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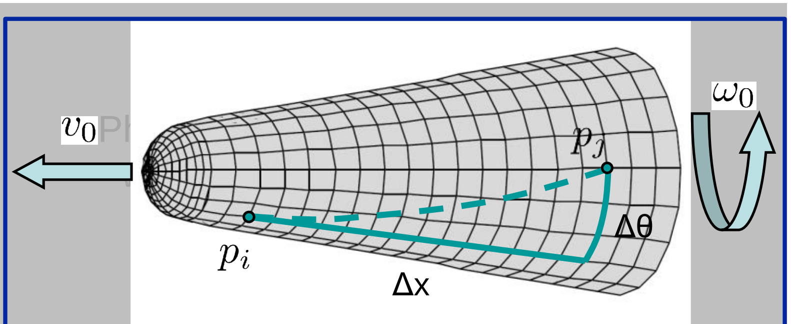

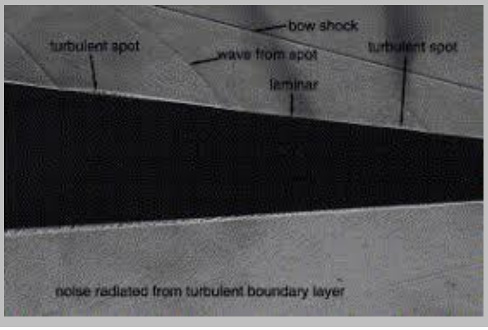

Consistent Turbulent Boundary Layer Wall Pressure Spectra and Coherence Functions

Lawrence DeChant and Justin Smith

Thursday, 11-January-2018; FD-53, Stability and Transition V: High-Speed Cones

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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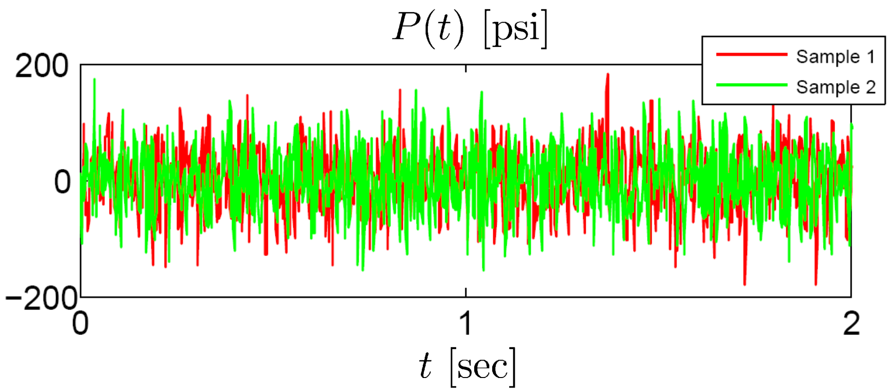
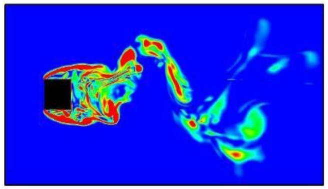
Outline

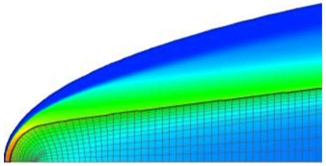
- Aerosciences at Sandia National Labs
  - Reentry Random Vibration
  - Pressure Fluctuation Loading
- Spatial Correlation/Coherence
- Hypersonic Sharp Cone
- Coherence: Linear System with Noise
- Conclusions
- Future Work

