SAND2018-5076C

Code Verification for a Solid Mechanics Code

-- Triumphs and Trials

Jim Cox

Sandia National Laboratories

ASME V&V Symposium
May 2018

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Acknowledgements

I had helpful discussions with David Lo, Brian Carnes, George Orient, and Kevin Dowding on various aspects of this work.

Preview

- Introduction
- Manufactured Solutions for Hyperelasticity
- Manufactured Solution for Hypoelasticity – work in progress
- Solution verification attempts to get closer to the problem space

The Trials

- Conclusions

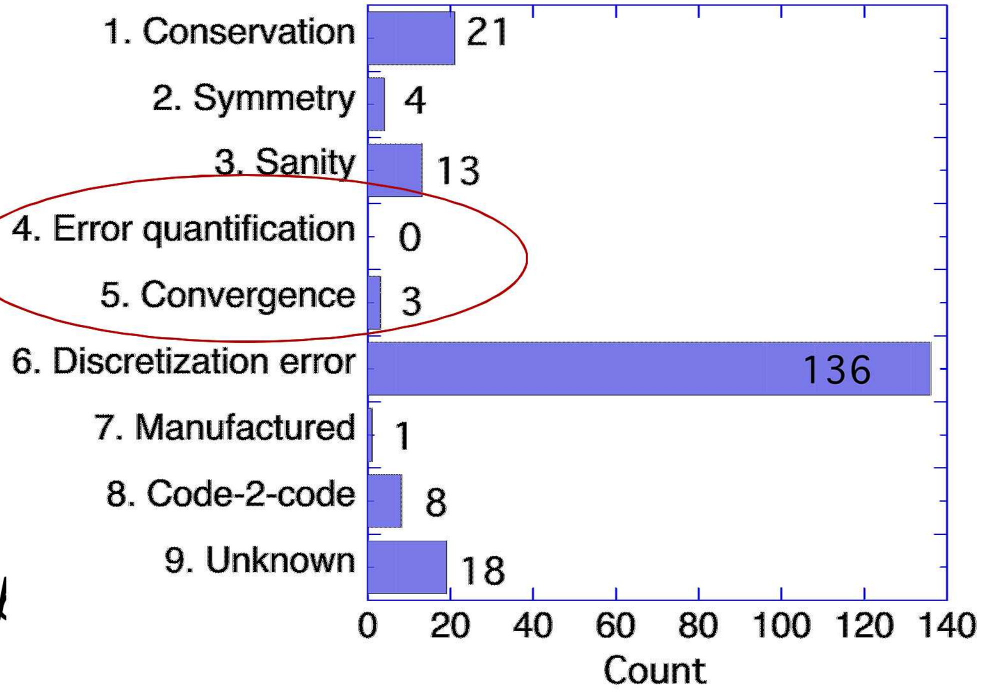
Introduction

- Motivation –
  - Sierra/Solid Mechanics (SM) is used in many high consequence applications.
  - Decisionmakers have increased their reliance upon simulation results.
  - Code verification is foundational to establishing the credibility of simulation results.

Introduction

- Background – Sierra/SM verification
  - 186 verification tests in 2012
  - Heavily weighted toward *discretization error* tests –1 analytical solution vs. 1 numerical solution
    - Natural starting place
  - No active *convergence* or *error quantification* tests
  - *Error quantification* test *aka order-of-accuracy test*
- *Sierra/SM solves nonlinear equations – no linear sub-capal*

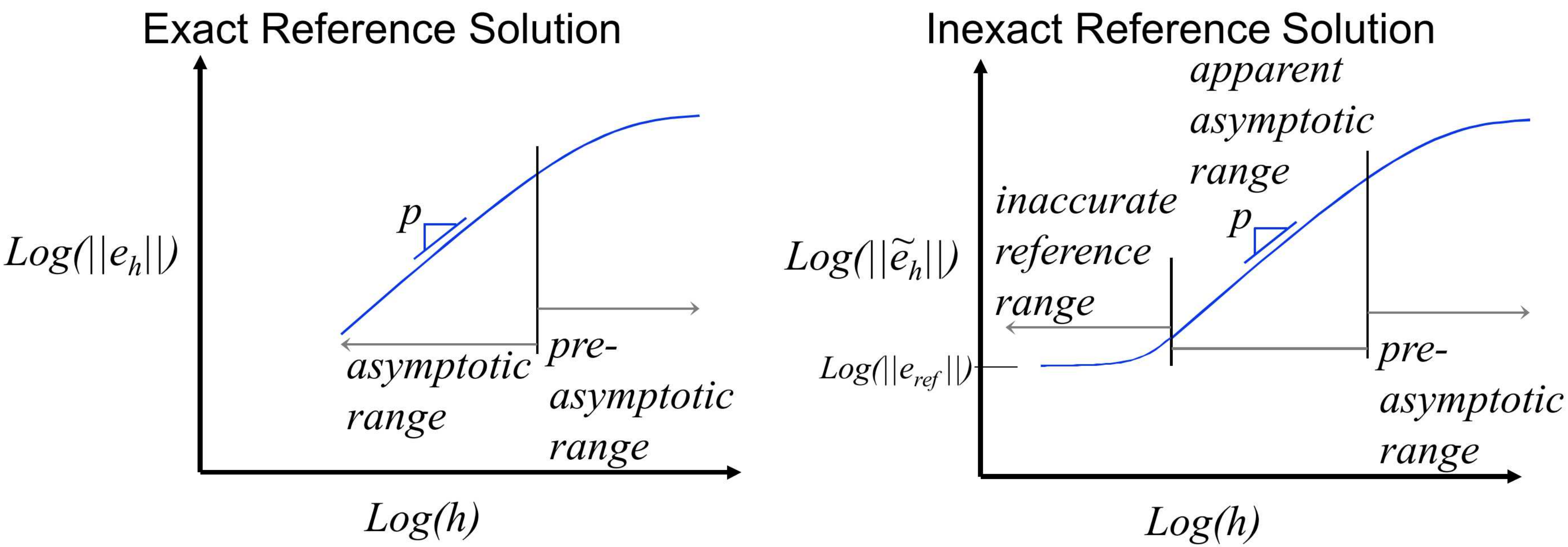
Verification Categories



Sandia National Laboratories

Introduction

- Classical "LE" vs. Manufactured -- Convergence Plots

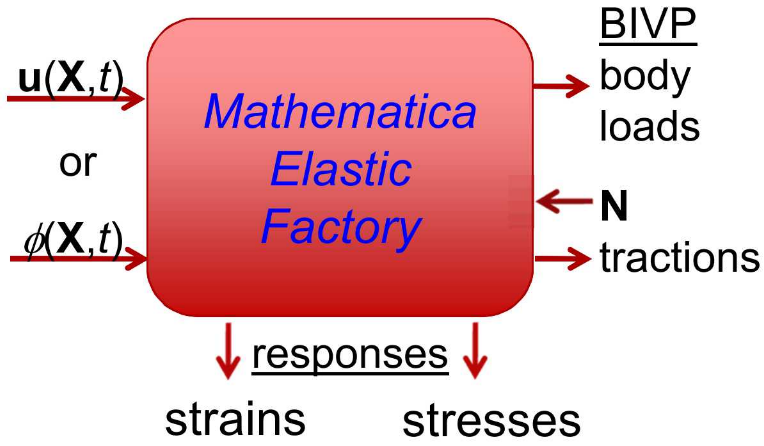


In the asymptotic range $e_h \approx ch^p$

In both cases we assume the code is working correctly, and that attributes of the plots are not due to errors in the code, only discretization errors.

