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Waveform Optimization For Resonantly Driven MEMS Switches Electrostatically Biased Near Pull-In

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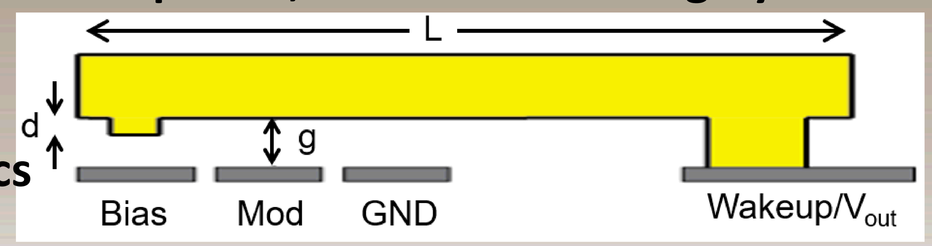
Introduction

- Biasing a MEMS switch close to static-pull in reduces the modulation amplitude necessary to achieve resonant pull-in, but results in a highly nonlinear system.
- We present a new methodology that captures the essential dynamics with an analytical expressions and provides a prescription for achieving the optimal drive waveform which significantly reduces the amplitude requirements of the modulation source.



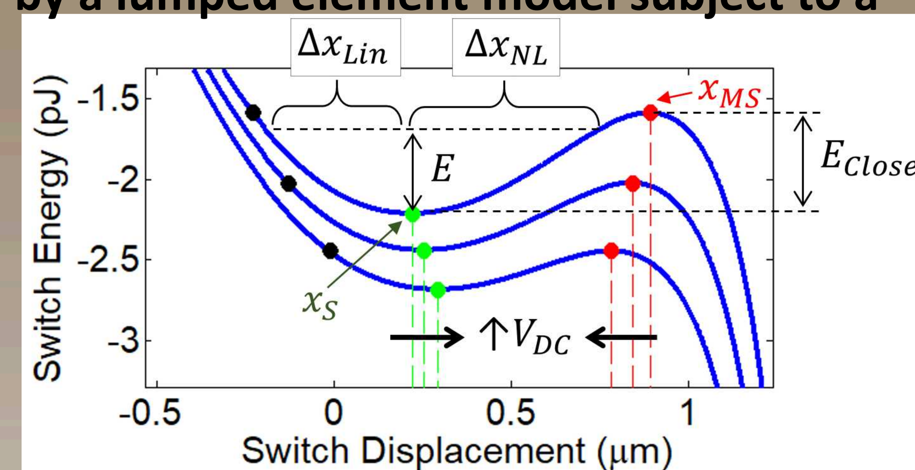
- Analytical results are validated numerically and experimentally.

Characterization In phase Space

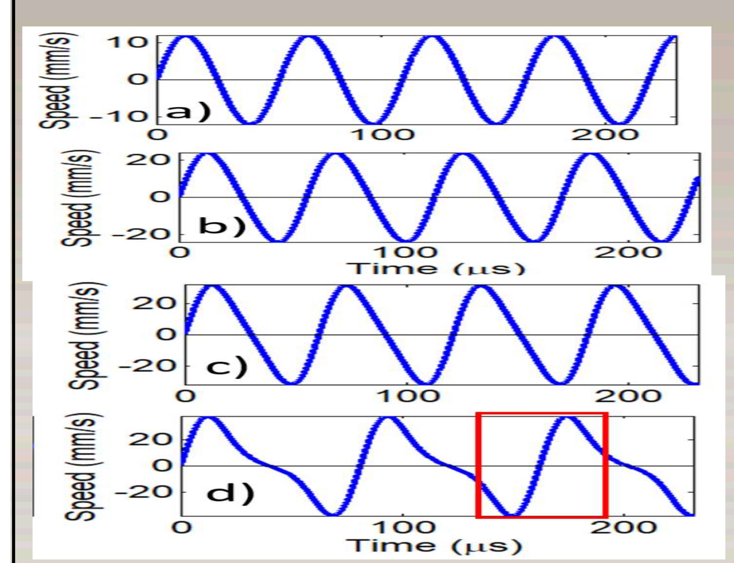
- Switch dynamics can be captured by a lumped element model subject to a conservative potential:

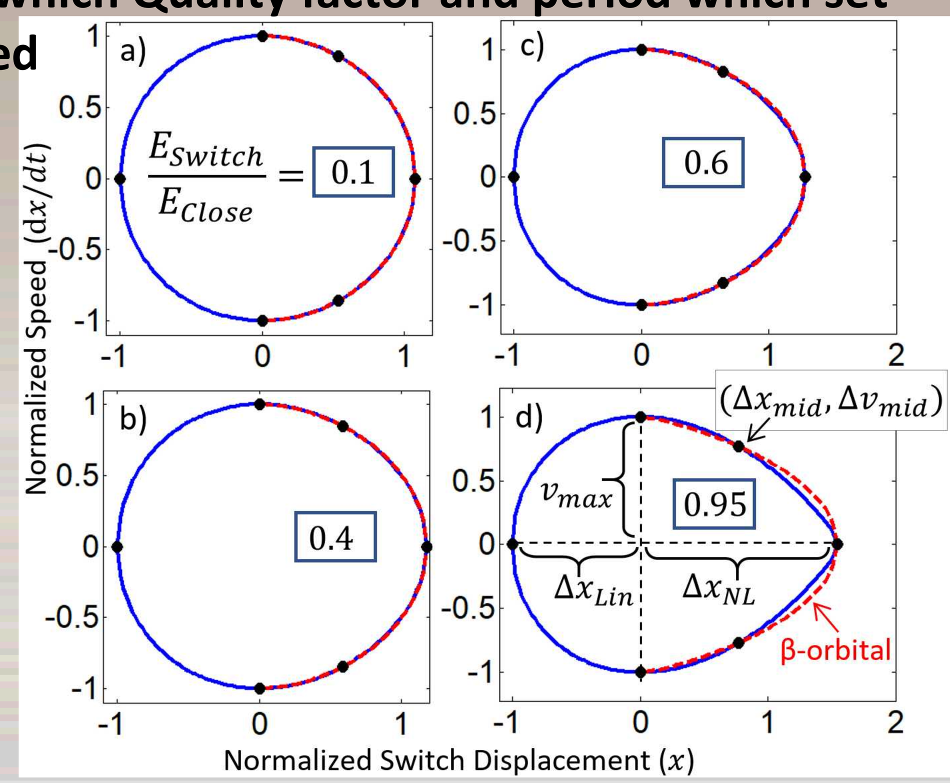
$$m\ddot{x} + b\dot{x} + Kx - \frac{\epsilon_0 A_{DC}}{2(g-x)^2} \times V_{DC}^2 = \frac{\epsilon_0 A_{AC}^2}{2(g-x)^2} \times \frac{V_{AC}^2(t)}{2}$$

$$U(x) = \frac{1}{2}Kx^2 - \frac{\epsilon_0 A_{DC}}{g-x} \times V_{DC}^2 - U_0$$



- Energy depend limit cycle trajectories can be visualized in phase space (speed vs displacement). From which Quality factor and period which set switch dynamics can be integrated





- Trajectories can be calculated from potential energy, but do not yield integratable solutions

$$T(E) = \int \frac{dx}{v(E)}; \quad \frac{Q_{NL}(E)}{2\pi} = \frac{E}{\int b \frac{dx}{dt} \cdot \frac{dx}{dt} dt} = \frac{E/b}{A_{LC}(E)}$$

Analytical Solution

- Beta parameter, β, yields tractable integrals for $T(E)$ and $Q_{NL}(E)$

For a given switch energy, E, we can solve for β :

$$v(x) = \begin{cases} v_{max}\left(1 - \left(\frac{x}{\Delta x_{LIN}}\right)^2\right)^{\frac{1}{2}}, & x \geq 0 \\ v_{max}\left(1 - \left(\frac{x}{\Delta x_{NL}}\right)^{2+\beta}\right)^{\frac{1}{2}}, & x < 0 \end{cases}$$