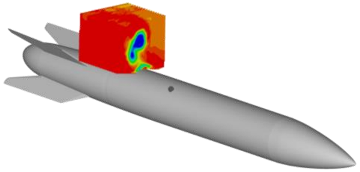


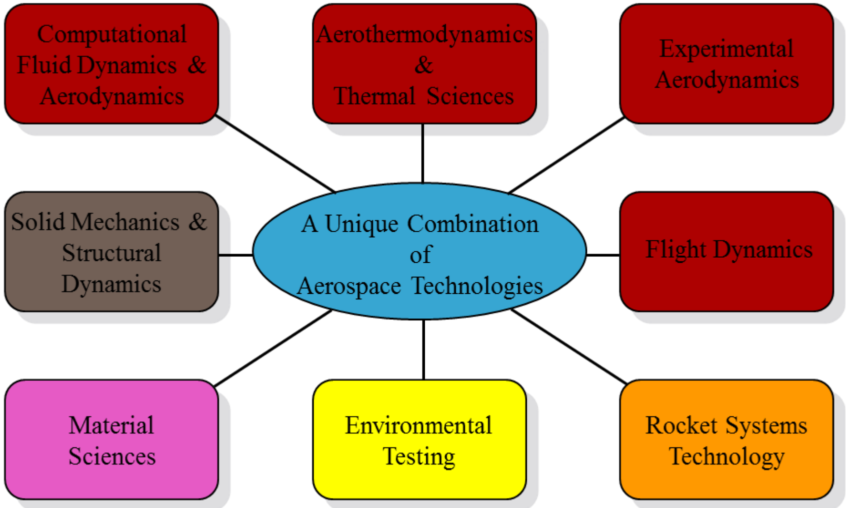
AeroSciences Sandia National Laboratories

- Only Aero Department in Engineering Sciences
  - 17 Staff spanning all areas
- Expertise
  - Both Computational and Experimental
  - Compressible Flow CFD
    - Numerical Methods, Turbulence Modeling, Software Development
    - Multi-Physics Modeling (FSI, Aero-Thermal Coupling)
    - Models for Re-Entry – Ablation, Random Vibration, Transition, Chemistry
  - Experimental Compressible Flows
    - Advanced Diagnostics – Laser Based, Surface Diagnostics, High Frequency Accelerometers
    - Experiments for Discovery and Validation
- Activities
  - Balance of Research, Development and Applications
  - Combination of Computations and Experiments - Lab-Scale & Full-scale



