A two-qudit operation in a 256-dimensional Hilbert space

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Abstract: By encoding two 16-dimensional qudits in the time and frequency degrees of freedom of a heralded single photon, we realize a deterministic photonic two-qudit SUM gate operating on a 256-dimensional Hilbert space.

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Quantum computations outperform their classical counterparts dramatically in some crucial algorithms [1]. Photonic quantum computing however, has the drawback of nondeterministic photon-photon operations with linear optics, making this platform challenging for large-scale operations. This problem has been addressed by encoding qubits in different degrees of freedom (DoFs) in a single photon [2,3], allowing deterministic qubit-qubit operations. These approaches, however, have been limited to relatively small total Hilbert spaces (8-dimensional in [3]). Here, we take advantage of high-dimensional DoFs in a single photon—namely time and frequency—in an independent fashion to encode two *qudits* in a photon, the Hilbert spaces of which can be extended to higher dimensions by using higher-dimensional time and frequency qudits. Recently, we have encoded two 3-dimensional qudits in a single photon using this method and demonstrated deterministic single- and two-qudit operations in this 9-dimensional Hilbert space [4].

Here, we significantly expand our previous approach and encode 16-dimensional qudits in both time and frequency bins, for an equivalent of 4 qubits in each DoF. We then demonstrate a 16×16 two-qudit SUM gate, where the qudit encoded in the frequency DoF is added modulo 16 to the qudit encoded in the time DoF. In this realization, an equivalent of 8 qubits are encoded in a single photon, showing the value of high-dimensionality in each DoF. The SUM gate operates on this 256-dimensional Hilbert space (2^8), showing the potential of our approach for encoding more information in a single photon. Integrating these deterministic operations within a photon with deterministic approaches to demonstrate photon-photon gates [5] would pave the way for using this photonic platform for near-term quantum computing purposes.

The experimental setup is depicted in Fig. 1. We use a continuous-wave laser with ~773 nm wavelength to pump a periodically poled lithium niobate (PPLN) crystal, generating a broadband spectrum of time-frequency entangled photons [6], with a bandwidth of ~5 THz centered around 1546 nm. After using filters to attenuate the pump frequency, an intensity modulator driven by an arbitrary waveform generator is used to carve out sixteen time bins with a full width at half maximum of ~200 ps and 1.2 ns spacing between them, to generate the time-bin qudits. Then, a pulse shaper is used to carve out the frequency of these entangled photons to generate sixteen 22 GHz wide frequency bins on both the signal and idler side of the spectrum, each spaced 75 GHz from each other. This pulse shaper is also used as a wavelength router to separate the signal and idler photons. The idler photons are then sent directly to a superconducting nanowire single-photon detector (SNSPD), to herald single photons in the signal arm. These heralded signal photons carry two 16-dimensional qudits in their time and frequency DoFs.

To operate the SUM gate on these heralded single photons, a dispersion module is used to delay adjacent

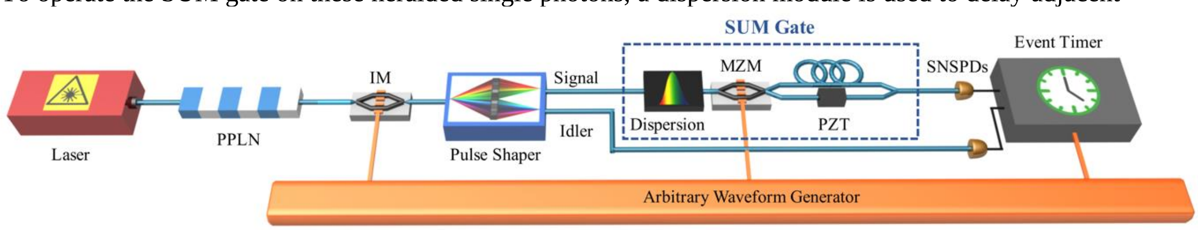

Fig. 1. The experimental setup. PPLN: periodically poled lithium niobate, IM: intensity modulator, PZT: piezo-electric phase shifter, MZM: Mach-Zehnder modulator, SNSPD: superconducting nanowire single-photon detector.

frequency bins by one time bin with respect to each other. We use a chirped fiber Bragg grating with a dispersion of −2 ns/nm, which puts a 1.2 ns delay on frequency bins separated by 75 GHz and adds the value of the frequency qudit to the time qudit. This leaves us with 31 time bins, half of which are now outside of our computational space. To demonstrate the modulo 16 additions, a 1×2 switch (a Mach-Zehnder modulator) is used to separate time bins that are now placed later than the 16^{th} time bin (which is the last bin in our computational space) and delay the computational space by 16 time bins with respect to these latter bins to bring them back into the computational space. In principle, recombination of these time bins can be done in a lossless way using switches and wavelength division multiplexers, but in this proof-of-principle experiment, we use a 50/50 beam combiner which probabilistically returns the spatially separated bins back into one spatial mode.

To verify the operation, we send in different two-qudit inputs, which can be in one of 256 different states, and measure the output after the gate. While this yields a total of 256×256 (2^{16}) computational input/output combinations to test, we have no active frequency-shifting elements in our setup, so we make the reasonable assumption that the frequency qudit remains unchanged through the operation. This is also enforced by the high extinction ratio of the pulse shaper (~40 dB), which blocks the unwanted frequency bins, allowing us to focus on results in the sixteen 16×16 transfer matrices measured in Fig. 2 (a subset with a total of 2^{12} input/output combinations). In each matrix, 16 different inputs with the same frequency and different time bins are sent into the SUM gate and the output time bins are measured without accidental subtraction. The average computational space fidelity of the whole process, with the assumption that frequencies do not leak into each other, can be calculated—using Bayesian estimation [4]—as $\bar{\mathcal{F}}_C = 0.9589 \pm 0.0005$, which shows the high performance of our operation. We note that the SUM gate operating on a 256-dimensional Hilbert space is a massive operation, and to perform this operation using two-qubit gates requires $15^2 = 225$ generalized controlled-NOT gates [7].

