2nd *International Conference on Impact Loading of Structures and Materials,*
Xi'an, China, May 7-11, 2018

SAND2018-4137C

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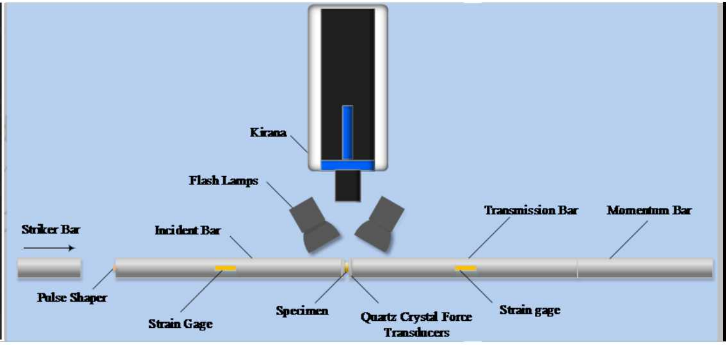

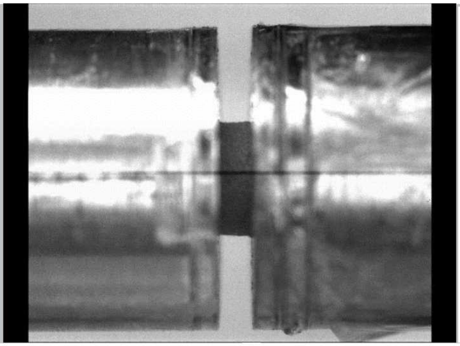

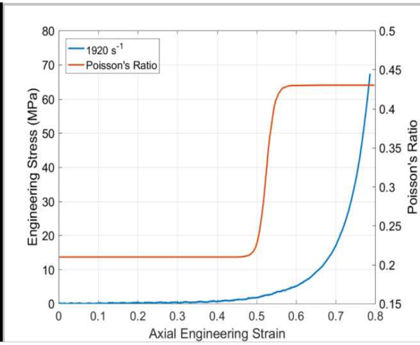

Poisson's Ratio Induced Radial Inertia Effect in Hyperelastic Foams During Dynamic Compression

Bo Song, Brett Sanborn, Wei-Yang Lu

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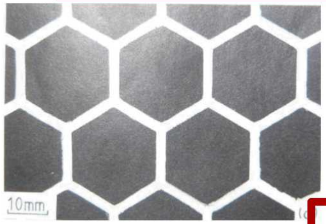
Background: Cellular and Polymeric Foams

Sandia National Laboratories

Cellular Materials

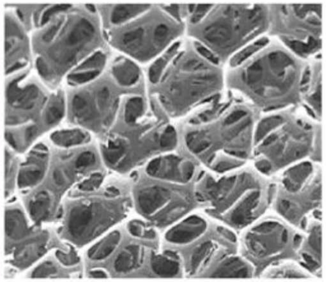
Honeycombs (2D)

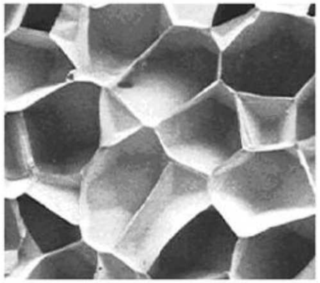


Foams (3D)

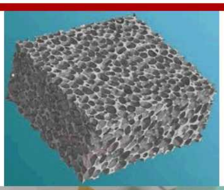
Open Cell

Closed Cell



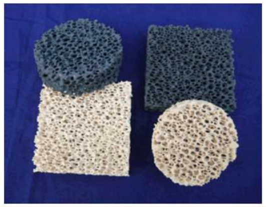


Metallic Foams



Polymeric Foams

Others
(Glass Foam, Ceramic Foam, Cement Foam, etc)



Elastomeric (hyperelastic) Foams

Silicone Foam

Elastic-plastic Foams

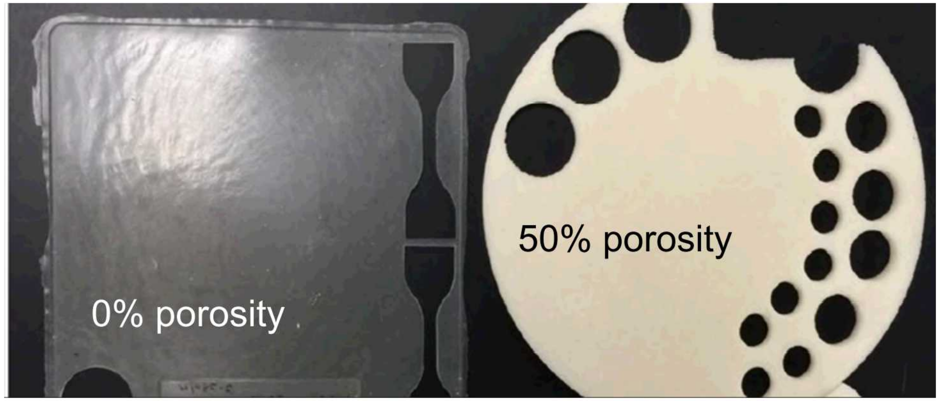
Epoxy, Polystyrene, Polyurethane-based Foams (TDI, PMDI, Tuf-Foam)