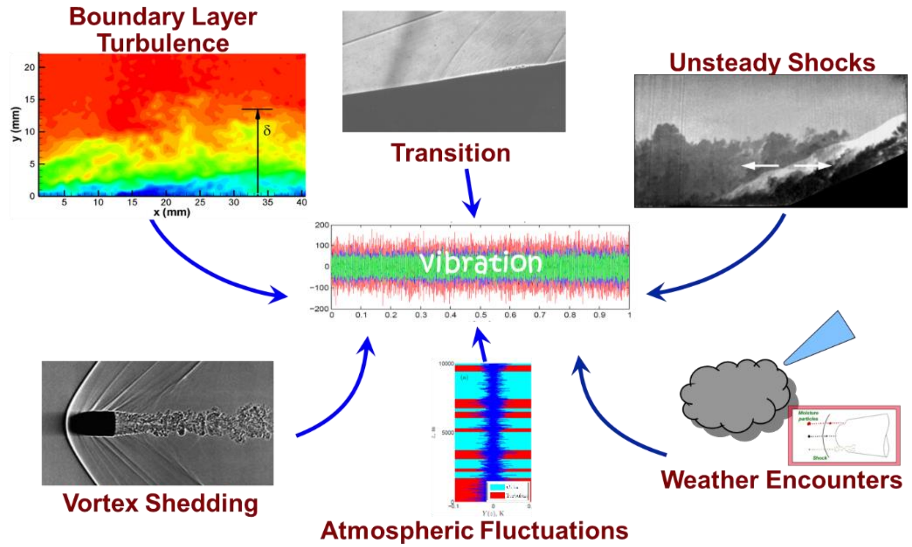


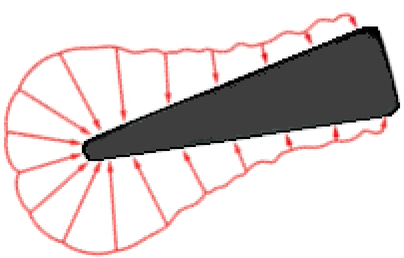


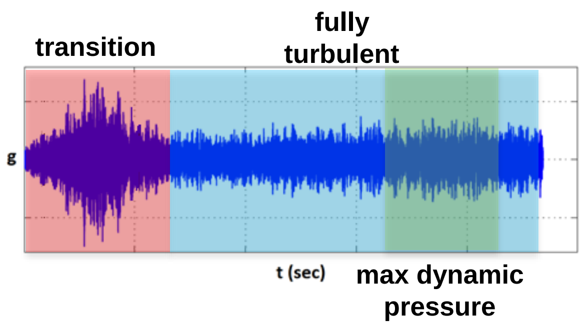


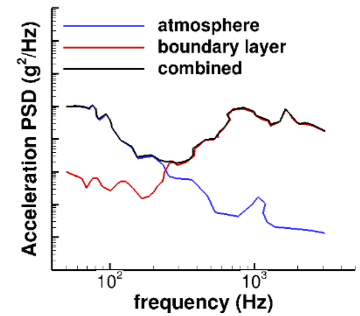
Reentry Random Vibration

- Reentry Random Vibration
  - Structural response of an RB and internal components
  - Random Aerodynamics Loads
    - Boundary Layer Transition
    - Turbulent Boundary Layer
    - Atmospheric Fluctuations
    - Weather
    - ...









Turbulent Pressure Fluctuations

- **Correlation**

$$R(\xi,\eta,\tau) \propto \int_0^\infty \int_0^\infty \int_0^\infty p(x,z,t)\, p(x+\xi, z+\eta, t+\tau)\, dt\, dx\, dz$$

- **Cross-spectral density (separable assumption)**

$$\Gamma(\xi,\eta,\omega) \propto \int_0^\infty R(\xi,\eta,\tau)\cos(\omega\tau)\, d\tau \qquad \Gamma(\xi,\eta,\omega) = \Phi(\omega) A(\xi,\omega) B(\eta,\omega) \exp(-i\alpha) \qquad \omega = \frac{\omega_D \delta^*}{U} \quad \xi = \frac{\xi_D}{\delta^*}$$

- **Corcos Models**

$$A(\xi,\omega) = \exp(-a_0 \xi \omega) \quad ; \quad B(\eta,\omega) = \exp(-b_0 \eta \omega)$$

- **Measurements (ASD/PSD, spatial correlation)**

$$\Phi(\omega) = \Gamma(\xi = 0, \eta = 0, \omega)$$

$$R_x(\xi) = R(\xi, \eta = 0, \tau = 0)$$
$$R_z(\eta) = R(\xi = 0, \eta, \tau = 0)$$

Longitudinal/Lateral Coherence

- **Streamwise (Longitudinal) Coherence**

$$R_x(\xi) \propto \int_0^\infty \Phi(\omega) A(\omega\xi)\cos(\omega\xi)\, d\omega$$

- **Lateral Coherence**

$$R_z(\eta) \propto \int_0^\infty \Phi(\omega) B(\omega\eta)\, d\omega$$

- **Solve Integral Equation=estimate**
- **Assume (Corcos) form and estimate parameters a0 and b0**

$$A(\xi,\omega) = \exp(-a_0\xi\omega) \quad ; \quad B(\eta,\omega) = \exp(-b_0\eta\omega)$$

- **Where coherence lengths are implicit, e.g.**

$$A(\xi,\omega) = \exp(-\frac{\xi}{\Lambda_\xi^*}\omega) \qquad \Lambda_\xi^* = \frac{L_\xi \omega_D}{U_D} \qquad \Lambda_\xi^* = \frac{1}{a_0}$$

- **But by assuming the functional form are we really solving the integral equation ?**

