



Engineering Sciences and National Security Applications at Sandia National Laboratories

Turbulent Jet-in-Crossflow Insights from Approximate Analytical Solution Methods

Lawrence J. DeChant

Aero Sciences Group; Engineering Sciences Center

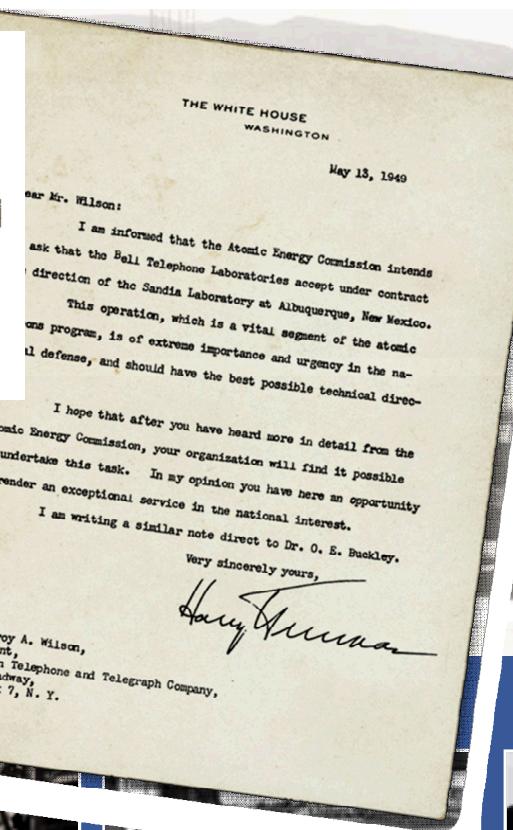
10/24/2017



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Sandia's History

Exceptional service in the national interest



Sandia's Governance Structure



Government-owned
contractor



Sandia Corporation

- AT&T / Bell Labs: 1949 – 1993
- Martin Marietta: 1993 – 1995
- Lockheed Martin: 1995 – April 30, 2017
- NTESS: May 1, 2017 -

Federally funded research
and development center



Sandia Addresses National Security Challenges



1950s

Nuclear weapons

Production and manufacturing engineering



1960s

Development engineering

Vietnam conflict



1970s

Multiprogram laboratory

Energy crisis



1980s

Missile defense work

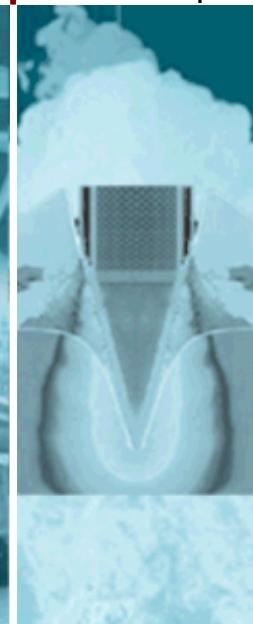
Cold War



1990s

Post-Cold War transition

Stockpile stewardship



2000s

START Post 9/11

National security



2010s

LEPs
Cyber, biosecurity
proliferation

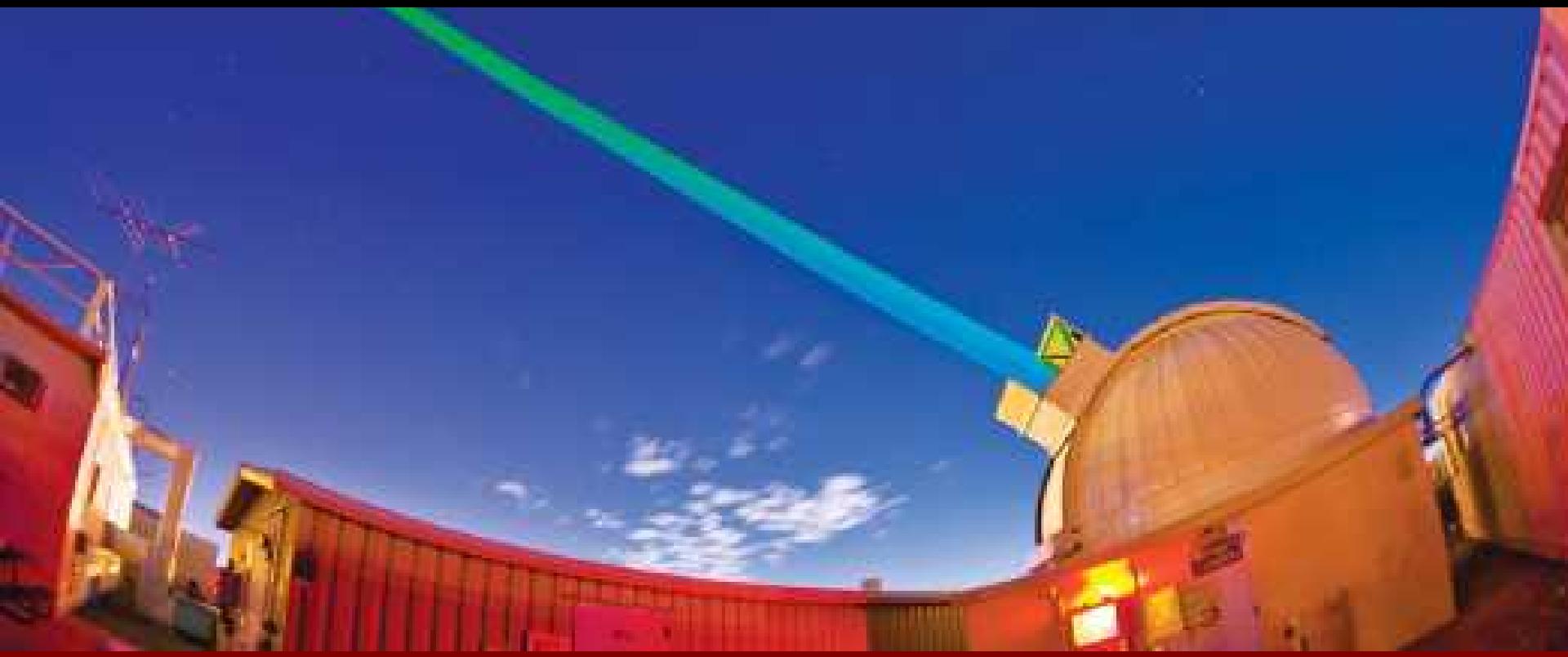
Evolving national security challenges



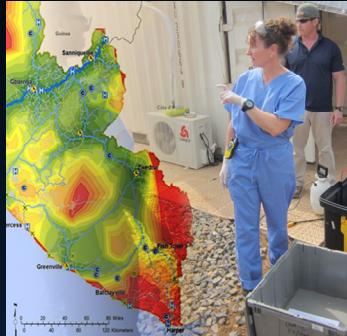
Sandia - Today



As a multi-faceted national security laboratory, Sandia has delivered essential science and technology for more than 60 years and plays a critical role in ensuring U.S. technical superiority.



Sandia's Impact



Ebola Outbreak

Sandia contributes to global response of Ebola outbreak by developing a sample delivery system cutting the wait time and potentially fatal exposure.



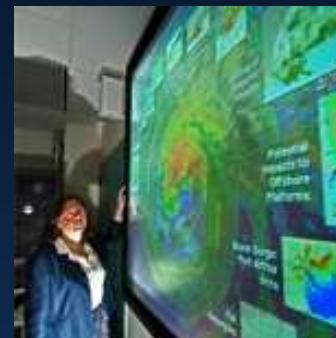
Detecting IEDs

Combat personnel now have a new tool for uncovering improvised explosive devices: Sandia's highly modified miniature synthetic aperture radar system, which is being transferred to the U.S. Army.



Cleanroom invented 1963

\$50 billion worth of cleanrooms built worldwide. It's used in hospitals, laboratories and manufacturing plants today.



Hurricane Katrina

Sandia is called to assess flooding and infrastructure failures.



Fukushima Quake

Sandia helps clean up radioactive wastewater.



9/11

Sandia sets contingency plans for release of materials and aircraft attacks on critical facilities immediately after 9/11. Search dogs are equipped with cameras for search and rescue K-9 handlers. The capability allowed search efforts to be carried out in spaces inaccessible to humans.