Manufactured Solutions

- Actually "manufacture the problem"
  - Start with displacements (**u**)
    or motion/configuration mapping (ϕ)
  - Two sources of input considered here:
    Polynomial description of **u**(**X**,*t*)
    Classical solutions for **u**(**X**,*t*)



- Manufacturing Scope
  - Currently limited to hyper-elasticity and hypo-elasticity
  - Results shown here are for the St. Venant-Kirchhoff material – first 3 problems

2nd implementation in stk_mms for hyperelastic (Brian Carnes)

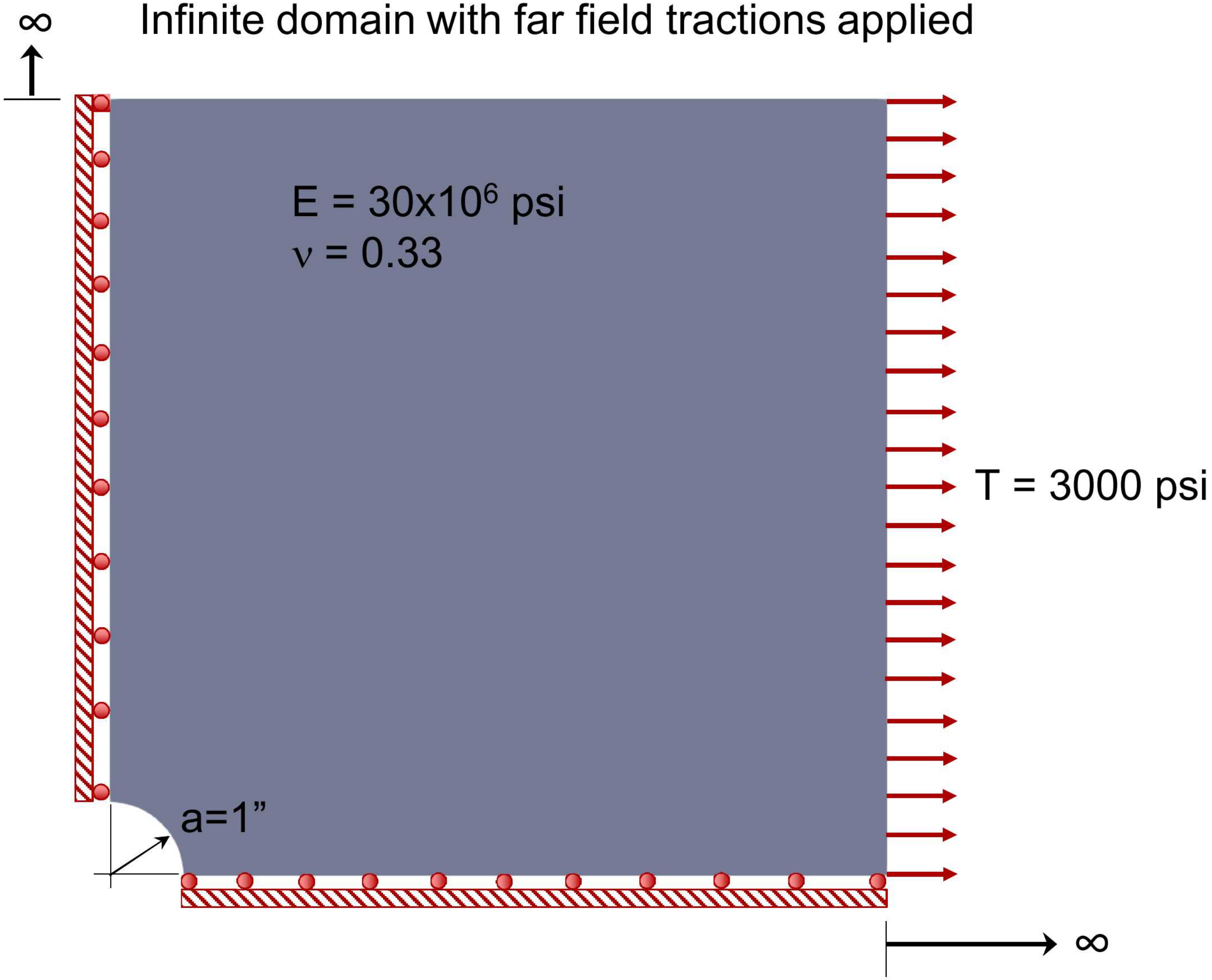
Manufactured Solutions for Hyperelasticity