"Guessing" Functional Forms=Traditional Galerkin

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- **Assuming a function for the coherence expressions is directly related to classical "traditional Galerkin method" for solving differential/integral equations:**
- **Consider:**

$$\frac{df}{dx} + f = 0 \quad ; \quad f(0) = 1$$

- **Analytical solution is trivial**
- **But "assume" a trial function with parameters: a_i**

$$f_t = 1 + a_1 x + a_2 x^2$$

- **Compute residual (bc→a_1=1)**

$$f(x) = e^{-x}$$

$$R(x, a_1, a_2) = \frac{df_t}{dx} + f_t = a_1 + (a_1 + 2a_2)x + a_2 x^2$$

- **Compute inner products (algebraic expressions for a_i)**

$$\int_0^1 R(x, a_1, a_2) w(x) dx = \int_0^1 (a_1 + (a_1 + 2a_2)x + a_2 x^2) dx = \frac{3}{2} a_1 + \frac{4}{3} a_2 + 1 = 0$$

$$\int_0^1 R(x, a_1, a_2) w(x) x dx = \int_0^1 (a_1 + (a_1 + 2a_2)x + a_2 x^2) x dx = \frac{5}{6} a_1 + \frac{11}{12} a_2 + \frac{1}{2} = 0$$

- **Solution:** $f_t = 1 + a_1 x + a_2 x^2 = 1 - \frac{18}{19} x + \frac{6}{19} x^2$
- **"Guessing solutions" is well found approach to solving governing equations**

Analytical Coherence Result

- Analytical solution for the longitudinal coherence is possible using a simplified (exponential) PSD

$$\Phi \propto \exp(-c_0\omega)$$

- Estimates for C_0 ? Match other models RMS $\int_0^\infty \Phi d\omega = \int_0^\infty \exp(-c_0\omega)d\omega = \frac{1}{c_0}$

- Houbolt: $\Phi \propto \frac{1}{1+\omega^2} \rightarrow c_0 = \frac{2}{\pi}$

- Lowson: $\Phi \propto \frac{1}{(1+\omega^2)^{3/2}} \rightarrow c_0 = 1$

- Robertson $\Phi \propto \frac{1}{(1+\omega^{0.9})^2} \rightarrow c_0 = (\frac{10}{9}\mathrm{B}(\frac{8}{9},\frac{10}{9}))^{-1} \approx 0.88$ B=beta function

- Integration gives:

$$R_x(\xi) \propto \int_0^\infty \exp(-c_0\omega)\exp(-a_0\omega\xi)\cos(\omega\xi)d\omega \rightarrow R_x(\xi) = \frac{c_0(a_0\xi + c_0)}{(a_0\xi + c_0)^2 + \xi^2}$$

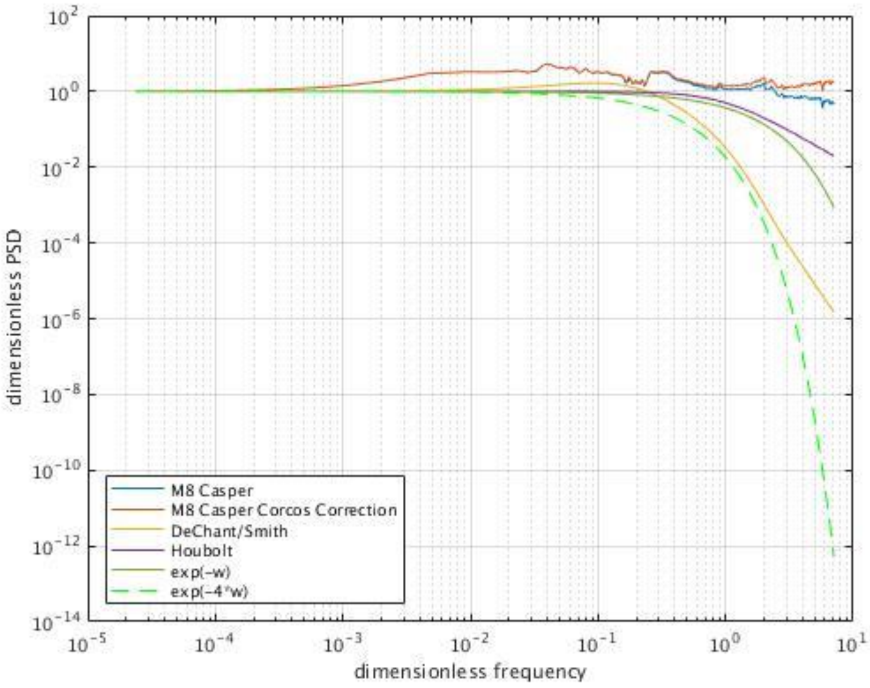
$$R_z(\eta) \propto \int_0^\infty \exp(-c_0\omega)\exp(-b_0\omega\eta)d\omega \rightarrow R_z(\eta) = \frac{c_0}{b_0\eta + c_0}$$

