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A Stable Low Frequency Time Domain EFIE with Weighted Continuity Equation

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***Abstract*—A new time domain electric field integral equation is proposed to solve low frequency problems. This new formulation uses the current and charge densities as unknowns, with a form of the continuity equation that is weighted by a Green's function as a second constraining equation. This equation can be derived from a scalar potential equivalence principle integral equation, which is in contrast to the traditional strong form of the continuity equation that has been used in an ad-hoc manner in the augmented EFIE. Numerical results demonstrate the improved stability of this approach, as well as the accuracy at low frequencies.**

I. Introduction

The marching on in time (MOT) algorithm has been used for many years to solve time domain integral equations (TDIEs), such as the time domain electric field integral equation (TD-EFIE). Unfortunately, the well-known low frequency breakdown of the EFIE is mirrored by a large time step breakdown in the TD-EFIE [1]. This precludes the use of traditional solvers based on the TD-EFIE in efficiently modeling a range of problems, including the broadband analysis of multiscale structures and scattering from subwavelength objects.

To overcome the effects of the low frequency breakdown in TDIEs, many of the frequency domain methods have been adapted to the time domain. This includes loop-tree decompositions [1], Calderón preconditioners [2], and the augmented EFIE (AEFIE) [3]–[4]. Although Calderón preconditioners have been the most successful at stabilizing the TD-EFIE, there is still interest in systems like the TD-AEFIE due to the relative ease of implementation.

This work presents a new TD-EFIE that uses a weighted continuity equation, termed the TD-WC-EFIE. Using this weighted continuity equation results in a provably stable system which also produces a symmetric matrix equation. This approach has implementation advantages over past TD-AEFIE systems and also directly links into a rigorous mathematical framework that can be used to prove that this system is well-posed [5]. Numerical results are presented which validate the accuracy and stability.

II. Formulation

The traditional TD-AEFIE system is formed by two equations. The first is the TD-EFIE, where the charge density is used as an explicit unknown, giving

$$\int_S \left[\mu \frac{\dot{\mathbf{J}}(\mathbf{r}',\tau)}{4\pi R} + \nabla \frac{\rho(\mathbf{r}',\tau)}{4\pi R\epsilon}\right] dS' = \mathbf{E}^{\mathrm{inc}}(\mathbf{r},t), \tag{1}$$

where $\mathbf{J}$ is the current density on the surface S, ρ is the charge density, $\tau = t - R/c$, and $\mathbf{E}^{\mathrm{inc}}$ is the incident field. Typically, the strong form of the current continuity equation,

$$\nabla \cdot \mathbf{J}(\mathbf{r},t) + \dot{\rho}(\mathbf{r},t) = 0, \tag{2}$$

is used as an ad-hoc constraining equation to yield a solvable system. However, this system can often lead to unstable numerical results, which have been compensated by using a filtering technique and a modal order reduction method [3], [6]. Another technique involves temporally integrating (2), however, this requires a more complicated implementation to be performed efficiently [4].

The approach advocated here is to derive a second constraining equation, as opposed to using an ad-hoc one. As a starting point, we use a type of scalar potential equivalence principle integral equation derived in [5], which is

$$\int_S \left[\frac{\nabla' \cdot \mathbf{J}(\mathbf{r}',\tau)}{4\pi R\epsilon} - \frac{\hat{n}' \cdot \ddot{\mathbf{A}}(\mathbf{r}',\tau)}{4\pi R}\right] dS' = \dot{\Phi}^{\mathrm{inc}}(\mathbf{r},t). \tag{3}$$

In (3), $\mathbf{A}$ is the vector potential and Φ^{inc} is the incident scalar potential. Using Huygens' principle for PEC objects,

$$\int_S \frac{\hat{n}' \cdot \nabla' \Phi(\mathbf{r}',\tau)}{4\pi R} dS' = \Phi^{\mathrm{inc}}(\mathbf{r},t), \tag{4}$$

(3) can be rewritten to give

$$\int_S \left[\frac{\nabla' \cdot \mathbf{J}(\mathbf{r}',\tau)}{4\pi R\epsilon} - \frac{\hat{n}' \cdot \ddot{\mathbf{A}}(\mathbf{r}',\tau) - \hat{n}' \cdot \nabla' \dot{\Phi}(\mathbf{r}',\tau)}{4\pi R}\right] dS' = 0. \tag{5}$$

