CONF-960238--3

SAND95-2479C

DAMAGE IDENTIFICATION WITH PROBABILISTIC NEURAL NETWORKS

RECEIVED
NOV 17 1995
OSTI

S. E. Klenke and T. L. Paez

Experimental Structural Dynamics Department
Sandia National Laboratories
Albuquerque, New Mexico 87185-0557

ABSTRACT.

Efforts to optimize the design of mechanical systems for preestablished use environments and to extend the durations of use cycles establish a need for in-service health monitoring. Numerous studies have proposed measures of structural response for the identification of structural damage, but few have suggested systematic techniques to guide the decision as to whether or not damage has occurred based on acquired data. Such techniques are necessary because in field applications the environments in which systems operate and the measurements that characterize system behavior are random.

This paper investigates the use of artificial neural networks (ANNs) to identify damage in mechanical systems. Two probabilistic neural networks (PNNs) are developed and used to judge whether or not damage has occurred in a specific mechanical system, based on experimental measurements. The first PNN is a classical type that casts Bayesian decision analysis into an ANN framework; it uses exemplars measured from the undamaged and damaged system to establish whether system response measurements of unknown origin come from the former class (undamaged) or the latter class (damaged). The second PNN establishes the character of the undamaged system in terms of a kernel density estimator of measures of system response; when presented with system response measures of unknown origin, it makes a probabilistic judgment whether or not the data come from the undamaged population. The physical system used to carry out the experiments is an aerospace system component, and the environment used to excite the system is a stationary random vibration. The results of damage identification experiments are presented along with conclusions rating the effectiveness of the approaches.

NOMENCLATURE.

ANN	: Artificial Neural Network
PNN	: Probabilistic Neural Network
PPC	: Probabilistic Pattern Classifier
VETO	: Virtual Environment for Test Optimization
X, Y	: vector of random variables with dimension n
$F_X(.)$	: cumulative distribution function estimator
H_X, H_Y	: a priori probabilities of X and Y
L_X, L_Y	: loss factors associated to sources Y and X
N	: number of measured data realizations
S	: covariance matrix
T	: a transform operator
$f_X(.)$, $f_Y(.)$	: estimated probability density functions
n	: dimensionality of a source of data
beta	: distance in standard normal space
u	: uniformly distributed random variable
w	: uncorrelated standard normal random variable
z	: datum of unknown source
$\Phi(.)$	: cumulative distribution function of a standard normal random variable
σ	: smoothing parameter

1. INTRODUCTION

Modern practice in structural design often dictates that systems be fabricated to minimum weight (and sometimes cost) specifications, and yet safely sustain the loads applied to them for a preestablished time duration. This is possible because great strides are being made in analysis, design and testing practice, but it is complicated by the fact that the loads applied to any real structure are unknown and the material properties and geometry of a structure are random. In view of this, the responses of structures must be monitored, and this information must be used to infer structural functionality and safety.

There are many frameworks that can be used to assess the relative health of a structure, and this paper presents two of them. They are the classical probabilistic neural network (PNN) of Specht [1], and a probabilistic pattern classifier (PPC) that we have developed. The former is an artificial neural network (ANN) implementation of the Bayes' decision analysis procedure. The latter is a transform procedure that permits us to judge the source of data of unknown origin.

The PNN requires data sets from two or more sources. For example, this particular technique is used in the assessment of structural damage when both normal and abnormal operating data are available from a structure. When presented with a datum of unknown source, the PNN judges which set of known data is the likeliest source of the unknown datum. The PNN implements Bayes' decision rule representing the probability density functions (pdf's) of the known data sets with kernel density estimators. These were first developed in the form in which they are used today by Parzen [2], and their form was later generalized to the multivariate case by Cacoullos [3]. A text that summarizes kernel density estimators is that of Silverman [4]. The PNN is briefly described in the following section.

The PPC requires a data set from one source. In this particular case, the change in health of a structure is determined using only normal operating data. When presented with a datum of unknown origin, the PPC judges whether the datum is a member, an outlier, or a nonmember of the set whose source is known. This tool also uses the pdf representation of Parzen and Cacoullos, but given that representation it utilizes a transform (see Rosenblatt, [5]) into the space of uncorrelated standard normal random variables. Data of unknown origin are transformed into this space, and a test of hypothesis is performed to judge the source of the data. The PPC is developed in a later section.

