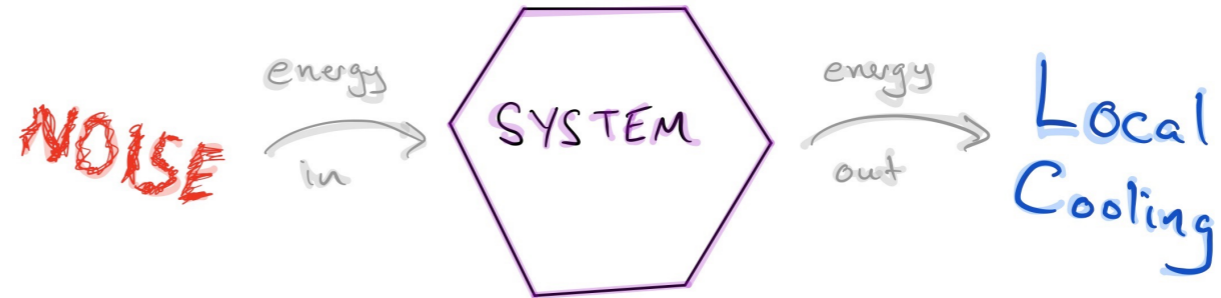


- This produces *fairly rapid thermalization to infinite temperature*.
- That's pretty bad for all types of simulation, so we'd like to fix it.
- **Note:** hereafter I will consider AQS that simulate *dynamics*, which is not obviously the same as “prepare ground/thermal states.”

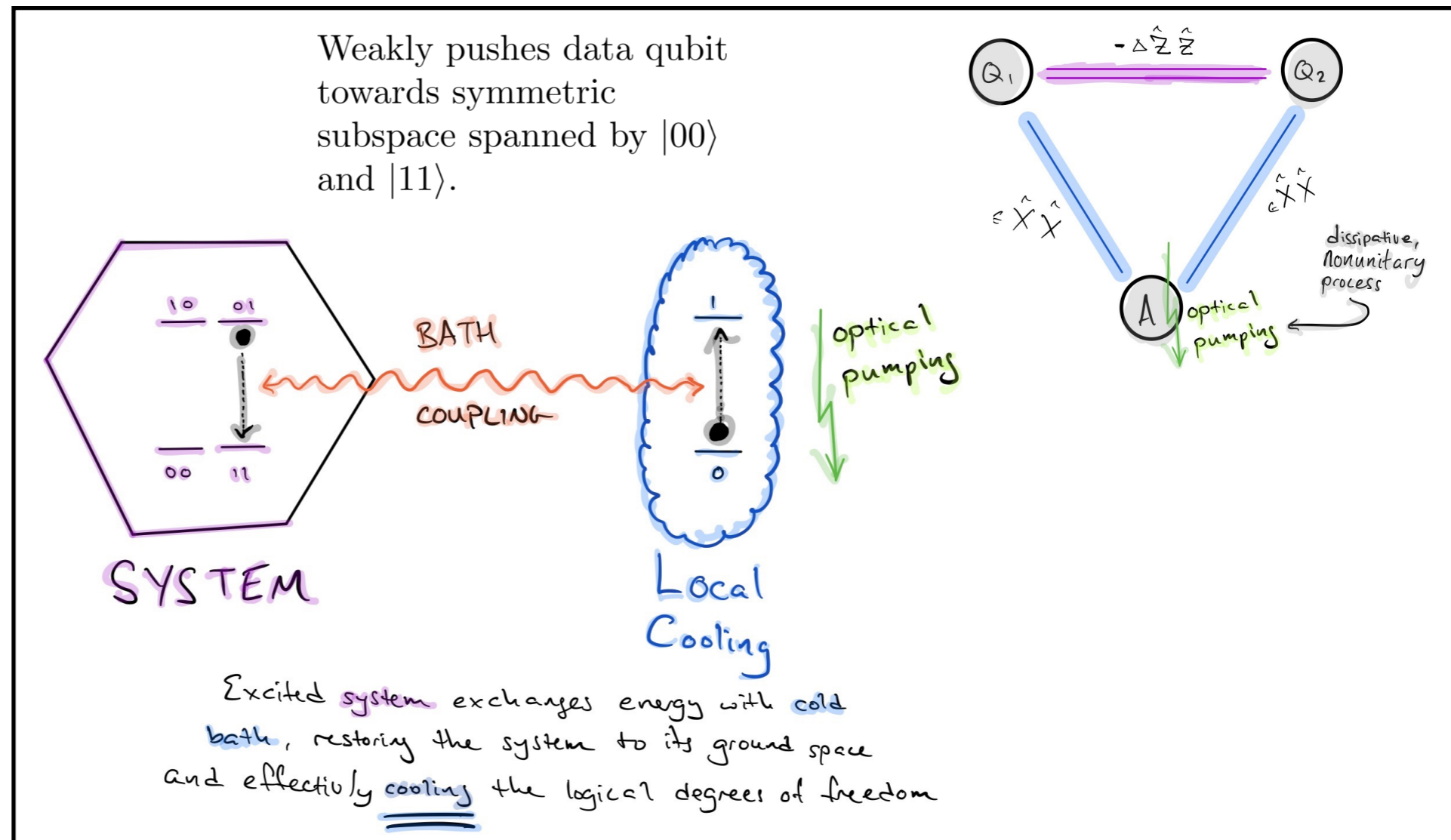
Active Logical Cooling: a fix?



