

# Engineering Sciences and National Security Applications at Sandia National Laboratories

## Turbulent Jet-in-Crossflow Insights from Approximate Analytical Solution Methods

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**Aero Sciences Group; Engineering Sciences Center**

**10/24/2017**

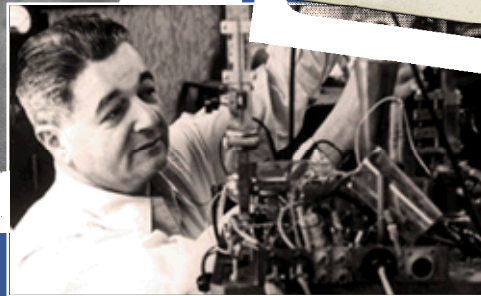
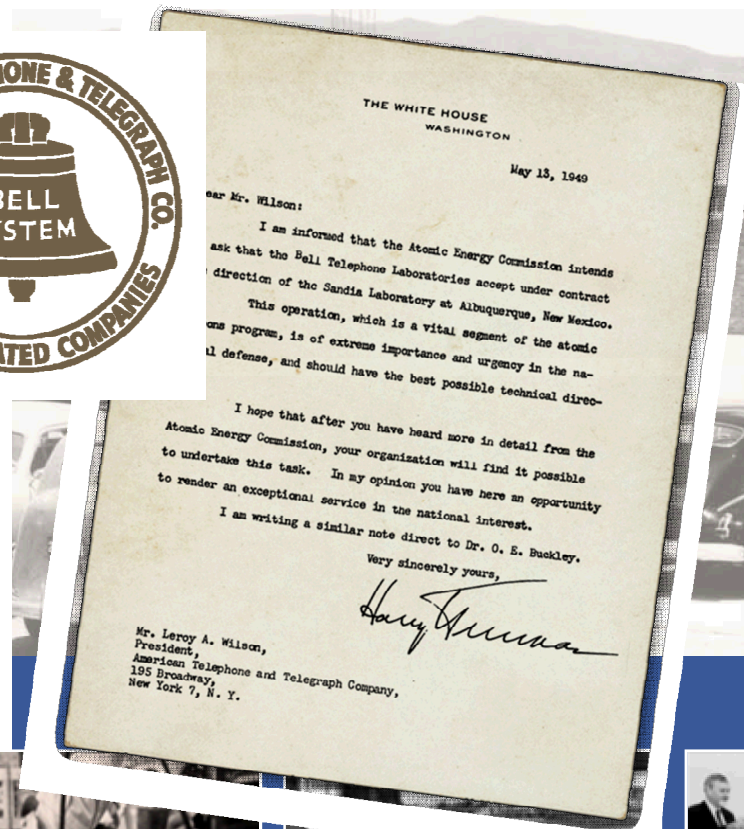


Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

# Sandia's History



*Exceptional service in the national interest*





# Sandia's Governance Structure



Government-owned  
contract



## Sandia Corporation

- AT&T / Bell Labs: 1949 – 1993
- Martin Marietta: 1993 – 1995
- Lockheed Martin: 1995 – April 30, 2017
- NTESS: May 1, 2017 -

## Federally funded research and development center



# Sandia Addresses National Security Challenges



**1950s**

Nuclear weapons

Production and  
manufacturing  
engineering



**1960s**

Development  
engineering

Vietnam conflict



**1970s**

Multiprogram  
laboratory

Energy crisis



**1980s**

Missile defense  
work

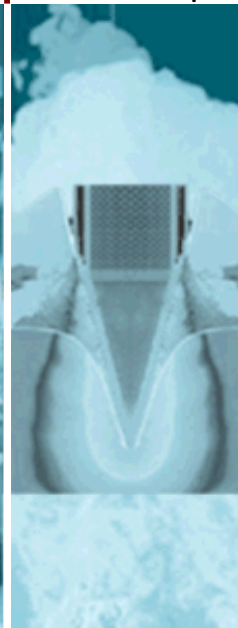
Cold War



**1990s**

Post-Cold War  
transition

Stockpile  
stewardship



**2000s**

START  
Post 9/11

National security



**2010s**

LEPs  
Cyber, biosecurity  
proliferation

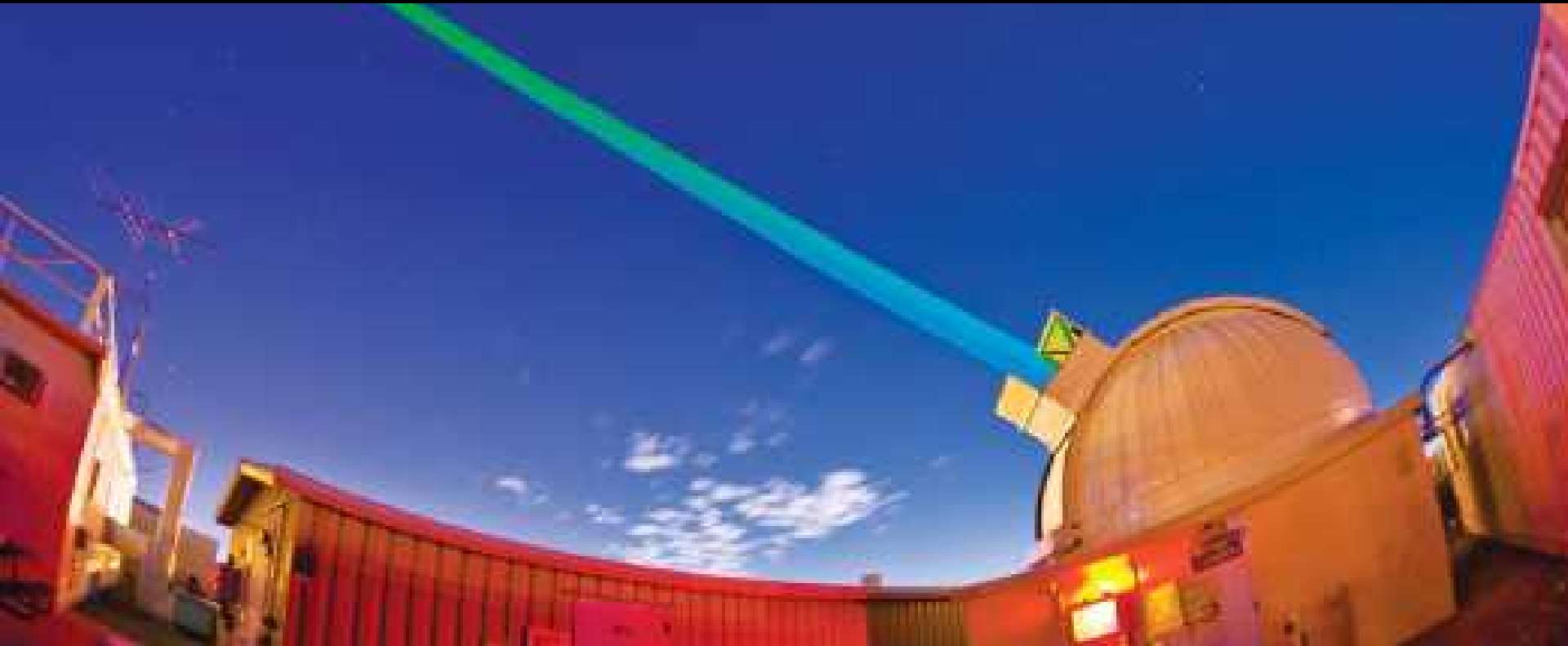
Evolving national  
security challenges



# Sandia - *Today*

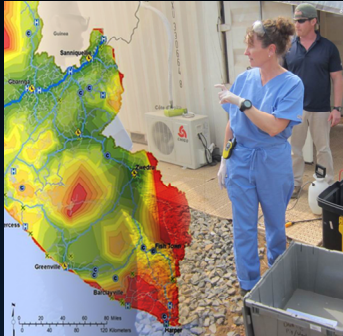


*As a multi-faceted national security laboratory, Sandia has delivered essential science and technology for more than 60 years and plays a critical role in ensuring U.S. technical superiority.*





# Sandia's Impact



## **Ebola Outbreak**

Sandia contributes to global response of Ebola outbreak by developing a sample delivery system cutting the wait time and potentially fatal exposure.



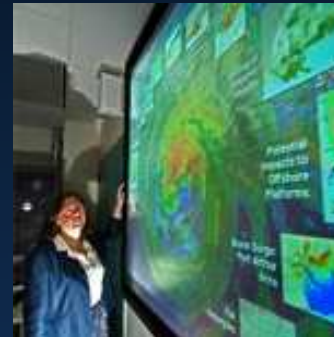
## **Detecting IEDs**

Combat personnel now have a new tool for uncovering improvised explosive devices: Sandia's highly modified miniature synthetic aperture radar system, which is being transferred to the U.S. Army.