The real test of a tool is its effectiveness in practical application. The two health monitoring tools considered in this study are applied to the monitoring of damage in a physical system. The system is a stereolithography model of an aerospace component. The system was tested using random vibration and its response measured and used to characterize the undamaged system. Next, a small amount of damage was introduced into the system, and it was tested and characterized again. This step was repeated four more times; each time incremental damage was introduced into the system before retesting. Finally, the PNN and PPC were used to determine whether the incremental damage could be recognized. The results were successful, and are presented in detail in a later section.

2. CLASSICAL PROBABILISTIC NEURAL NETWORK THEORY

The classical probabilistic neural network (PNN) uses the Bayesian decision analysis cast into an Artificial Neural Network (ANN) framework to judge the origin of datum z given that data from two random variable sources, X and Y, are known. The known data are denoted $x_i, y_i, i = 1,...,N$. The sources X and Y are assumed to be vector random variables with dimension n, and their corresponding realizations are also assumed to be vectors. For the two-source case, the origin of z is determined based on the following Bayesian decision rule

$$\begin{aligned} z \in X \quad &\text{if} \quad H_X L_X f_X(z) > H_Y L_Y f_Y(z) \\ z \in Y \quad &\text{if} \quad H_X L_X f_X(z) < H_Y L_Y f_Y(z) \end{aligned} \quad (1)$$

Where $f_X(z)$ and $f_Y(z)$ are the estimated probability density functions for the sources X and Y, respectively; H_X and H_Y are the a priori probabilities of sources X and Y; and L_X and L_Y are the losses resulting from incorrect decisions that the sources are Y and X, respectively. Often the a priori probabilities can be determined for the source data, however, the loss factors do require some subjective evaluation based on the application from which the source data have come. The key to using equation (1) is the ability to estimate the probability density functions $f_X(z)$ and $f_Y(z)$ based on experimental data. These joint probability density functions (pdf) can be approximated using the kernel density estimator (see Parzen [2], Cacoullos [3] and Silverman [4]). The kernel density estimator (kde) is a data based estimator and one form is

$$\hat{f}_X(z) = \frac{1}{N(2\pi)^{n/2}|S|^{1/2}} * \sum_{j=1}^{N} exp\left(-\frac{1}{2}\left(z - x_j\right)^T S^{-1}\left(z - x_j\right)\right) \quad (2)$$

Of course, the kernel in this expression, is a multivariate normal pdf. The kernel density estimator is a superposition of N multivariate normal densities centered at each measured realization of X. This summation is normalized so that its hyperspace volume equals one. S is the covariance matrix for the kernel. This matrix can conveniently be approximated by the general form

$$S = \sigma^2 I \quad (3)$$

where I is the identity matrix and σ^2 is the smoothing parameter of the kde. A small smoothing parameter can cause the estimated density function to show distinct modes at the locations of the training data, while a large value of σ provides greater smoothing or interpolation between points in the density estimation. The following was utilized in the multivariate normal kde of source X

$$\sigma = 0.9 * \{4/(n+2)\}^{1/(n+4)} * \sqrt{\{\sum_i std(x_i)\}^2} * N^{-1/(n+4)} \quad (4)$$

where $std(x_i)$ refers to the standard deviation of the random variable source X, and the other parameters were previously described.

3. PROBABILISTIC PATTERN CLASSIFIER THEORY

The probabilistic pattern classifier (PPC) is similar to the PNN in that it seeks to distinguish the source of a datum of unknown origin. However, the PPC differs from the PNN in that the PPC seeks to answer the question: Is the datum of unknown origin a member of the data set of interest, or is it an outlier, or is it a nonmember? It answers this question by: (1) characterizing the data set of interest using the kernel density estimator of Eq. (2), (2) using this expression to develop a Rosenblatt transformation (see Rosenblatt, [5]) to the space of uncorrelated standard normal random variables, then (3) transforming the datum of unknown origin to the standard normal space where we perform a test of hypothesis to judge its membership. This transformation is used to cast the data into a form that will allow us to easily make a quantifiable decision about the membership of the datum of unknown origin.