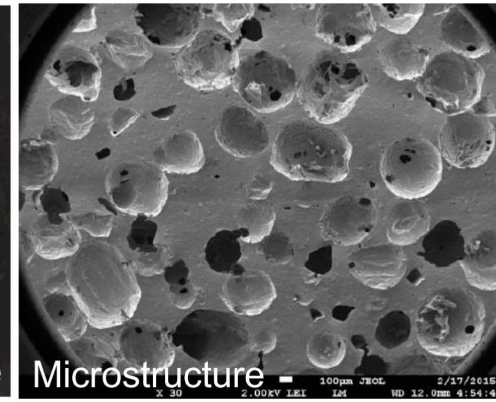
Elastic-brittle Foams

Syntactic Foam

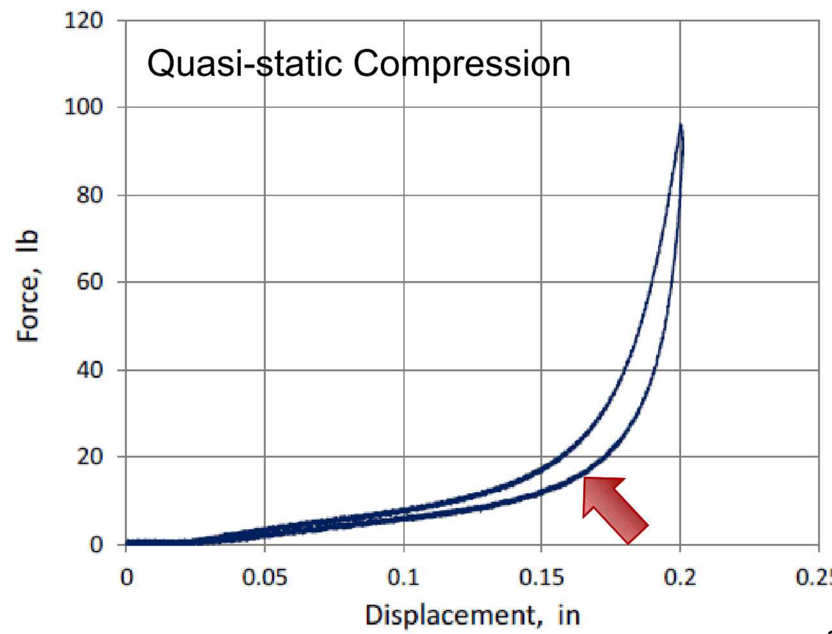
Gibson, L.J. and Ashby, M.F. (1999) Cellular solids, structure, and properties, 2nd ed., Cambridge.

Hyperelastic Foam (Silicone Foam)





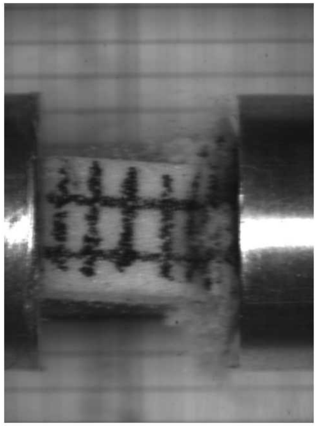
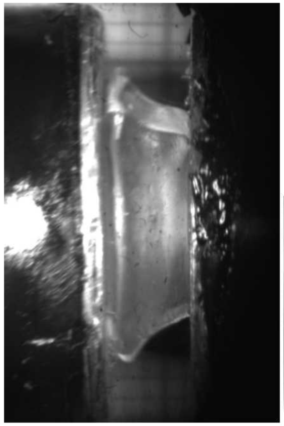
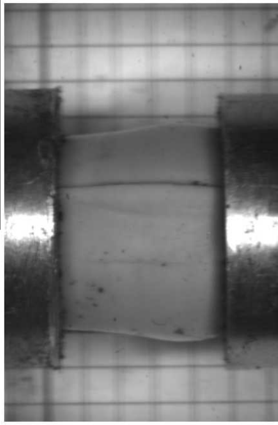
- **<u>Light</u> weight (low density)**
  - Good for transportation
- **<u>Soft</u> - Excellent vibration damper**
  - Comfort
  - Protection (sensitive devices)
- **Excellent shock/impact mitigation**
  - Protection (human, sensitive devices, hazard materials, etc)



Lu, W.-Y., (2015) Compression of silicone foams. 2015 SEM Annual Conference and Exposition on Experimental and Applied Mechanics. June 8-11, 2015, Costa Mesa, CA, USA.

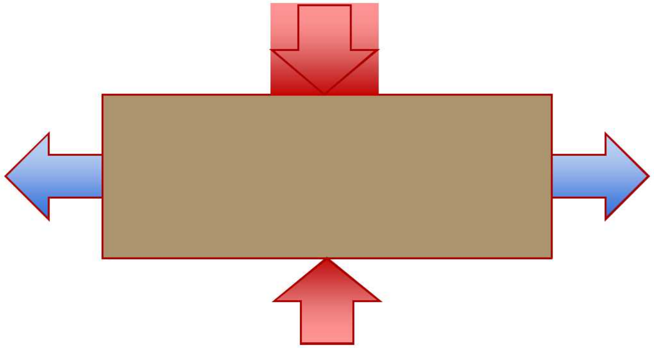
Experimental Challenges in Dynamic Compression of Hyperelastic Foams

- Dynamic stress equilibrium/uniform deformation
  - ✓ Thinner specimen
  - ✓ Pulse shaping

- Weak transmitted signal
  - ✓ Sensitive transmission bar/strain gage or force transducers

- Radial inertia
  - ➢ Poisson's effect
    - ➢ Poisson's ratio
    - ➢ Loading condition
    - ➢ Specimen geometry



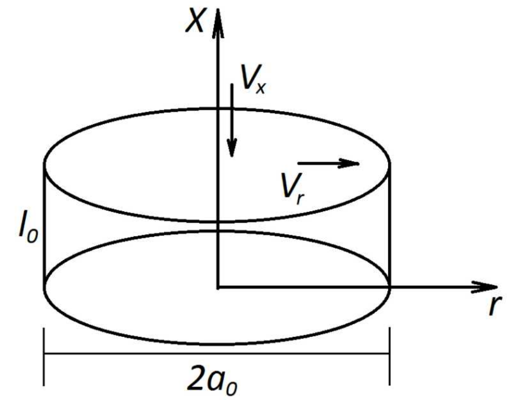
Chen, W., and Song, B., (2011) Split Hopkinson (Kolsky) Bar: Design, Testing and Applications. Springer, New York.

Radial Inertia

Inertia ⟷ *Acceleration* ⟷ *Inertial Force*

- Dharan and Hauser (1970) $$V_r(t) = \frac{r}{2l} V_x(t)$$

    - In a compression test with a constant axial velocity, the lateral velocity increases with increasing strain (or decreasing specimen thickness).



