

Analytical modelling of propagation velocity in nanolaminates

In non-stoichiometric, shock impacted, and electron transparent systems

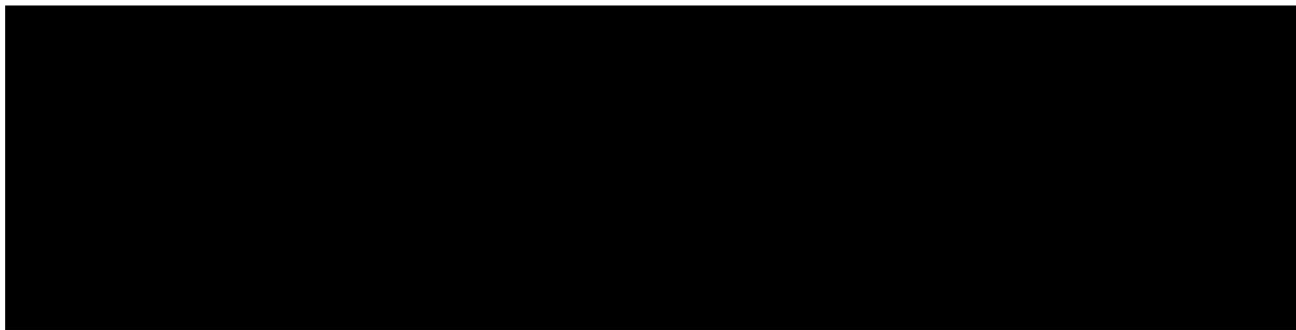
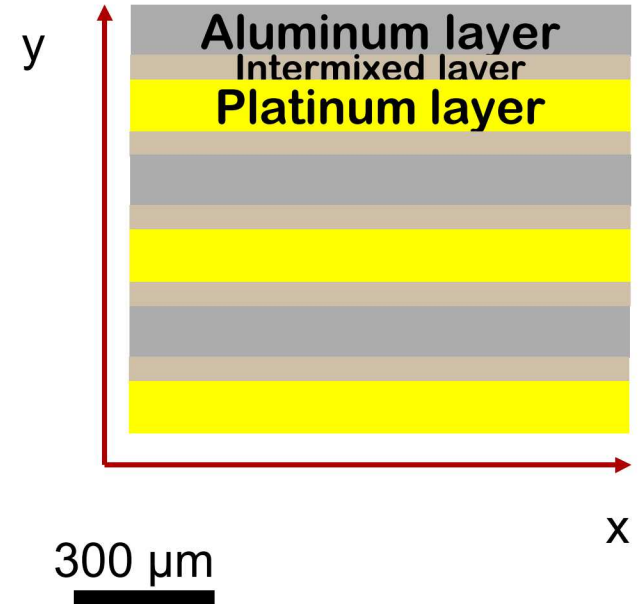
Michael J. Aberer and David P. Adams

Video of Laser Ignition and Self-Propagating Reaction in Al/Pt Multilayer

760 W/cm²

164 nm Bilayer, 31 m/s

1.8 μ s per frame, played at 4 fps



The relationship between physical parameters and propagation velocity

From Mann, Gavens, Reiss, Van Heerden, Bao, and Weihs, J. Appl. Phys. 1997:

Fick's Second Law

$$\frac{dC}{dt} - \nabla \cdot D \nabla C = 0$$

Heat Equation

$$c_p \rho \frac{dT}{dt} - c_p \rho \lambda \nabla^2 T = \frac{dQ}{dt}$$

Solved for the case of a
moving wave

$$v_x^2 = \left(\sum_{n=odd} \frac{k_n^2}{\alpha_n^2} \right)^{-1} \frac{4\lambda_x^2 R T_f^2 D}{E_A \lambda_y (T_{ad} - T_0)} \exp \left(\frac{-E_A}{R T_f} \right)$$