Noting that the charge density may be defined in terms of the potentials as

$$\rho(\mathbf{r},t) = -\epsilon \hat{n}' \cdot \big[\mathbf{A}(\mathbf{r},t) + \nabla' \Phi(\mathbf{r},t)\big], \tag{6}$$

we arrive at a form of the continuity equation that uses the free space Green's function as a weighting function, given by

$$\int_S \left[\frac{\nabla' \cdot \mathbf{J}(\mathbf{r}',\tau)}{4\pi R} + \frac{\dot{\rho}(\mathbf{r}',\tau)}{4\pi R}\right] dS' = 0. \tag{7}$$

The TD-WC-EFIE proposed in this work solves (1) and (7) together. This results in a system that can be proven to be stable, and yields stable results numerically when discretized with appropriate basis and testing functions. No additional modifications are required to produce stable results, such as the filtering or modal order reduction methods that were required in the past. Further, this system can be made symmetric after scaling (7) with ϵ^{-1}.

III. Numerical Results

A key component to achieving stable results with TDIEs is to use appropriate basis and testing functions [5]. For the TD-WC-EFIE proposed in this work, the spatial discretization can be performed by expanding $\mathbf{J}$ with RWG functions and ρ with pulse basis functions (constant over a triangle). The testing is then performed with RWG functions for (1) and pulse functions for (7). One appropriate temporal basis function for this formulation is a triangle function, with the testing performed with a delta function, as is typically done in the MOT procedure.

To demonstrate the stability and accuracy of this method two numerical examples are performed. The incident field is a plane wave, with temporal dependence given by a modulated Gaussian pulse. The scattering from a 1 meter radius PEC sphere is calculated for a variety of different center frequencies and bandwidths of the incident pulse. To demonstrate the stability, an eigenvalue stability analysis (see [3] for more details) is performed on the MOT system when using an incident pulse with a 30 MHz center frequency, 29 MHz bandwidth, and 0.847 ns time step. This is shown in Fig. 1 for the traditional TD-AEFIE and the TD-WC-EFIE proposed in this work. The presence of eigenvalues outside of the unit circle for the TD-AEFIE demonstrates its instability, while the TD-WC-EFIE has all eigenvalues inside the unit circle (i.e., it is stable). The accuracy is demonstrated in Fig. 2. The error in the RCS is calculated with

$$\text{Error} = \frac{||\text{RCS}_{\text{TDIE}} - \text{RCS}_{\text{Mie}}||}{||\text{RCS}_{\text{Mie}}||}, \tag{8}$$

for a sequence of simulations with center frequencies from 1 Hz to 1 MHz, and bandwidths set to half the center frequency. It is seen that the TD-EFIE suffers a catastrophic low frequency breakdown, while the TD-WC-EFIE achieves a constant level of accuracy.

IV. Conclusion

A new TD-EFIE, termed the TD-WC-EFIE, was proposed in this work that uses a Green's function weighted form of the continuity equation. Numerical results demonstrated the superior stability of this method compared to the TD-AEFIE. The accuracy was also demonstrated to be good, even to very low frequencies.

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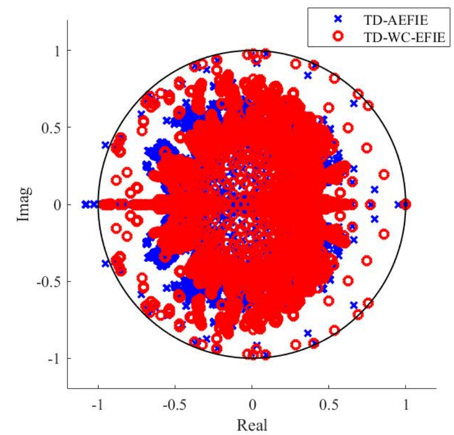

Fig. 1: Results of the eigenvalue stability analysis for the TD-AEFIE and the TD-WC-EFIE.

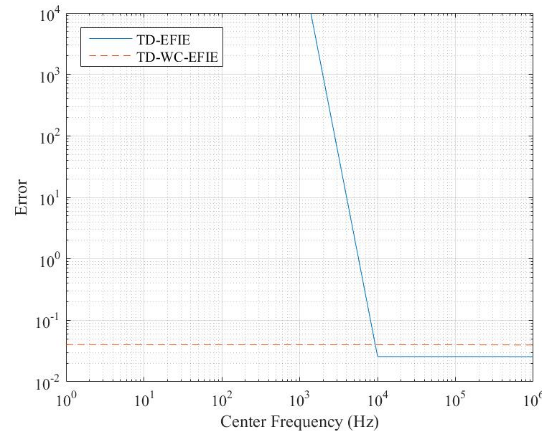

Fig. 2: Error at low frequencies for the EFIE and the TD-WC-EFIE.

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