PSD/ASD Power Spectral Density

- Coherence estimates are directly related to PSD/ASD
- We can use simple analytical models

$$\Phi \propto \exp(-c_0\omega)$$

- More complex theory-based models e.g. Dechant-Smith (2015)
- Or experimental measurements e,g, Casper et. al. (2016)
- Models are largely equivalent for low frequency



Classical Low Speed Results

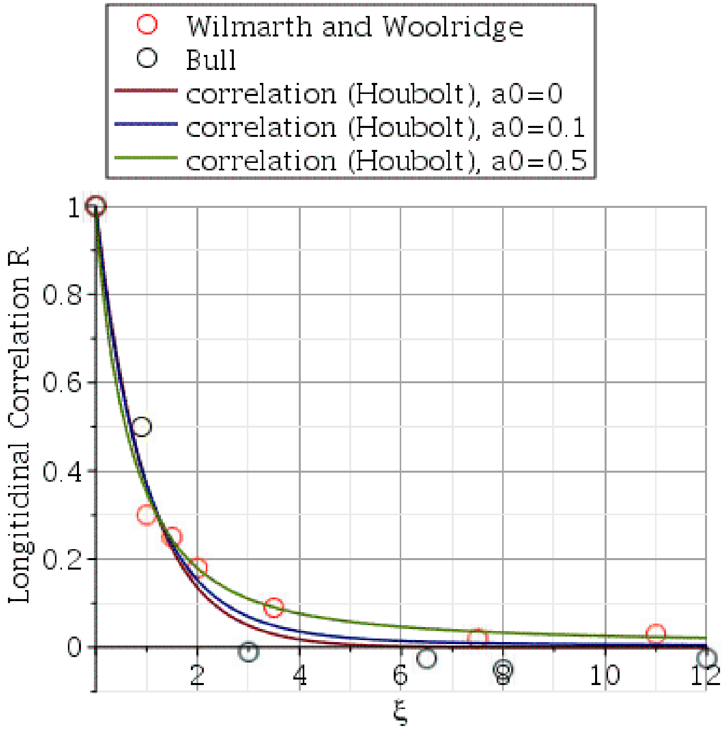
- simple results are possible using a PSD (Houbolt)

$$\Phi \propto \frac{1}{1+\omega^2}$$

- Good agreement for a_0=O(0.1)
- Notice that for a_0=0 we recover classical Taylor Hypothesis equivalency between space and time in convected field.

$$R(\xi) \propto \int_0^\infty \Phi(\omega)\cos(\omega\xi)d\omega$$

$$\Phi(\omega) \propto \int_0^\infty R(\xi)\cos(\omega\xi)d\xi$$



Mach 5-8 Sharp Cone Coherence

- Casper 2016 sharp cone measurements (correlation and PSD)
- Compute longitudinal and lateral coherence
- Error=degree to which correlation computed using coherence expression (see below) matches measurements