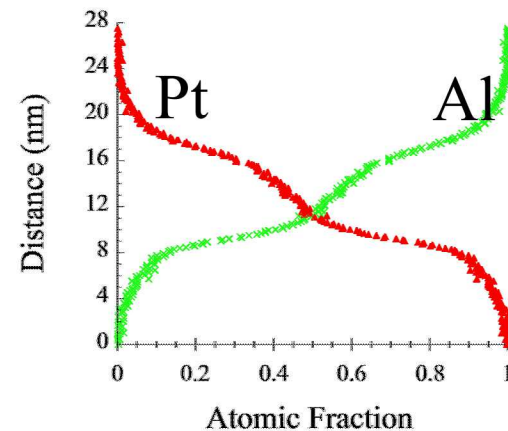
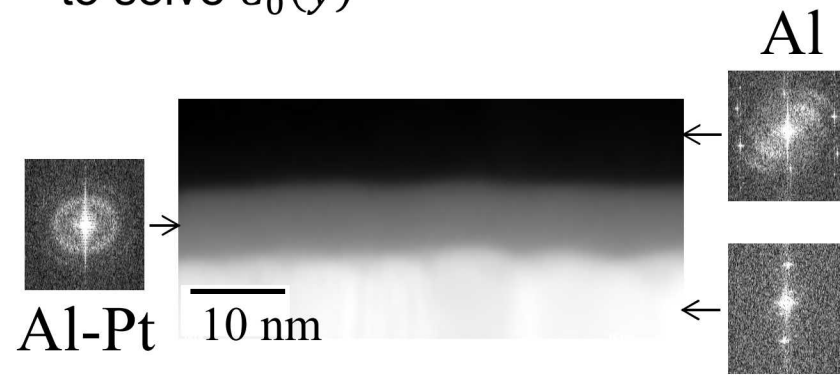
Values for model parameters are informed by experimental data

$$v_x^2 = \left(\sum_{n=odd} \frac{k_n^2}{\alpha_n^2} \right)^{-1} \frac{4\lambda_x^2 R T_f^2 D}{E_A \lambda_y (T_{ad} - T_0)} \exp\left(\frac{-E_A}{R T_f}\right)$$

Series of eigenvalues, α_n , and Fourier coefficients, k_n , account for the potential energy lost to premixed material as a function of initial concentration, $C_0(y)$, for quarter period δ .

$$\alpha_n = \frac{n\pi}{2\delta} \quad k_n = \frac{1}{2\delta} \int_{-2\delta}^{2\delta} C_0(y) \sin(\alpha_n y) dy$$