Removing energy fixes errors
Ground space is **SPECIAL**



2+1 qubit toy example



Where I'm going with this

- I'm going to dive into the technical content of my talk shortly, by comparing active logical cooling to QEC. But first... a summary.
 - Eventually, we intend to analyze cooling (and its effects) in detail. Today, a more basic question: **“What would success look like?”**
 - AQS and QEC are probably incompatible
 - ==> unrealistic to expect the same results as QEC...
 - ...but we need to see *something* useful achieved by it!
 - Focus: (1) what could active logical cooling accomplish?
(2) how could this enhance computational utility of AQS
- Technical contributions:
- (1) steps toward theory of *approximately preserved information*.
 - (2) small step toward understanding [potential] power of AQS.

QEC vs. Active Logical Cooling

- Quantum error correction is also basically cooling. However:

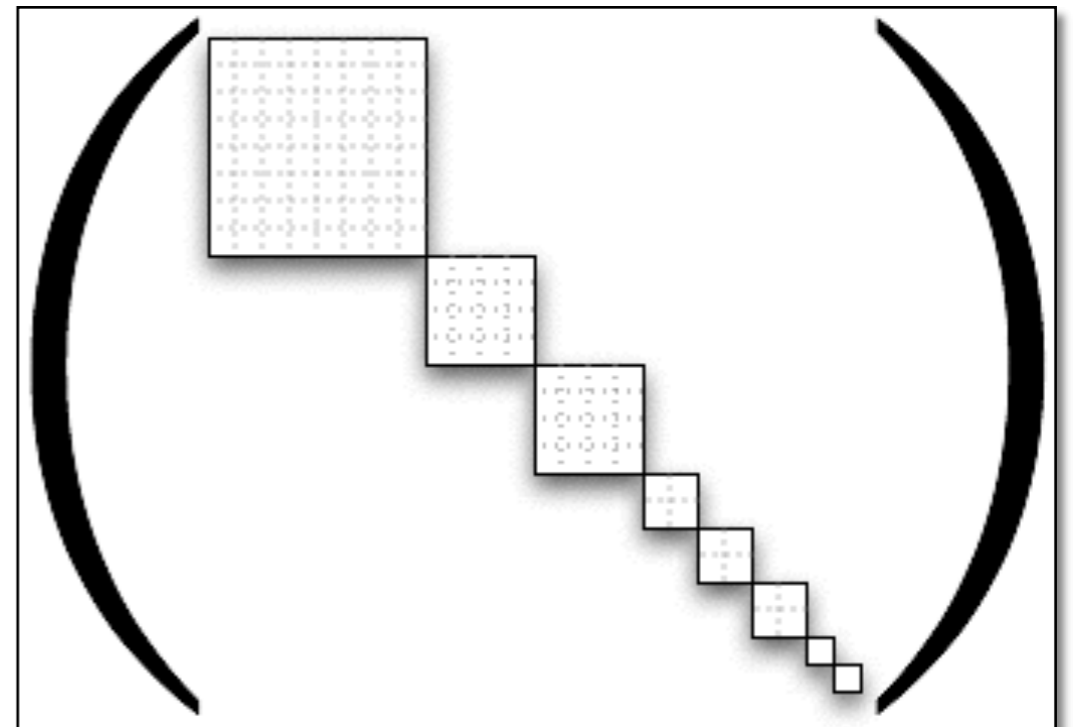
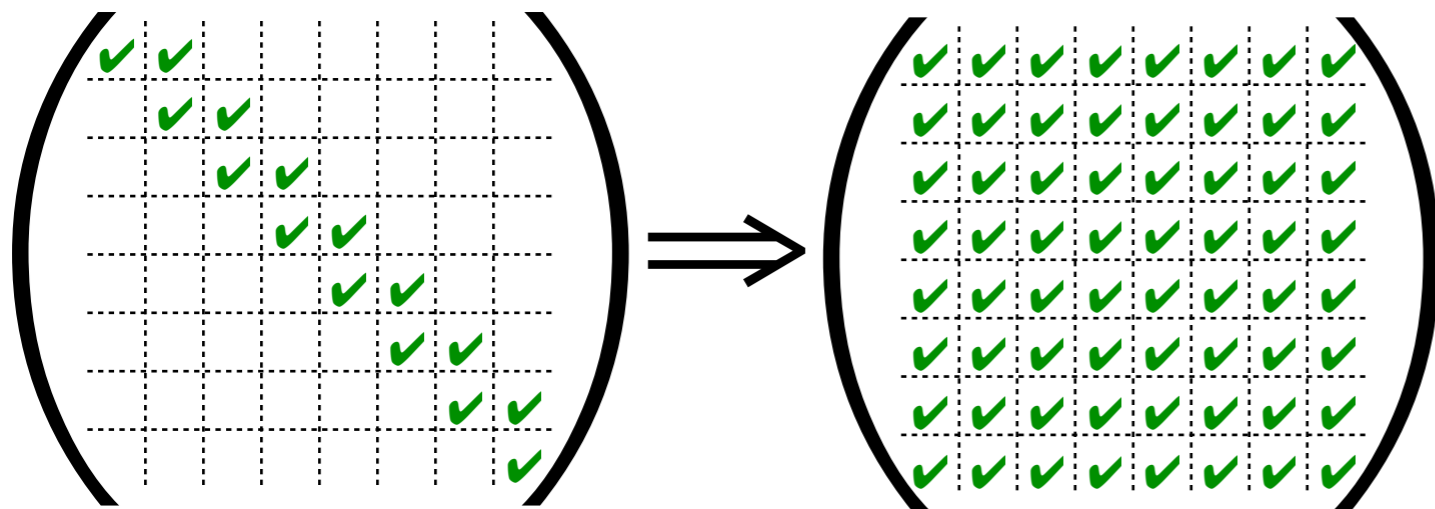
- QEC is cooling w/r.t. a very weird “Hamiltonian” defined by a complicated syndrome extraction circuit.