- Stress concentration problem
- Quadratic displacement fields

Hole in a Plate

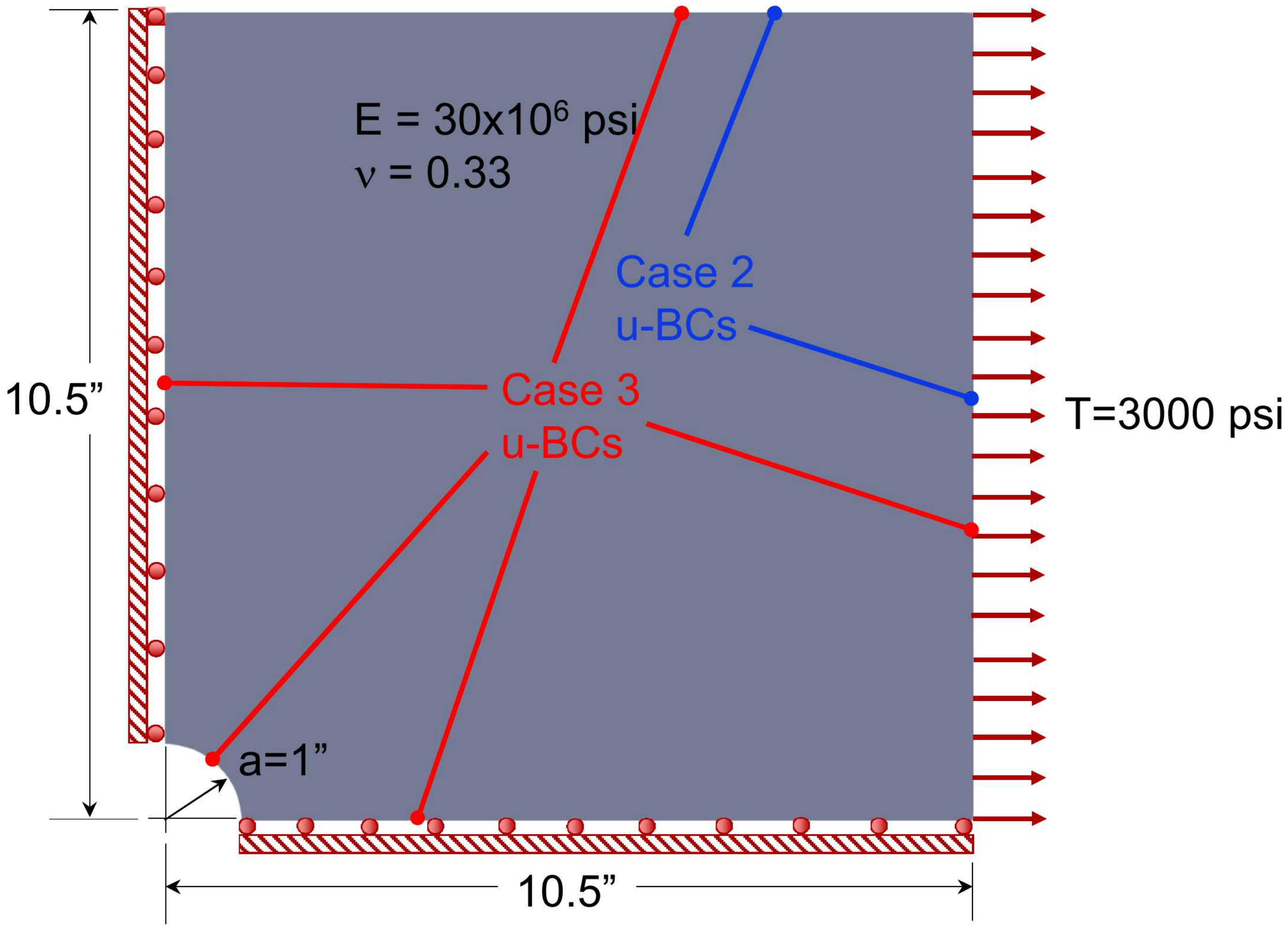
LE Stress Concentration Problem

Infinite domain with far field tractions applied



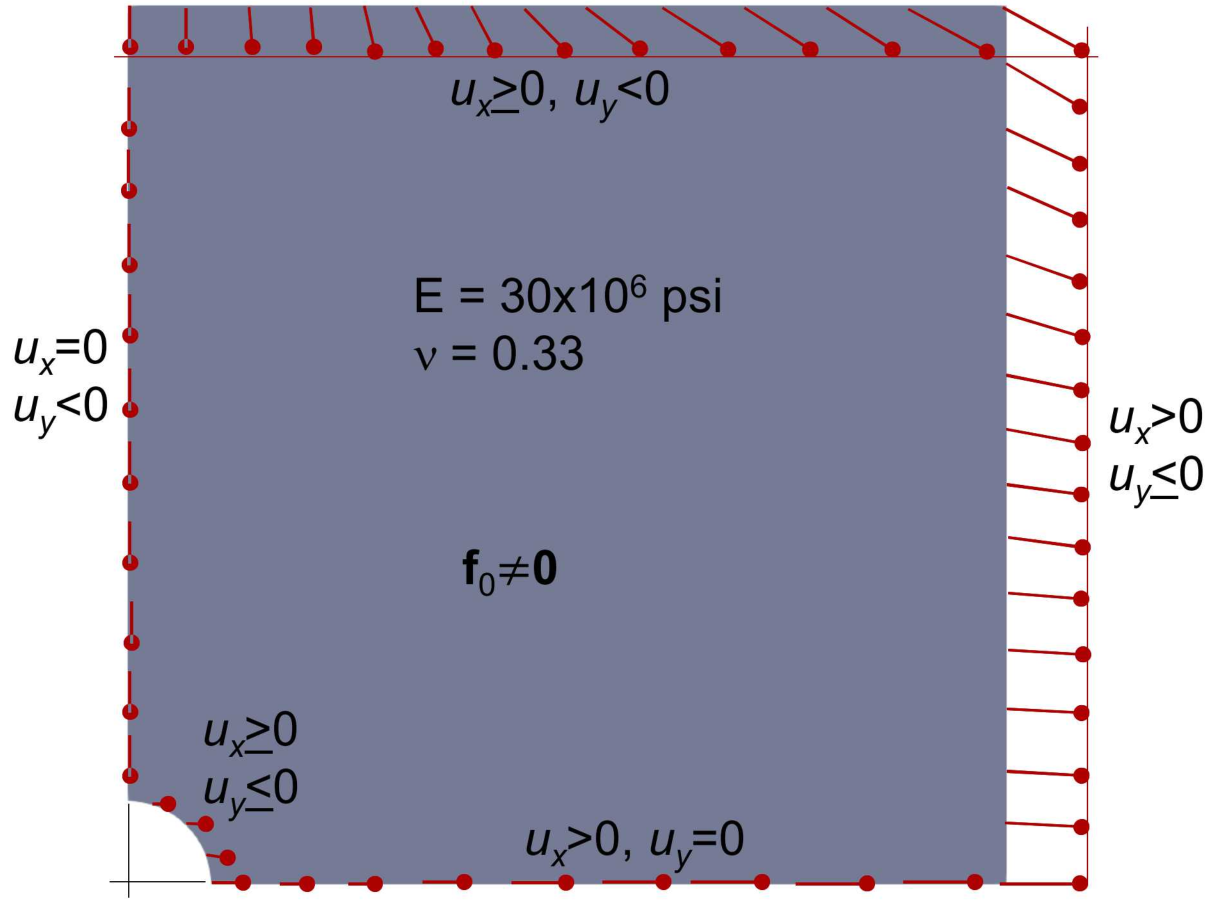
Classical Numerical Solution (Case 1)

Far field tractions applied to top and right side surfaces, but with a finite domain.

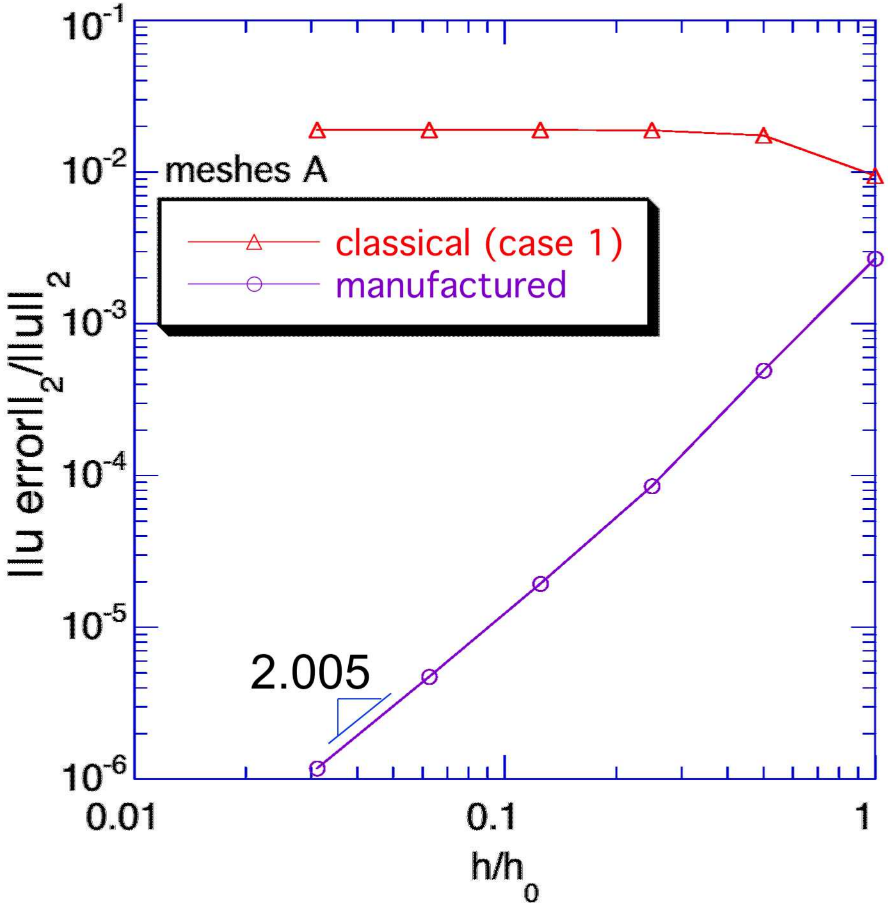


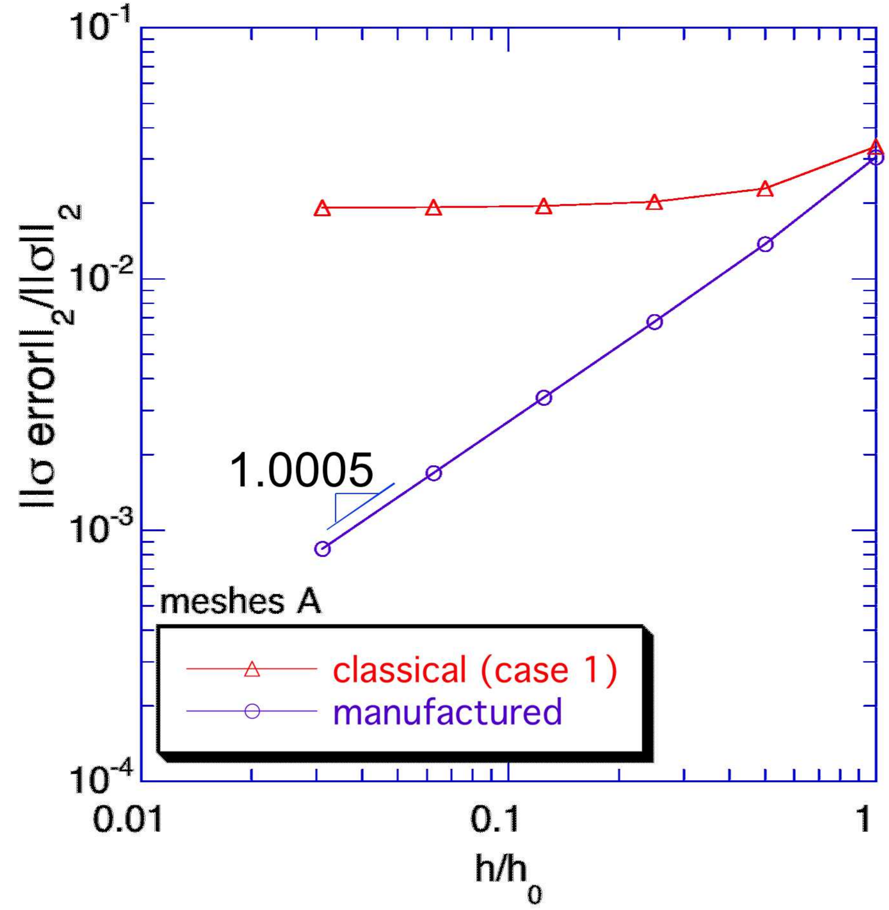
Manufactured Solution

Displacement Version – All boundary displacements defined by the classical solution