## **Cleanroom invented 1963**

\$50 billion worth of cleanrooms built worldwide. It's used in hospitals, laboratories and manufacturing plants today.



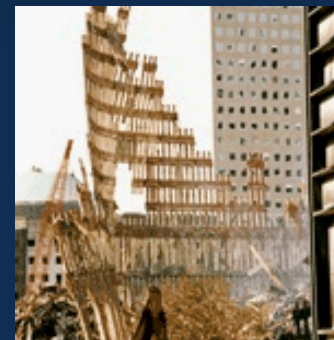
## **Hurricane Katrina**

Sandia is called to assess flooding and infrastructure failures.



## **Fukushima Quake**

Sandia helps clean up radioactive wastewater.



## **9/11**

Sandia sets contingency plans for release of materials and aircraft attacks on critical facilities immediately after 9/11. Search dogs are equipped with cameras for search and rescue K-9 handlers. The capability allowed search efforts to be carried out in spaces inaccessible to humans.

# Sandia has two main locations



Science labs

Nuclear energy lab



Fossil energy lab

Energy efficiency and  
renewable energy lab

# Sandia Sites

U.S. Department of Energy  
Sandia National Laboratories  
Livermore, CA 94550  
Tel: 925/294-2000  
Fax: 925/294-2000  
http://www.sandia.gov

*Albuquerque, New Mexico*



*Livermore, California*



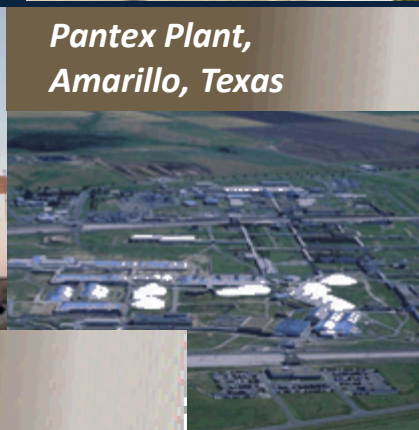
*Kauai, Hawaii*



*Waste Isolation Pilot Plant,  
Carlsbad, New Mexico*



*Pantex Plant,  
Amarillo, Texas*

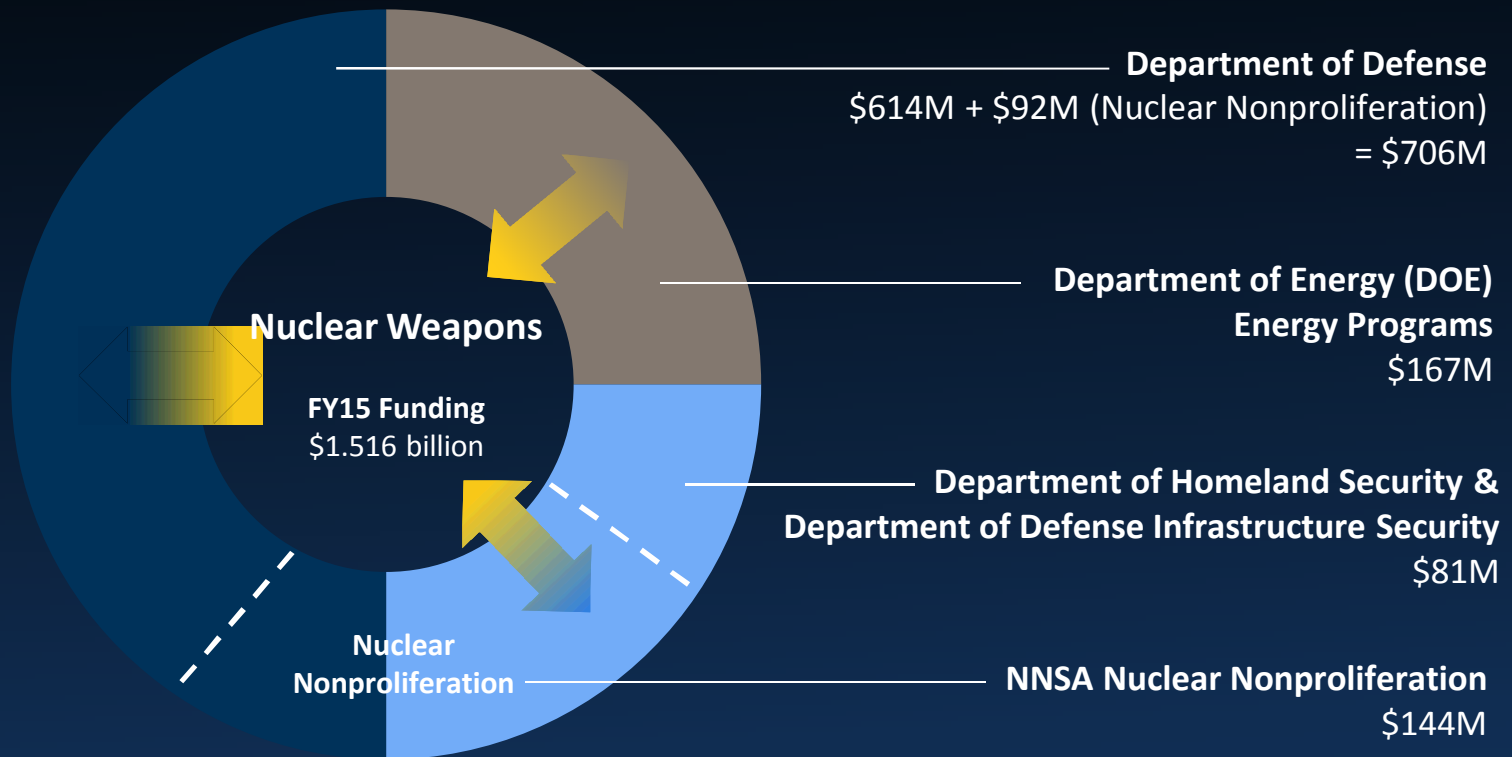


*Tonopah,  
Nevada*





# Sandia's Funding - ~\$2.8 Billion



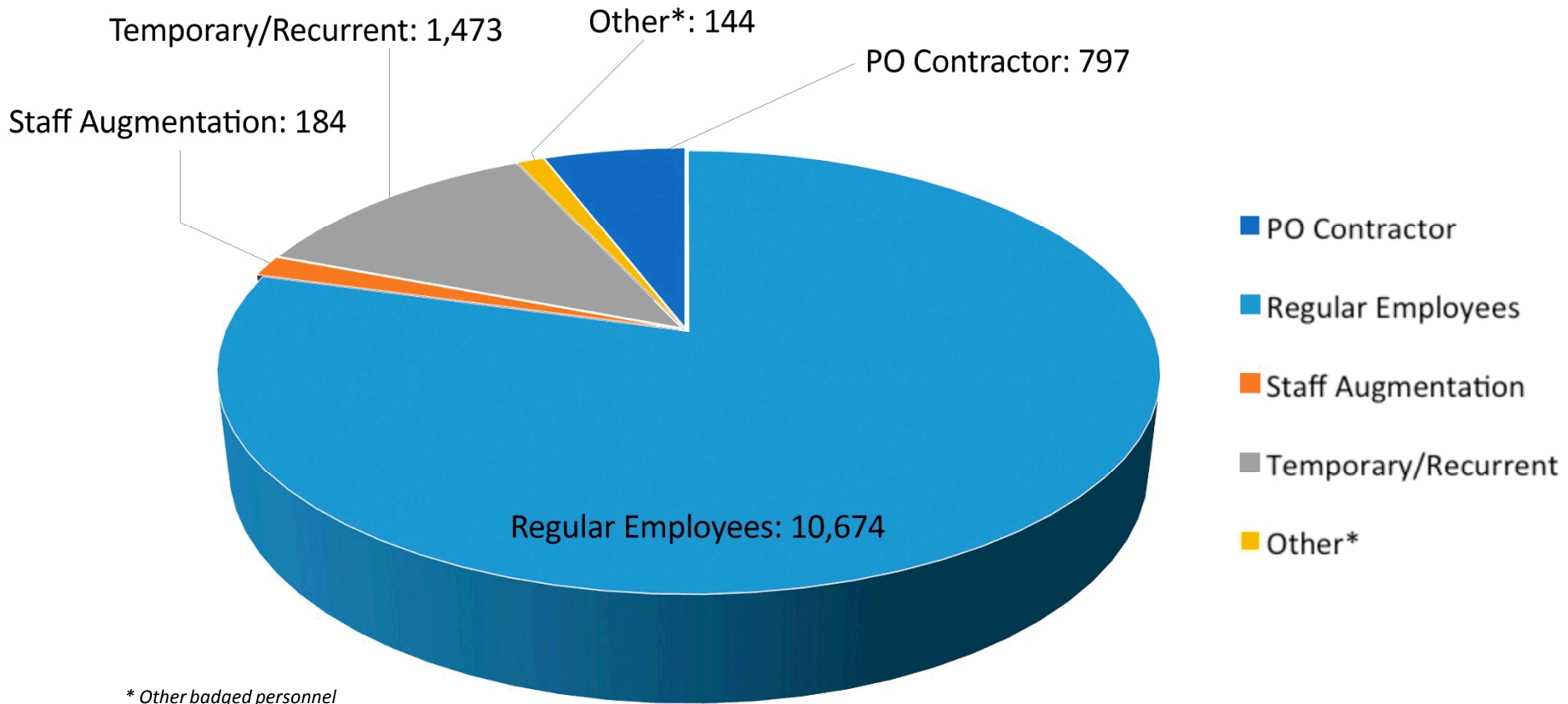
**Note: Other DOE and non-DOE Funding**  
\$195M