- QEC *requires* digital encodings that break topology of “easy” transitions. \implies dynamics in protected space is incredibly restricted (Eastin-Knill).
- QEC is generic — it protects all conceivable computations. \implies very very powerful, but also very expensive (overhead).
- Most importantly for us: **QEC seeks to preserve quantum information perfectly (or arbitrarily close to it).**

Perfectly Preserved Information

- **Q:** Suppose that a quantum dynamical process (e.g., noise + error correction) preserves information about some property *perfectly*. What do we know about the structure of the preserved information?
- **A:** States and/or observables corresponding to perfectly preserved information must form a C^* -*algebra* — they're closed under addition and multiplication. This implies rigid *structure theorems*.



Pretty well-preserved information

- In AQS, we can't hope for *perfect* preservation. We'll settle for *approximately* preserved information... but what does *that* look like?
- Not yet obvious what best definition of “approximately preserved” is.
- Central tool for bounds is *operator Schwarz inequality* (Kadison, Choi...):

$$\Phi(A^\dagger A) - \Phi(A)^\dagger \Phi(A) \geq 0 \quad \text{if } \Phi \text{ is a unital CP map}$$

- Useful example: *weak dephasing*

$$\rho \rightarrow \rho \circ \begin{pmatrix} 1 & .99 & .96 & .91 & \dots \\ .99 & 1 & .99 & .96 & .91 & \dots \\ .96 & .99 & 1 & .99 & .96 & .91 \\ .91 & .96 & .91 & 1 & .99 & .96 \\ \dots & .91 & .96 & .91 & 1 & .99 \\ \dots & \dots & .91 & .96 & .91 & 1 \end{pmatrix}$$

$$|k\rangle\langle k + \Delta| \rightarrow (1 - \epsilon\Delta^2) |k\rangle\langle k + \Delta|$$

The “shape” of preserved info can look like a band, rather than a kite.

Lifetimes of coherences can vary by d^2 within a d -dimensional system.

(Preservation is like Logic)

<https://www.scottaaronson.com/blog/?p=232>



Religion's rules of inference

I hereby wish to propose a different theory of fundamentalist psychology. My theory is this: *fundamentalists use a system of logical inference wherein you only have to apply the inference rules two or three times before you stop.* (The exact number of inferences can vary, depending on how much you like the conclusion.) Furthermore, this system of "bounded inference" is actually the *natural* one from an evolutionary standpoint. It's we — the scientists, mathematicians, and other nerdy folk — who insist on a bizarre, *unnatural* system of inference, one where you have to keep turning the modus ponens crank whether you like where it's taking you or not.