Radial Inertia

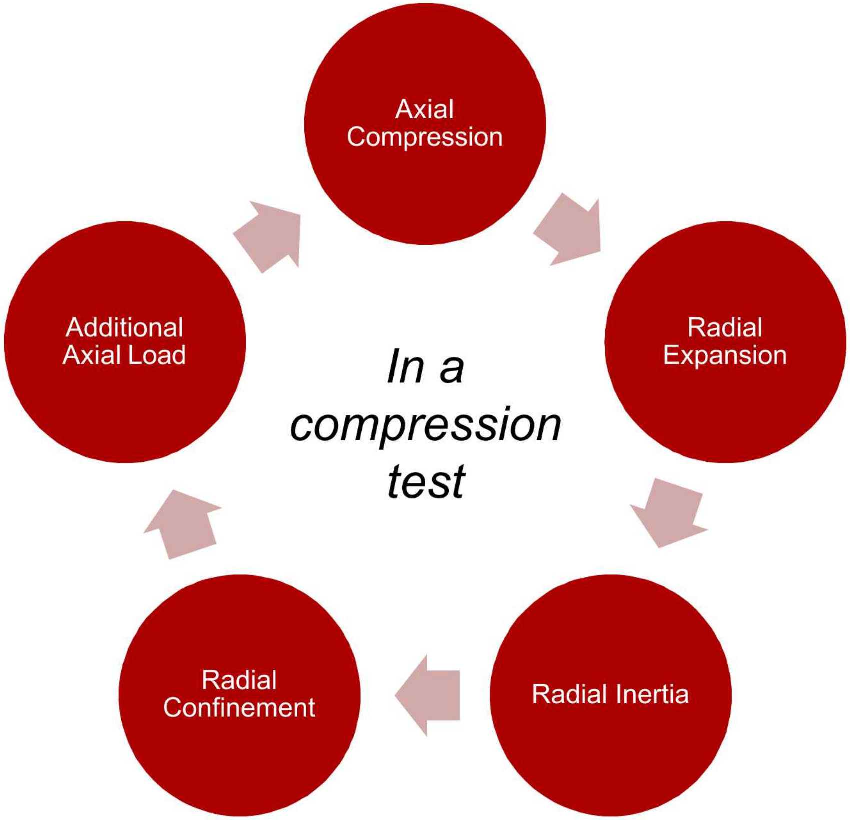
Temporal:

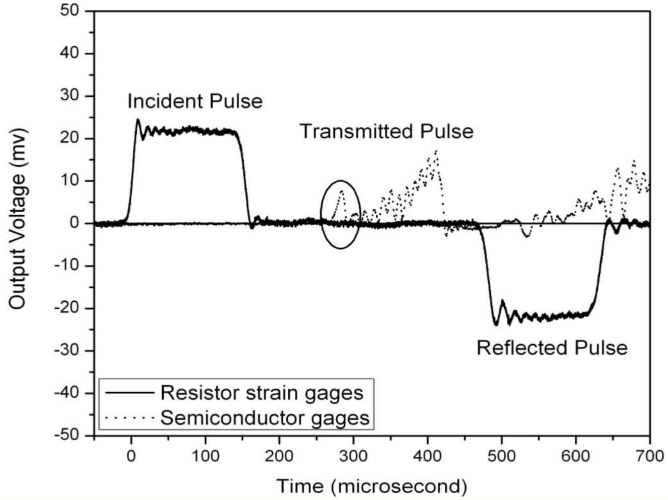
Radial Acceleration ⟷ *Radial Inertial Force*

Spatial:

Velocity Gradient ⟷ *Radial Confinement*

Consequence of Radial Inertia





An abnormal stress peak that may overcome the intrinsic material stress-strain response.

The radial inertia becomes more significant when characterizing

- ***Soft materials***
  - ***Song et al. 2007***
- ***Brittle materials***
  - ***Li and Meng, 2003; Forrestal et al. 2007***

- *Song, B., Chen, W.W., Ge, Y., Weerasooriya, T., (2007) Radial inertia effects in Kolsky bar testing of extra-soft materials. Experimental Mechanics, 47:659-670.*
- *Li, Q.M., and Meng, H. (2003) About the dynamic strength enhancement of concrete-like materials in a split Hopkinson pressure bar test. International Journal of Solids and Structures. 40:343-360.*
- *Forrestal, M.J., Wright, T.W., and Chen, W., (2007) The effect of radial inertia on brittle samples during the split Hopkinson pressure bar test. International Journal of Impact Engineering. 34:405-411.*

Radial Inertia Induced Axial Stress

- Kolsky (1949, Proc. Royal Soc.)

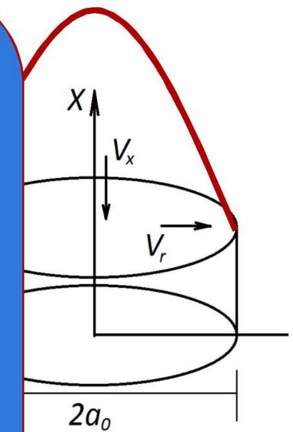
$$\sigma_z = \frac{\nu^2 a_0^2 \rho_0}{\cdots} \cdot \ddot{\varepsilon}$$

- Forre…
- Dhar…
- Warren and Forrestal (2010, EM)

$$\sigma_z = \frac{\rho}{4\left(1-\varepsilon_x^2\right)}\left[\frac{3\dot{\varepsilon}_x^2}{2\left(1-e_x\right)}+\ddot{\varepsilon}_x\right]\left(a_0^2-r^2\right)$$

✓ Incompressible Solid
✓ Small deformation

$$\bar{\sigma}_z = \frac{1}{\pi a^2}\int_0^a \int_0^{2\pi} \sigma_z(r)\, r\, dr\, d\theta$$