High reliability, high consequence of failure, challenging environments, and technology solutions

# Our Workforce

- Total Sandia workforce: 13,332
- Regular employees: 10,574
- Advanced degrees: 6,085 (57%)

*Data as of January 31, 2017*



# Fulfilling Our National Security Mission



*Nuclear Weapons*



*Global Security*



*Energy & Climate*

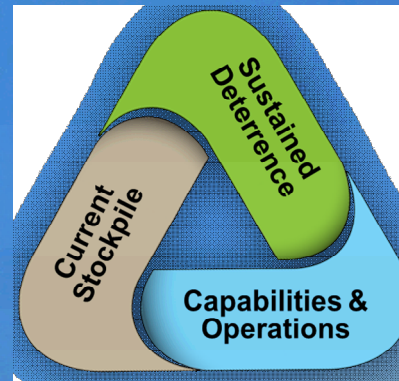


*Defense Systems & Assessments*



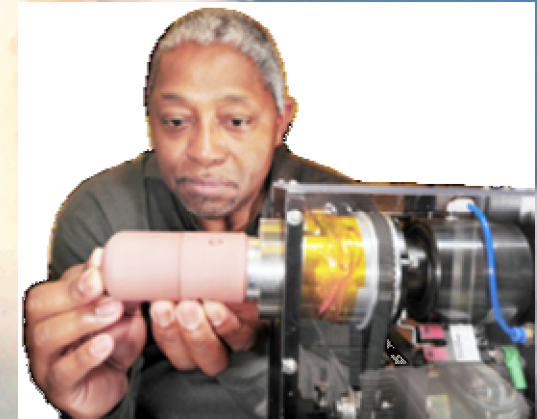
# Nuclear Weapons

*Sandia assumes an increasingly pivotal role in sustaining the nation's nuclear deterrent.*



Roles:

- Design
- Production
- Warhead systems engineering



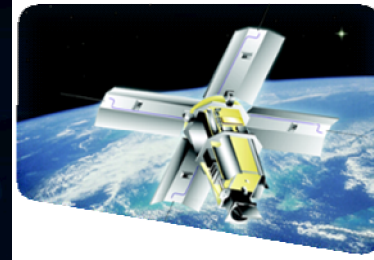
# Global Security

*Nonproliferation*

*Critical Asset Protection*

*Global Security*

*Remote Sensing*





# Energy & Climate



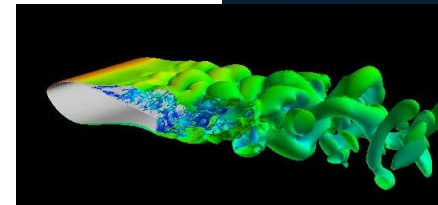
Energy Security

Climate Security

Structure Security

g Capabilities

ewable Energy



# Defense Systems & Assessments

*We support our troops and help to keep them safe*





# Our Foundations in Research

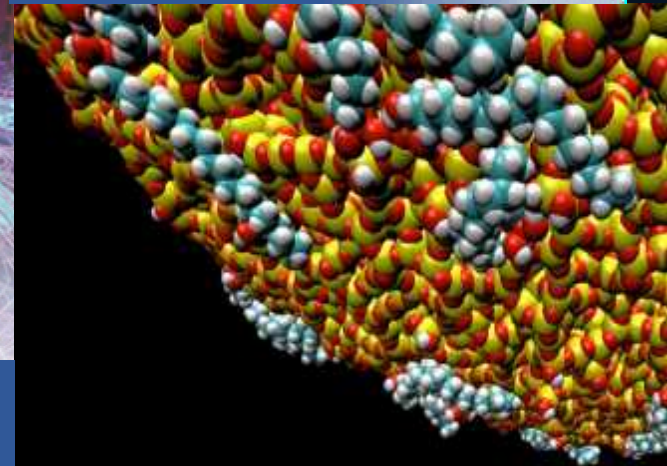
*We support essential research-and-discovery activities that translate into invention, innovation, entrepreneurship, economic opportunity, and public benefit.*

## Computing & Information Sciences

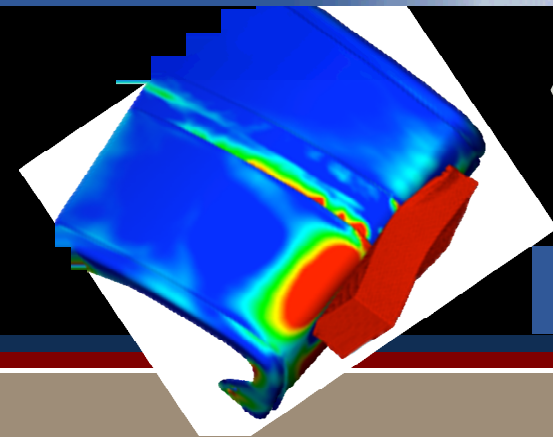


## Radiation Effects & High Energy Density Science

## Materials Science

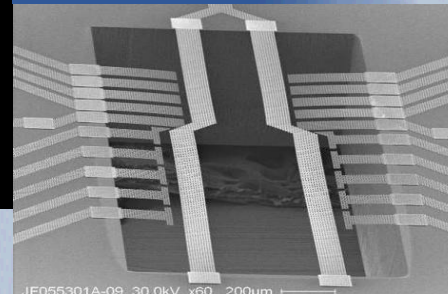


## Engineering Sciences



## Geoscience

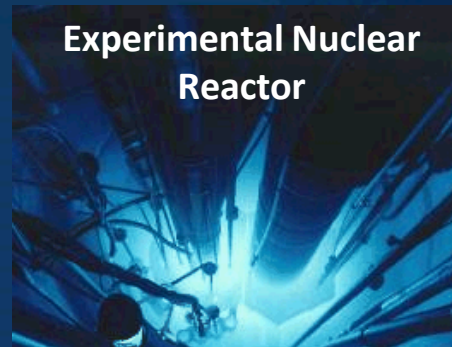
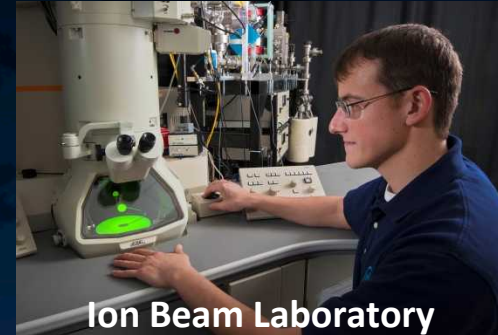
## Nanodevices & Microsystems



## Bioscience



# We Steward Key R&D Facilities for the Nation

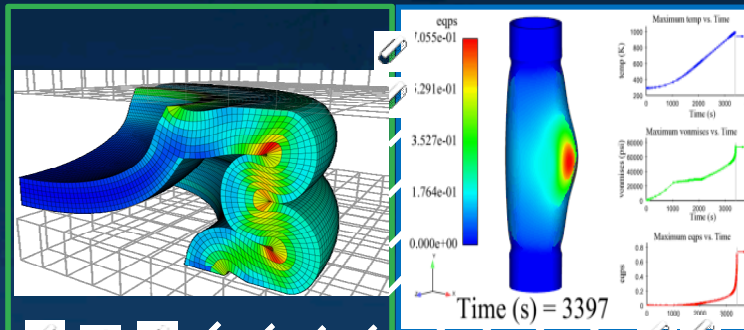


# Engineering Sciences Center

**Mission:** *To provide validated, science-based engineering expertise and solutions across the life cycle of products to inform engineering decisions.*

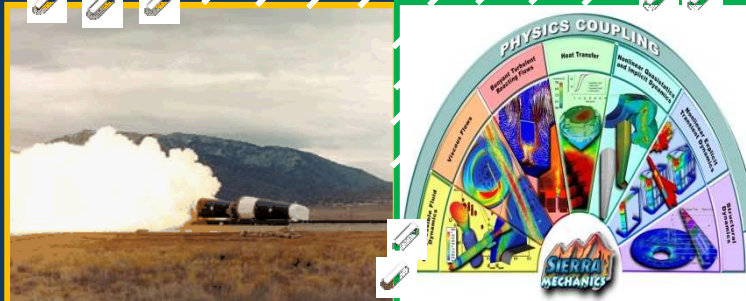
- We integrated theory, computational simulation and experimental discovery/validation across length and time scales is critical to develop the technical basis for complex systems.
- We partner internally and externally to advance our knowledge base and tool sets to provide physical and computational simulation capabilities that support the development and deployment of innovative, mission-driven products and services.