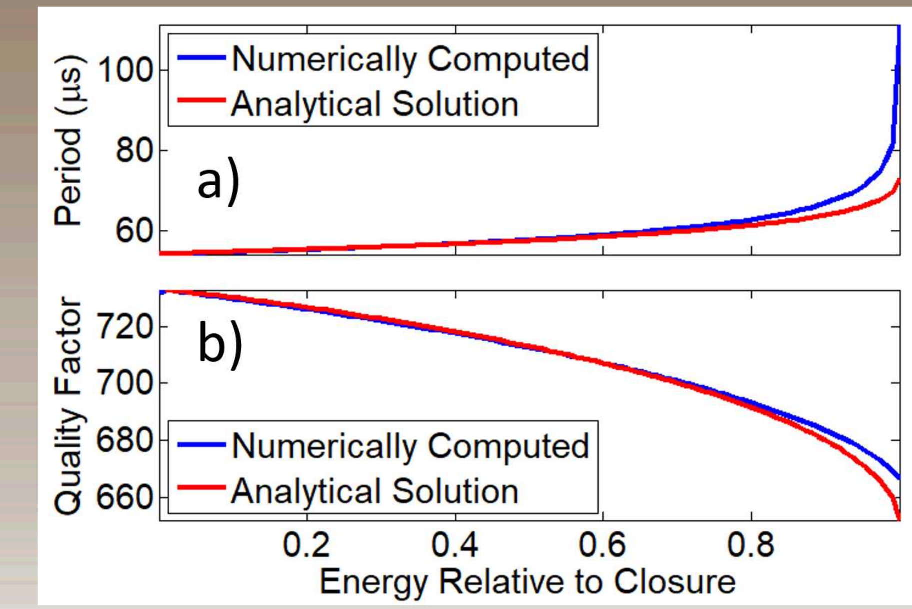
- For a given switch energy we solve for β as:

1. Find v_{max} by using : $E = mv_{max}^2/2$
2. Solve for the stable point, x_S, by solving the cubic expression: $\frac{dU(x_S)}{dx} = 0$.
3. Solve for Δx_{LIN}, Δx_{NL}, by solving the cubic: $U(x) = E$.
4. Solve for the point (x_{mid}, v_{mid}) by solving the relation: $U(x_{mid}) = E/4$.

Analytical Solution

- Following outlined steps and solving only cubic polynomials yields the following solutions in terms of the Gamma Function, $\Gamma(z)$:

$$\beta(E) = \frac{\log\left(1 - \left(\frac{v_{mid}}{v_{max}}\right)^2\right)}{\log\left(\left(\frac{x_{mid} - x_S}{\Delta x_{NL} - x_S}\right)\right)} - 2$$



$$\frac{Q(E)}{2\pi} = \frac{\frac{1}{2} m\, v_{max}{}^2/b}{\Delta x_{NL} v_{max} \frac{\sqrt{\pi}\Gamma\left(1+\frac{1}{2+\beta}\right)}{2\Gamma\left(\frac{3}{2}+\frac{1}{2+\beta}\right)} + \frac{\pi}{2}\Delta x_{Lin} v_{max}}$$