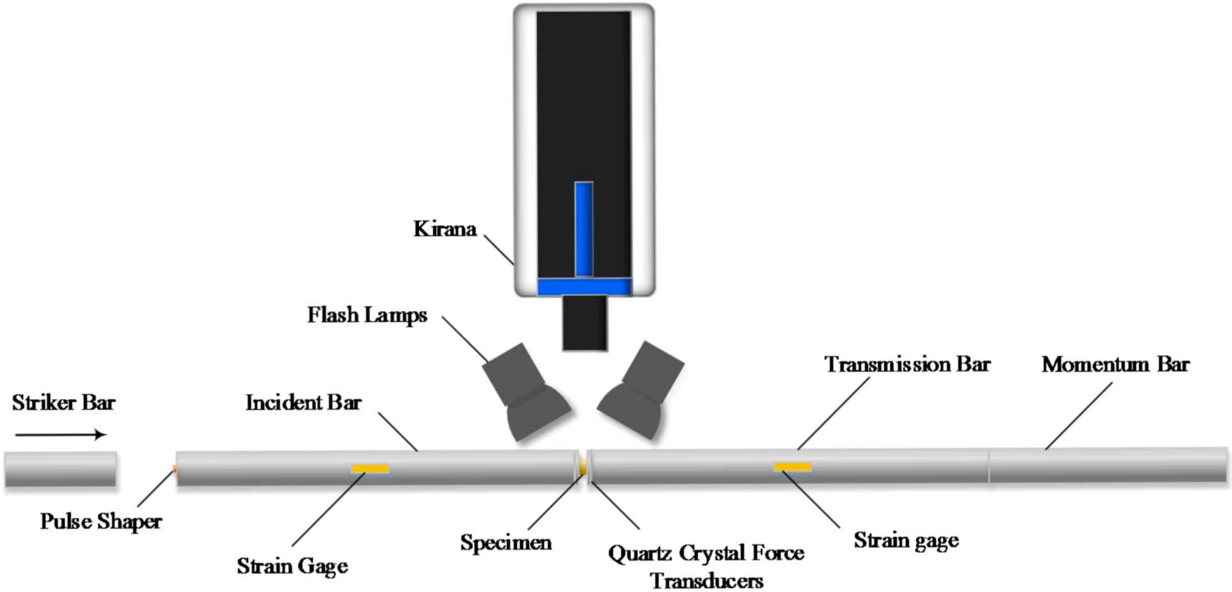


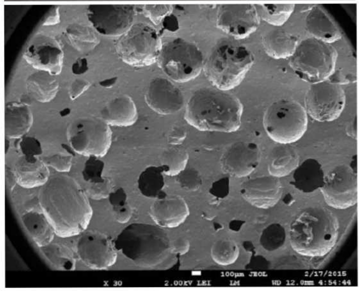
✓ Incompressible Solid
✓ Large deformation
✓ Stress distribution

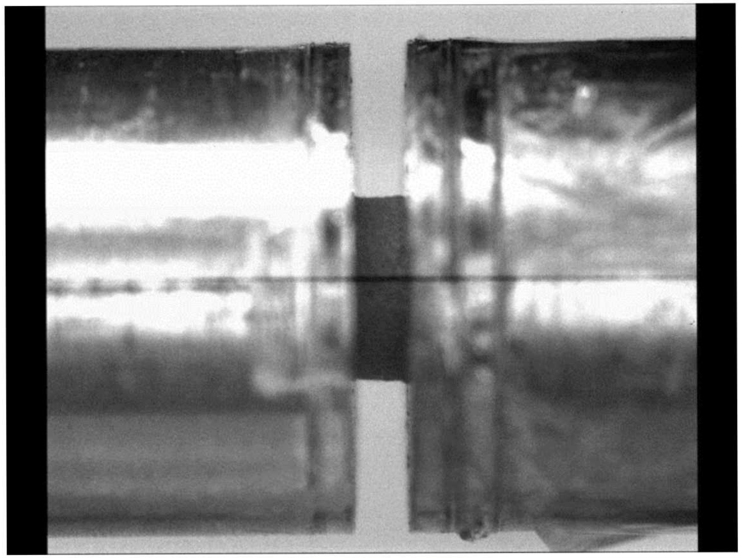
All current analyses are based on constant Poisson's ratio.

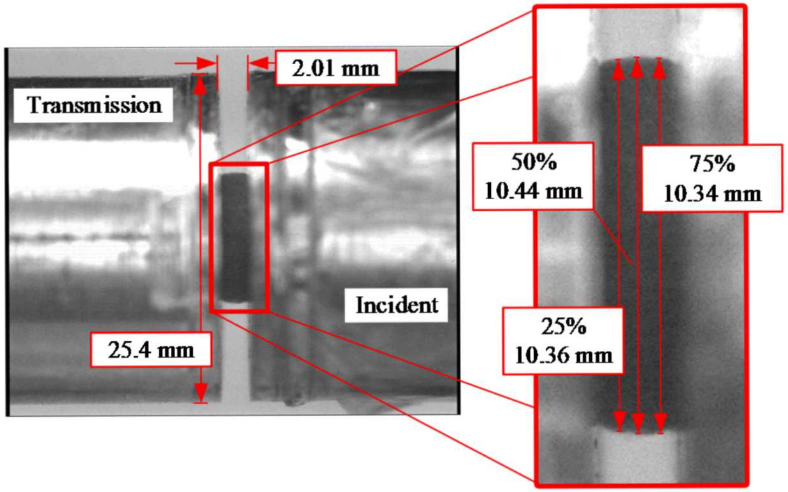
What will happen if Poisson's ratio is no longer a constant?

Poisson's Ratio in a Silicone Foam









Dynamic Poisson's Ratio in a Silicone Foam

Poisson's ratio calculation of a foam material is tricky:

- Large deformation
- Non-linear response

$$\nu = -\frac{d\varepsilon_r}{d\varepsilon_x}$$

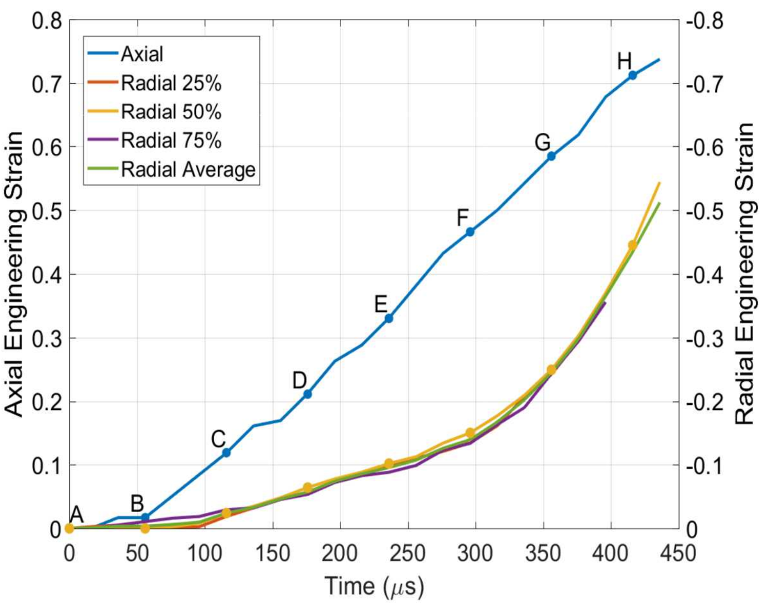
ε_r and ε_x are true strains along radial and axial directions, respectively.