Engineering  
Analysis



Engineering Science  
Physical Phenomena

Environmental  
Simulation & Test



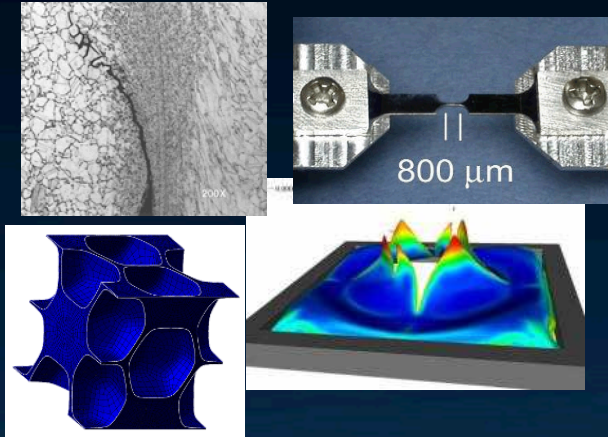
Computational  
Simulation Technology



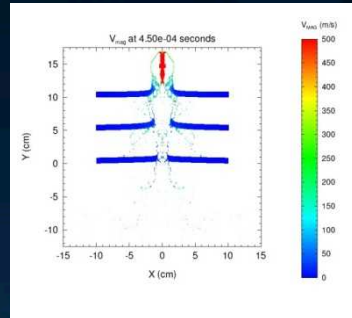
# Engineering Sciences Core Technical Areas