(Preservation is like Logic)

- Algebraic structure of *perfectly* preserved information is like logic:
 If $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$.
 If $|0\rangle\langle 1|$ and $|1\rangle\langle 2|$ are preserved, then $|0\rangle\langle 2|$ is too.
- In the presence of noise, long chains of logic (coherence) are unreliable
- What's remarkable here is just *how* rapidly the unreliability grows — not linearly (which is what you'd expect in [classical] logic), but quadratically.
- This turns out to have some interesting consequences...

$$\rho \rightarrow \rho \circ \begin{pmatrix} 1 & .99 & .96 & .91 & \dots \\ .99 & 1 & .99 & .96 & .91 & \dots \\ .96 & .99 & 1 & .99 & .96 & .91 \\ .91 & .96 & .91 & 1 & .99 & .96 \\ \dots & .91 & .96 & .91 & 1 & .99 \\ \dots & \dots & .91 & .96 & .91 & 1 \end{pmatrix} \quad |k\rangle\langle k + \Delta| \rightarrow (1 - \epsilon\Delta^2) |k\rangle\langle k + \Delta|$$

Implications of weak dephasing

- Do you want a nice classification of approximately preserved information, that directly generalizes algebraic structure?

Or a powerful relaxation of the complementarity condition that says “If a qubit channel transmits X and Z, it also transmits Y”?

Then quadratic decay of coherence is depressing.

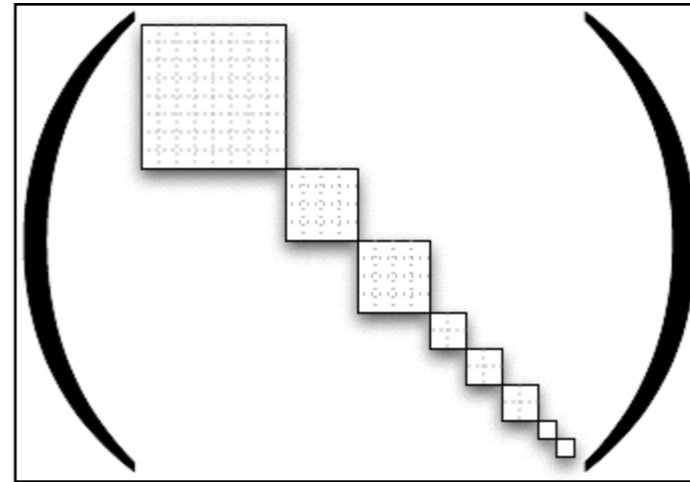
- But... there seems to be something genuinely “quantum” going on here, which might leave room for interesting physics. Why?

Algebra of classical observables is entirely Hermitian, and for *Hermitian* operators, the OSI implies linear decay: $\Phi(A^2) - \Phi(A)^2 \geq 0$

But quantum coherence operators aren't Hermitian, which allows them to behave more weirdly. Can we leverage this into something interesting/relevant?

Semiclassical information

- Perfectly preserved info must take the shape of an algebra \Rightarrow boring hybrids of quantum + classical codes.

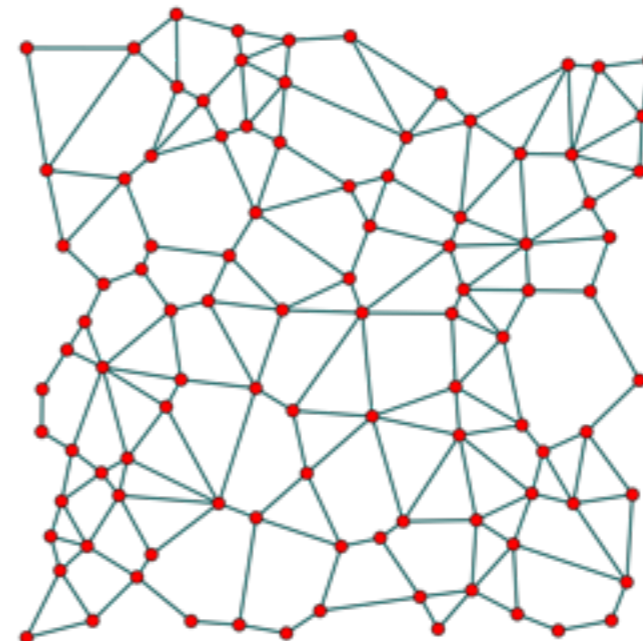


- But *approximate* preservation admits at least one new structure, which I'm uncreatively calling "semiclassical info".
(yes, this idea is 100 years old!)