Sandia has two main locations



Science labs



Nuclear energy lab



Fossil energy lab



Energy efficiency and
renewable energy lab

Sandia Sites



Albuquerque, New Mexico



Livermore, California

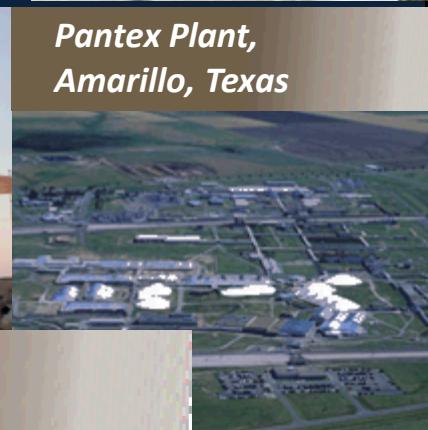


Kauai, Hawaii



*Waste Isolation Pilot Plant,
Carlsbad, New Mexico*

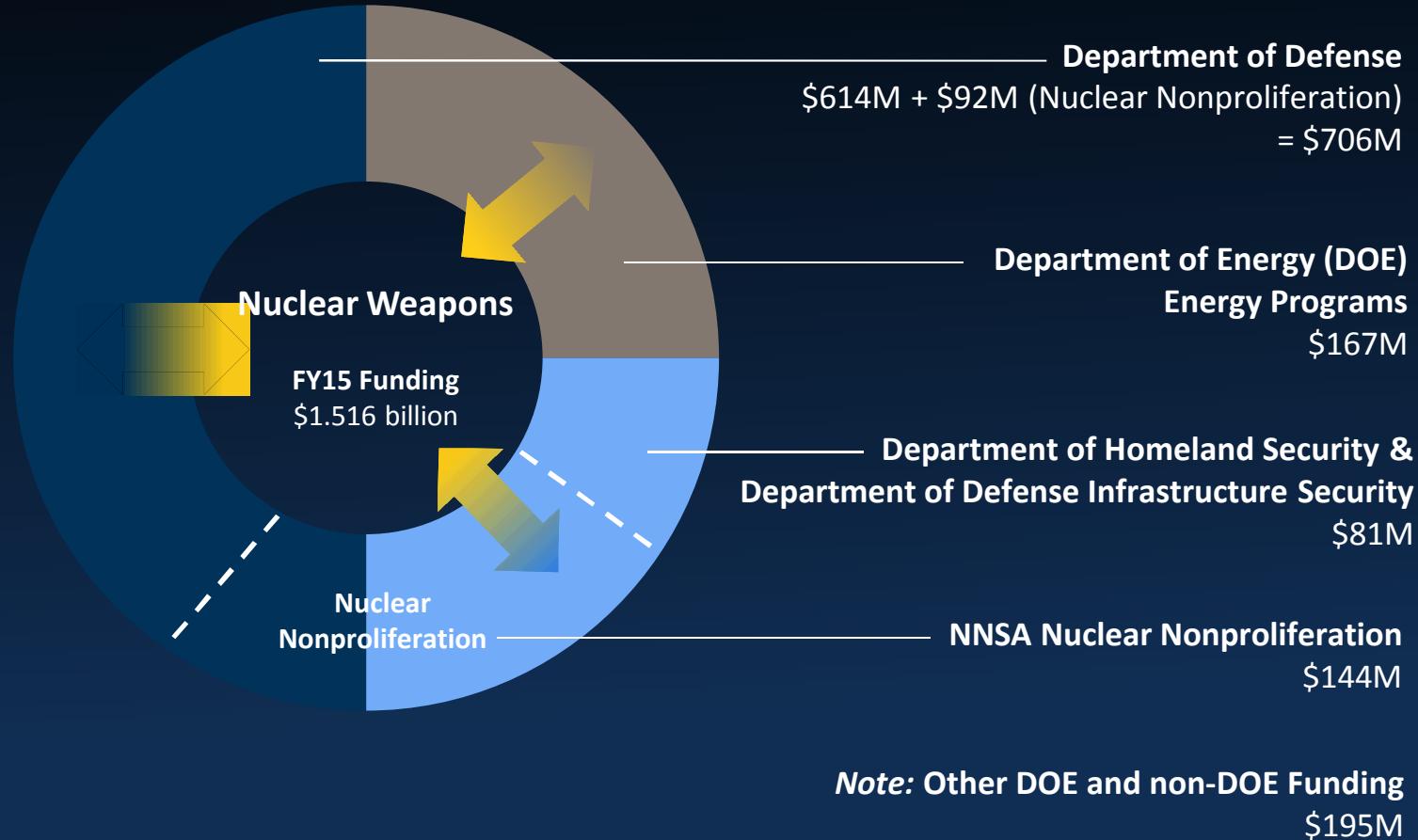
*Pantex Plant,
Amarillo, Texas*



*Tonopah,
Nevada*



Sandia's Funding - ~\$2.8 Billion



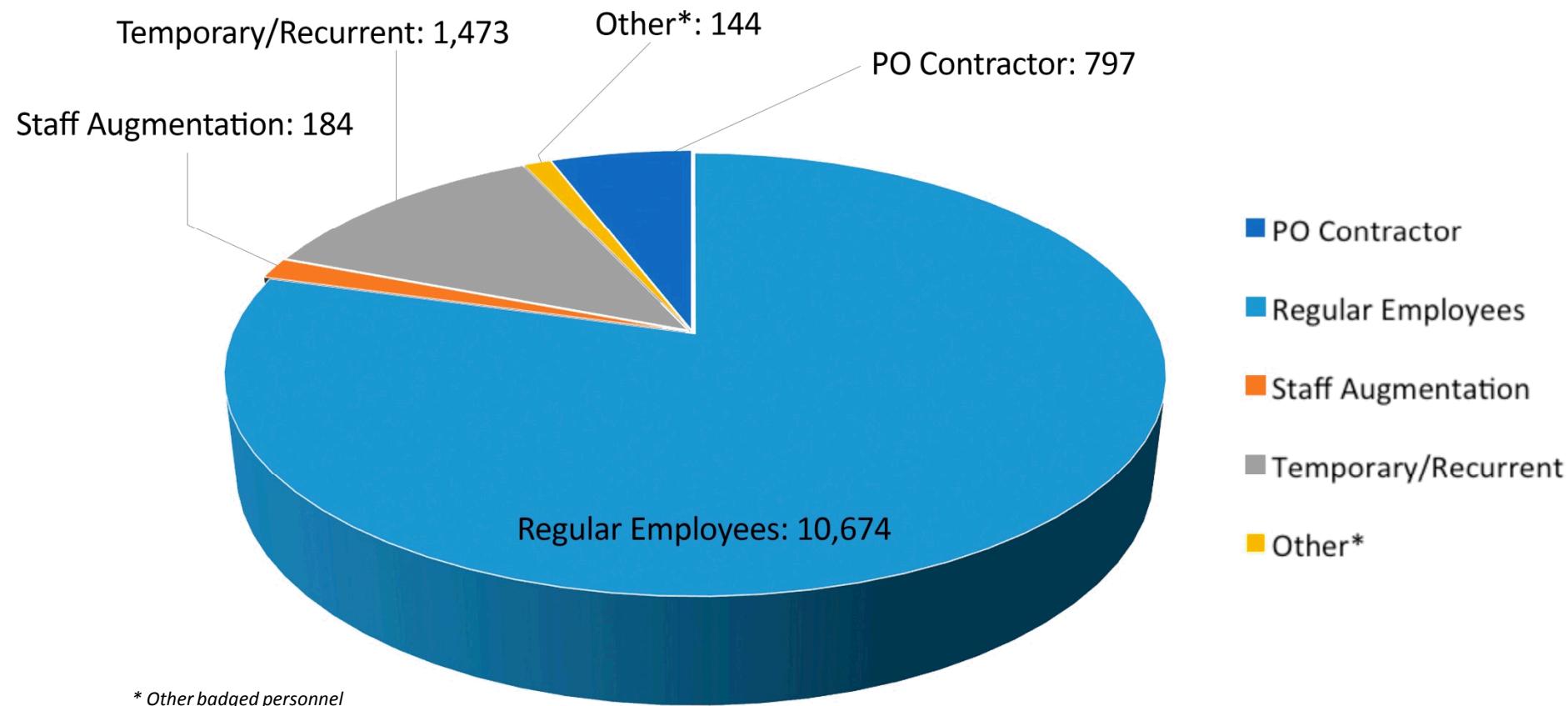
High reliability, high consequence of failure, challenging environments, and technology solutions

Our Workforce



- Total Sandia workforce: 13,332
- Regular employees: 10,574
- Advanced degrees: 6,085 (57%)