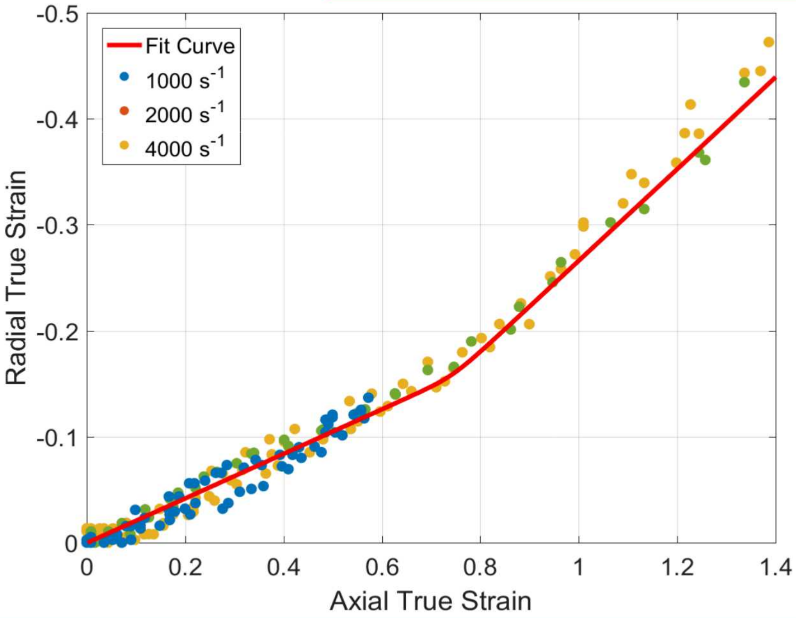
VS

Classical Calculation

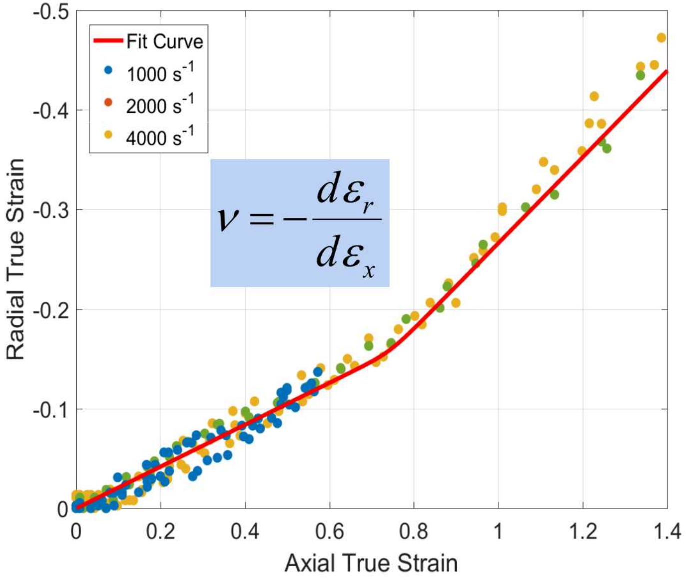
e_r and e_x are engineering strains along radial and axial directions, respectively.