We commence the development by assuming that a random variable X is characterized by a collection of data denoted $x_j, j = 1,...,N$. The source and the data it produces may be vector quantities. The kernel density estimator for the data is given by Eq. (2). We seek a transformation from the space of X to the space of uncorrelated standard normal random variables. Such a transformation can be developed using the Rosenblatt transformation.

The Rosenblatt transformation is a unique and invertible mapping that permits the conversion of vector realizations of random variables with arbitrary joint probability distribution to vector realizations of independent, uniformly distributed random variables on the interval [0,1]. To develop the transformation, note that there is a cumulative distribution function (cdf) estimator that corresponds to the kde in Eq. (2). It is easy to obtain because of the form of the covariance matrix in Eq. (3), and is given by

$$F_X(\xi) = \int_{-\infty}^{\xi_1} d\alpha_1 \ldots \int_{-\infty}^{\xi_n} d\alpha_n f_{\{X\}}(\{\alpha\}) = \frac{1}{N}\sum_{j=1}^{N}\prod_{k=1}^{n}\Phi\left(\frac{\xi_k - x_{kj}}{\sigma}\right) \quad (5)$$

where ξ is the variate vector and ξ_k is its kth element, x_{kj} is the kth vector element in the jth data point x_j, $\Phi(.)$ is the cdf of a standard normal random variable and the other quantities in the expression are defined following Eq. (2). This is the joint cdf of all the random variables $X_k, k=1,...,n$, in the vector $\boldsymbol{X}$. From this function all the lower order joint cdf's (including marginal cdf's) and conditional cdf's can be developed. The Rosenblatt transformation takes the form

$$\begin{aligned} u_1 &= F_{X_1}(\xi_1) \\ u_2 &= F_{X_2|X_1}(\xi_2 \mid \xi_1) \\ &\ldots \\ u_n &= F_{X_n|X_{n-1},...,X_1}(\xi_n \mid \xi_{n-1},...,\xi_1) \end{aligned} \quad (6)$$

where the $u_j, j=1,...,n$, are realizations of independent, uniformly distributed random variables on [0,1], the $\xi_j, j=1,...,n$, are elements of the vector ξ, and the functions on the right hand side are one marginal (the first equation) and several conditional cdf's obtained from Eq. (5). The following shorthand notation can be adopted for Eqs. (6).

$$u = T(\xi) \quad (7)$$

where $\boldsymbol{u}$ is the vector of elements $u_k, k=1,...,n$ and ξ is the vector of elements $\xi_k, k=1,...,n$.

Because the cdf defined in Eq. (5) is monotone increasing (The standard normal cdf, $\Phi(.)$ is a monotone increasing function.), the transformation of Eqs. (6) and (7) is invertible, therefore,

$$\xi = T^{-1}(u) \quad (8)$$

Because we can define the forward and inverse transformations in Eqs. (6) through (8) for a vector of random variables $\boldsymbol{X}$ with arbitrary distribution, we can also define the transformation for a vector of random variables $\boldsymbol{W}$ that are uncorrelated with standard normal distribution (i.e., each element of $\boldsymbol{W}$ is normally distributed with mean zero and unit variance.). The forward and inverse transformations may be denoted

$$u = T_{sn}(w) \qquad w = T_{sn}^{-1}(u) \quad (9)$$

where the subscript "*sn*" refers to the fact that these are transformations to and from the standard normal space.

The existence of the transformation in Eq. (7) and the second transformation in Eq. (9) implies that a transformation from a realization of a vector random variable with arbitrary joint probability distribution to a realization of a vector of uncorrelated standard normal random variables can be defined. In terms of the notation in Eqs. (7) and (9), it is

$$w = T_{sn}^{-1}(T(\xi)) \quad (10)$$

This transformation, developed using the detailed forms of Eqs. (5) and (6), forms the basis of the PPC. The transformation reflects the character of the data source $\boldsymbol{X}$ via its measured realizations $x_j, j=1,...,N$, because the cdf's in Eq. (6) come from Eq. (5), and Eq. (5) involves the $x_j, j=1,...,N$.