Data as of January 31, 2017



Fulfilling Our National Security Mission



Nuclear Weapons



Global Security



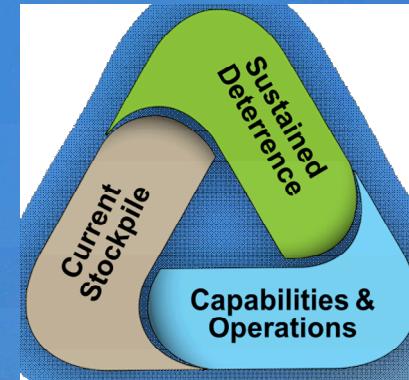
Energy & Climate



Defense Systems & Assessments

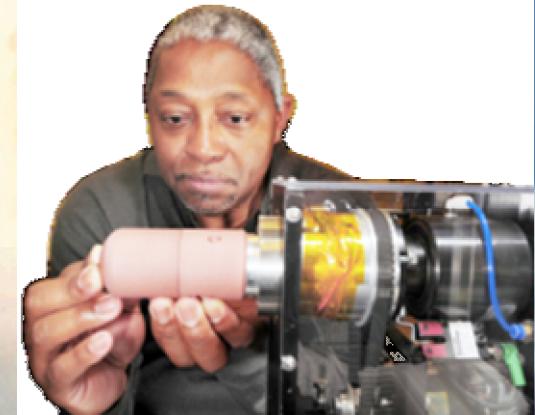
Nuclear Weapons

Sandia assumes an increasingly pivotal role in sustaining the nation's nuclear deterrent.



Roles:

- Design
- Production
- Warhead systems engineering



Global Security

Nonproliferation

Critical Asset Protection

Global Security

Remote Sensing

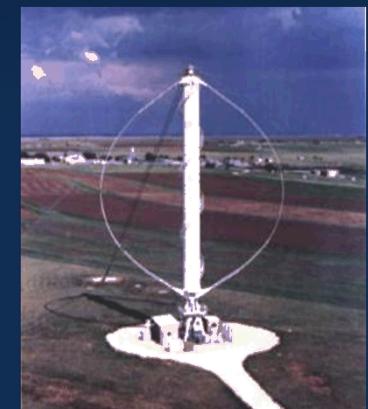
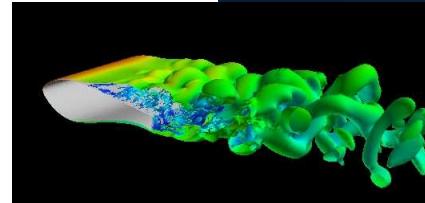


Energy & Climate



Energy Security
Climate Security

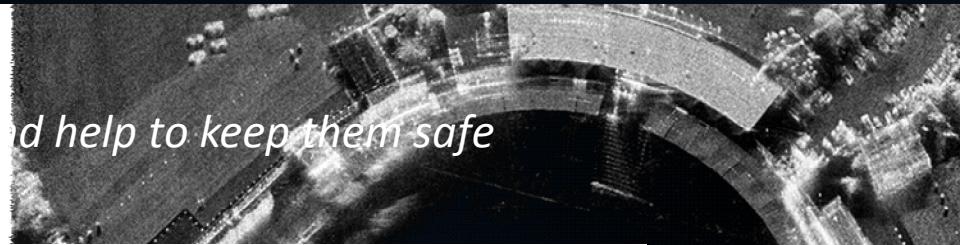
Nature Security
Testing Capabilities
Renewable Energy



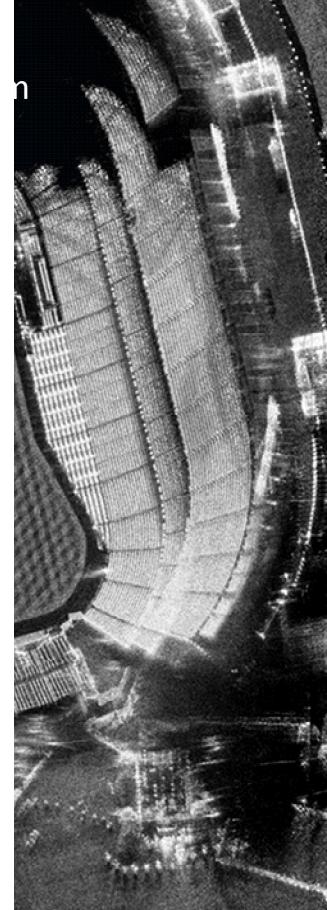
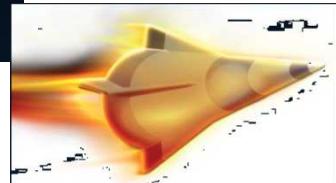
Defense Systems & Assessments



We support our troops around the world



and help to keep them safe



Our Foundations in Research

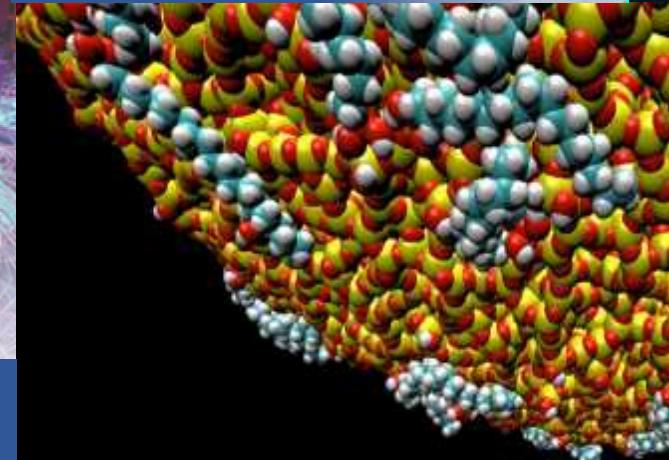
We support essential research-and-discovery activities that translate into invention, innovation, entrepreneurship, economic opportunity, and public benefit.



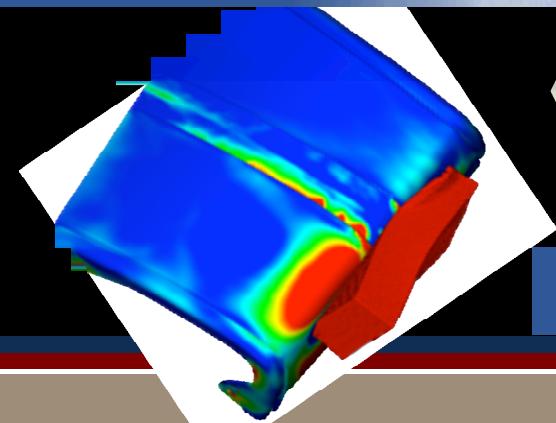
Computing & Information Sciences



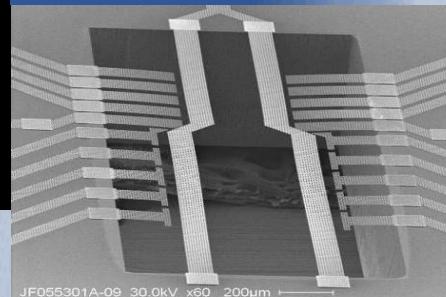
Materials Science



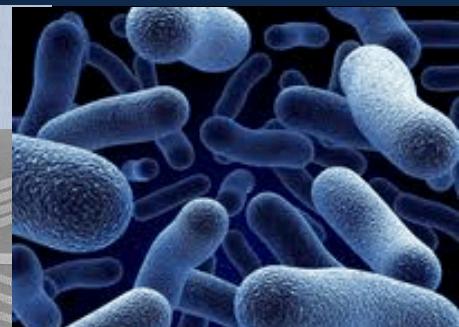
Engineering Sciences



Nanodevices & Microsystems



Geoscience



Bioscience

We Steward Key R&D Facilities for the Nation



Combustion Research Facility



High Performance Computing



Environmental Testing



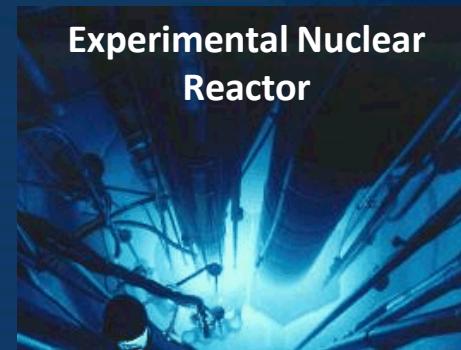
Ion Beam Laboratory



Microelectronics Fabrication



Z Machine



Experimental Nuclear Reactor



Center for Integrated Nanotechnologies (CINT)