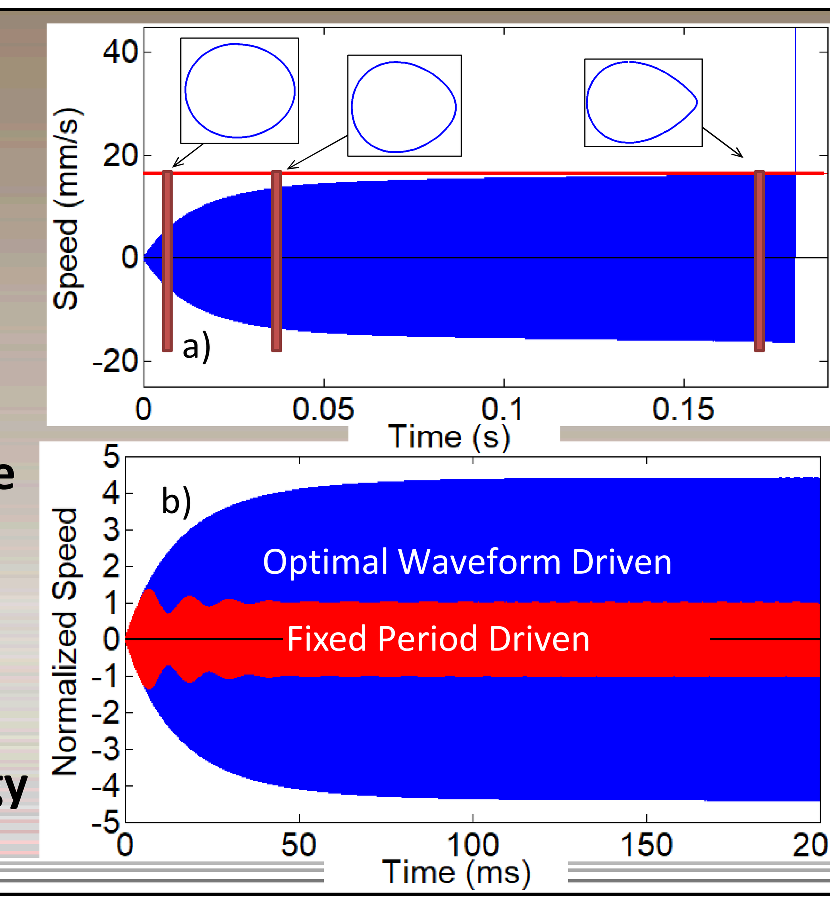
$$T(E) = \frac{\pi \Delta x_{Lin}}{v_{max}} + \frac{\Delta x_{NL}}{v_{max}} \frac{2\sqrt{\pi}\Gamma\left(1+\frac{1}{2+\beta}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2+\beta}\right)}$$

Optimal Waveform Drive

- Optimal waveform transfers power when switch is moving forward

$$V_{AC}(t) = V_0 \times \{v(E(t)) > 0\}$$

- Since the energy transfer per cycle is small the switch will sample all possible waveforms.
- We simulated optimal vs fixed period drive
- Optimal waveform achieves greater energy transfer since it tracks the energy dependent switch period.



Transient dynamics

- Switch energy growth is mediated by momentum transfer and energy dissipation:

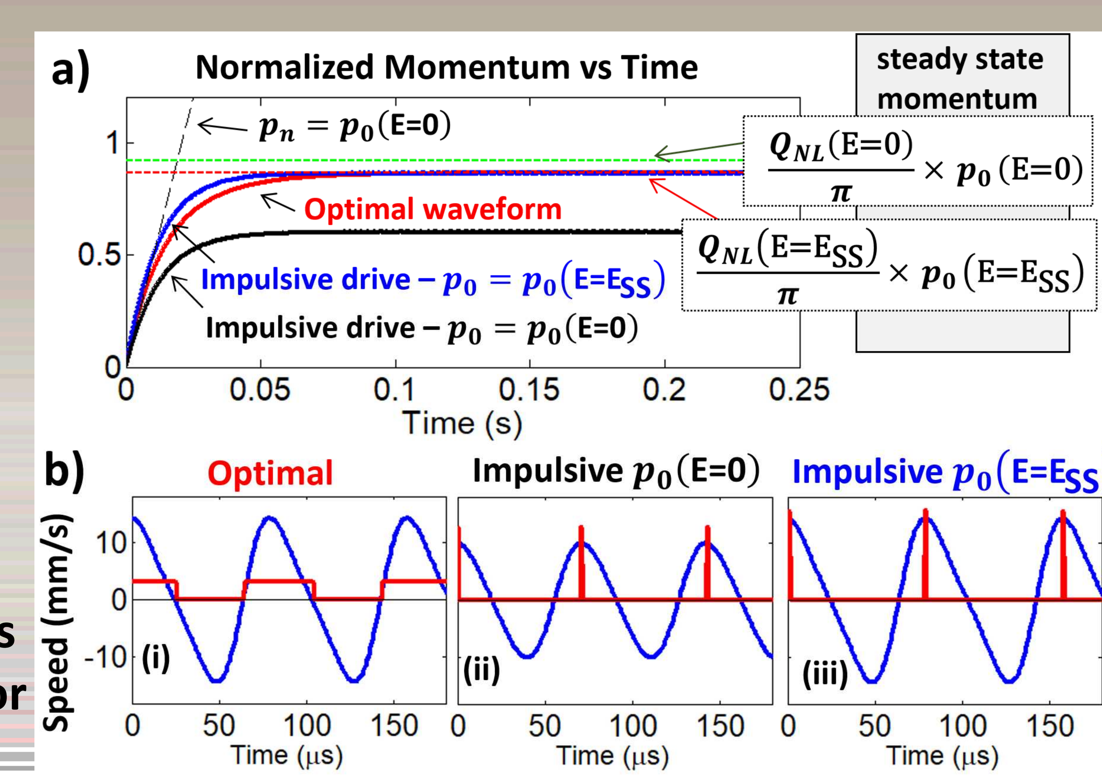
Momentum transfer per cycle

$$E_{n+1} - E_n = p_0(E_n)\sqrt{2m^{-1}}\sqrt{E_n} - \frac{E_n}{Q_{NL}(E_n)/2\pi}$$

$$E_{SS} = \frac{1}{2m}\left(\frac{Q_{NL}}{\pi} \times p_0\right)^2$$

Steady state energy

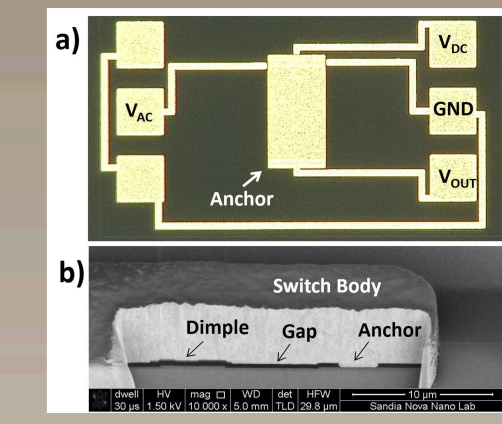
- Interplay of nonlinear momentum transfer and nonlinear Quality factor impact energy transfer
  - Optimal wave form
  - Impulsive drive force with: $p_0(E_n = 0)$
  - Impulsive drive force with: $p_0(E_n = E_{SS})$
- Increase in momentum transfer per cycle overcomes the decrease in quality factor



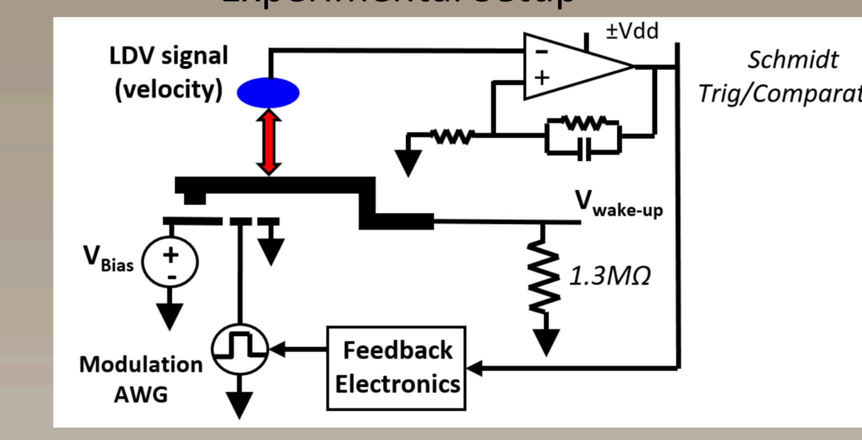
Experimental Validation

- Fabricated switches were measured with a laser Doppler velocimeter (LVD). The switch is driven initially with a constant period drive waveform to build up to a switch velocity detectable by the LDV system and subsequently a comparator-based feedback loop is implemented so that the waveform tracks the period