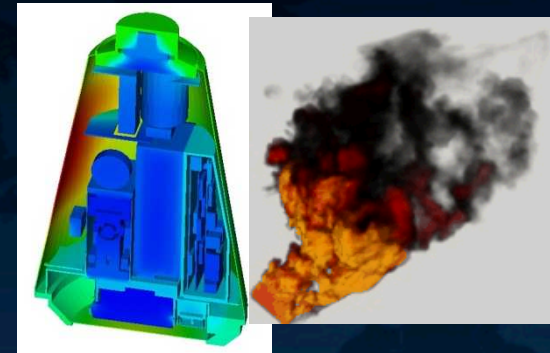
## Solid Mechanics



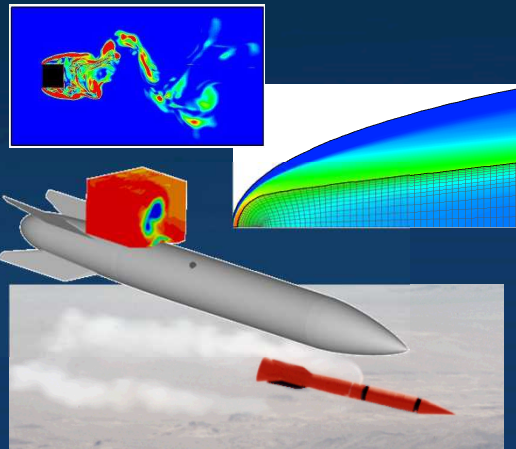
## Shock Physics and Energetics



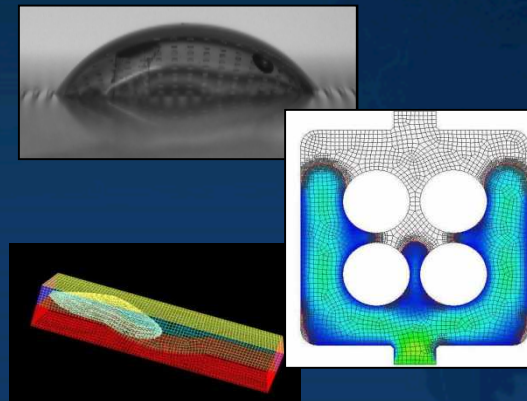
## Thermal and Combustion Sciences



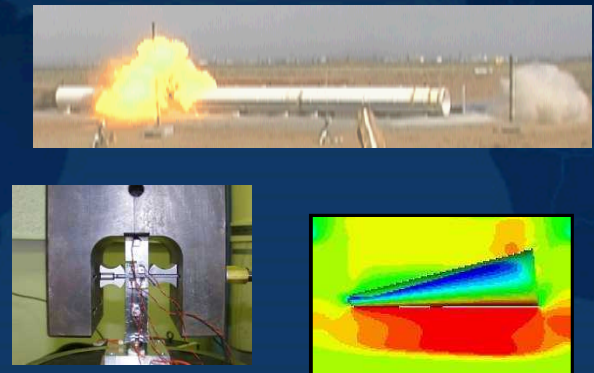
## Aerosciences



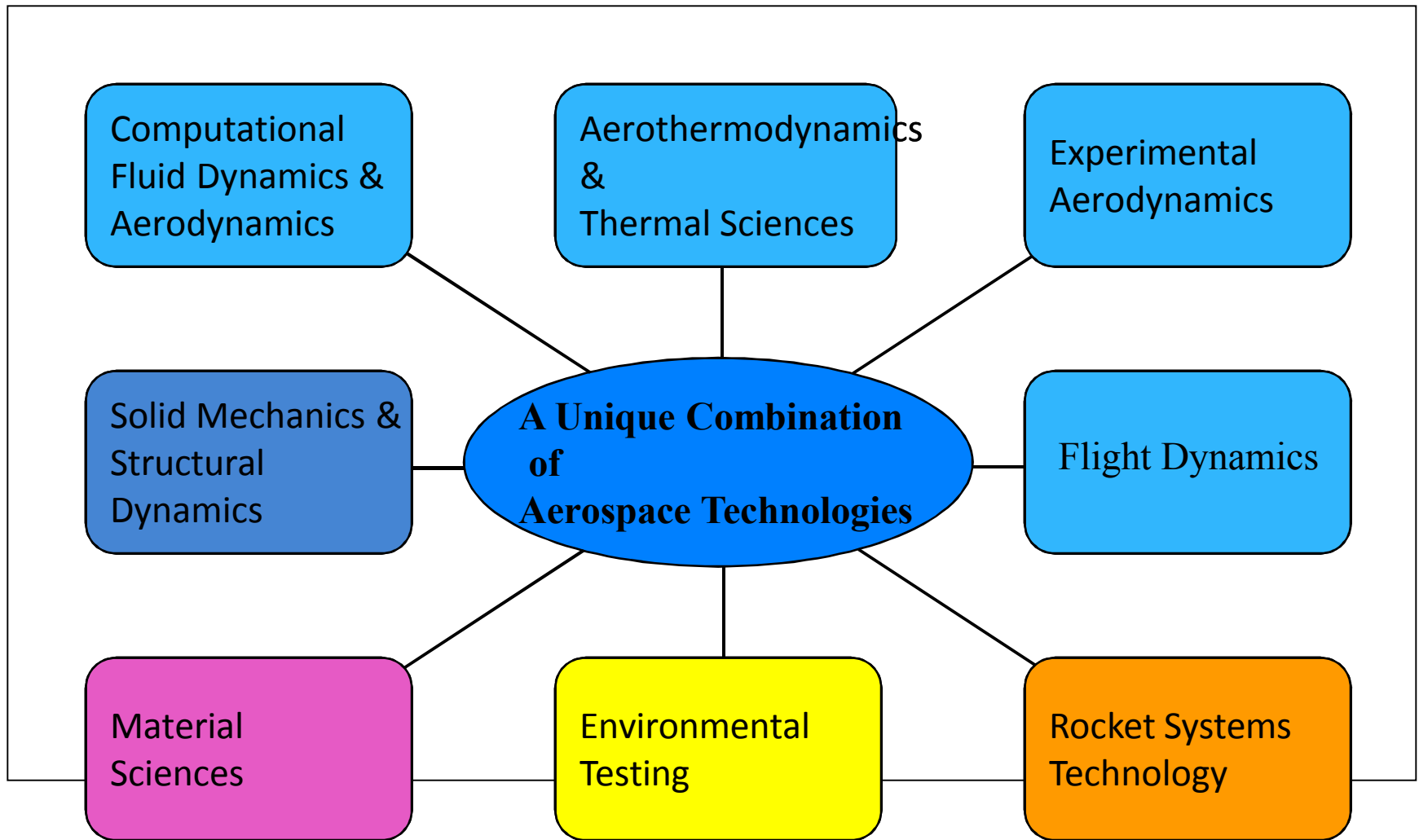
## Fluid Mechanics



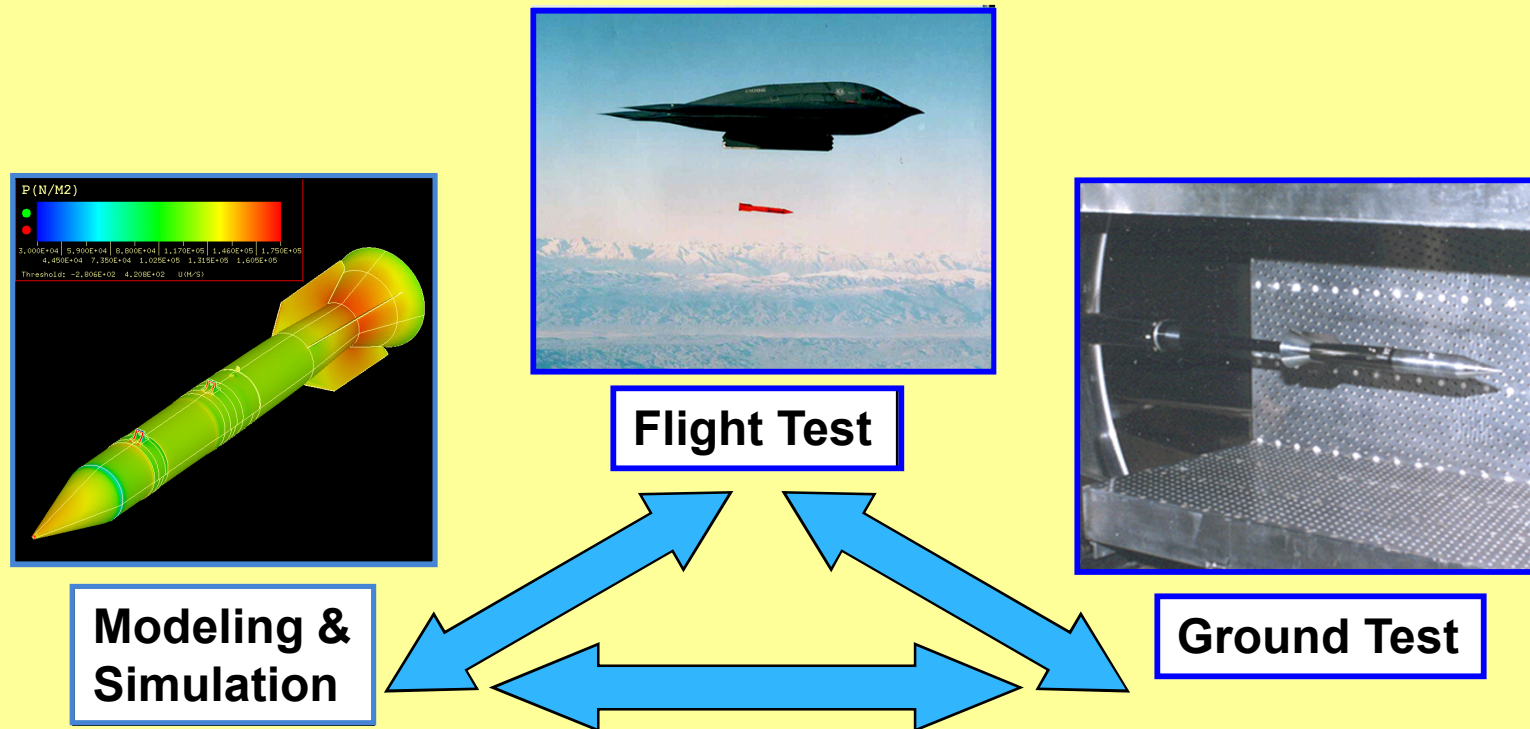
## Structural Dynamics



# Aerosciences and *Aerospace Technologies* at Sandia



# Aeroscience at Sandia



## Engineering Solution to Complex Problems:

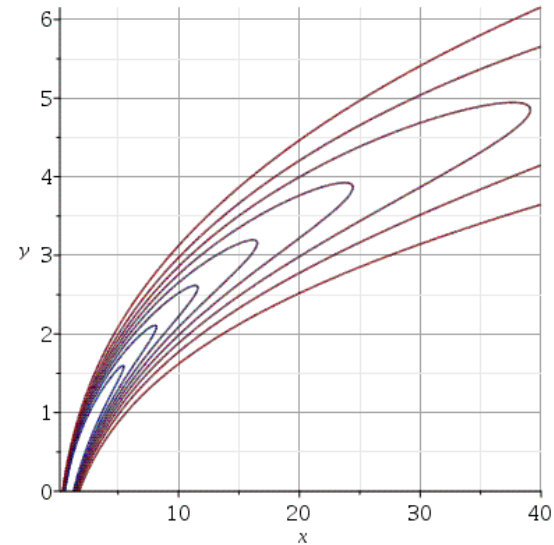
- Through a sustained R->D->A effort
- By applying combination of ground testing, modeling and simulation and flight testing
- **Mod-Sim:**
- **High Performance Computing Fluid Dynamic Simulation**
- **Classical Fluid Mechanics**



# Analytical Methods in Fluid Mechanics

## ■ Fluid Dynamics Problems

- Experimental Measurements
- Computational Fluid Dynamics (CFD)
- Analytical methods

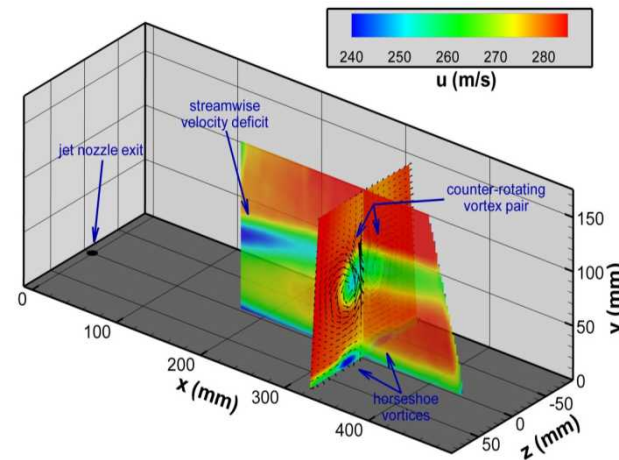
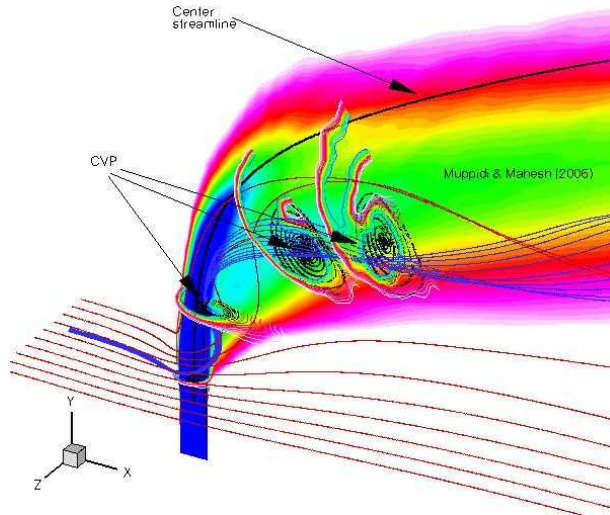


## ■ Analytical Methods

- Unique type of insight
- Limited applicability
- Often asymptotic
- Virtually always approximate in terms of formulation, solution or (likely) both

$$\frac{V(x, y)}{V_j} = \frac{1}{2} \left[ \operatorname{erf} \left( \frac{2x^{1/3} - y - 2\left(\frac{1}{2}\right)^{1/3}}{(2K)^{1/2} x^{2/3}} \right) - \operatorname{erf} \left( \frac{2x^{1/3} - y - 2\left(\frac{3}{2}\right)^{1/3}}{(2K)^{1/2} x^{2/3}} \right) \right]$$

# Jet-in-Crossflow: Turbulence Parameters



- Calibration of turbulence model parameters using data (computational/measurements) can be a highly effective procedure (Ray et. al. 2016)
- Turbulence model parameters have traditionally been estimated by demanding recovering of simplified/canonical flows
- Here we utilize a classical self-similar, axisymmetric wake/jet solution to provide estimates for a  $k$ - $\epsilon$  turbulence model implementation
- **Simplified methods interacting with computational approaches**