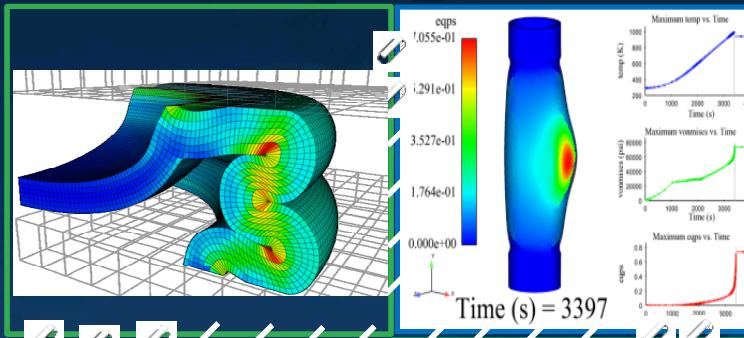
Engineering Sciences Center



Mission: *To provide validated, science-based engineering expertise and solutions across the life cycle of products to inform engineering decisions.*

- We integrated theory, computational simulation and experimental discovery/validation across length and time scales is critical to develop the technical basis for complex systems.
- We partner internally and externally to advance our knowledge base and tool sets to provide physical and computational simulation capabilities that support the development and deployment of innovative, mission-driven products and services.

Engineering Analysis



Engineering Science Physical Phenomena

Environmental Simulation & Test

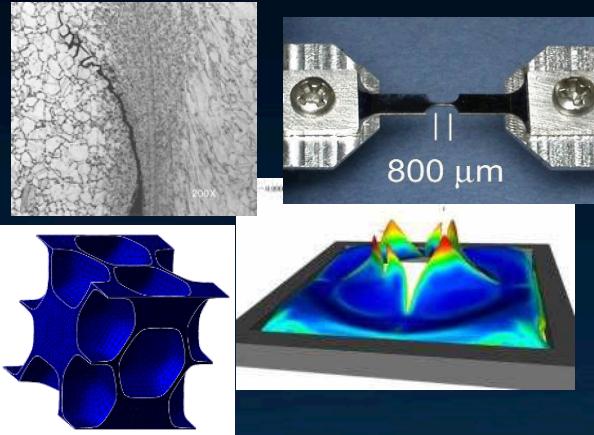


Computational Simulation Technology

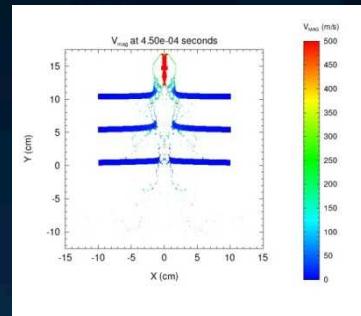
Engineering Sciences Core Technical Areas