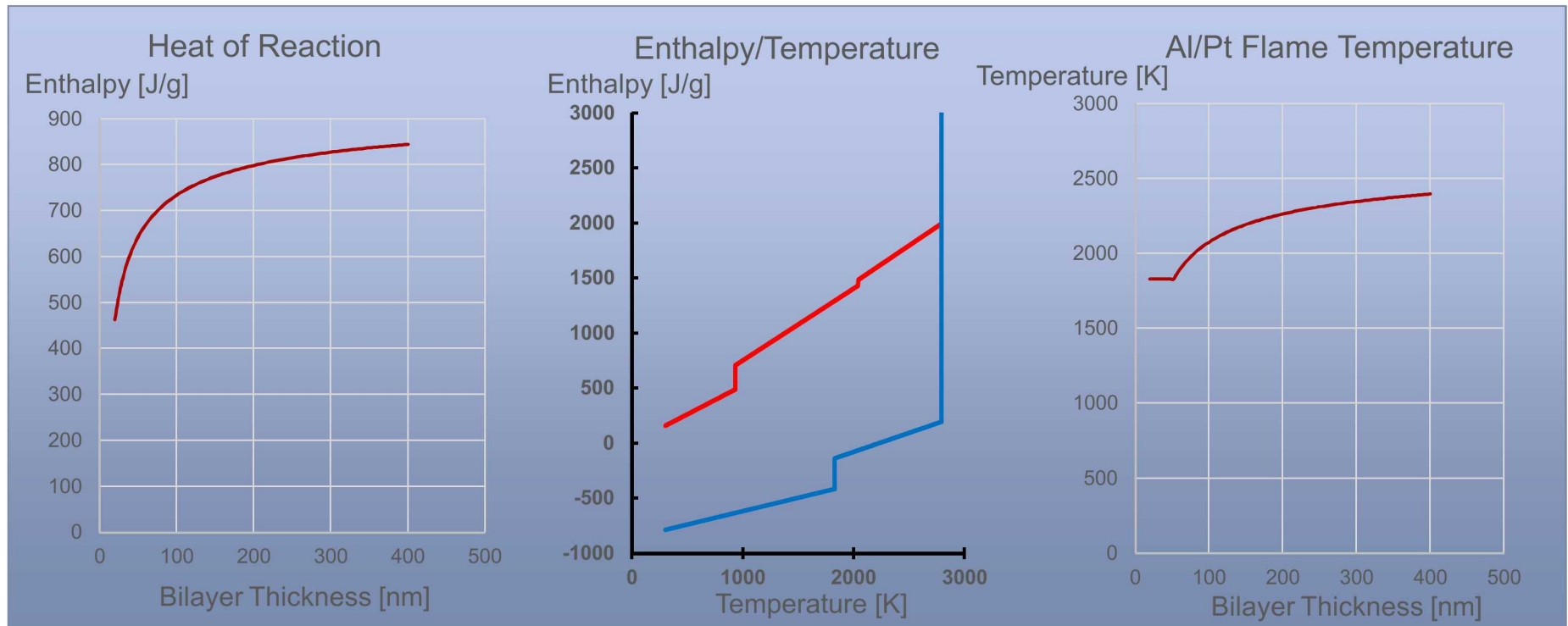
Concentration at each 1 Å wide mesh point comes from STEM-EDS to solve $C_0(y)$



Values for model parameters are informed by experimental data

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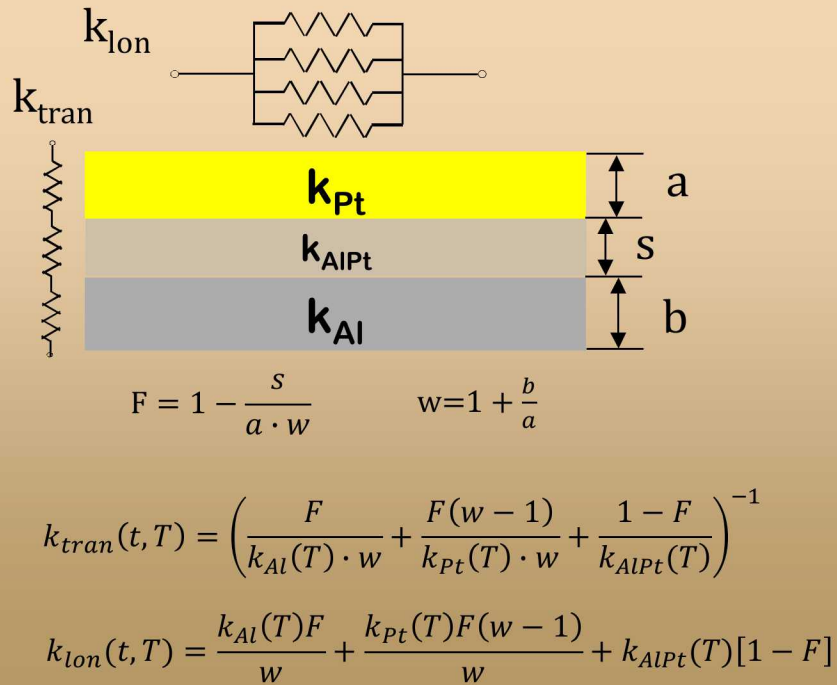
Heat of reaction from calorimetry and enthalpy-temperature diagrams allow for the calculation of adiabatic and flame temperatures



