Bayesian machine learning of frequency-bin CNOT

Hsuan-Hao Lu[1], Joseph M. Lukens[2], Brian P. Williams[2], Poolad Imany[1], Nicholas A. Peters[2], Andrew M. Weiner[1], and Pavel Lougovski[2,*]

[1] *School of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana 47907, USA*
[2] *Quantum Information Science Group, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
**lougovskip@ornl.gov*

Abstract: We analyze the first experimental two-photon frequency-bin gate: a coincidence-basis CNOT. A novel characterization approach based on Bayesian machine learning is developed to estimate the gate performance with measurements in the logical basis alone.

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Photonic quantum information processing (QIP) encoded in narrow frequency bins has shown strong potential in recent years, thanks to the advance of on-chip quantum frequency comb sources, as well as single-photon quantum gates based on both nonlinear [1, 2] and electro-optical approaches [3, 4]. However, a two-photon *entangling* gate, an essential building block for universal QIP, has only just been realized on frequency-bin qubits [5]. In this work, we describe this first two-photon frequency-bin gate, a coincidence-basis controlled-NOT (CNOT). Expanding on the preliminary results in [5], we introduce a tailored Bayesian machine learning approach to validate high-fidelity gate performance with measurements in the two-photon computational basis alone [6].

The implementation of our CNOT gate follows the architecture of spectral linear-optical quantum computation (spectral LOQC)—a universal QIP scheme utilizing a sequence of electro-optic phase modulators (EOMs) and line-by-line pulse shapers (PSs) to perform coherent operations on frequency-bin qubits [7]. The experimental configuration for spectral LOQC, termed the "quantum frequency processor" (QFP), has previously been utilized to demonstrate high-fidelity, parallel *single-photon* operations in a low-noise fashion [3, 4], but not yet a *two-photon* entangling gate. Theoretically, unheralded control gates can be realized with no ancillas and $\mathscr{P} = 1/9$ [8, 9] by verifying the presence of photons in the control and target output modes (the so-called coincidence basis). Following the optimization approach in [3, 7], we numerically have found sets of phase patterns for our previously developed 2EOM/1PS QFP circuit, capable of realizing a coincidence-basis CNOT with $\mathscr{F} = 0.9999$ and $\mathscr{P} = 0.0445$.

Figure 1(a) provides a schematic of the setup. The CNOT operation is programmed onto the central QFP, in which both EOMs are driven by a 25 GHz sinusoidal voltage, while the PS applies the numerically obtained phase patterns onto each spectral bin. Since the complexity of a two-qubit gate using an EOM/PS sequence highly depends on how computational modes are assigned to specific frequency bins, this particular mode placement scheme shown in Fig. 1(b) follows our intuition: control mode 0 (C_0) should be spectrally isolated, while control mode 1 (C_1) is packed with both target modes (T_0, T_1). The coherent-state-based characterization approach, which utilizes an electro-optic frequency comb to predict the performance of a linear-optical gate [3, 10], further illustrates our CNOT gate design. The experimentally obtained mode transformation matrix V [Fig. 1(c)] shows the isolated C_0 has negligible overlap with other bins. On the other hand, C_1 is coupled to both target bins with equal strength. From this matrix V, we can compute the equivalent two-photon state transformation matrix W [7], plotted for the coincidence basis in Fig. 1(d). We obtain the *inferred* fidelity $\mathscr{F}_{\text{inf}} = 0.9947 \pm 0.0008$ and success probability $\mathscr{P}_{\text{inf}} = 0.045 \pm 0.001$ with respect to the ideal CNOT. This indirect coherent-state estimate provides strong initial evidence for proper operation of our gate.

To test our gate with truly quantum states, we prepare a biphoton frequency comb (BFC) by pumping a periodically poled lithium niobate (PPLN) waveguide with a continuous-wave Ti:sapphire laser, followed by filtering the broadband biphotons with a 25 GHz spaced etalon. By translating the pump frequency to four different values [as shown in