Two-qudit operations are necessary elements for qudit-based quantum operations, where increased amount of quantum information can be encoded and processed in a single photon [8]. Here, we have demonstrated a two-qudit SUM gate with two 16-dimensional qudits encoded in the time and frequency degrees of freedom of a single photon. These demonstrations extend the amount of quantum information in single photons dramatically, showing the potential of our scheme to contribute to near-term quantum computing protocols.

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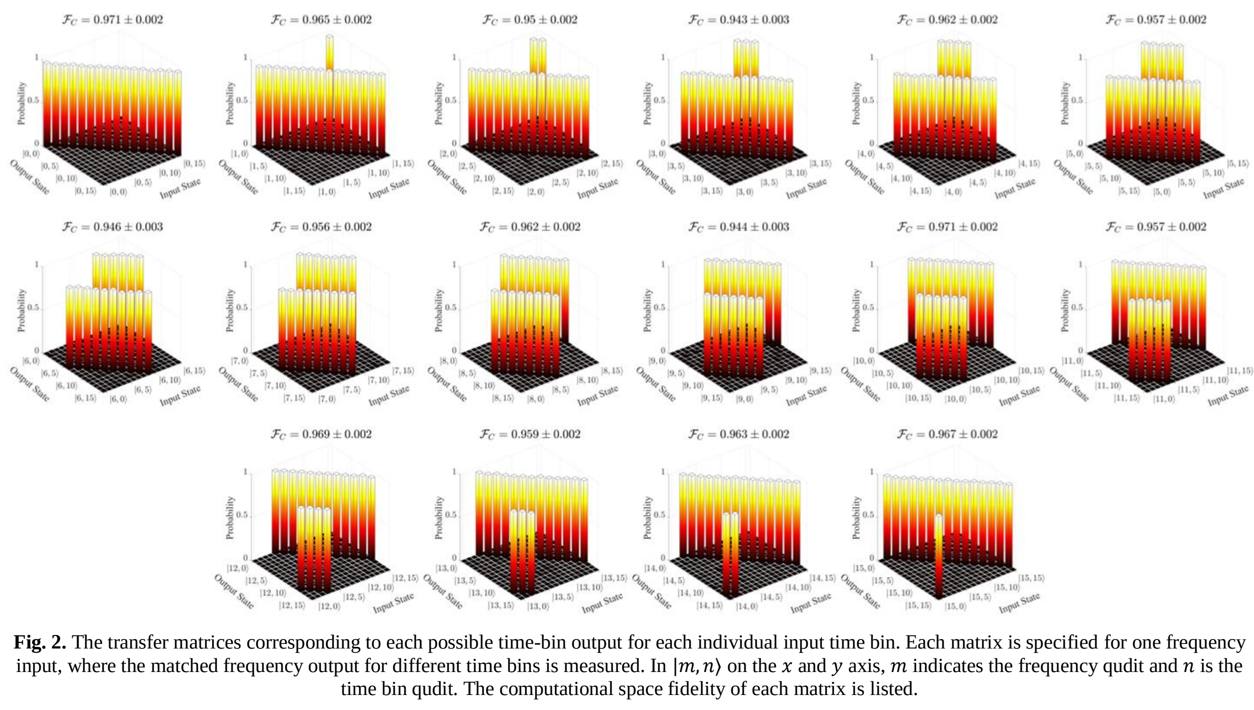

Fig. 2. The transfer matrices corresponding to each possible time-bin output for each individual input time bin. Each matrix is specified for one frequency input, where the matched frequency output for different time bins is measured. In $|m, n\rangle$ on the x and y axis, m indicates the frequency qudit and n is the time bin qudit. The computational space fidelity of each matrix is listed.

References

[1] M. Nielsen and I. L. Chuang. "Quantum Computation and Quantum Information." (Cambridge University Press, 2010).
[2] M. Fiorentino, and F. N. C. Wong. "Deterministic controlled-NOT gate for single-photon two-qubit quantum logic." *Physical Review Letters* **93**, 070502 (2004).
[3] K. H. Kagalwala et al. "Single-photon three-qubit quantum logic using spatial light modulators." *Nature Communications* **8**, 739 (2017).
[4] P. Imany et al. "Deterministic optical quantum logic with multiple high-dimensional degrees of freedom in a single photon." arXiv preprint arXiv:1805.04410 (2018).
[5] B. Hacker et al. "A photon-photon quantum gate based on a single atom in an optical resonator." *Nature* **536**, 193–196 (2016).
[6] P. Imany et al. "Characterization of coherent quantum frequency combs using electro-optic phase modulation." *Physical Review A* **97**, 013813 (2018).
[7] Y.-M. Di, and H.-R. Wei. "Synthesis of multivalued quantum logic circuits by elementary gates." *Physical Review A* **87**, 012325 (2013).
[8] M. Erhard et al. "Twisted photons: new quantum perspectives in high dimensions." *Light: Science & Applications* **7**, 17146 (2018).