Thermal Conductivity in a three-parameter fit of velocity data

$$v_x^2 = \left(\sum_{n=odd} \frac{k_n^2}{\alpha_n^2} \right)^{-1} \frac{4\lambda_x^2 R T_f^2 D}{E_A \lambda_y (T_{ad} - T_0)} \exp \left(\frac{-E_A}{R T_f} \right)$$

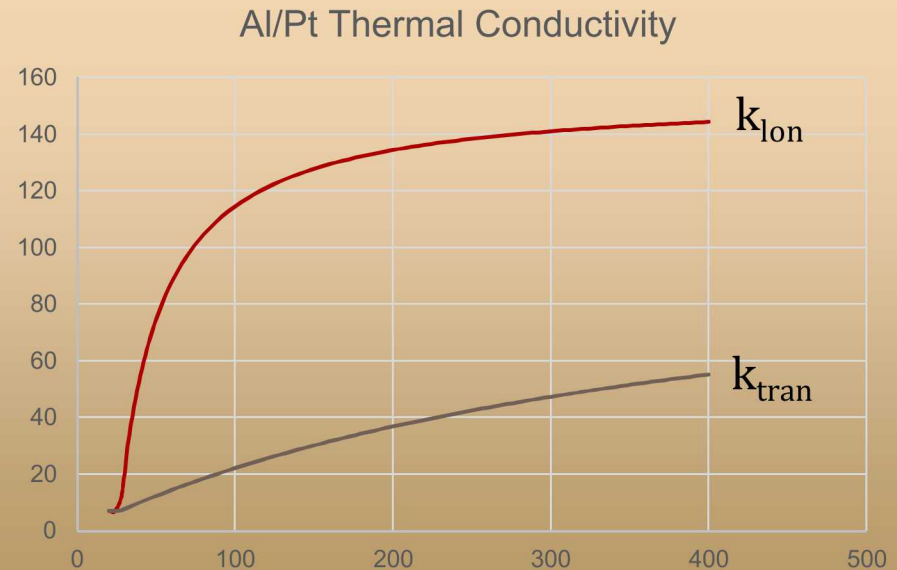
Heat of reaction from calorimetry and enthalpy-temperature diagrams allow for the calculation of adiabatic and flame temperatures



$$F = 1 - \frac{s}{a \cdot w} \quad w = 1 + \frac{b}{a}$$

$$k_{tran}(t, T) = \left(\frac{F}{k_{Al}(T) \cdot w} + \frac{F(w-1)}{k_{Pt}(T) \cdot w} + \frac{1-F}{k_{AlPt}(T)} \right)^{-1}$$