# JIC Simplified Governing Equations

- **Approximate axisymmetric wake Model (Following Tennekes-Lumley, 1972)**

$$U \frac{\partial u}{\partial x} = \frac{1}{y} \frac{\partial}{\partial y} \left( y C_\mu \frac{k^2}{\varepsilon} \frac{\partial u}{\partial y} \right)$$

- **Turbulence model (k-ε)**

$$U \frac{\partial k}{\partial x} = \frac{1}{y} \frac{\partial}{\partial y} \left( y C_\mu \sigma_k \frac{k^2}{\varepsilon} \frac{\partial k}{\partial y} \right) + C_\mu \frac{k^2}{\varepsilon} \left( \frac{\partial u}{\partial y} \right)^2 - \varepsilon$$

$$U \frac{\partial \varepsilon}{\partial x} = \frac{1}{y} \frac{\partial}{\partial y} \left( y C_\mu \sigma_\varepsilon \frac{k^2}{\varepsilon} \frac{\partial \varepsilon}{\partial y} \right) + C_{\varepsilon 1} C_\mu k \left( \frac{\partial u}{\partial y} \right)^2 - C_{\varepsilon 2} \frac{\varepsilon^2}{k}$$

- **Momentum constraint**

$$\begin{aligned} \frac{\pi}{4} \rho_j V_j^2 d^2 &= const = 2\pi \rho_\infty \int_0^\infty U u y dy \\ &= 2\pi \rho_\infty U u_s l^2 \int_0^\infty f \xi d\xi \end{aligned}$$

# Classical Self-Similar Solution: Far-Field

- Traditional length and velocity scales (axisymmetric wake analogy)

$$u_s = A^* U d^{2/3} x^{-2/3} \quad ; \quad l = B^* d^{2/3} x^{1/3}$$

- Connection to turbulence model

$$k_s = C A^{*2} U^2 d^{4/3} x^{-4/3} \quad \varepsilon_s = D \frac{A^{*3}}{B^*} \frac{U^3}{d} d^{7/3} x^{-7/3}$$

- Where  $k = k_s(x)g(\xi)$  ;  $\varepsilon = \varepsilon_s(x)h(\xi)$   $u = u_s(x)f(\xi)$

- Tennekes and Lumley solution for  $f(\xi)$  assume  $\frac{g^2}{h} = 1$  define  $\alpha = 1$

$$\alpha \left( 2f + \xi \frac{df}{d\xi} \right) = \frac{d}{d\xi} \left( \frac{g^2}{h} \frac{df}{d\xi} \right) \quad f = \exp\left(-\frac{1}{2} \xi^2\right)$$



# Classical Self-similar Solution: Far-Field

- Using empirical input (effective viscosity)

$$\text{Re}_T = \frac{u_s l}{\nu} = 14.1$$

- Compute length(spreading ) and velocity decay

$$\frac{u_s}{U} = A^* \left(\frac{d}{x}\right)^{2/3} \approx \frac{1}{2} \left(\frac{\text{Re}_T}{3}\right)^{2/3} J^{1/3} \left(\frac{d}{x}\right)^{2/3}$$

$$\frac{l}{d} = B^* \left(\frac{x}{d}\right)^{1/3} = \frac{1}{2} \left(\frac{3}{\text{Re}_T}\right)^{1/3} J^{1/3} \left(\frac{x}{d}\right)^{1/3}$$

- These are axisymmetric wake scaling laws

# Jet Trajectory: Classical 1/3 law scaling

- Counter-rotating vortex pair CVP induce velocity

$$W = \frac{dz}{dt} = \frac{\Gamma}{4\pi L}$$

- Jet impulse (circulation)

$$\frac{\pi}{4} \rho_j V_j^2 d^2 = 2 \rho_\infty U \Gamma L$$

- Combine with spreading rate:  $\frac{l}{d} = 0.3 J^{1/3} \left(\frac{x}{d}\right)^{1/3}$

$$\left(\frac{z}{d}\right) = \begin{cases} 3(0.54)J^{1/3} \left(\frac{x}{d}\right)^{1/3} = 1.62J^{1/3} \left(\frac{x}{d}\right)^{1/3} & ; \quad L = 0.8l \\ 3(0.71)J^{1/3} \left(\frac{x}{d}\right)^{1/3} = 2.13J^{1/3} \left(\frac{x}{d}\right)^{1/3} & ; \quad L = 0.7l \end{cases}$$

- Good agreement with measurement and theory: Broadwell and Breidenthal<sup>12</sup>, Greitzer, et. al.<sup>19</sup> and Durando<sup>20</sup>

# Jet-in Crossflow: Near Field

- Classical far-field behavior is well modeled via axisymmetric wake scaling

$$u_s = A^* U d^{2/3} x^{-2/3} \quad ; \quad l = B^* d^{2/3} x^{1/3}$$

- But a different near-field scaling has also been observed (Hasslebrink and Mungal (2001))

$$u_s = A^* U d^{1/2} x^{-1/2} \quad ; \quad l = B^* d^{1/2} x^{1/2}$$

- This law reverts to the wake scaling far-downstream
- How can we reconcile two different scaling laws in a self-similar analysis?
- **We can compute turbulence equation parameters (constants) that honor near field**
- **But solve the flow problem using the classical far-field**

# Near Field: K-ε

- Use near field scaling

$$u_s = A^* U d^{1/2} x^{-1/2} \quad ; \quad l = B^* d^{1/2} x^{1/2}$$

- TKE and ε scaling

$$k_s = C A^{*2} U^2 d^1 x^{-1} \quad \varepsilon_s = D \frac{A^{*3}}{B^*} \frac{U^3}{d} d^2 x^{-2}$$

- Where

$$k = k_s(x) g(\xi) \quad ; \quad \varepsilon = \varepsilon_s(x) h(\xi)$$

- Effective viscosity constraints

$$\frac{g^2}{h} \approx const \approx 1$$

$$\frac{D}{C_\mu C^2} = \text{Re}_T = const$$



# Near Field: K-ε Equations and Constraints

- Substitute into k-ε

$$\alpha(2g + \xi \frac{dg}{d\xi}) + \frac{1}{\xi} \frac{d}{d\xi} (\xi \frac{dg}{d\xi}) + \frac{1}{C} \left( \frac{df}{d\xi} \right)^2 - \text{Re}_T \frac{D}{C} h = 0$$

$$\alpha(4h + \xi \frac{dh}{d\xi}) + \frac{1}{\xi} \frac{d}{d\xi} (\xi \frac{dh}{d\xi}) + \frac{C_{\varepsilon 1}}{C} \left( \frac{df}{d\xi} \right)^2 - C_{\varepsilon 2} \text{Re}_T \frac{D}{C} \frac{h^2}{g} = 0$$

- To obtain bounded solution for  $\xi \rightarrow \infty$  require that dissipation terms cancel homogeneous term so that:

$$f(M)g_0 \text{Re}_T \frac{D}{C} = 2$$

$$C_{\varepsilon 2} \text{Re}_T \frac{D}{C} g_0 = 4$$

- Where we have used the linearization

$$\frac{h^2}{g} = \frac{g_0 g}{g} h \approx g_0 h$$

# K-ε Solutions and Constraints

- With constraint TKE equations can be solved:

$$g = \frac{1}{2C} \left( 2E_{i-1}\left(\frac{1}{2}\xi^2\right) - 2E_{i-1}(\xi^2) - \exp(-\xi^2) \right)$$

- Where  $E_{i-1}$  is the exponential integral  $E_{i-1}(x) = \int_x^\infty \frac{e^{-u}}{u} du$
- Solution permits us to estimate  $g_0$  used previously

$$g_0 = \frac{1}{2} (g(0) + g(\infty)) = \frac{1}{4C} (2\ln(2) - 1)$$

- Algebraic constraints; solve for  $C_\mu$

$$\frac{D}{C_\mu C^2} = \text{Re}_T \quad ; \quad \frac{2\ln(2)-1}{4C} \text{Re}_T \frac{D}{C} = 2$$

# K-ε Parameter Estimates

- solve for  $C_\mu$

$$C_\mu = \frac{8}{(2\ln(2)-1)} \text{Re}_T^{-2} = 0.1$$

- Dissipation constraint  $C_{\varepsilon 2} \text{Re}_T \frac{D}{C} g_0 = 4$       lets us estimate  $C_{\varepsilon 2}$

$$C_{\varepsilon 2} = 2$$

- Finally solve dissipation to give:

$$C_{\varepsilon 1} = g_0 = \left( \frac{2\ln(2)-1}{4(0.072)} \right) \approx 1.34$$

- Where the Bradshaw constraint:  $u'v' = \frac{2}{3}(0.45)k_s = 0.3k_s$       has been used.