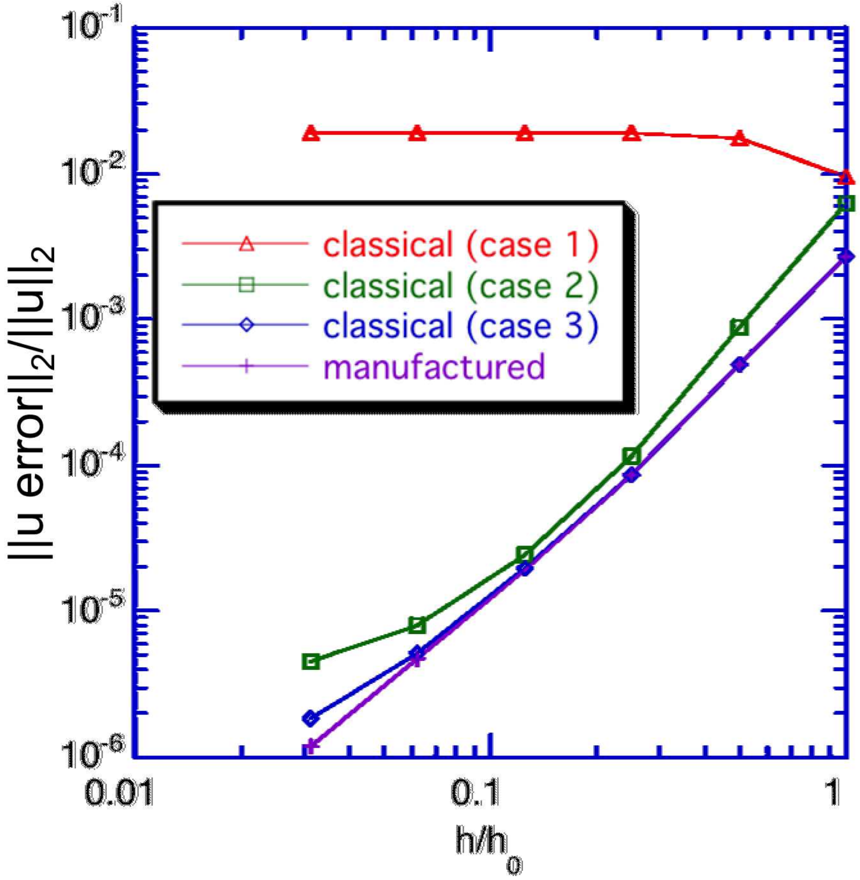


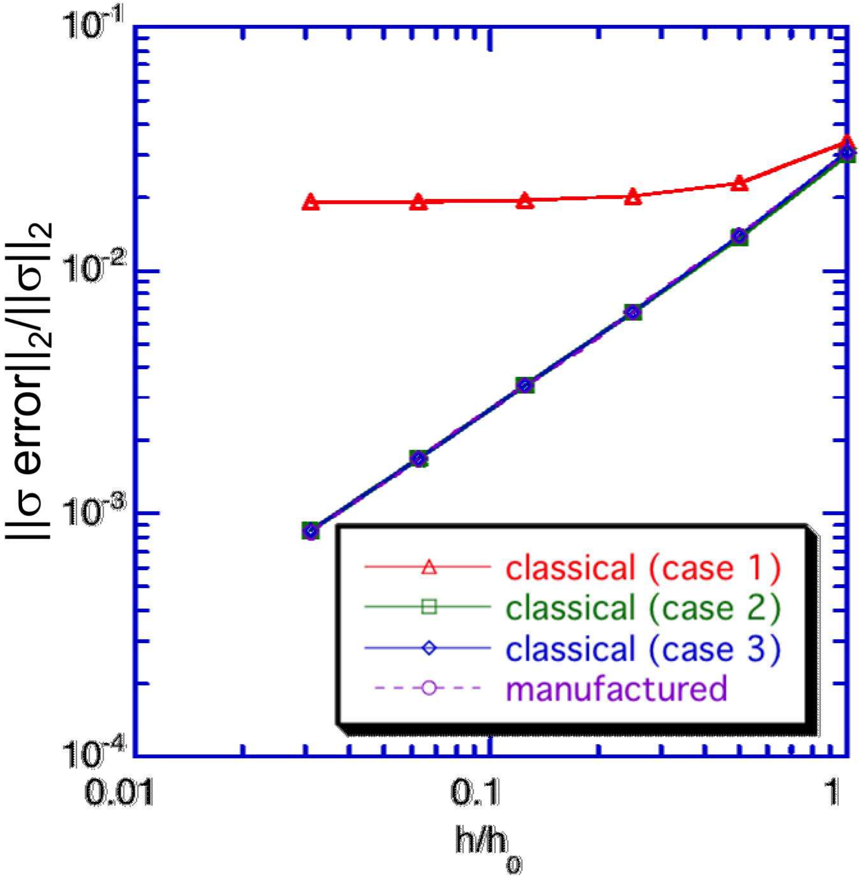
Initial Classical vs. Manufactured





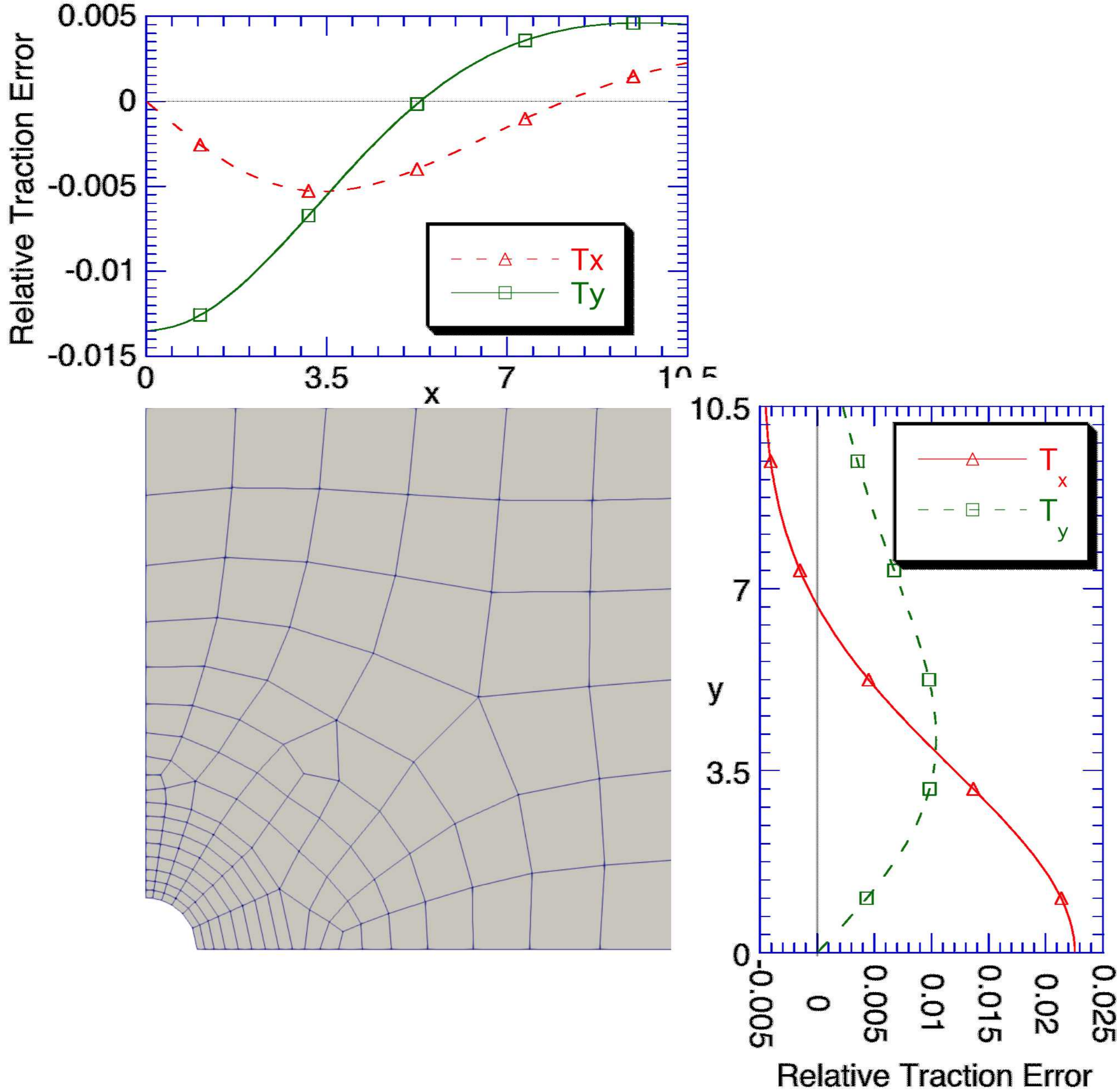
All Classical vs. Manufactured





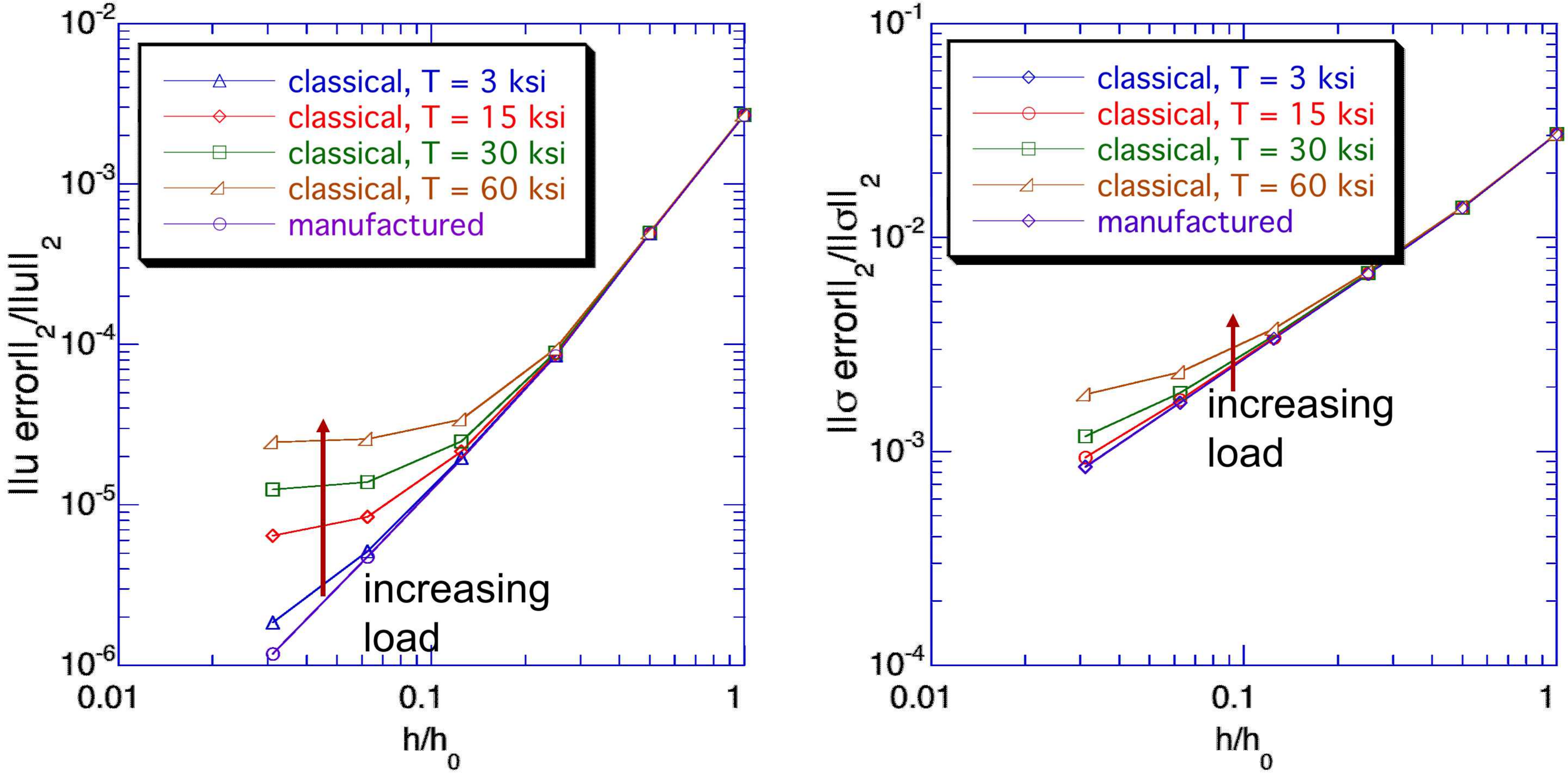
Sandia National Laboratories

Traction Errors



Classical vs. Manufactured

At 4 Load Levels, classical (case 3)



Manufactured Solutions for Hyperelasticity

- Stress concentration problem
- Quadratic displacement fields (plus a twist)

Quadratic Displacement Field plus a Twist

More complete Test

- **u** 2nd order (patch tests + 1)
- 30 constants -> 9 (test poly_2h)
- Displacements and motion maps

$u_x = u_y = u_z = taR^2$ *(prior to including the rotation)*

$$\phi_{deform}[\mathbf{X}, t] = \left\{t\left(aX_1^2 + aX_2^2 + aX_3^2\right) + X_1, t\left(aX_1^2 + aX_2^2 + aX_3^2\right) + X_2, t\left(aX_1^2 + aX_2^2 + aX_3^2\right) + X_3\right\}$$