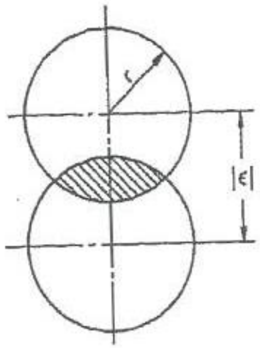
$$R_x(\xi) \propto \int_0^{\infty} \Phi(\omega) A(\omega\xi) \cos(\omega\xi) d\omega$$

- Notice relatively poor agreement

Spectral Density Φ	Longitudinal a_0	RMS error	Lateral b_0	error
Measurement	0.75	2.7E-2	3	4.2E-4
Measurement Corcos attenuation correction (see appendix)	0.5	4.3e-2	1.5	2.3E-4
DeChant-Smith	4	1.5E-3	20	1.4E-4
Houbolt; $\Phi \propto \frac{1}{1+\omega^2}$	0.75	2.1E-2	4	7.9E-4
Exponential: $\Phi \propto \exp(-\omega)$	0.75	1.1E-2	5.5	6.7E-4
Exponential: $\Phi \propto \exp(-4\omega)$	3.75	5.4E-4	22	6.7E-4

Conical Flow Sensor Attenuation

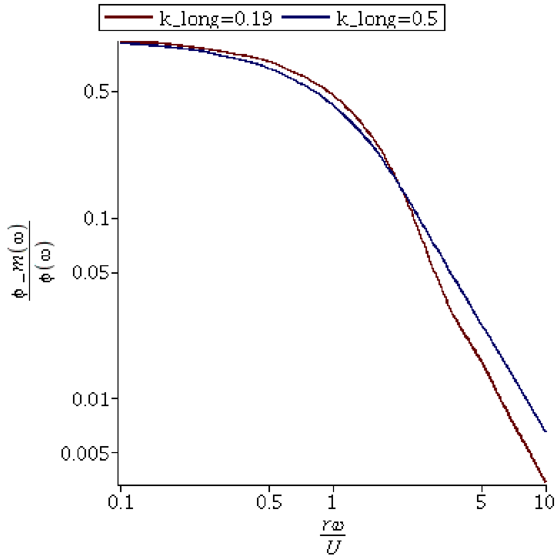
- Sensor attenuation correction can be important for some measurements
- Corcos pioneered a plausible correction model for flat plates based upon a geometric offset "ε" correlation



$$\frac{\phi_m}{\phi} = \int_0^{2\pi}\int_0^{2} \theta(\frac{\varepsilon}{r}) A\left((\frac{r\omega}{U})(\frac{\varepsilon}{r})\cos\theta) \right) B\left((\frac{r\omega}{U})(\frac{\varepsilon}{r})\sin\theta \right) \cos\left((\frac{r\omega}{U})(\frac{\varepsilon}{r})\cos\theta) \right) (\frac{\varepsilon}{r}) d(\frac{\varepsilon}{r}) d\theta$$

$$\theta = \frac{2}{\pi^2}\left(\arccos(\frac{1}{2}(\frac{\varepsilon}{r})) - \frac{1}{2}(\frac{\varepsilon}{r})\sqrt{1-\frac{1}{4}\left(\frac{\varepsilon}{r}\right)^2} \right)$$

- Perform a similar analysis using cone coherence closures
- Conical flow modification to attenuation is virtually same as flat plate