$$\nu = -\frac{e_r}{e_x}$$

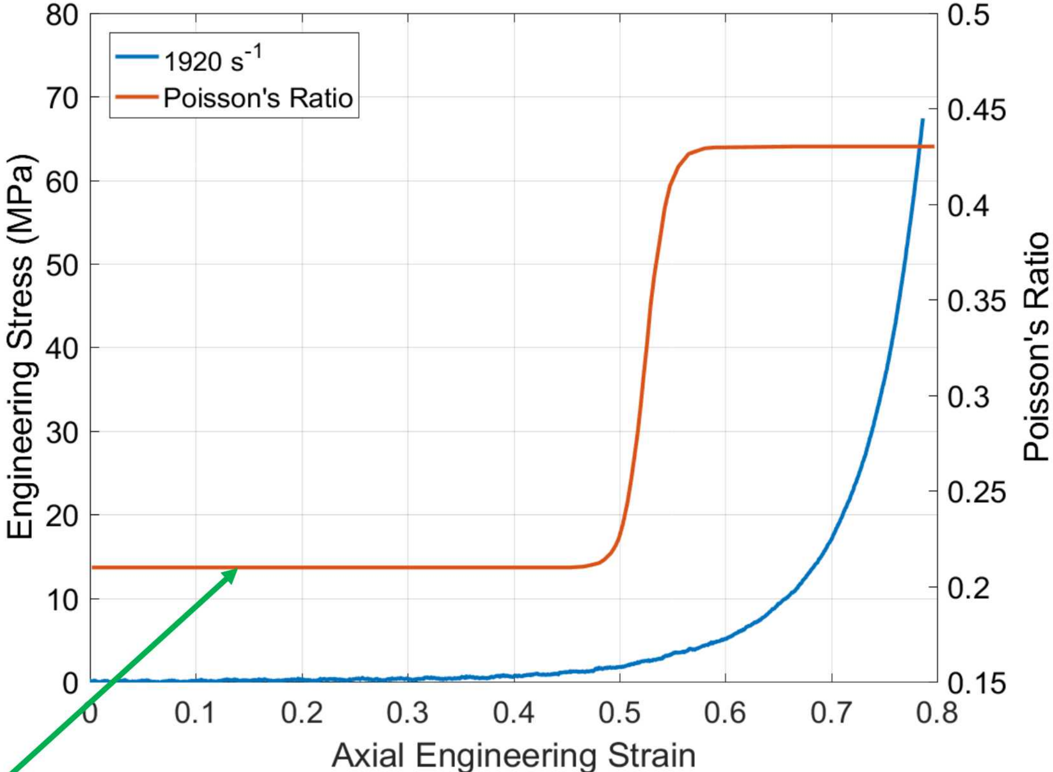




Dynamic Poisson's Ratio in a Silicone Foam



$$\nu = -\frac{d\varepsilon_r}{d\varepsilon_x}$$



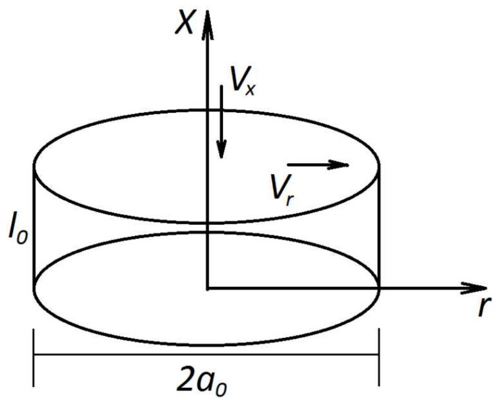
Boltzmann fitting

$$\nu(e_x) = \frac{\nu_1 - \nu_2}{1 + \exp\left(\frac{e_x - e_{x0}}{\delta}\right)} + \nu_2$$

$\nu_1 = 0.21$ $\quad e_{x0} = 0.525$

$\nu_2 = 0.43$ $\quad \delta = 0.01$

Comprehensive Radial Inertia Analysis



Mass conservation:

$$\frac{1}{\rho(t)}\cdot\frac{d\rho(t)}{dt}=-\left(\frac{V_x(t)}{l(t)}+2\frac{V_r(r,t)}{r(t)}\right)$$

Momentum conservation:

$$\frac{\partial\sigma_r(r,t)}{\partial r}=-\rho(t)\left(\frac{\partial V_r(r,t)}{\partial t}+V_r(r,t)\frac{\partial V_r(r,t)}{\partial r}\right)$$

$$\frac{\partial\sigma_r(r,t)}{\partial r}=-\rho(t)\cdot\frac{r}{1-e_x(t)}\cdot\left[\nu(t)\ddot{e}_x(t)+\nu(t)\cdot\left(\nu(t)+1\right)\cdot\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{e}_x^2(t)\right]$$

$$\sigma_r(r,t)=\rho(t)\cdot\frac{a^2(t)-r^2(t)}{2\left(1-e_x(t)\right)}\cdot\left[\nu(t)\ddot{e}_x(t)+\nu(t)\cdot\left(\nu(t)+1\right)\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{e}_x^2(t)\right]$$

Understanding Radial Inertia

$$\sigma_r(r,t) = \rho(t)\cdot\frac{a^2(t)-r^2(t)}{2\left(1-e_x(t)\right)}\cdot\left[\nu(t)\ddot{\varepsilon}_x(t)+\nu(t)\cdot\left(\nu(t)+1\right)\frac{\dot{\varepsilon}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{\varepsilon}_x^2(t)\right]$$

Radial inertia effect:

- Strain acceleration
- Specimen strain (deformation)
- Change of Poisson's ration during deformation

Radial Inertia in a solid with a constant Poisson's ratio

$$\frac{d\nu}{de_x}=0$$

$$\sigma_r(r_0,t) = \rho_0\cdot\frac{a_0^2-r_0^2(t)}{2\left(1-e_x(t)\right)^2}\cdot\left[\nu\ddot{\varepsilon}_x(t)+\nu\cdot(\nu+1)\frac{\dot{\varepsilon}_x^2(t)}{1-e_x(t)}\right]$$

$$\nu = 1/2$$

Warren, T.L., and Forrestal, M.J., (2010) Comments on the effect of radial inertia in the Kolsky bar test for an incompressible material. Experimental Mechanics. 50:1253-1255.

$$\sigma_r(r_0,t) = \rho_0\cdot\frac{a_0^2-r_0^2}{4\left(1-e_x(t)\right)^2}\cdot\left[\ddot{\varepsilon}_x(t)+\frac{3}{2}\cdot\frac{\dot{\varepsilon}_x^2(t)}{1-e_x(t)}\right]$$

Exactly same as that given by Warren and Forrestal (2010) for an incompressible solid

Poisson's Ratio Effect on Radial Inertia in a Silicone Foam

Radial inertia in a silicone foam with varying Poisson's ratio

$$\nu(e_x) = \frac{\nu_1 - \nu_2}{1 + \exp\left(\dfrac{e_x - e_{x0}}{\delta}\right)} + \nu_2$$

$$\sigma_r(r,t) = \rho(t) \cdot \frac{a^2(t) - r^2(t)}{2\left(1 - e_x(t)\right)} \cdot \left[\nu(t)\ddot{e}_x(t) + \nu(t)\cdot\left(\nu(t)+1\right)\frac{\dot{e}_x^2(t)}{1 - e_x(t)} + \frac{d\nu(e_x)}{de_x}\dot{e}_x^2(t) \right]$$

$$\sigma_r(r_0,t) = \rho_0 \cdot \frac{a_0^2 - r_0^2}{2\left(1 - e_x(t)\right)^2} \cdot \left[\begin{array}{l} \ddot{e}_x(t) \cdot \left(\nu_2 + \dfrac{\nu_1 - \nu_2}{1 + \exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)} \right) \\ + \left(\nu_2 + \dfrac{\nu_1 - \nu_2}{1 + \exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)} \right) \cdot \left(1 + \nu_2 + \dfrac{\nu_1 - \nu_2}{1 + \exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)} \right) \cdot \dfrac{\dot{e}_x^2(t)}{1 - e_x(t)} \\ + \dot{e}_x^2(t) \cdot \dfrac{\nu_2 - \nu_1}{\delta} \cdot \dfrac{\exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)}{\left[1 + \exp\left(\dfrac{e_x(t) - e_{x0}}{\delta}\right)\right]^2} \end{array} \right]$$