Add a rotation about a fixed axis (60 degrees)

$$\boldsymbol{x} = \phi(\boldsymbol{X}, t) = \phi_{rotate}\left[\phi_{deform}(\boldsymbol{X}, t), t\right]$$

Example displacement component

$$u_x = \frac{1}{3}\left(3atX_1^2 + 3atX_2^2 + 3atX_3^2 + \sqrt{3}\,(X_3 - X_2)\,s + (2X_1 - X_2 - X_3)\,c - 2X_1 + X_2 + X_3\right)$$

where $c = \cos\left(\frac{\pi t}{3}\right) \quad s = \sin\left(\frac{\pi t}{3}\right)$

Deformation gradient

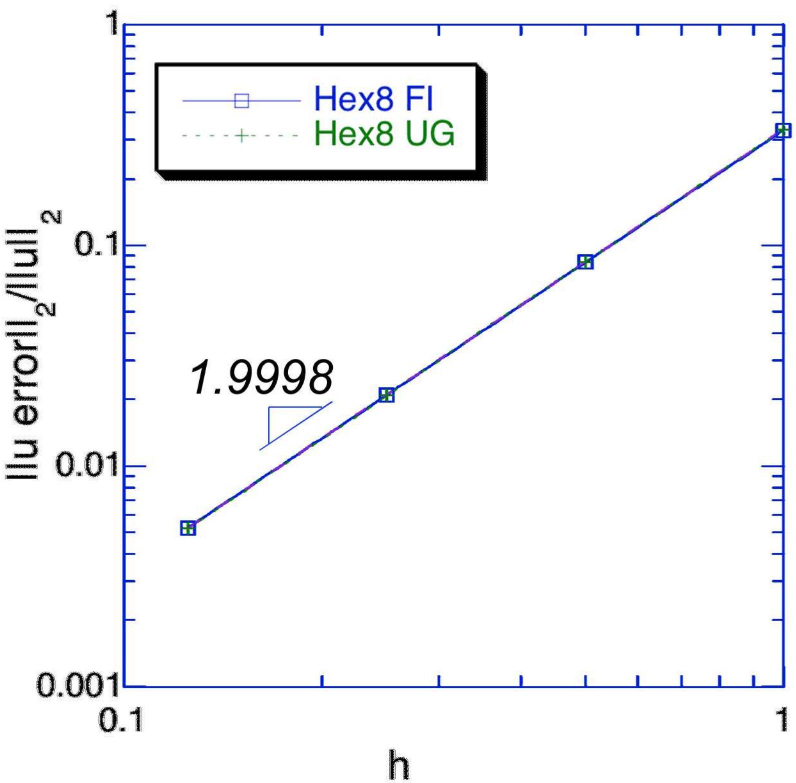
$$\boldsymbol{F} = \begin{bmatrix} \frac{1}{3}(2c + 6atX_1 + 1) & \frac{1}{3}\left(-c - \sqrt{3}s + 6atX_2 + 1\right) & \frac{1}{3}\left(-c + \sqrt{3}s + 6atX_3 + 1\right) \\ \frac{1}{3}\left(-c + \sqrt{3}s + 6atX_1 + 1\right) & \frac{1}{3}(2c + 6atX_2 + 1) & \frac{1}{3}\left(-c - \sqrt{3}s + 6atX_3 + 1\right) \\ \frac{1}{3}\left(-c - \sqrt{3}s + 6atX_1 + 1\right) & \frac{1}{3}\left(-c + \sqrt{3}s + 6atX_2 + 1\right) & \frac{1}{3}(2c + 6atX_3 + 1) \end{bmatrix}$$