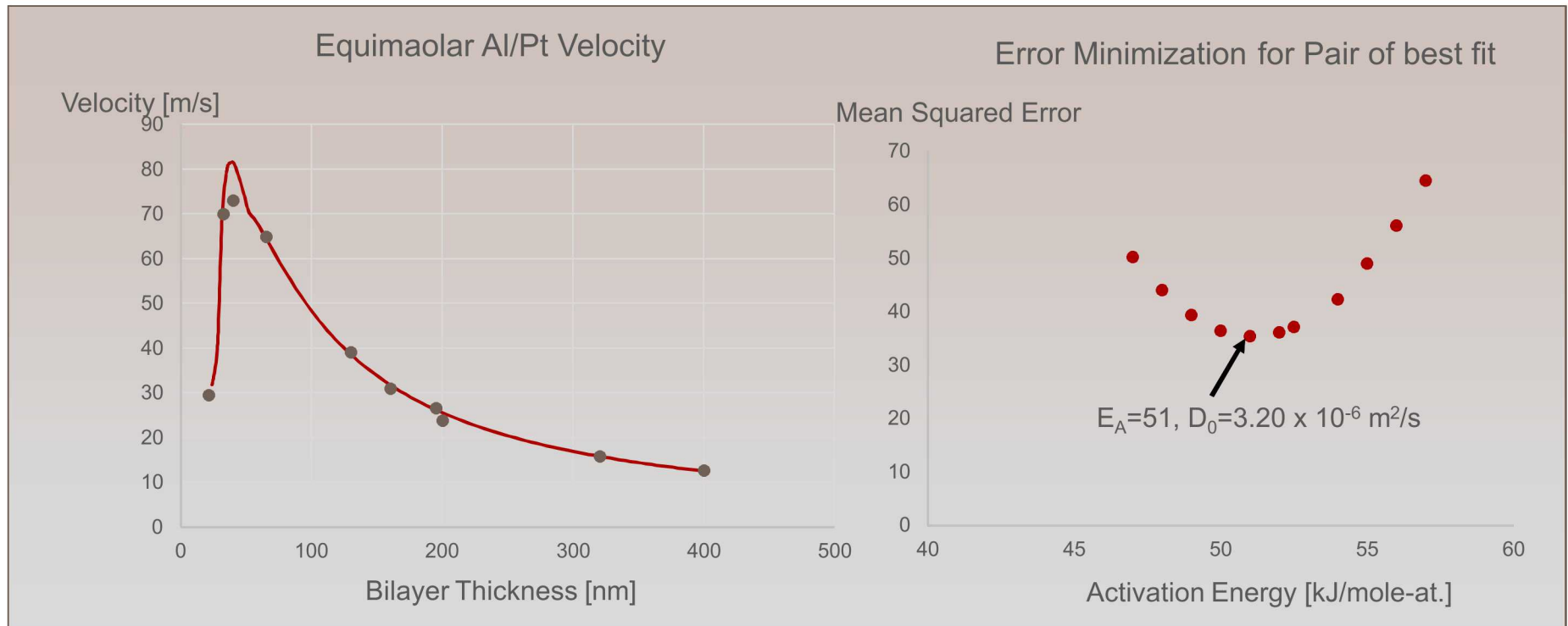
$$k_{lon}(t, T) = \frac{k_{Al}(T)F}{w} + \frac{k_{Pt}(T)F(w-1)}{w} + k_{AlPt}(T)[1-F]$$



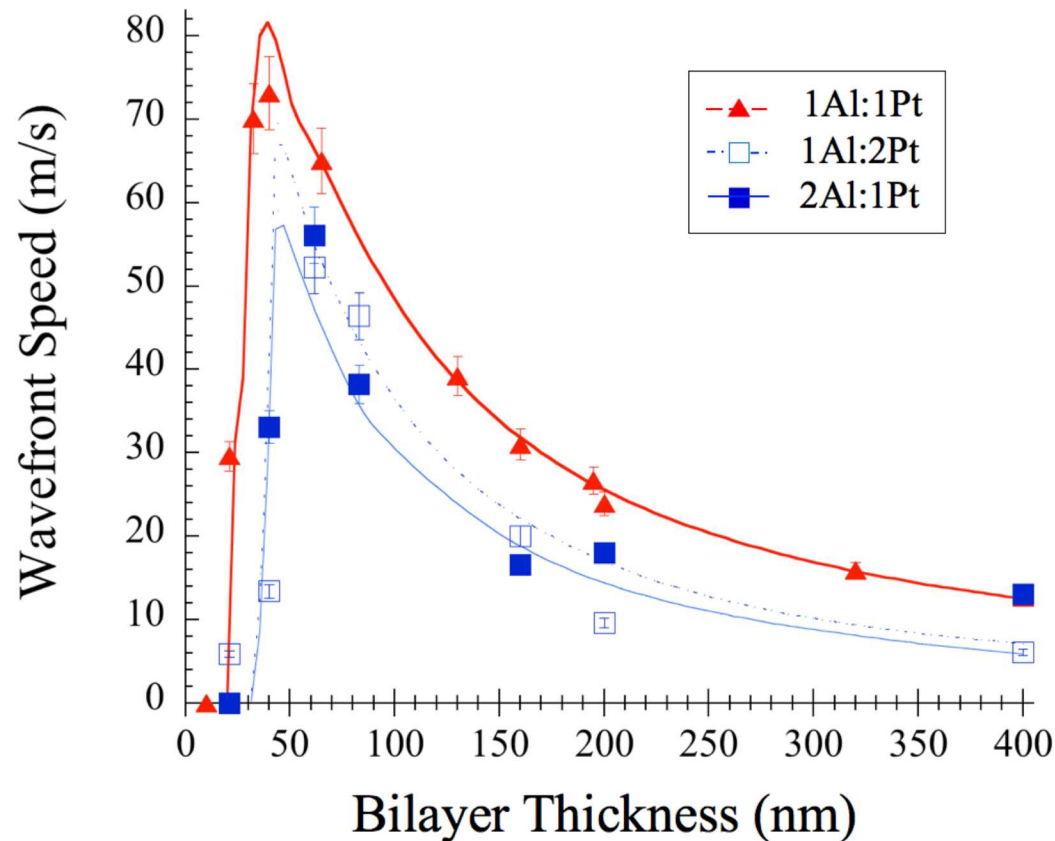
Pair of best fit for activation energy and diffusion coefficient