The PPC operates on the following basis. We consider a datum $\boldsymbol{z}$ of unknown origin, and make the hypothesis that it comes from the random source $\boldsymbol{X}$. We transform $\boldsymbol{z}$ to the space of realizations of uncorrelated standard normal random variables using Eq. (10). The operation yields

$$w_z = T_{sn}^{-1}(T(z)) \quad (11)$$

Note that the distance from the origin of a random vector in uncorrelated standard normal space is related to the chi squared distribution. Specifically, the square of the distance from the origin of a random vector with dimension n, whose components are standard normal random variables, is chi squared distributed with n degrees of freedom. In view of this, the hypothesis specified above is accepted at the $\alpha \times 100\%$ level of significance if the norm of w_z (i.e., $\|w_z\|$) falls in the interval $\left[0, \sqrt{\chi^2_{n,1-\alpha}}\right]$, where

$$F_{\chi^2_n}\left(\chi^2_{n,1-\alpha}\right) = 1 - \alpha \quad (12)$$

and is the cdf of a chi squared distributed random variable with n degrees of freedom. Otherwise, the hypothesis is rejected.

In summary, we transform the datum $\boldsymbol{z}$ using Eq. (11), compute the norm of w_z, then observe whether $\|w_z\|$ falls within $\left[0, \sqrt{\chi^2_{n,1-\alpha}}\right]$. If it does, then we conclude that $\boldsymbol{z}$ is a realization of the random variable $\boldsymbol{X}$; otherwise, we conclude that it is not. It is anticipated that, on average, $(1-\alpha) \times 100\%$ of the realizations $\boldsymbol{z}$ that come from the random source $\boldsymbol{X}$ will fall in the interval. When we perform a test under practical conditions, we will often set the significance level in the range 0.1% through 5%. In a heuristic sense, we can conclude that when $\|w_z\|$ is outside the interval $\left[0, \sqrt{\chi^2_{n,1-\alpha}}\right]$, but not too much greater than

$\sqrt{\chi^2_{n,1-\alpha}}$, then z may simply be an outlier of the random variable X. When $\|w_z\|$ is much greater than $\sqrt{\chi^2_{n,1-\alpha}}$, then we conclude that z did not arise from the random source X.

4. APPLICATION OF PROBABILISTIC NEURAL NETWORKS TO STRUCTURAL HEALTH MONITORING

The current research effort has focused on the development of two PNN software codes (the classical probabilistic neural network and the probabilistic pattern classifier) to address the health of mechanical structures based on experimental data. These neural network approaches use system response data to accurately and efficiently model the dynamic behavior of a component under different structural health conditions, both undamaged and damaged. Once these complex models have been developed with measured response data, there are numerous ways in which the models can be used to enhance or improve the decision making process related to the health of the structure. On-line measurements of both inputs and responses of an operating system, such as equipment on a production or manufacturing line, can be used to train a neural network to model normal response behavior of that system. Different types of structural response measures can be used in the neural network training process to help assure that change in structural response or structural damage is clearly detected. The structural response of the system can be monitored to determine when it deviates from the established model of normal behavior. These techniques not only have an impact in the area of structural health monitoring but also in the areas of experimental modeling and experimental test simulations.

There are several key elements that are required to develop a useful PNN. First, the selection of a kernel density estimator (kde) plays an important role in the neural network development process. The kde is an estimator of the probability density function required in the decision analysis. Second, the selection of appropriate measures of structural response are needed that help to clearly reveal structural damage. These elements are a critical part of the development of a probabilistic neural network that can be used to establish a measure of system health. Additionally, the PNNs clearly offer the potential to more accurately model complex nonlinear systems that have traditionally been modeled with linear structural dynamic techniques.

There are, however, some limitations to using these neural networks. Care needs to be taken when calculating multivariate density estimates. The size of the exemplar or training set needed in kernel density estimation increases dramatically as the order or dimensionality of the density estimation increases (Silverman, [4]). Thus, the requirement for large amounts of experimental data in estimating the probability densities might cause some limitations of these neural network techniques. Also, these two techniques are currently limited to assessing whether damage has occurred in a structure and they do not provide a method for determining the location or extent of the damage in the structure. In addition, the type of smoothing chosen in the kernel density estimation could limit not only the accuracy but also the computational speed of the estimation. Also, when the sample set is large, the choice of kernel estimator may also be very important in reducing the computation time of the probability density estimation (Silverman, [4]).