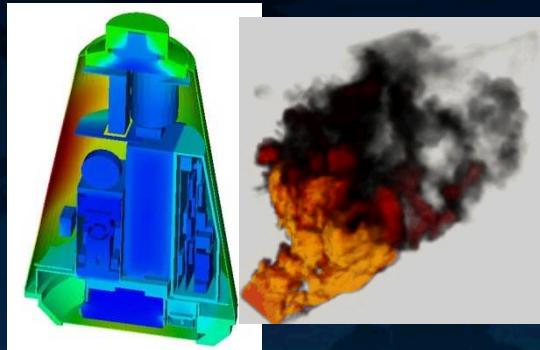
Solid Mechanics



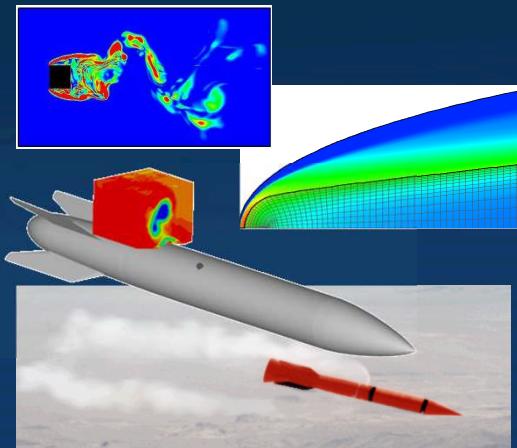
Shock Physics and Energetics



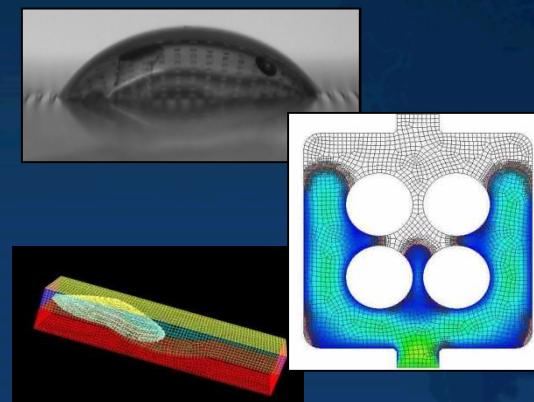
Thermal and Combustion Sciences



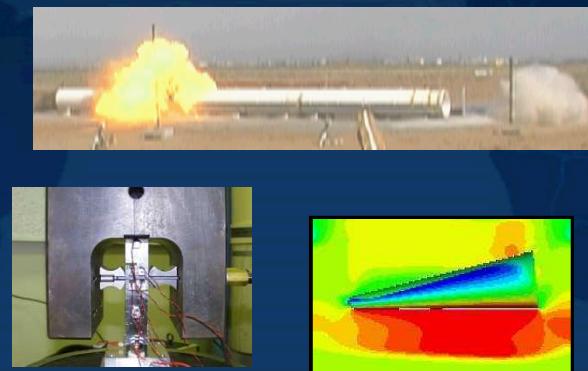
Aerosciences



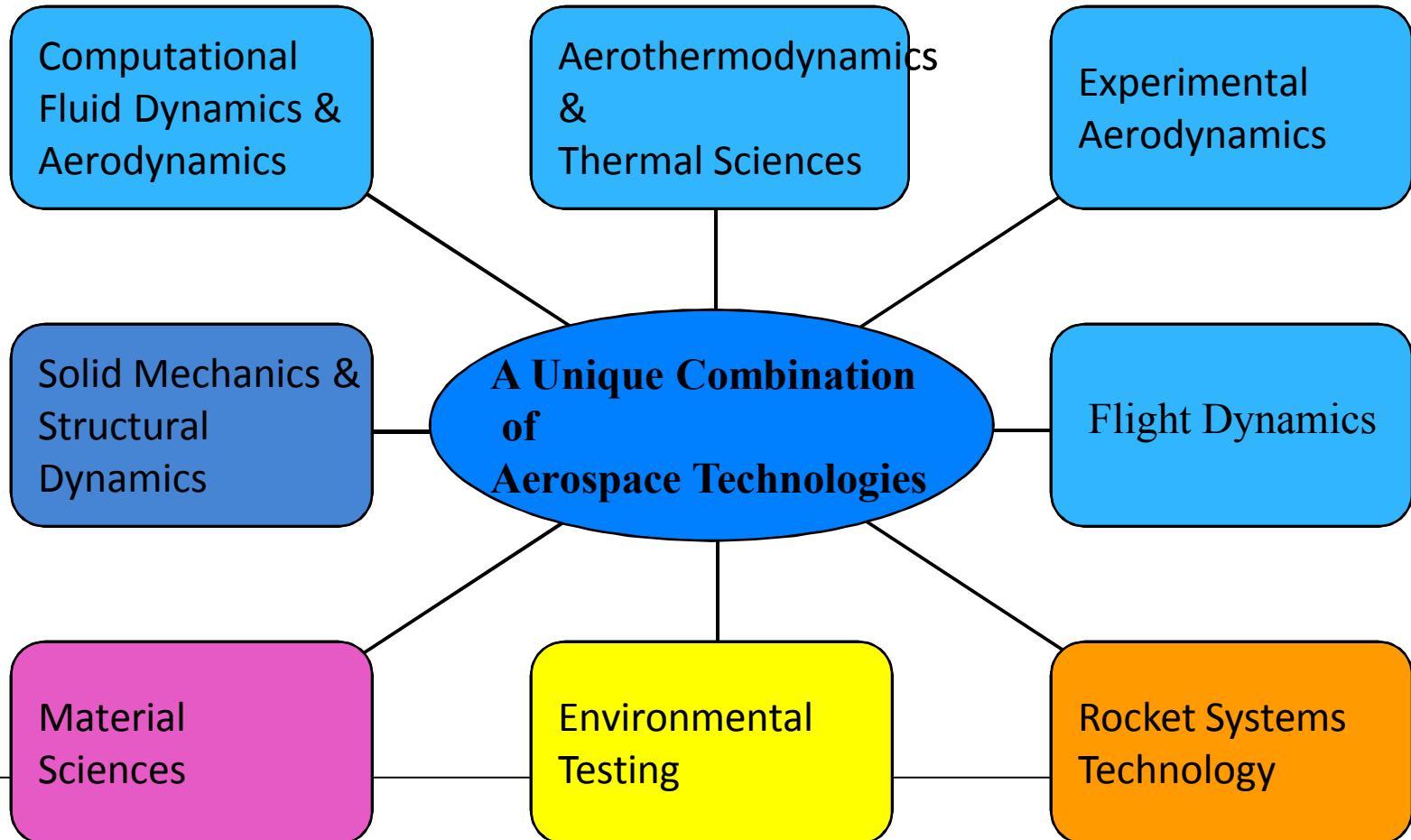
Fluid Mechanics



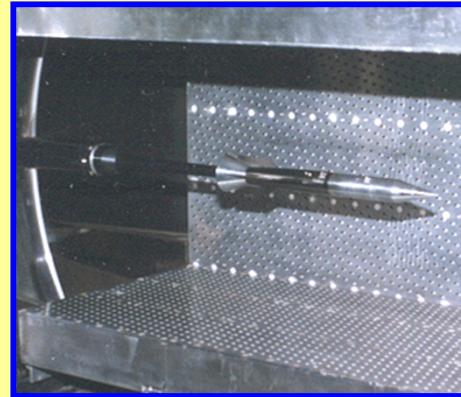
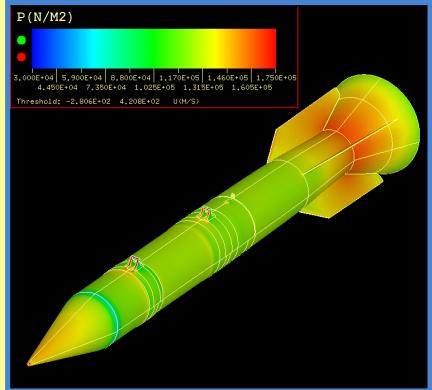
Structural Dynamics



Aerosciences and Aerospace Technologies at Sandia



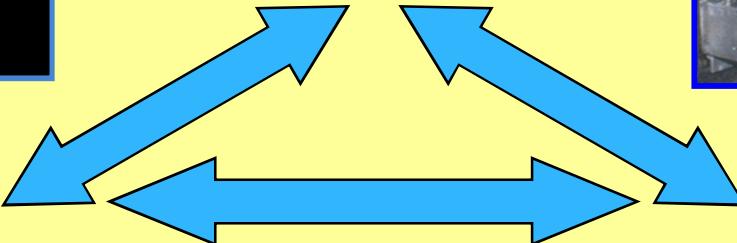
Aeroscience at Sandia



Modeling & Simulation

Flight Test

Ground Test



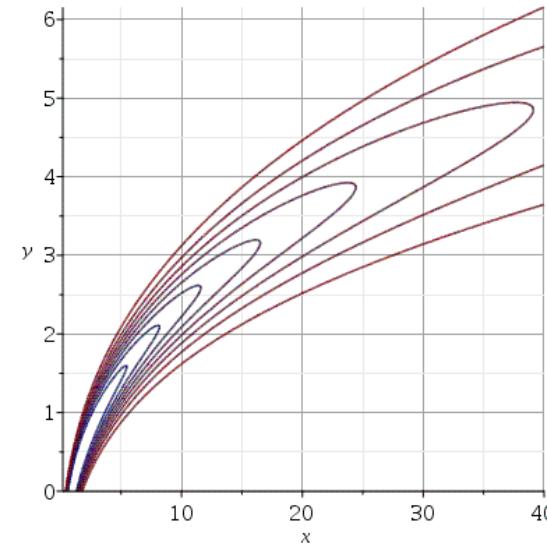
Engineering Solution to Complex Problems:

- Through a sustained R->D->A effort
- By applying combination of ground testing, modeling and simulation and flight testing
- **Mod-Sim:**
- High Performance Computing Fluid Dynamic Simulation
- Classical Fluid Mechanics

Analytical Methods in Fluid Mechanics

■ Fluid Dynamics Problems

- Experimental Measurements
- Computational Fluid Dynamics (CFD)
- Analytical methods

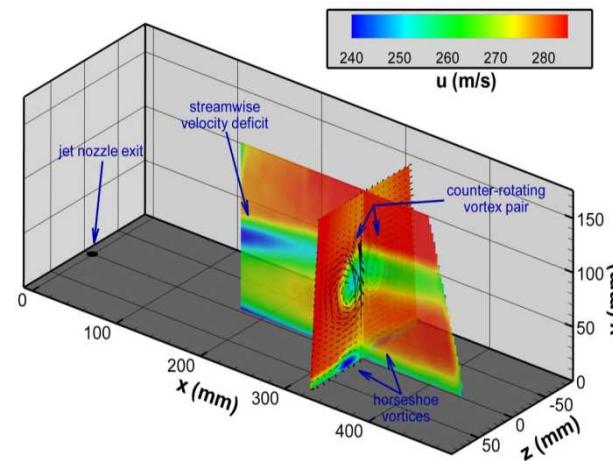
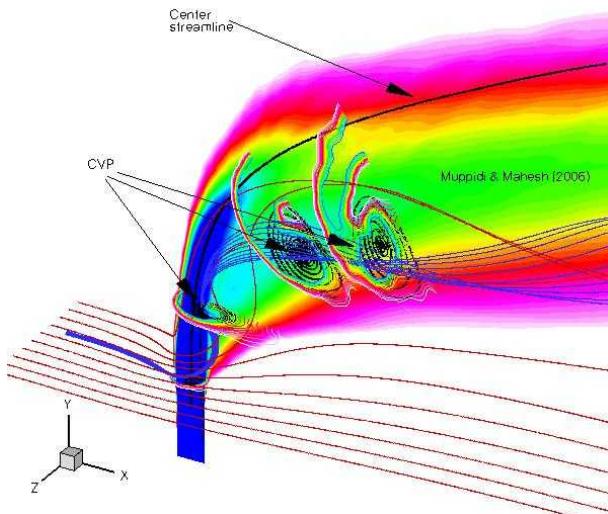


■ Analytical Methods

- Unique type of insight
- Limited applicability
- Often asymptotic
- Virtually always approximate in terms of formulation, solution or (likely) both

$$\frac{V(x,y)}{V_j} = \frac{1}{2} \left[\operatorname{erf} \left(\frac{2x^{1/3} - y - 2\left(\frac{1}{2}\right)^{1/3}}{(2K)^{1/2} x^{2/3}} \right) - \operatorname{erf} \left(\frac{2x^{1/3} - y - 2\left(\frac{3}{2}\right)^{1/3}}{(2K)^{1/2} x^{2/3}} \right) \right]$$

Jet-in-Crossflow: Turbulence Parameters



- Calibration of turbulence model parameters using data(computational/measurements) can be a highly effective procedure (Ray et. al. 2016)
- Turbulence model parameters have traditionally been estimated by demanding recovering of simplified/canonical flows
- Here we utilize a classical self-similar, axisymmetric wake/jet solution to provide estimates for a $k-\epsilon$ turbulence model implementation
- Simplified methods interacting with computational approaches

JIC Simplified Governing Equations

- **Approximate axisymmetric wake Model (Following Tennekes-Lumley, 1972)**

$$U \frac{\partial u}{\partial x} = \frac{1}{y} \frac{\partial}{\partial y} \left(y C_\mu \frac{k^2}{\varepsilon} \frac{\partial u}{\partial y} \right)$$

- **Turbulence model (k- ε)**

$$U \frac{\partial k}{\partial x} = \frac{1}{y} \frac{\partial}{\partial y} \left(y C_\mu \sigma_k \frac{k^2}{\varepsilon} \frac{\partial k}{\partial y} \right) + C_\mu \frac{k^2}{\varepsilon} \left(\frac{\partial u}{\partial y} \right)^2 - \varepsilon$$

$$U \frac{\partial \varepsilon}{\partial x} = \frac{1}{y} \frac{\partial}{\partial y} \left(y C_\mu \sigma_\varepsilon \frac{k^2}{\varepsilon} \frac{\partial \varepsilon}{\partial y} \right) + C_{\varepsilon 1} C_\mu k \left(\frac{\partial u}{\partial y} \right)^2 - C_{\varepsilon 2} \frac{\varepsilon^2}{k}$$