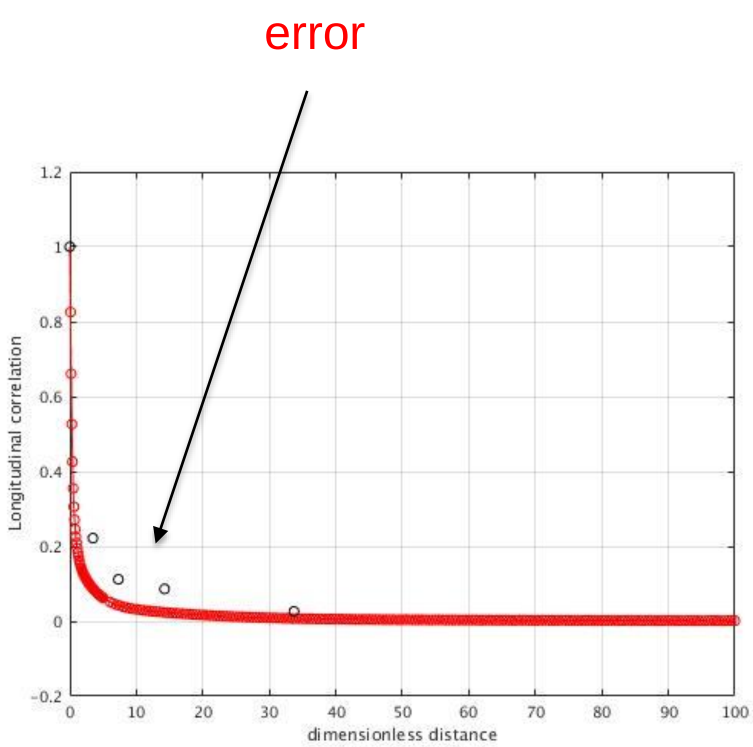
Cone Coherence Model Error

- Classical coherence models give poor match to supersonic conical flow correlation no matter what parameters I use
- Why?
- Regardless of parameters some curve shapes cannot be matched
- Generalize Corcos model… e.g. Efimstov

$$A(\xi) = \exp\left(-\frac{\xi}{\Lambda^*_{\xi e}}\right)$$

$$\Lambda^*_{\xi e} = \left[\left(\frac{a_1 Sh}{U/v^*}\right)^2 + \frac{a_2^2}{Sh^2 + (a_2/a_3)^2}\right]^{-1/2}$$

- Useful for low frequency
- Reverts to classical Corcos model for Sh>>1



Coherence Model Error: Noise?

- Longitudinal coherence =1 for noise free linear system, e.g. Taylor-hypothesis
- Coherence is degraded by system noise
- Analyze coherence for linear system with noise. PSD

$$\Phi_{pp} = \Phi_{pp\_0}(\omega) + N(\omega)$$
$$\Phi_{p'p'} = \Phi_{p'p'\_0}(\omega) + N(\omega)$$

- Coherence; Relate noise to coherence deviation

$$\gamma^2_{pp'} = \frac{\Phi^2_{pp'}}{(\Phi_{pp\_0} + N)(\Phi_{p'p'\_0} + N)} = \frac{\gamma^2_{pp'\_0}}{(1+\frac{N}{\Phi_{pp\_0}})(1+\frac{N}{\Phi_{p'p'\_0}})}$$

$$N = \left(\frac{\gamma_{pp'\_0}}{\gamma_{pp'}} - 1\right)\Phi_{pp\_0}$$

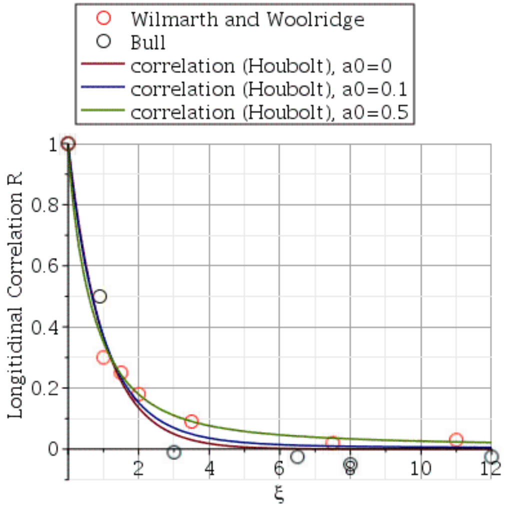
- Assume Corcos and relate Corcos deviation to RMS noise