5. NUMERICAL EXAMPLE

An aerospace housing component was selected as test case hardware for generating experimental data where the health of the system could be monitored under different structural conditions. A test design tool called the Virtual Environment for Test Optimization (VETO), Klenke [6], was used to design an optimal experiment for this housing. The VETO software simulation tool allows the user to combine the analytical model of the structure being tested with theoretically or experimental derived models of test equipment and instrumentation to simulate the complete test environment. The simulation was performed with the assistance of advanced visualization software within a computer environment before testing the actual hardware. The goal of performing this test design optimization was to select an appropriate sensor and actuator set to be placed on the structure to maximize the dynamic response data over a particular frequency band, up to 400 Hz. This frequency band of interest was selected to include the first five vibration modes of the structure. A solids model of the aerospace housing component was used to generate a rapid prototype component through a stereolithography process. The testing was performed on this plastic or stereolithography component based on the VETO test design. Figure 1 shows a test setup photo based on the VETO design.

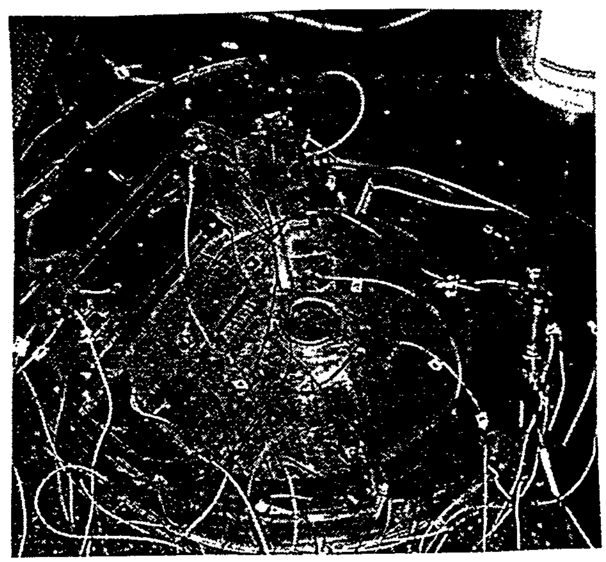

Figure 1. Experimental Test Setup

The outcome of the VETO test design was to excite the structure with stationary random vibration and to measure 55 responses on the housing component to characterize the behavior of the system. Using the visualization software within the VETO environment, two separate locations on the housing structure were selected for the introduction of damage. The basis for the selection of these locations was made by animating the vibration modes of interest while observing maximum strain energy density on the structure. Five separate damage cuts were introduced at two locations with high strain energy density to produce a detectable change in structural response measurements needed in the PNN analysis. The first damage location included three successive quarter inch cuts into the outer flange near the raised portion of the housing. The second selected damage location on the housing included two successive cuts (for a total of five cuts or damage cases) into the flange adjacent to the dome and opposite the raised portion of the housing.

The selection of independent response measures for training the PNN was a very important factor in developing a useful tool to measure the health of the housing component. The goal in choosing these measures was to reduce the dimension of the neural network (from 55 measures of response to 5 measures) while preserving or amplifying the response differences as damage was introduced into the structure. After some discussions with researchers in the area of damage detection, it was determined that static flexibility would be a good measure to show damage in a structure. Measures of static flexibility at five locations on the housing component were used to train the neural networks to assist in detecting structural damage. Selecting static flexibility as the measure of structural response to use in the neural network applications did require some analysis to be completed on the experimental data. Large sample sets of data were collected from input as well as for each of these response locations on the structure. Thirty-nine frequency response function (frf) realizations were calculated using smaller blocks of this large sample set of input and response data. These frf calculations were completed for the one undamaged case and the five damaged cases. An approximation of the static flexibility was calculated for each of these frf realizations. The method for estimating the static flexibilities was to average the low frequency frf behavior to asymptotically approximate these measures. The difficulty in determining these estimates was in selecting an appropriate frequency range to make the calculations. The frequency range selected, 50 to 60 Hz, was above the rigid body modes at 10 and 14 Hz and below the first elastic mode at 115 Hz. A typical frequency response function measure is shown in Figure 2.