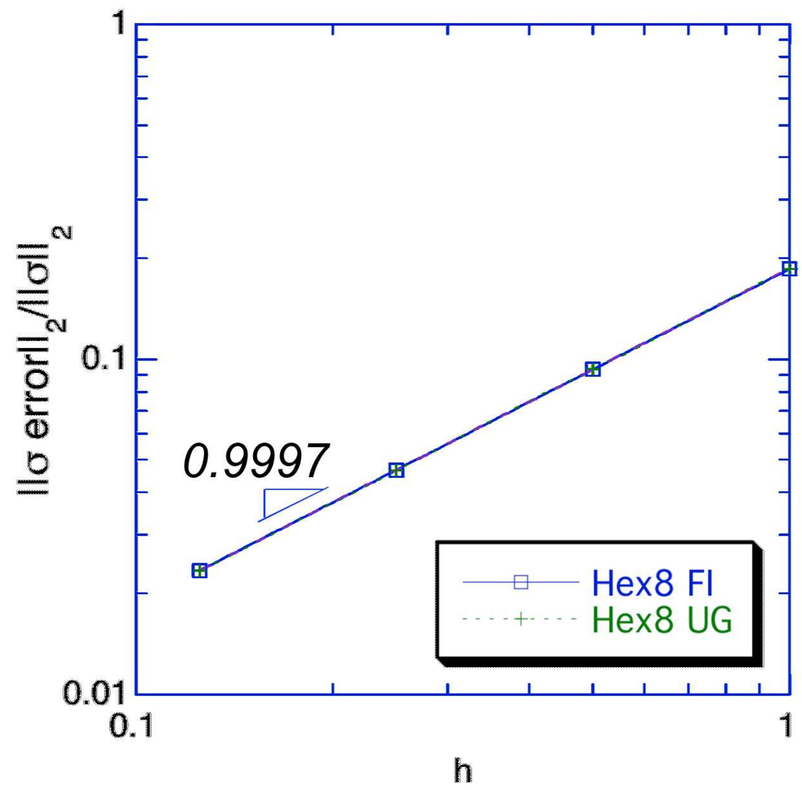
Quadratic Displacement Field Plus a Twist

Animation for O(200%) Strain Level

Quadratic Displacement Field Plus a Twist

Convergence





FI ~ fully-integrated
UG ~ uniform gradient – rates shown for this case

Manufactured Solution for Hypoelasticity

Hypoelastic: Quadratic Displacement Field plus a Twist

The plot thickens -- work in progress

- First attempt based on last problem. Assume the hypoelastic model commonly used in Sierra/SM of the form

  $$\overset{o}{\sigma} = \mathbf{C} : \boldsymbol{d}^e$$

  where $\mathbf{C}$ ~ isotropic tensor, and $\boldsymbol{d}^e$ ~ rate of deformation tensor

- To get σ we must integrate. Assume no initial stresses then

  $$\sigma_t = \mathbf{R}_t \left[\int_0^t \mathbf{R}_\tau^T \mathbf{C} : \boldsymbol{d}_\tau^e \mathbf{R}_\tau \, d\tau \right] \mathbf{R}_t^T$$

  Could not get the polar decomposition ($\mathbf{F}$=$\mathbf{RU}$).

- Instead construct the problem to obtain $\mathbf{F}$=$\mathbf{RU}$ from

  $$\boldsymbol{x} = \phi(\boldsymbol{X}, t) = \phi_{rotate}\left[\phi_{deform}(\boldsymbol{X}, t), t\right]$$

- Now let ϕ_{rotate} be the rigid body rotation used previously.
- We can obtain ϕ_{deform} from a cubic potential function (Ω) as

  $$\begin{aligned} x_{deform} &= \phi_{deform}(\boldsymbol{X}, t) \\ &= \nabla_0 \Omega(\boldsymbol{X}, t) + \boldsymbol{X} \end{aligned}$$

  then

  $$\boldsymbol{F}_{deform} = \frac{\partial \phi_{deform}(\boldsymbol{X}, t)}{\partial \boldsymbol{X}} = \nabla_0^2 \Omega + I$$

Hypoelastic: Quadratic Displacement Field plus a Twist

- Integral form $\sigma_t = \mathbf{R}_t \left[\int_0^t \mathbf{R}_\tau^T \mathbf{C} : \boldsymbol{d}_\tau^e \mathbf{R}_\tau \, d\tau \right] \mathbf{R}_t^T$ reduces to integration of a rational function, solvable but yields complicated expressions.
- First attempt -> analytical expressions for the stress divergence
- To debug, considered getting divergence from central difference estimate Tested idea (called it *semi-manufactured*) with previous hyperelastic problem. For finest two meshes:

Divergence Calc	*u* conv. rate	σ conv. rate
Analytical	1.9998	0.9997
Central Difference	1.9998	0.9997

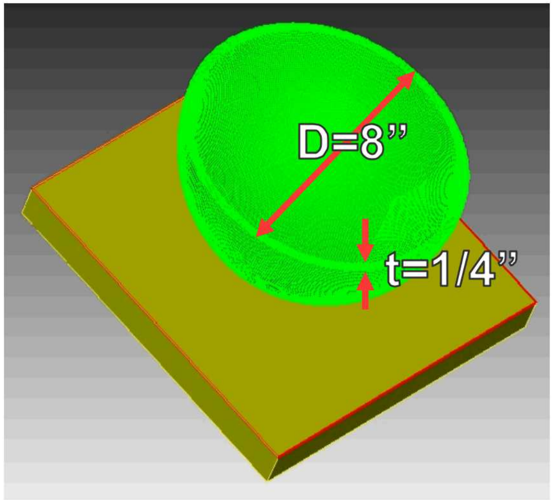
- Thus may only use the semi-manufactured results.
- Third complication: change of integration result as $f(\boldsymbol{X})$. Symbolic calculations didn't recognize this.
- To be continued …

Hemisphere-Plate contact –

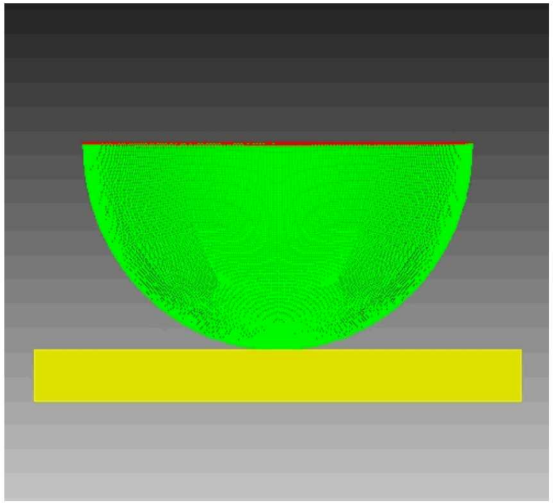
Solution verification attempts to get closer to the problem space