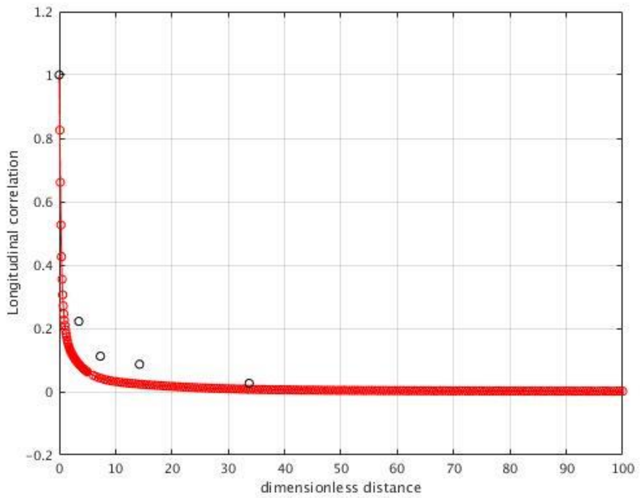
$$N_{rms} = \left(\frac{\varepsilon_0 \xi}{1-\varepsilon_0 \xi}\right)^{1/2} \approx (\varepsilon_0 \xi)^{1/2} \qquad \gamma_{pp'} = \exp(-a_0 \omega \xi) \qquad a_0 = a_{0\_0} + \varepsilon_0$$

- But this implies that noise is very large O(30%) of measurement!
- Unlikely; implies that system is inherently nonlinear for M>>1

Conclusions

- Coherence is an essential portion of pressure fluctuation loading for reentry vehicles
- Coherence related to correlation and PSD
- Classical: Corcos model works well for many flows
- Classical coherence models (Corcos) give poor match to supersonic conical flow correlation
- Examine noise in linear (Taylor-hypothesis) systems
- Noise needed to support Corcos modifications is larger than plausible measurement estimates
- **Supersonic conical flows nonlinear; simple Corcos models inadequate**

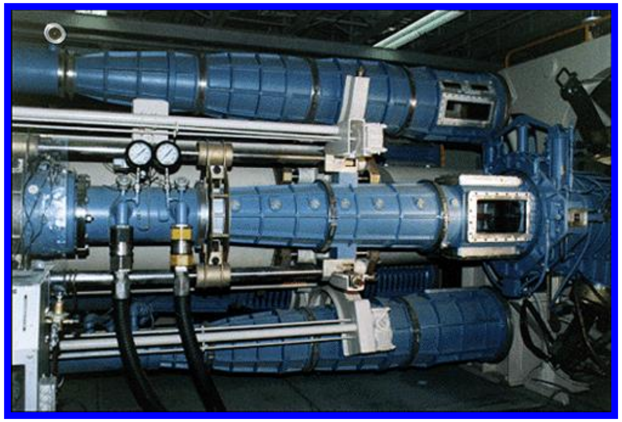


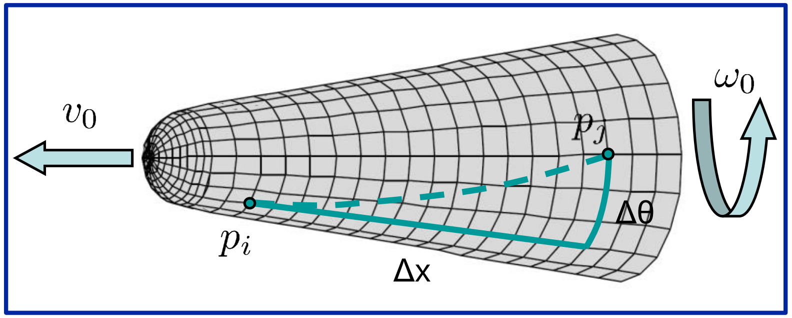


Further Work

- Examine reasons for deviation of conical hypersonic flows from classical models by:
  1. Broaden coherence model analytical bases (other than Corcos); Bayesian data fit J. Ray (2018)
  2. New measurements in Purdue (Chynoweth et. al.) quiet tunnels (sharp cone and flare cone)
  3. DNS simulation (Sandia Bitter et. al.)

- Characterization hypersonic pressure field essential to describe reentry behavior





Acknowledgements/Questions

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Questions???