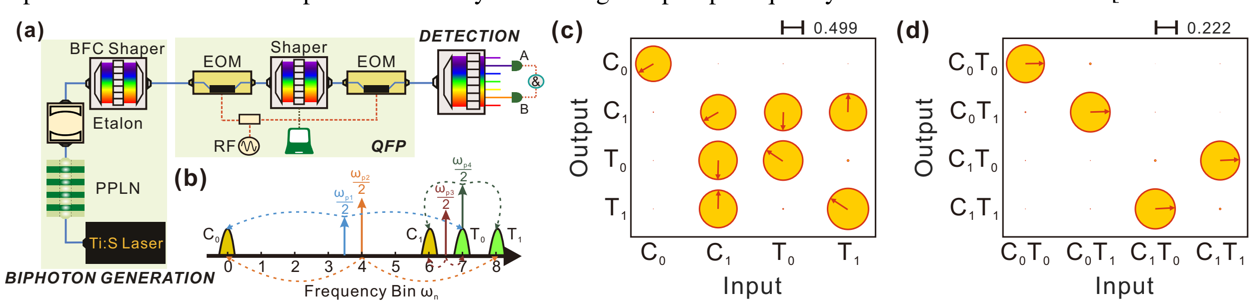

Fig. 1. (a) Experimental setup. (b) Mode placement. (c) Experimentally obtained mode transformation V. (d) Inferred two-photon transformation W. We use phasor notation to represent the complex elements in (c) and (d); the radius represents the amplitude, and the arrow marking out the phase.

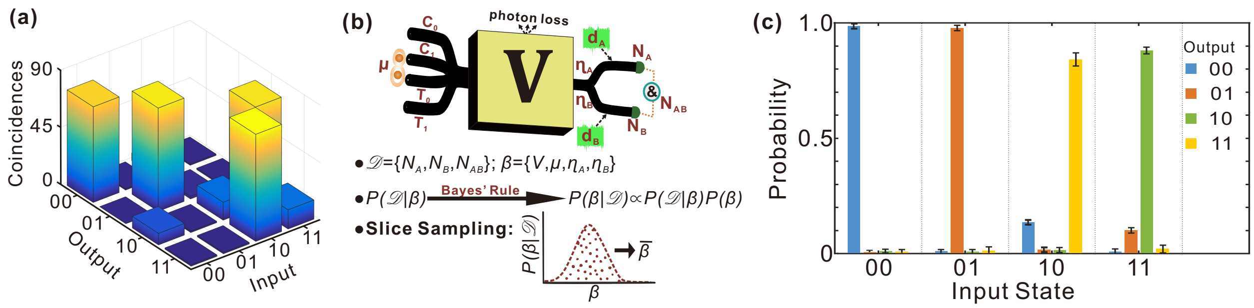

Fig. 2. (a) Measurements of frequency-bin CNOT in the logical basis. (b) Outline of our Bayesian machine learning approach. (c) Output state probabilities retrieved from BME.

Fig. 1(b)] and selecting the desired modes using the BFC shaper, we can prepare all four computational-basis inputs. After the gate operation, the output photons are frequency-demultiplexed, as we send control (target) photon bins to detector A (B) for coincidence measurement. Figure 2(a) plots the coincidences for all input/output mode combinations (600 s integration time; no accidentals subtraction). The results indicate our gate performs largely as designed—the quantum state holds with an input photon in C_0, while a photon in C_1 flips the target qubit.

While previously such truth-table measurements have enabled extraction of only amplitudes in the state transformation matrix, we develop a novel machine learning technique for numerical inference based on Bayesian mean estimation (BME), allowing us to retrieve full complex matrix values with measurements in the logical basis only. In our model [Fig. 2(b)], the unknown parameters of interest β include the mode matrix V, the pair generation probability μ, and the system efficiencies η_A and η_B. Incorporating knowledge from registered single counts on each detector (N_A, N_B) as well as the coincidences (N_{AB}) in M detection frames, we can construct a multinomial likelihood function:

$$P(\mathscr{D}|\beta) = (p_A - p_{AB})^{N_A - N_{AB}} (p_B - p_{AB})^{N_B - N_{AB}} p_{AB}^{N_{AB}} (1 - p_A - p_B + p_{AB})^{M - N_A - N_B + N_{AB}}, \tag{1}$$

where probabilities for a click on either detector (p_A, p_B) and a coincidence (p_{AB}) are functions of β and the dark count probabilities (d_A, d_B) [6]. We assume a uniform prior distribution $P(\beta)$ and obtain the posterior probability distribution $P(\beta|\mathscr{D})$ from Bayes' rule. Employing slice sampling on this posterior for machine learning of the model parameters, we retrieve 4096 samples of all 28 unknowns in β, converging to a Bayesian fidelity estimate $\mathscr{F}_{\mathrm{BME}} = 0.91 \pm 0.01$. Figure 2(c) depicts how this $\mathscr{F}_{\mathrm{BME}}$ translates into the output state probabilities for logical-basis inputs.

In conclusion, we have analyzed the first entangling gate for frequency-bin qubits, and confirmed its high-fidelity operation with measurements using coherent states and photon pairs in the computational basis. Our newly developed Bayesian characterization approach showcases the potential of machine learning in extracting parameters and quantifying uncertainty in quantum systems.

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