Hemisphere-Plate contact

- Previous Hertz-like contact convergence tests required Richardson Extrapolation to get convergence rate estimates.
- Many application relevant contact problems are not Hertzian in nature.
- Some of the test cases here included: explicit dynamics, elasto-plasticity, finite deformations, and large contact areas.
- “System model like meshes” – more uniform, not refined *a priori* for contact.
- 3-Parameter Richardson Extrapolation used to estimate convergence rate
- QoI: Reaction Force (F) on cut surface and Equivalent plastic strain

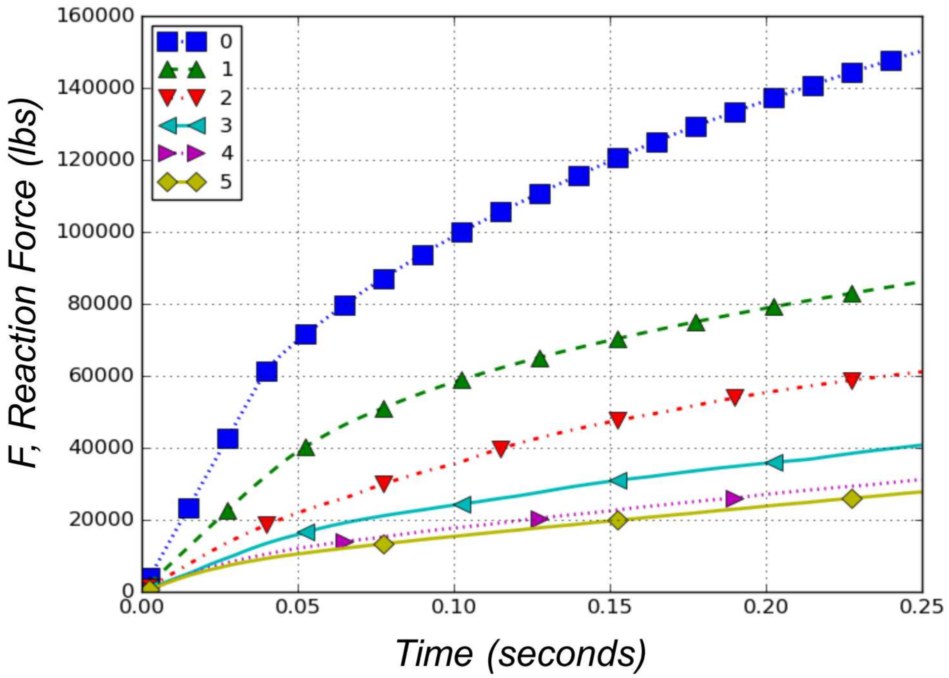


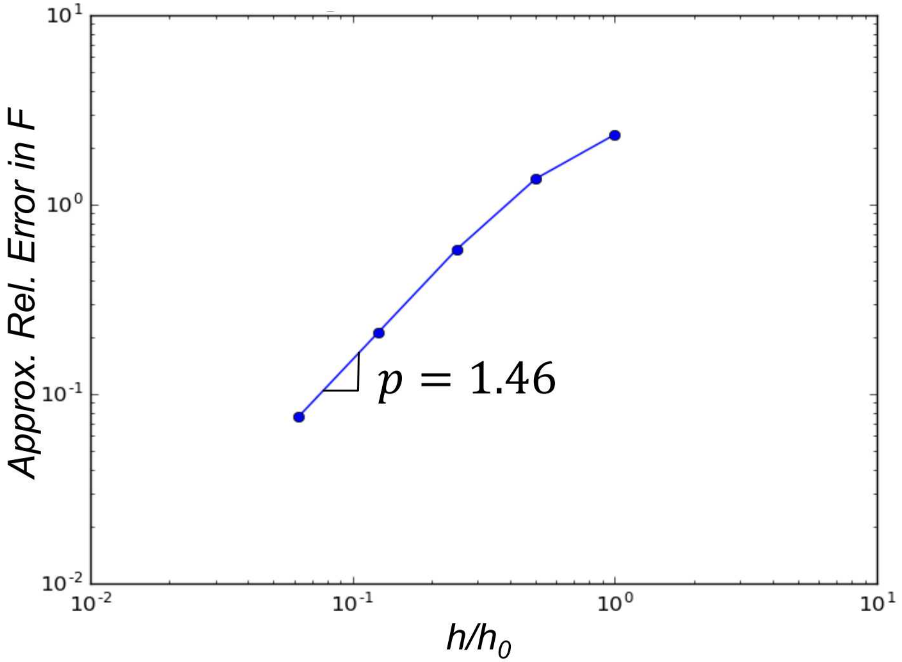
- Aluminum hemisphere
- Rigid plate
- Quasistatic & dynamic
- v_n= 47 mph
- Normal & oblique impact