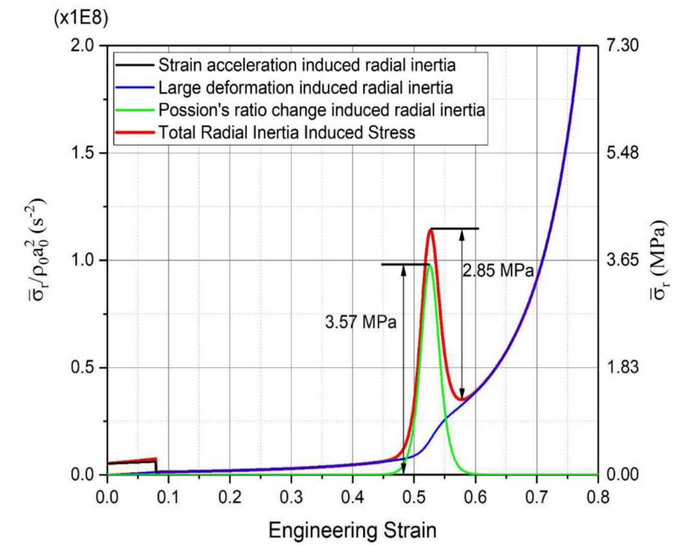
Poisson's Ratio Effect on Radial Inertia in a Silicone Foam

Radial inertia in a silicone foam with varying Poisson's ratio

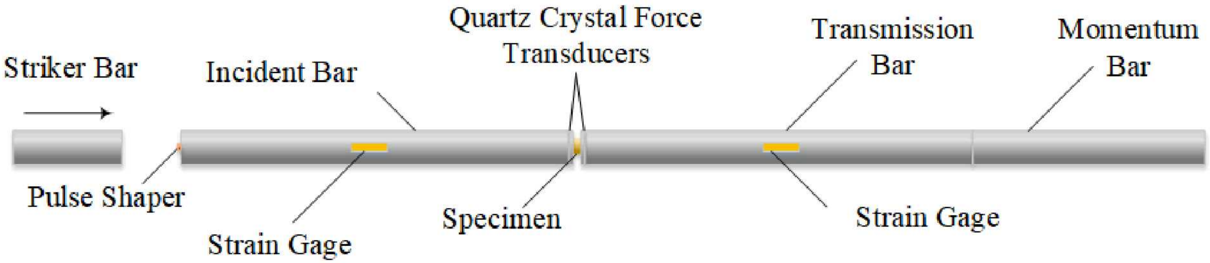
$$\bar{\sigma}_z = \frac{1}{\pi a^2}\int_0^a\int_0^{2\pi}\sigma_z(r)\,r\,dr\,d\theta$$

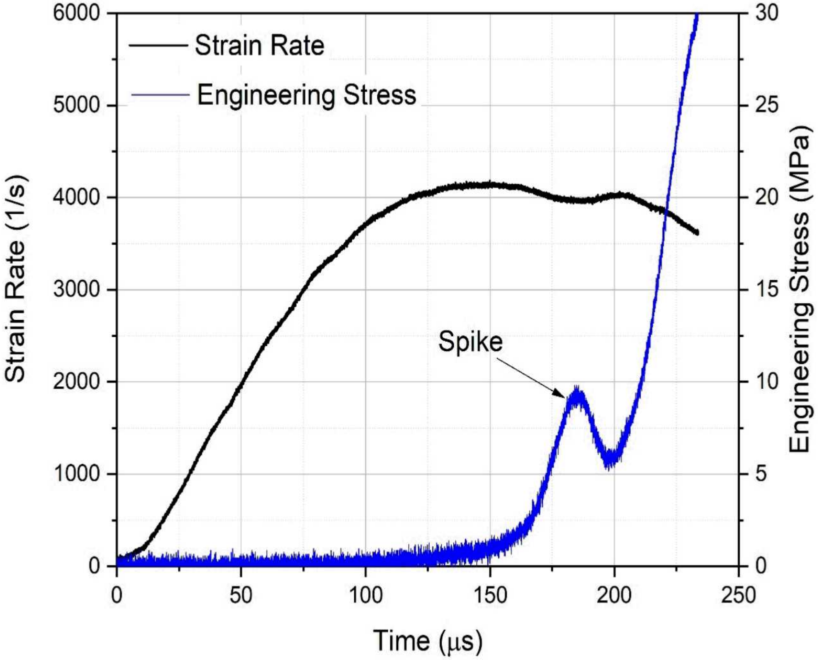
$$\sigma_r(r_0,t) = \rho_0\cdot\frac{a_0^2-r_0^2}{2\left(1-e_x(t)\right)^2}\cdot\left[\begin{array}{l}\ddot{e}_x(t)\cdot\left(\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\\ +\left(\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\left(1+\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\dfrac{\dot{e}_x^2(t)}{1-e_x(t)}\\ +\dot{e}_x^2(t)\cdot\dfrac{\nu_2-\nu_1}{\delta}\cdot\dfrac{\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}{\left[1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)\right]^2}\end{array}\right]$$