# K- $\epsilon$ Turbulence Model Parameter Estimates

- Parameter estimates

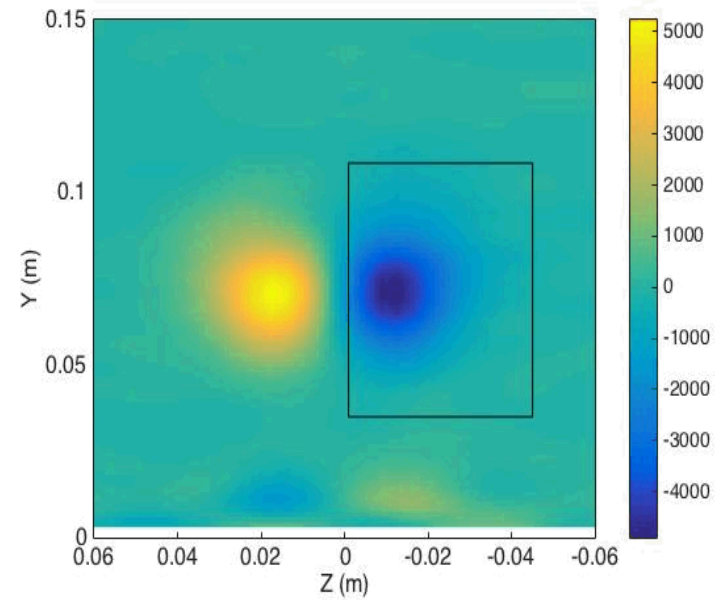
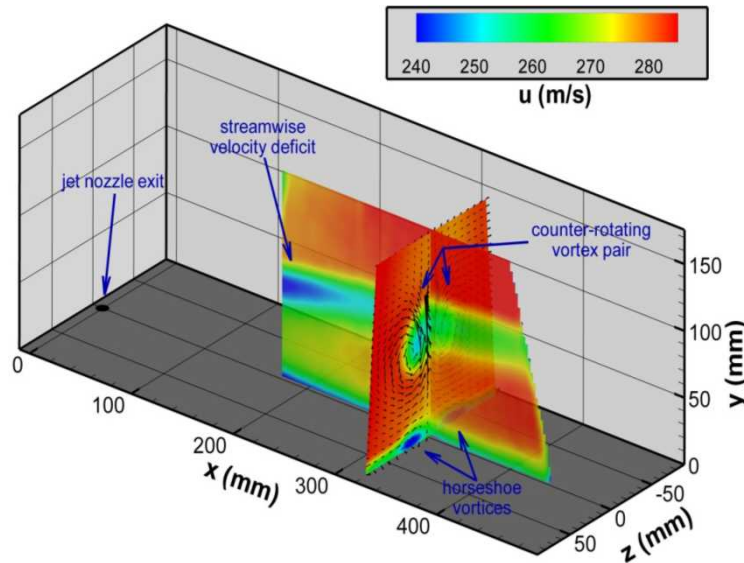
Self-Similar Model (current) (M=0)	0.10	1.34	2.00
Bayesian Model (M=0.8)	0.10	1.42	2.10
Relative Error ( $SS_{M=0}-B$ )/B(100%)	0%	-6%	-5%
Nominal	0.09	1.44	1.92
Relative Error (SS-N)/N(100%)	11%	-7%	4%

- Analytical solution based parameter estimates agree well with Bayesian calibration approach.
- Compressibility appears to have minimal effect



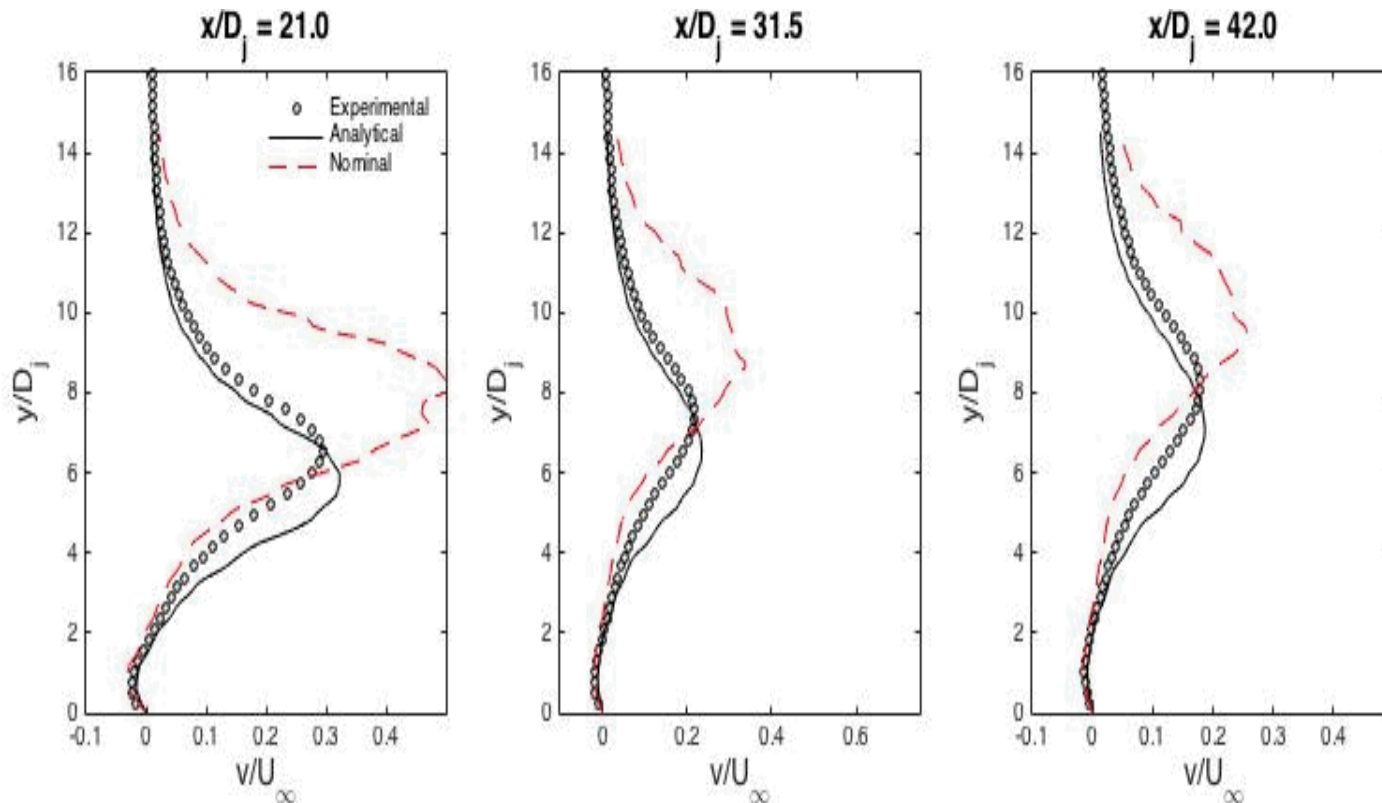
# Computation Using Analytical Parameters

- JIC RANS computation Sigma; J. Ray, S. Lefantzi, S. Arunajatesan
- Measurements: Beresh et. al. (2005)
- “v” velocity; Improvement over nominal