- **Momentum constraint**

$$\frac{\pi}{4} \rho_j V_j^2 d^2 = \text{const} = 2\pi \rho_\infty \int_0^\infty U u y dy$$

$$= 2\pi \rho_\infty U u_s l^2 \int_0^\infty f \xi d\xi$$

Classical Self-Similar Solution: Far-Field

- Traditional length and velocity scales (axisymmetric wake analogy)

$$u_s = A^* U d^{2/3} x^{-2/3} \quad ; \quad l = B^* d^{2/3} x^{1/3}$$

- Connection to turbulence model

$$k_s = C A^{*2} U^2 d^{4/3} x^{-4/3} \quad \varepsilon_s = D \frac{A^{*3}}{B^*} \frac{U^3}{d} d^{7/3} x^{-7/3}$$

- Where $k = k_s(x)g(\xi)$; $\varepsilon = \varepsilon_s(x)h(\xi)$ $u = u_s(x)f(\xi)$

- Tennekes and Lumley solution for $f(\xi)$ assume $\frac{g^2}{h} = 1$ define $\alpha = 1$

$$\alpha \left(2f + \xi \frac{df}{d\xi} \right) = \frac{d}{d\xi} \left(\frac{g^2}{h} \frac{df}{d\xi} \right) \quad f = \exp\left(-\frac{1}{2} \xi^2\right)$$

Classical Self-similar Solution: Far-Field

- Using empirical input (effective viscosity)

$$\text{Re}_T = \frac{u_s l}{\nu} = 14.1$$

- Compute length(spreading) and velocity decay

$$\frac{u_s}{U} = A^* \left(\frac{d}{x} \right)^{2/3} \approx \frac{1}{2} \left(\frac{\text{Re}_T}{3} \right)^{2/3} J^{1/3} \left(\frac{d}{x} \right)^{2/3}$$

$$\frac{l}{d} = B^* \left(\frac{x}{d} \right)^{1/3} = \frac{1}{2} \left(\frac{3}{\text{Re}_T} \right)^{1/3} J^{1/3} \left(\frac{x}{d} \right)^{1/3}$$

- These are axisymmetric wake scaling laws

Jet Trajectory: Classical 1/3 law scaling

- Counter-rotating vortex pair CVP induce velocity

$$W = \frac{dz}{dt} = \frac{\Gamma}{4\pi L}$$

- Jet impulse (circulation)

$$\frac{\pi}{4} \rho_j V_j^2 d^2 = 2 \rho_\infty U \Gamma L$$

- Combine with spreading rate:

$$\frac{l}{d} = 0.3 J^{1/3} \left(\frac{x}{d}\right)^{1/3}$$

$$\left(\frac{z}{d}\right) = \begin{cases} 3(0.54) J^{1/3} \left(\frac{x}{d}\right)^{1/3} = 1.62 J^{1/3} \left(\frac{x}{d}\right)^{1/3} & ; \quad L = 0.8l \\ 3(0.71) J^{1/3} \left(\frac{x}{d}\right)^{1/3} = 2.13 J^{1/3} \left(\frac{x}{d}\right)^{1/3} & ; \quad L = 0.7l \end{cases}$$

- Good agreement with measurement and theory: Broadwell and Breidenthal¹², Greitzer, et. al.¹⁹ and Durando²⁰

Jet-in Crossflow: Near Field

- Classical far-field behavior is well modeled via axisymmetric wake scaling

$$u_s = A^* U d^{2/3} x^{-2/3} ; \quad l = B^* d^{2/3} x^{1/3}$$

- But a different near-field scaling has also been observed (Hasslebrink and Mungal (2001))

$$u_s = A^* U d^{1/2} x^{-1/2} ; \quad l = B^* d^{1/2} x^{1/2}$$

- This law reverts to the wake scaling far-downstream
- How can we reconcile two different scaling laws in a self-similar analysis?
- We can compute turbulence equation parameters (constants) that honor near field**
- But solve the flow problem using the classical far-field**

Near Field: K- ε

- Use near field scaling

$$u_s = A^* U d^{1/2} x^{-1/2} \quad ; \quad l = B^* d^{1/2} x^{1/2}$$

- TKE and ε scaling

$$k_s = C A^{*2} U^2 d^1 x^{-1} \quad ; \quad \varepsilon_s = D \frac{A^{*3}}{B^*} \frac{U^3}{d} d^2 x^{-2}$$

- Where

$$k = k_s(x) g(\xi) \quad ; \quad \varepsilon = \varepsilon_s(x) h(\xi)$$

- Effective viscosity constraints

$$\frac{g^2}{h} \approx const \approx 1$$

$$\frac{D}{C_\mu C^2} = \text{Re}_T = const$$

Near Field: K- ε Equations and Constraints

- **Substitute into k- ε**

$$\alpha(2g + \xi \frac{dg}{d\xi}) + \frac{1}{\xi} \frac{d}{d\xi}(\xi \frac{dg}{d\xi}) + \frac{1}{C} \left(\frac{df}{d\xi} \right)^2 - \text{Re}_T \frac{D}{C} h = 0$$

$$\alpha(4h + \xi \frac{dh}{d\xi}) + \frac{1}{\xi} \frac{d}{d\xi}(\xi \frac{dh}{d\xi}) + \frac{C_{\varepsilon 1}}{C} \left(\frac{df}{d\xi} \right)^2 - C_{\varepsilon 2} \text{Re}_T \frac{D}{C} \frac{h^2}{g} = 0$$

- **To obtain bounded solution for $\xi \rightarrow \infty$ require that dissipation terms cancel homogeneous term so that:**

$$f(M)g_0 \text{Re}_T \frac{D}{C} = 2$$

$$C_{\varepsilon 2} \text{Re}_T \frac{D}{C} g_0 = 4$$

- **Where we have used the linearization**

$$\frac{h^2}{g} = \frac{g_0 g}{g} h \approx g_0 h$$

K- ε Solutions and Constraints

- With constraint TKE equations can be solved:

$$g = \frac{1}{2C} \left(2E_{i-1}\left(\frac{1}{2}\xi^2\right) - 2E_{i-1}(\xi^2) - \exp(-\xi^2) \right)$$

- Where E_{i-1} is the exponential integral $E_{i-1}(x) = \int_x^{\infty} \frac{e^{-u}}{u} du$
- Solution permits us to estimate g_0 used previously

$$g_0 = \frac{1}{2}(g(0) + g(\infty)) = \frac{1}{4C}(2 \ln(2) - 1)$$

- Algebraic constraints; solve for C_{μ}

$$\frac{D}{C_{\mu}C^2} = \text{Re}_T \quad ; \quad \frac{2 \ln(2) - 1}{4C} \text{Re}_T \frac{D}{C} = 2$$

K- ε Parameter Estimates

- solve for C_μ

$$C_\mu = \frac{8}{(2 \ln(2) - 1)} \text{Re}_T^{-2} = 0.1$$

- Dissipation constraint $C_{\varepsilon 2} \text{Re}_T \frac{D}{C} g_0 = 4$ lets us estimate $C_{\varepsilon 2}$

$$C_{\varepsilon 2} = 2$$

- Finally solve dissipation to give:

$$C_{\varepsilon 1} = g_0 = \left(\frac{2 \ln(2) - 1}{4(0.072)} \right) \approx 1.34$$

- Where the Bradshaw constraint: $u'v' = \frac{2}{3}(0.45)k_s = 0.3k_s$ has been used.