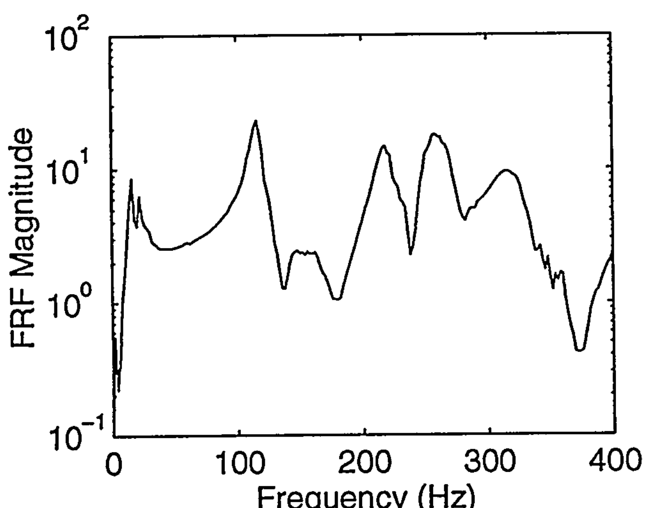

Figure 2. Typical Frequency Response Function

The first case study utilized these measures of static flexibility at the five selected locations on the housing structure as input to the classical PNN. Operation of the classical PNN requires data from two known sources; one set of static flexibilities from the undamaged case and one set of static flexibilities from the group of damaged cases. When the classical PNN was presented with data from an unknown source (this unknown data was taken from the sample set of undamaged or damaged flexibilities and was subsequently not used as PNN training data), the neural network would judge the origin of that data based on the Bayesian decision criterion shown in Eq. (1). The a priori probabilities given the two known sources of data were 0.5 or 50% and the loss factors were set to 1. The results from the classical PNN study were outstanding with the code predicting the correct origin of an unknown source 78 out of a possible 78 times in all damage cases. Because of the obvious difficulties in graphically presenting the results of a five-dimensional density, two of the five locations on the housing structure were arbitrarily selected for displaying results from the classical PNN. Figure 3 shows the two-dimensional scatter plot of the static flexibilities plotted against one another for the undamaged (o) and five successive damaged cases (+). Each (o) and (+) represents a single realization (total of 39/case) of these two flexibilities.

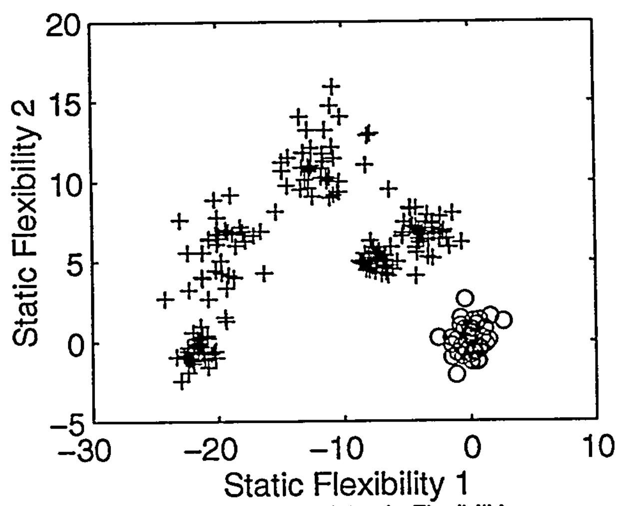

Figure 3. Scatter Plot of Static Flexibilities

The differences between the undamaged and damaged cases for these two static flexibility measures are quite apparent enabling the PNN to easily detect the origin of an unknown source. The classical PNN was able to distinguish the damaged from the undamaged data in all cases, including the most lightly damage case.