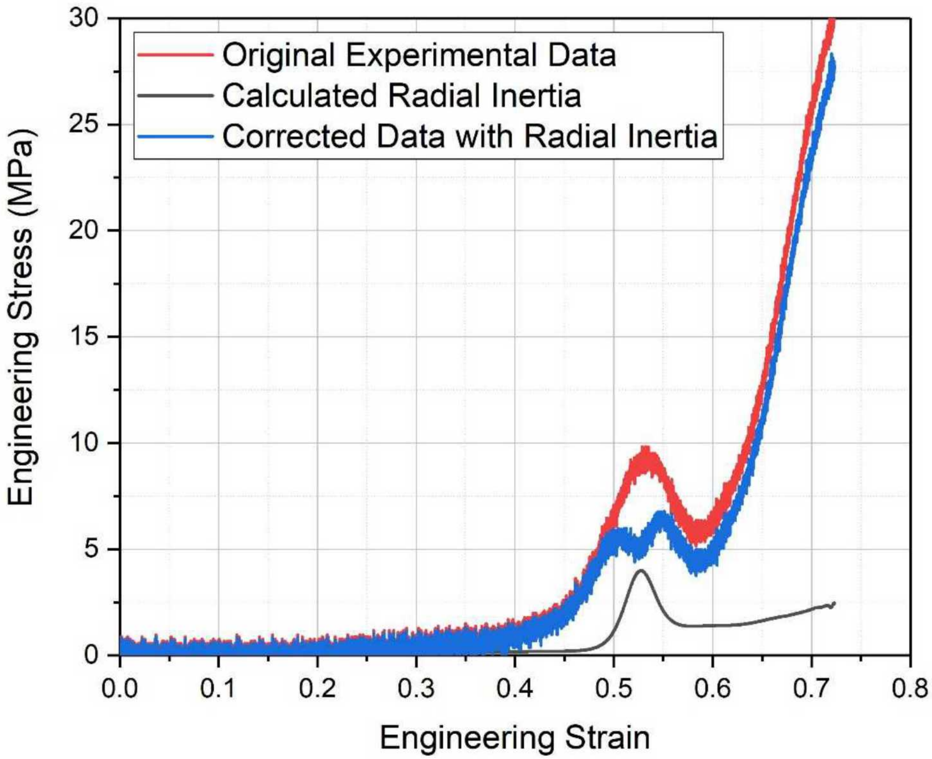
$$\bar{\sigma}_r(t) = \frac{\rho_0\cdot a_0^2}{4\left(1-e_x(t)\right)^2}\cdot\left[\begin{array}{l}\ddot{e}_x(t)\cdot\left(\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\\ +\left(\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\left(1+\nu_2+\dfrac{\nu_1-\nu_2}{1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\dfrac{\dot{e}_x^2(t)}{1-e_x(t)}\\ +\dot{e}_x^2(t)\cdot\dfrac{\nu_2-\nu_1}{\delta}\cdot\dfrac{\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)}{\left[1+\exp\left(\dfrac{e_x(t)-e_{x0}}{\delta}\right)\right]^2}\end{array}\right]$$



Experimental Verification





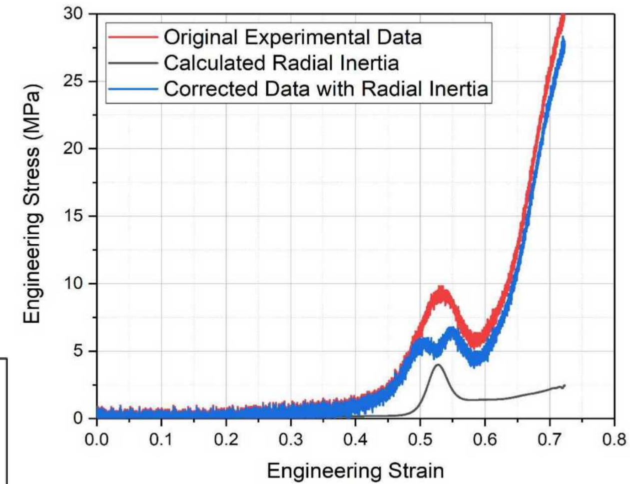


Potential Solutions

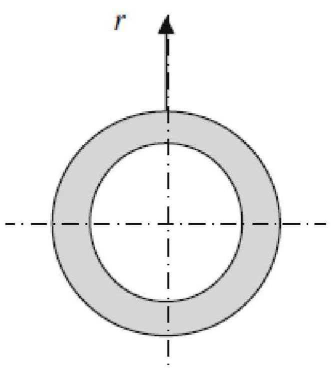
- ❑ Numerical correction

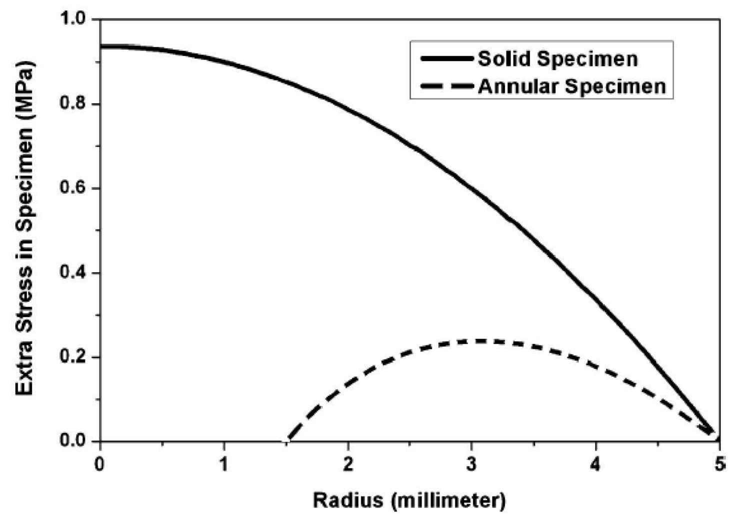
$$\sigma_r(r,t)=\rho(t)\cdot\frac{a^2(t)-r^2(t)}{2\left(1-e_x(t)\right)}\cdot\left[\nu(t)\ddot{\varepsilon}_x(t)+\nu(t)\cdot\left(\nu(t)+1\right)\frac{\dot{\varepsilon}_x^2(t)}{1-e_x(t)}+\frac{d\nu(e_x)}{de_x}\dot{\varepsilon}_x^2(t)\right]$$

  - ➢ <u>Challenge</u>: *it is highly sensitive to strain-dependent Poisson's ratio (differentiation)*



- ❑ Experimental correction
  - ➢ Using an annular specimen





The radial inertia effect becomes more significant when charactering soft materials.

Song, B., Chen, W.W., Ge, Y., Weerasooriya, T., (2007) Radial inertia effects in Kolsky bar testing of extra-soft materials. Experimental Mechanics, 47:659-670.

Conclusions

- Unique Poisson's ratio of hyperelastic foams, i.e., silicone foam in this study
  - Experimental characterization of Poisson's ratio of hyperelastic foams
  - Poisson's ratio varies with deformation
    - Drastic change before and after desification
- Radial inertia when dynamically characterizing hyperelastic foams
  - Key factors
    - Strain acceleration
    - Large deformation
    - Poisson's ratio change
  - More significant in soft materials
- Suggested solutions
  - Numerical correction
  - Experimental remedy