K- ε Turbulence Model Parameter Estimates

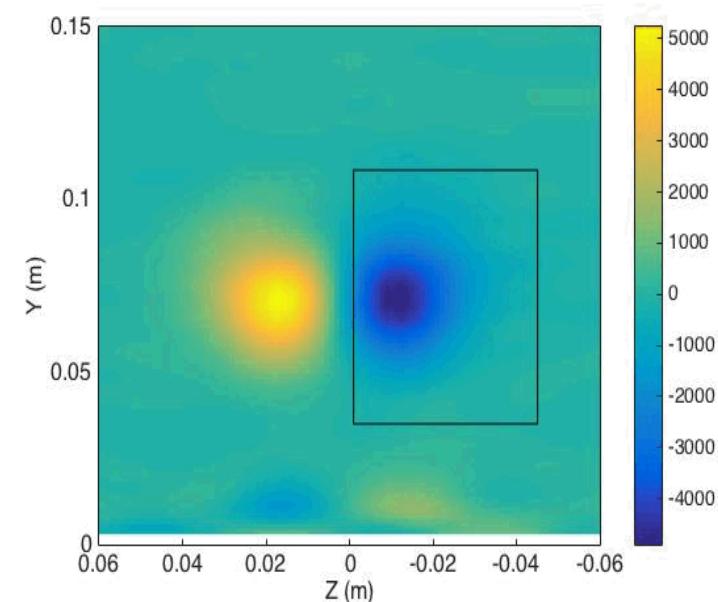
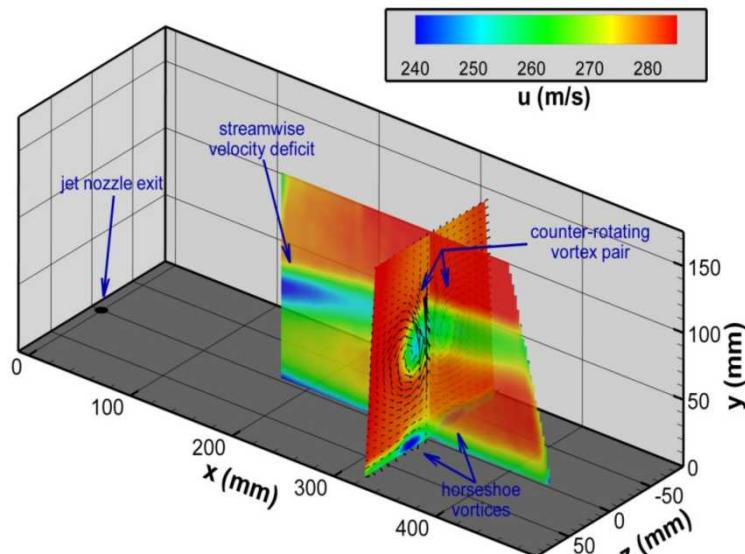
- Parameter estimates

Self-Similar Model (current) (M=0)	0.10	1.34	2.00
Bayesian Model (M=0.8)	0.10	1.42	2.10
Relative Error ($SS_{M=0}-B$)/B(100%)	0%	-6%	-5%
Nominal	0.09	1.44	1.92
Relative Error ($SS-N$)/N(100%)	11%	-7%	4%

- Analytical solution based parameter estimates agree well with Bayesian calibration approach.
- Compressibility appears to have minimal effect

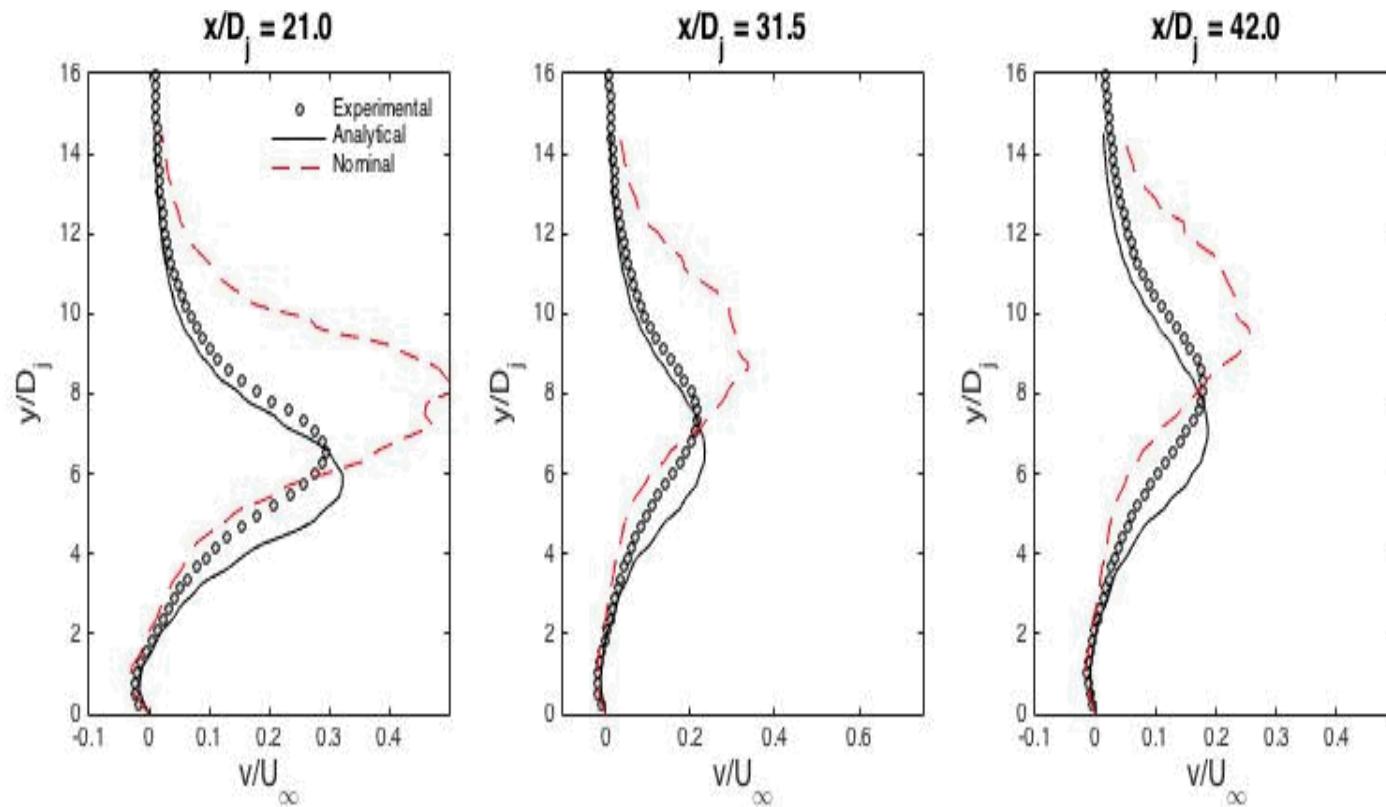
Computation Using Analytical Parameters

- JIC RANS computation Sigma; J. Ray, S. Lefantzi, S. Arunajatesan
- Measurements: Beresh et. al. (2005)
- “v” velocity; Improvement over nominal