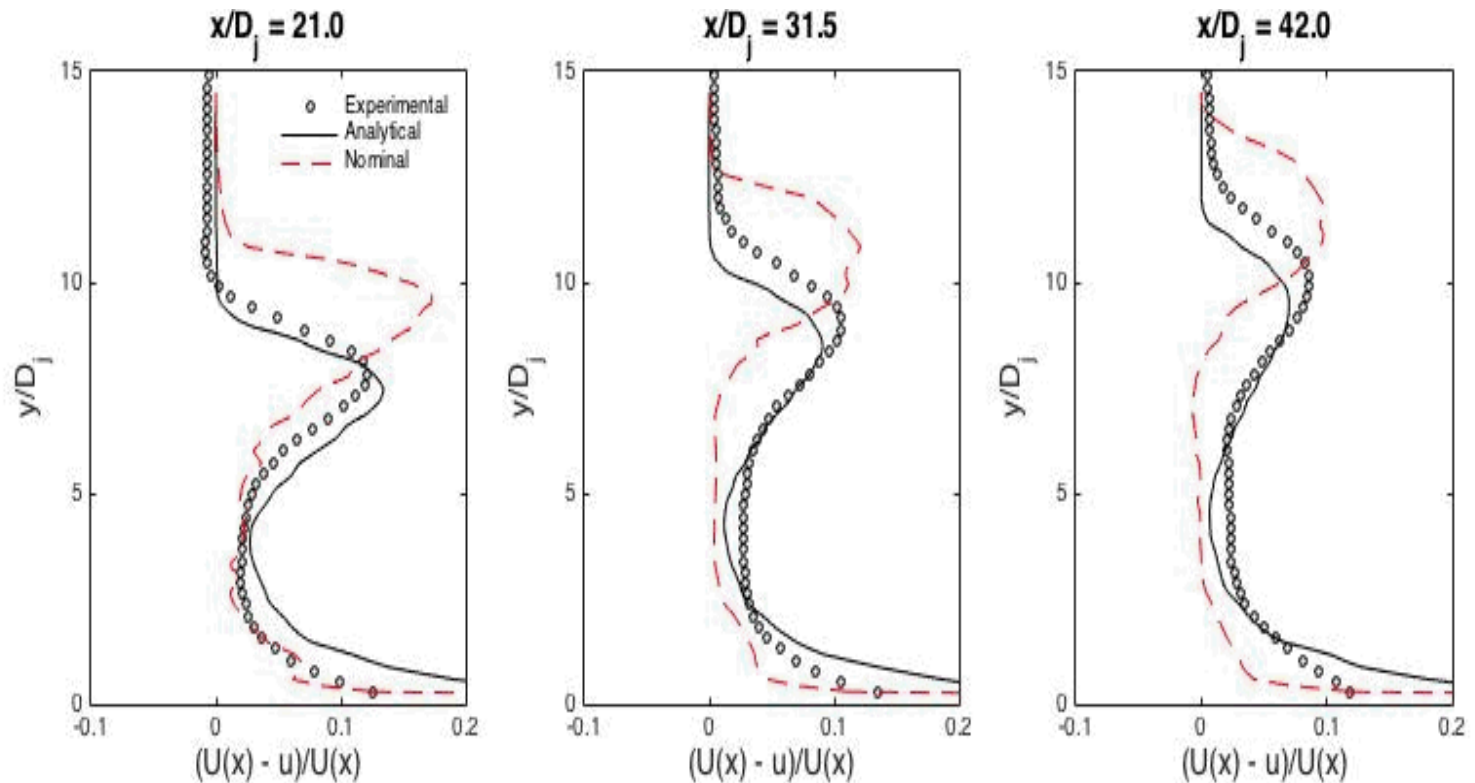
# Computation Using Analytical Parameters

- JIC RANS computation Sigma; J. Ray, S. Lefantzi, S. Arunajatesan
- Measurements: Beresh et. al.
- “v” velocity,  $M=0.8$ ; Improvement over nominal



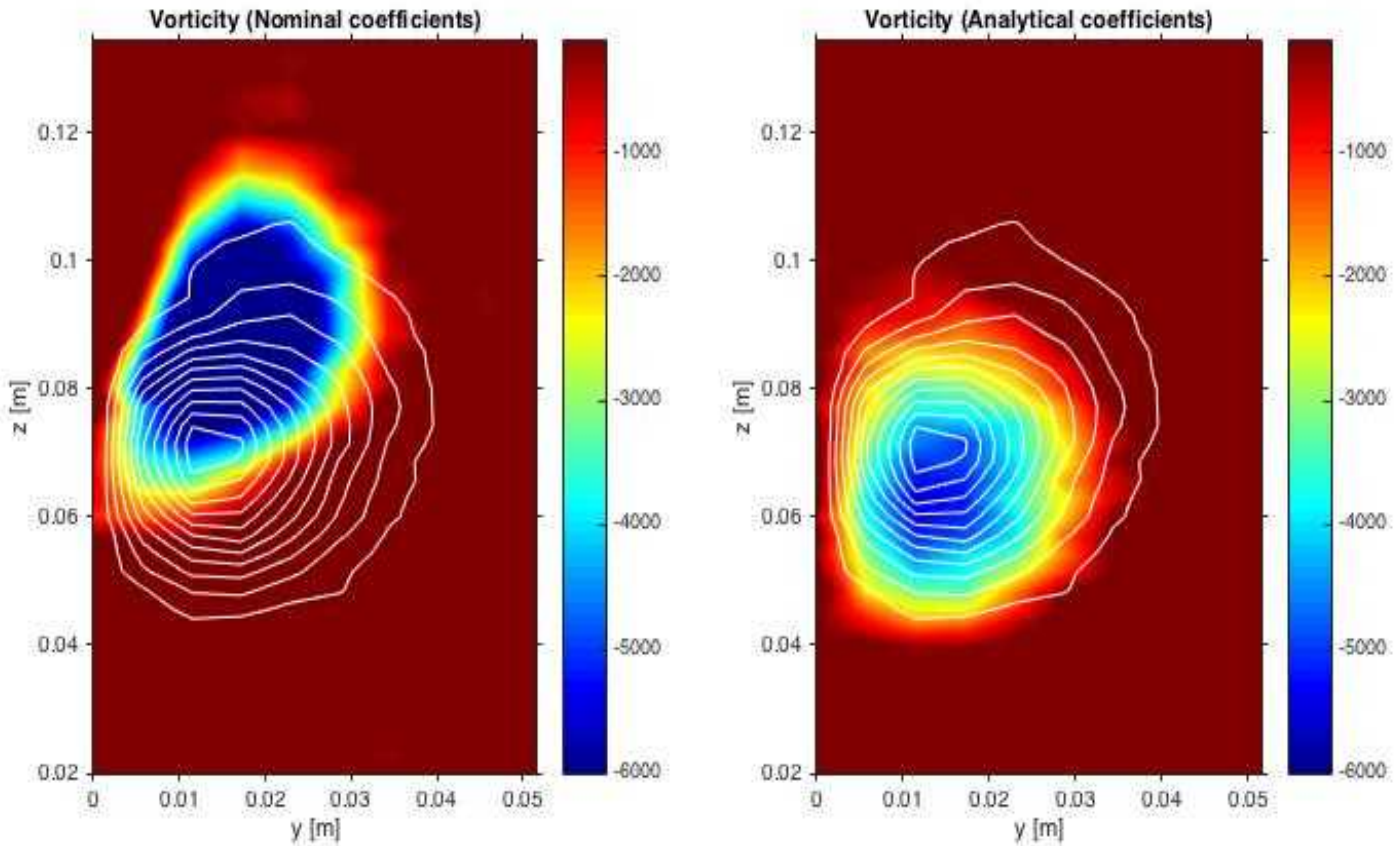
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- JIC RANS computation Sigma; J. Ray, S. Lefantzi, S. Arunajatesan
- Measurements: Beresh et. al.
- Velocity Deficit,  $M=0.8$ ; Improvement over nominal



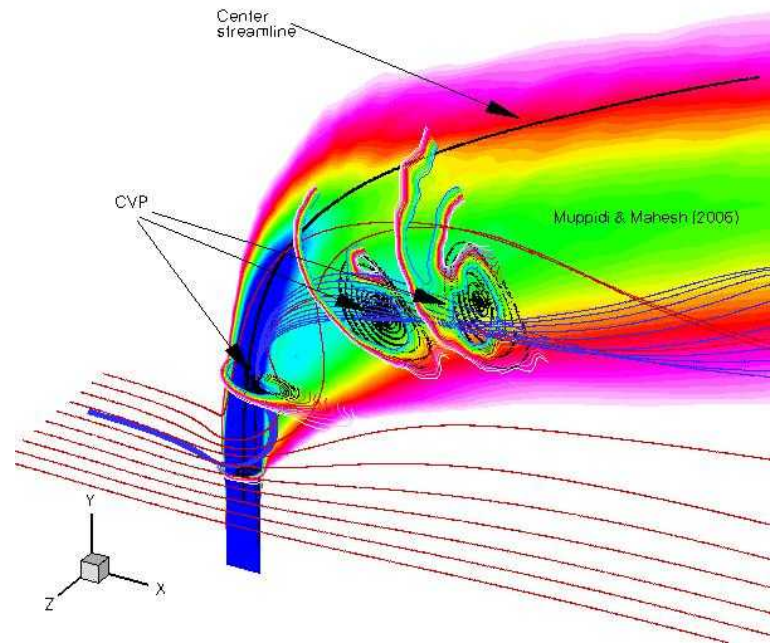
# Computation Using Analytical Parameters

- **Vorticity location:**



# Jet-in-Crossflow: Turbulence Parameters

- **Comments:**
- **Selection of jet-in-crossflow turbulence model parameters can be improved by utilization of approximate analytical, axisymmetric wake/jet solution**
- **Estimates support Bayesian turbulence model calibration (Ray et. al. 2016)**
- **Analytical modeling supports computational modeling**





# Conclusions

- **Jet-in-Crossflow/Analytical Models**
  - Examined approximate analytical models for turbulent flow problems
  - Analytical approaches can provide a useful role in fluid dynamic modeling
  - They support experimental and computational studies
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- **National Nuclear Security Laboratory**
- **“Exceptional Service in the National Interest”**
- **Please consider us!**
- <http://www.sandia.gov/careers/>