Convergence

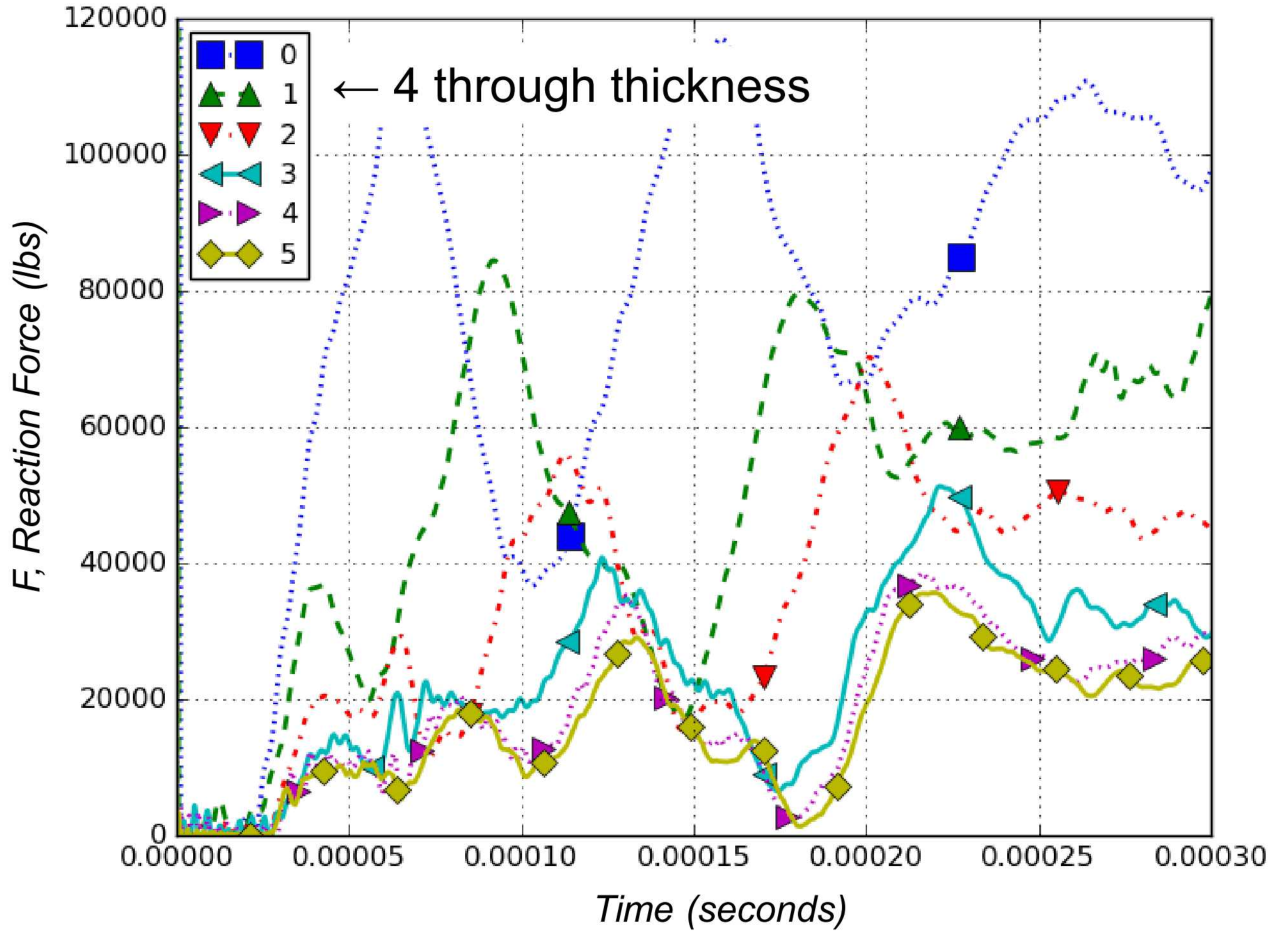
Quasi-static Elastoplastic





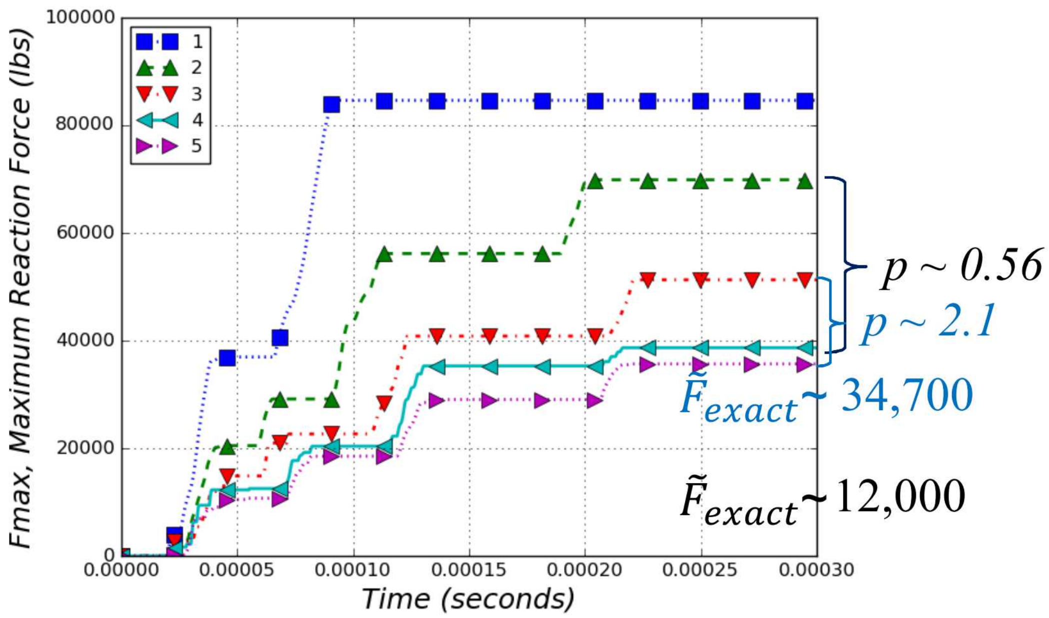
Convergence: *Explicit Elastoplastic*

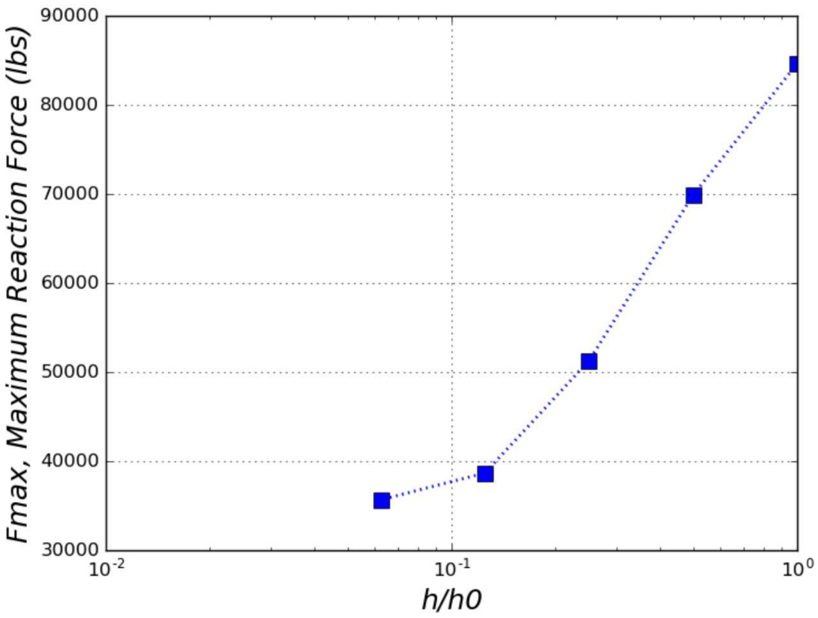
Analyzing Reaction Force



Convergence with Differing Elements

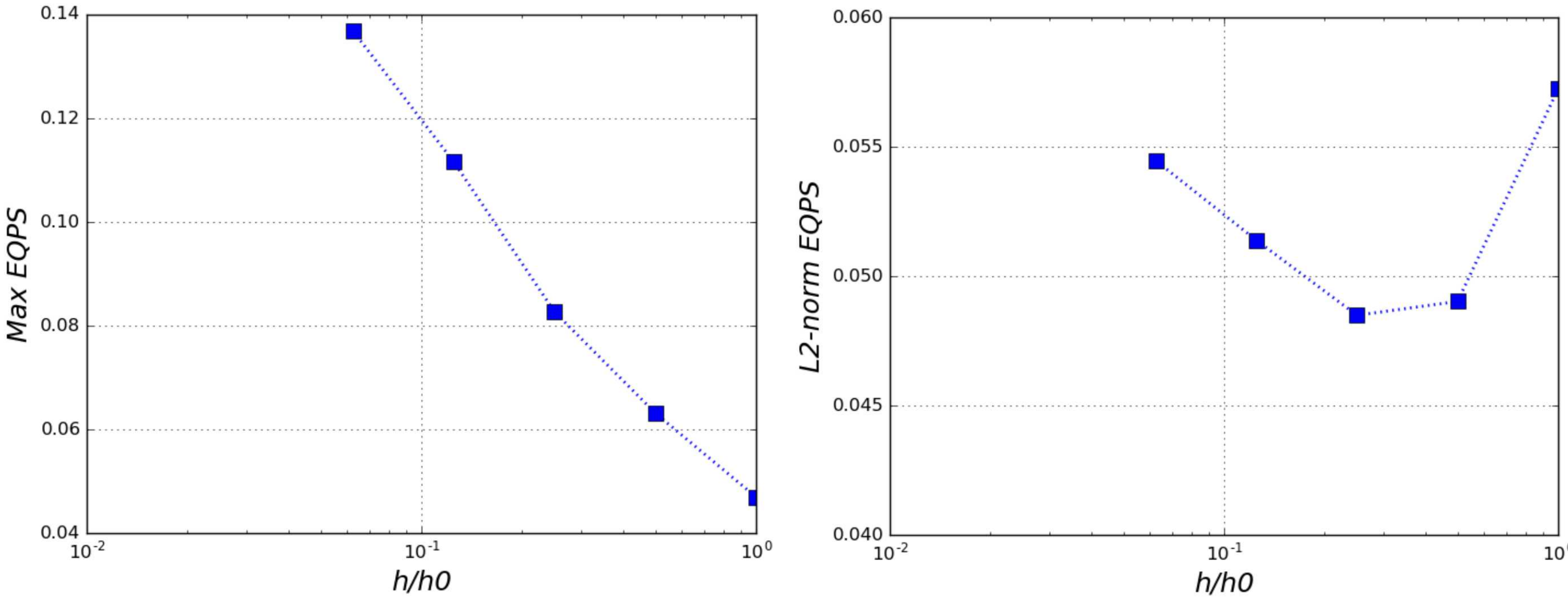
Explicit Dynamic Elastoplastic, Frictionless





Convergence with Different EQPS Measures

Explicit Dynamic, Elastoplastic



EQPS ~ Equivalent plastic strain

Neither L_2-norm or maximum of EQPS yield a significant convergence trend

Conclusions

- Manufactured solutions for hyperelastic problems give strong verification evidence for the elements
- Hole-in-plate problem demonstrates:
  - issues with using an inexact reference solution
  - issues can be "manufactured away"
- Manufactured solution for hypoelasticity is:
  - problematic for an arbitrary case
  - but possible with some simplifications to the problem and approach
- Hemisphere-plate contact problem
  - closer to problem space
  - shows tendency to converge for some QoI -- but not all
  - have not accurately measured convergence rates in dynamic cases