The second case study utilized the same measures of static flexibility as input to the PPC. In this case, the PPC requires data from only a single source, such as the undamaged set of flexibilities, and seeks to judge whether or not the data from an unknown origin comes from that source. The Rosenblatt transformation was used to map the static flexibility data from the space of the kernel density estimator into the space of uncorrelated, standard normal random variables. This transformation was also used to transform the data from an unknown source, static flexibility data from the damage cases, into the standard normal space. A distance, beta, from the origin was used as criterion to judge whether the data from the unknown source (data from successive damage cases) came from the known undamaged source. An acceptance region, distances from the origin considered as part of the undamaged source, was established based on the use of the chi square distribution. A chi square random variable with five degrees of freedom has a 99.9% probability of a distance from the origin less than 4.53. The results for the five damage cases input into the probabilistic pattern classifier are shown in Figure 4 as well as the maximum distance from the origin in standard normal space at which a datum could be considered a realization of a standard normal random variable (4.53). This figure shows the trend that as damage increases in the structure the distance measure in standard normal space also increases. The data near beta = 12 correspond to the first damage case. The data near beta = 50, 90 (smoother curve), and 110, correspond to the second, third, and fourth level damage cases, respectively. The data near beta = 90 (more erratic curve) correspond to the fifth level damage case. At this time it is not clear to us why the fifth level damage case yields lower beta values than the fourth level damage case.

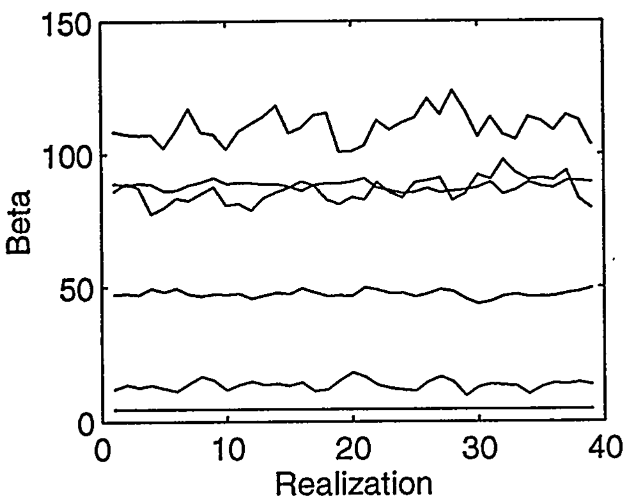

Figure 4. Plot of Distance in Standard Normal Space

6. CONCLUSIONS

The results of using both the classical PNN and the PPC were quite successful. The damage in the aerospace housing component was identified, even in the most lightly damaged case, using both of these techniques. These neural networks clearly offer a robust method for assisting in the identification of damage in structures. The use of the Virtual Environment for Test Optimization did assist in the neural networks ability to identify the damage in the aerospace housing structure. The capability to design the experiment within the computer environment and to choose the locations to place the actuator and sensors did make it possible to identify all the vibration modes of interest during the experiment. Also, VETO was used to help identify locations with high strain energy density in which to damage the structure. The use of this experimental design tool did provide some important insight into the testing of the aerospace housing which provided beneficial inputs for the neural networks.

There were, however, a number of limitations in using these neural network techniques. The first is the limitation of these methods to provide or determine the location and extent of the structural damage. Further research in these neural networks will explore the combining of these techniques with data condensation methods to assist identifying the location and ultimately the extent of the structural damage. Another limitation is the software's inability to run "real-time". Current efforts are under way to rewrite the codes in order to increase their computational speed. Some additional research will focus on the sensitivity of these neural networks to boundary conditions. Studies will be done to assess the effects that changing test configurations might have on the neural network results.

7. ACKNOWLEDGEMENT

This work was supported by the United States Department of Energy under Contract DE-AC04-94AL85000.

8. REFERENCES

[1] Specht, D.F., Probabilistic Neural Networks. *Neural Networks*, Vol. 3, pp. 109-118, 1990.

[2] Parzen, E., On Estimation of a Probability Density Function and Mode. *The Annals of Mathematical Statistics*, Vol. 33, pp. 1065-1076, 1962.

[3] Cacoullos, T., Estimation of a Multivariate Density. *Annals of the Institute of Statistical Mathematics* (Tokyo), Vol. 18(2), pp. 179-189, 1966.

[4] Silverman, B.W., Density Estimation for Statistics and Data Analysis. Chapman and Hall, New York, 1986.

[5] Rosenblatt, M., Remarks on a Multivariate Transformation, *The Annals of Mathematical Statistics*, Vol. 23, pp. 470-472, 1952

[6] Klenke, S., Reese, G., Schoof, L. and Shierling, C., Modal Test Optimization Using VETO (Virtual Environment for Test Optimization), Sandia Report SAND95-1565, 1995.