$$\rho \rightarrow \rho \circ$$

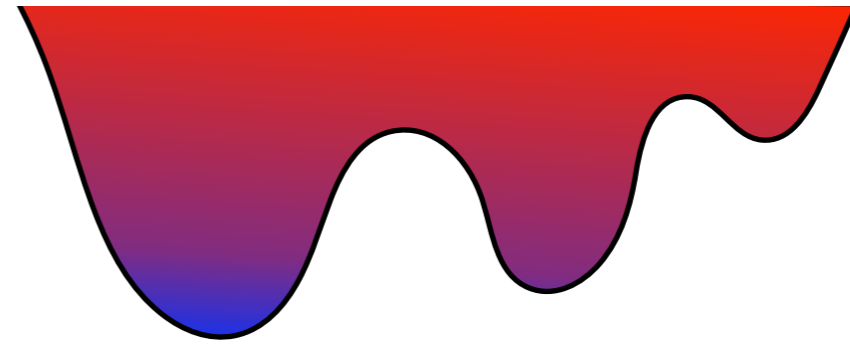
1	.99	.96	.91	...	
.99	1	.99	.96	.91	...
.96	.99	1	.99	.96	.91
.91	.96	.91	1	.99	.96
...	.91	.96	.91	1	.99
...	.91	.96	.91	1	

- "Semiclassical codes" support a separation of timescales between lifetime of *local* coherence (by some definition of locality) and the lifetime of *global* coherence.



Semiclassical computation?

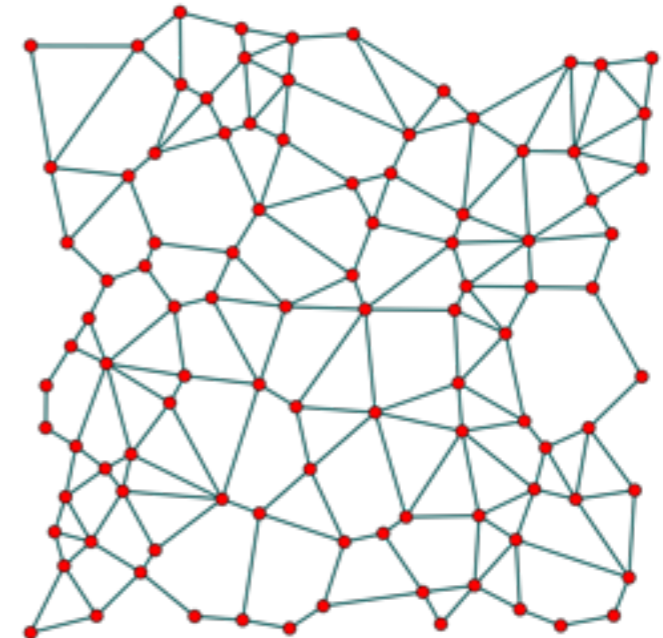
- Cooling can keep the simulation in the right phase indefinitely... but it can't eliminate or reduce the fundamental rate of decoherence/errors.
- In other words, if the heating rate is 10^{-3} , then cooling at the same rate can stabilize the simulator's global behavior — but the coherence/correlation *timescale* will still be limited to 10^3 timesteps.
- Does this leave room for any interesting computational utility?
- Interesting observation about Google search hits:
 - ☞ “quantum computing”: 556,000 hits
 - ☞ “classical computing”: 28,000 hits
 - ☞ “semiclassical computing”: 22 hits (1 publication, July 2017)



A candidate example



This sort of potential landscape...



...on a high-dimensional graph.

- Large system with a global potential encouraging transport.
- Many local “wells” separated by barriers that can be tunneled through.
- Tunneling is relatively *local* and *fast* — so it can survive noise/ decoherence that crush *global* and *long-lived* quantum coherence.
- Not a standard “quantum speedup” because limited coherence means this could be simulated on a cluster... if we knew how to decompose it!
- Similar to many strong-coupling problems — problem is finding a basis.

Conclusions

- As promised: this is all $<50\%$ baked! But...
 - ☞ plausibility argument for utility of analog simulation.
 - ☞ some direction for where to keep looking (and where not to).
- Cooling might help some problems, but won't eliminate errors that preclude true “quantum computational speedup” — coherence length time will be strictly limited \Rightarrow potential classical simulation.
- But a lot of many-body problems would be much easier if we knew how to decompose them... so there's room for computational utility.
- If this works, I call dibs on the term *semiclassical computing*. :)