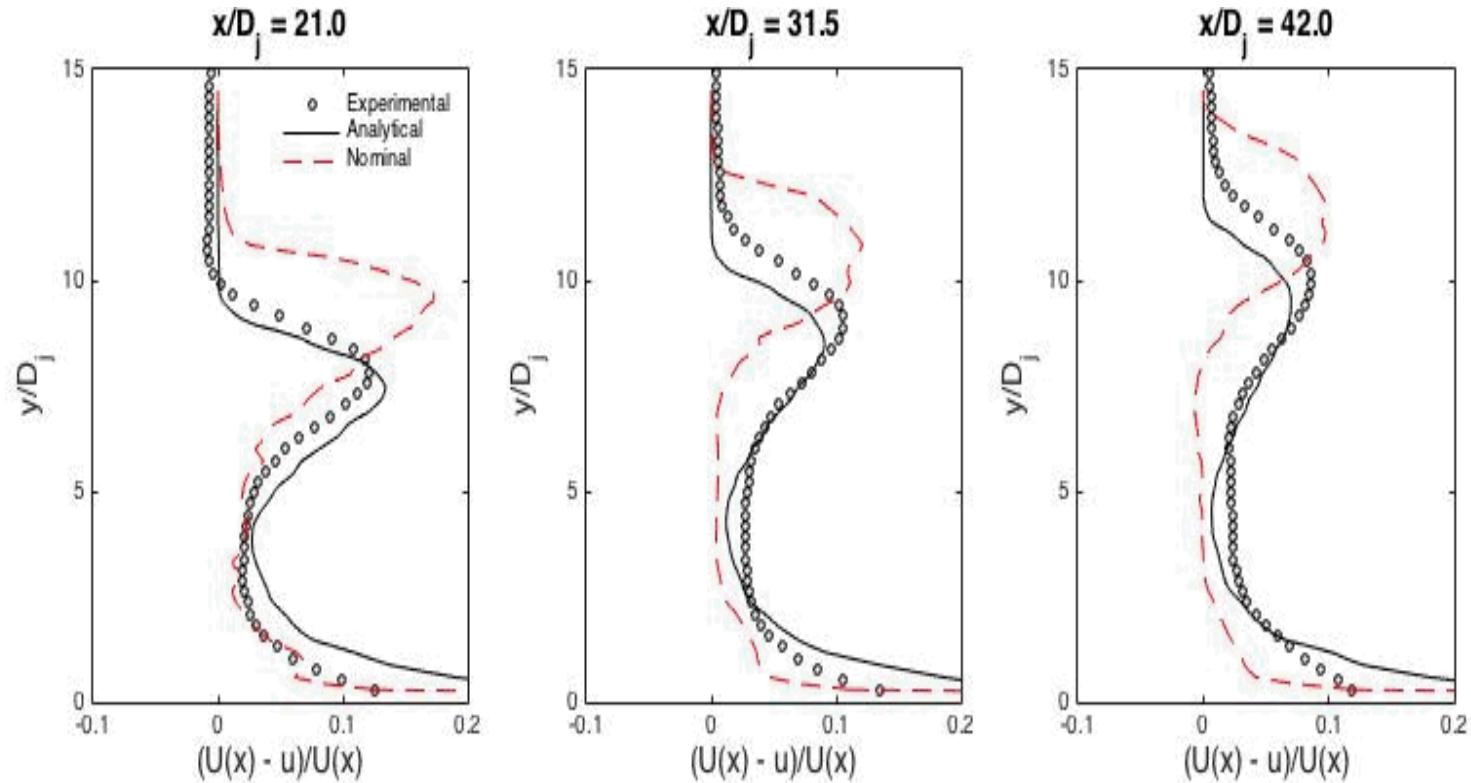
Computation Using Analytical Parameters

- JIC RANS computation Sigma; J. Ray, S. Lefantzi, S. Arunajatesan
- Measurements: Beresh et. al.
- “v” velocity, M=0.8; Improvement over nominal



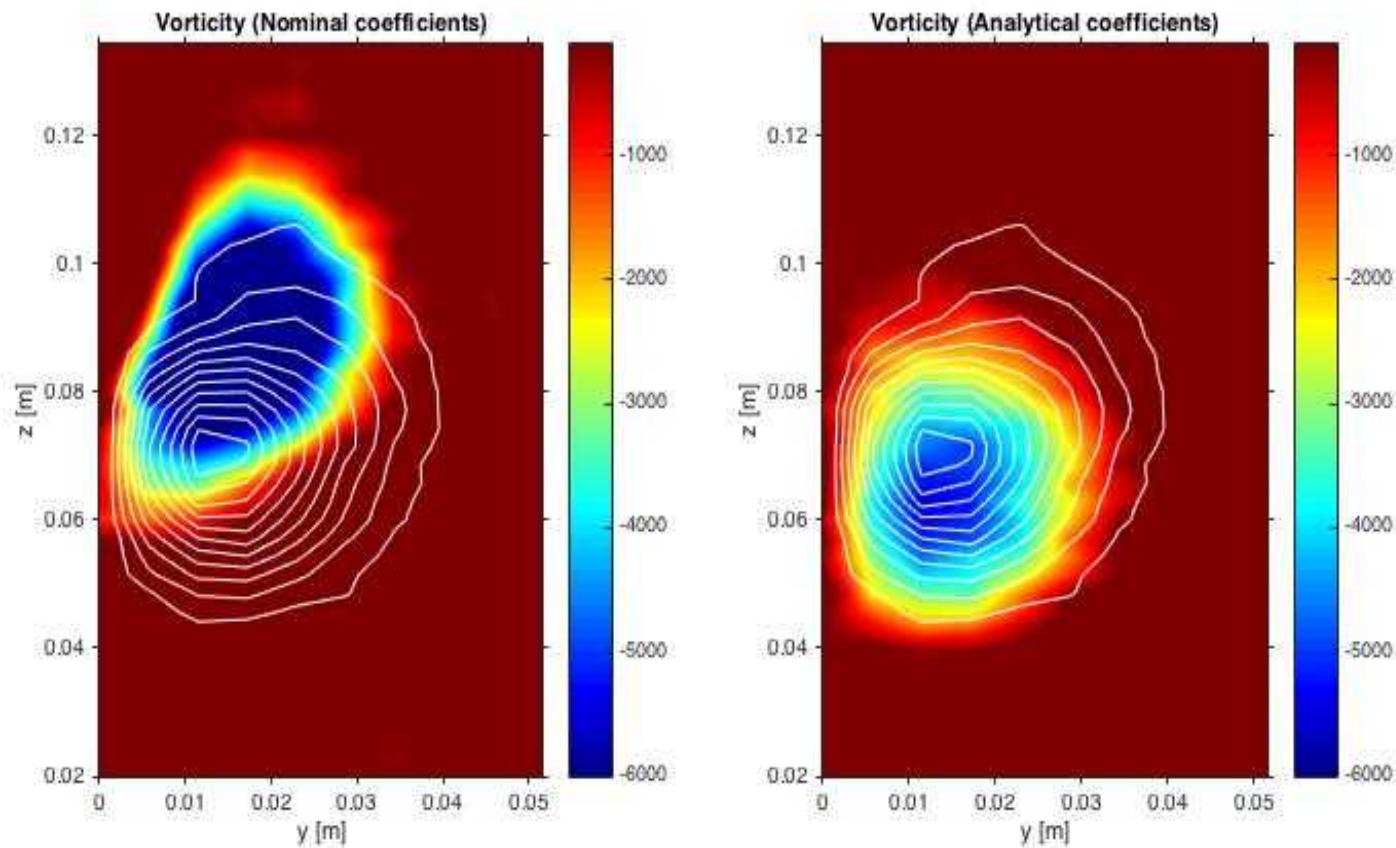
Computation Using Analytical Parameters

- JIC RANS computation Sigma; J. Ray, S. Lefantzi, S. Arunajatesan
- Measurements: Beresh et. al.
- Velocity Deficit, $M=0.8$; Improvement over nominal



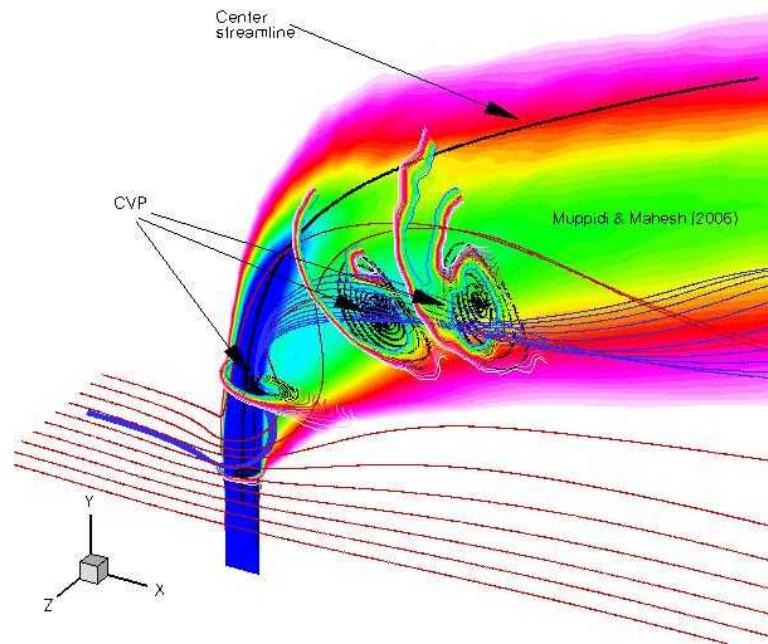
Computation Using Analytical Parameters

- **Vorticity location:**



Jet-in-Crossflow: Turbulence Parameters

- **Comments:**
- **Selection of jet-in-crossflow turbulence model parameters can be improved by utilization of approximate analytical, axisymmetric wake/jet solution**
- **Estimates support Bayesian turbulence model calibration calibration (Ray et. al. 2016)**
- **Analytical modeling supports computational modeling**



Conclusions

- **Jet-in-Crossflow/Analytical Models**
 - Examined approximate analytical models for turbulent flow problems
 - Analytical approaches can provide a useful role in fluid dynamic modeling
 - They support experimental and computational studies
- **Sandia National Laboratories**
- **National Nuclear Security Laboratory**
- **“Exceptional Service in the National Interest”**
- **Please consider us!**
- <http://www.sandia.gov/careers/>