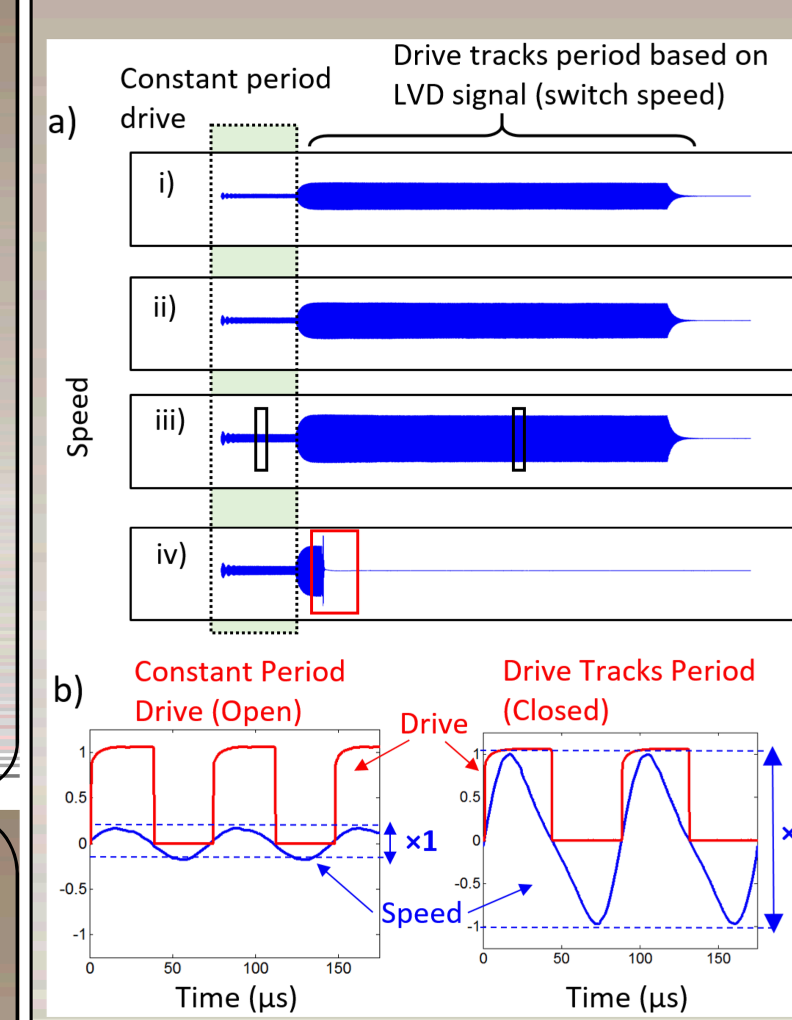
Switch Fabrication

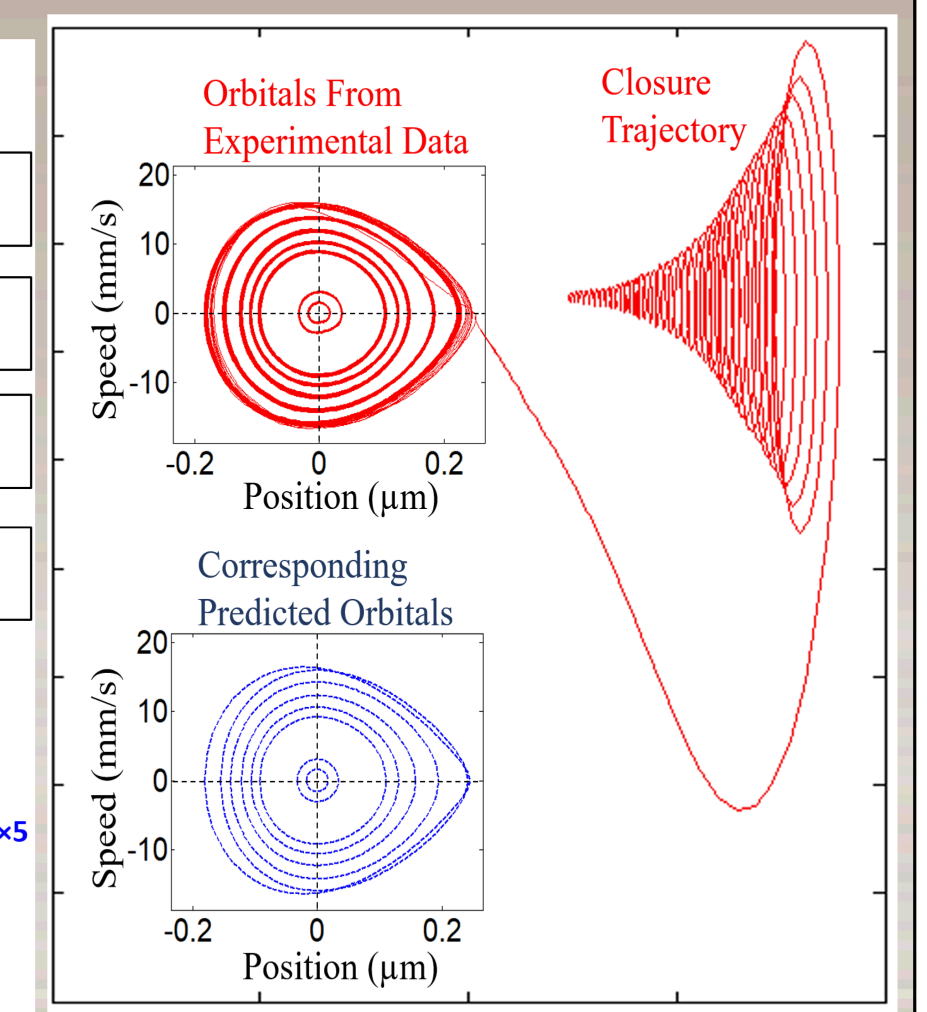


Experimental Setup

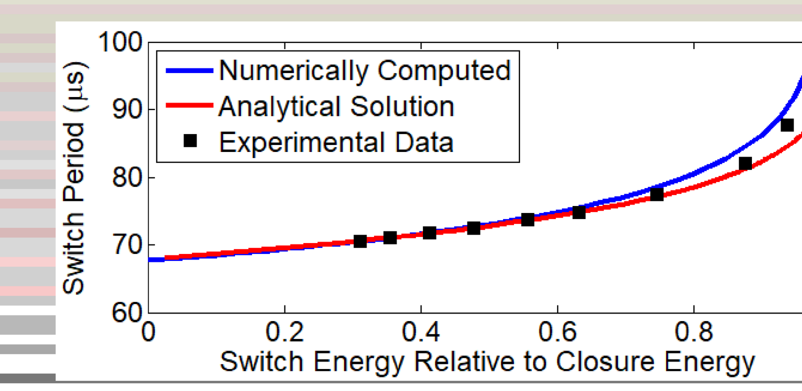


- We demonstrated that the optimal waveform is more effective at delivering energy to the switch resulting in nearly 5× times more speed





- Detailed analysis of experimental trajectories in phase space closely match the predicated values as did switch period vs switch energy



Conclusions and Future Work

- We presented a hybrid dynamic static solution where the switch motion is strongly influenced by electrostatic forces.
  - Theoretical frame work
  - Analytical solutions validated both numerically and experimentally
  - This work can potentially be exploited for low power signal processing applications requiring a wakeup or switch closure

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