$$v_x^2 = \left(\sum_{n=odd} \frac{k_n^2}{\alpha_n^2} \right)^{-1} \frac{4\lambda_x^2 R T_f^2 D}{E_A \lambda_y (T_{ad} - T_0)} \exp\left(\frac{-E_A}{R T_f}\right)$$

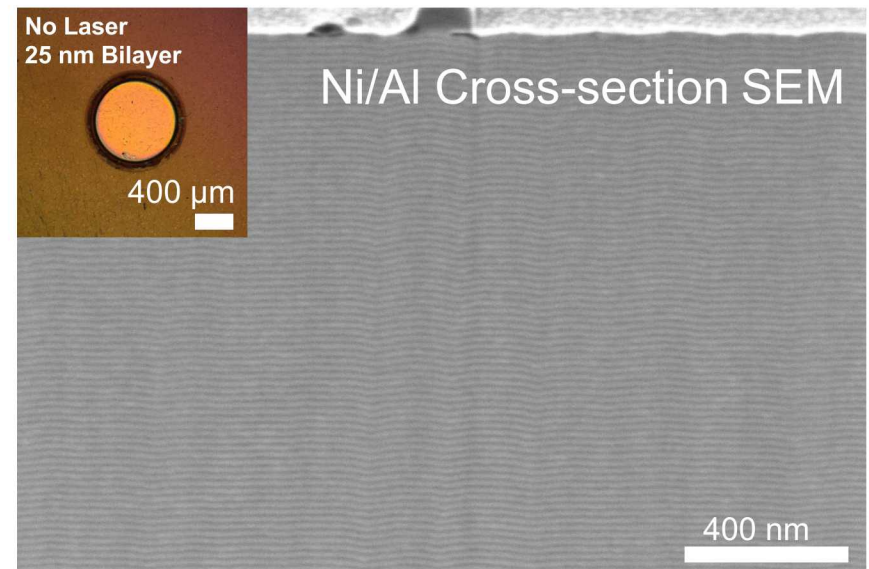
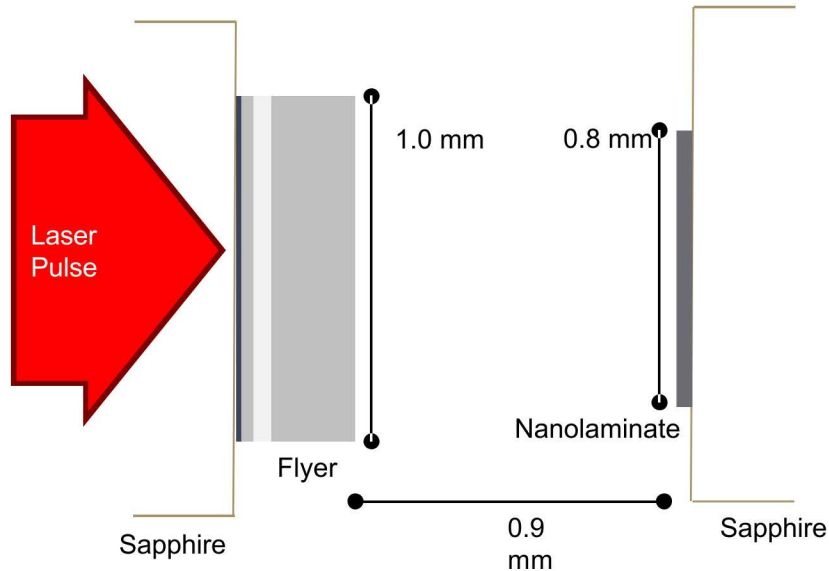
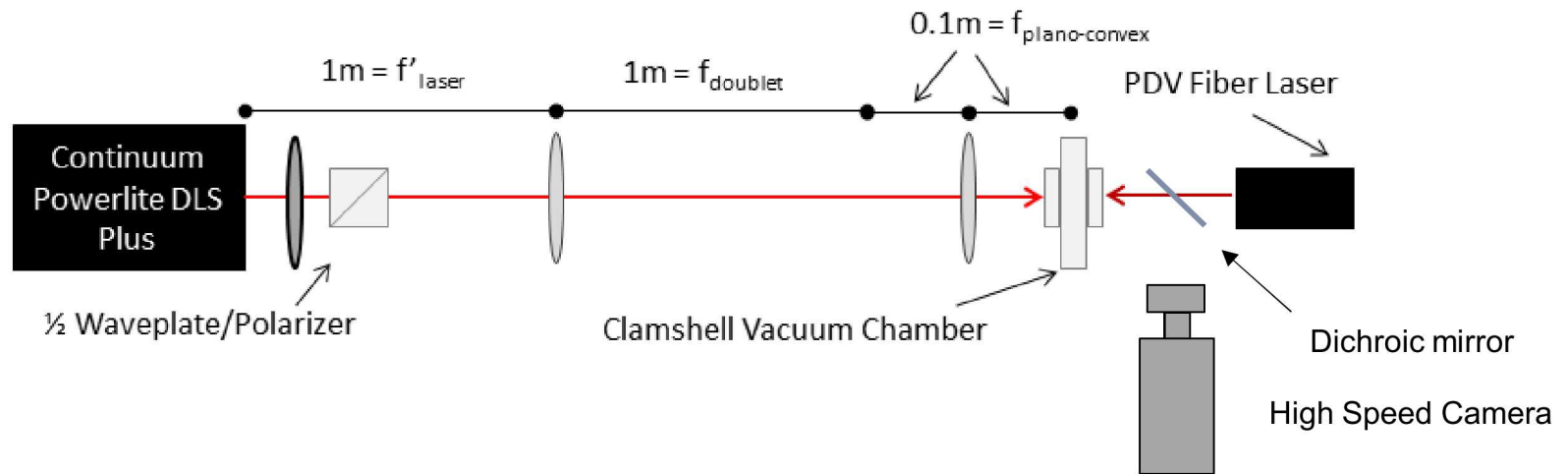
Heat of reaction from calorimetry and enthalpy-temperature diagrams allow for the calculation of adiabatic and flame temperatures



Material parameters remain constant for fitting off-stoichiometric compositions of Al/Pt

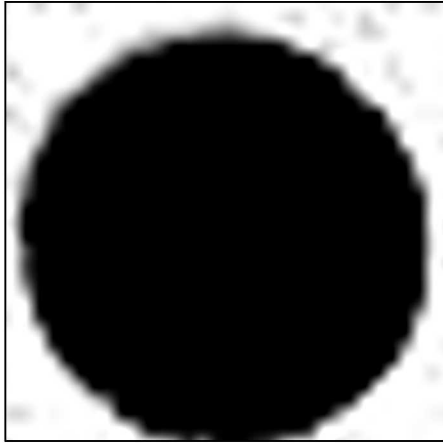


Laser flyer plate impact of Ni/Al

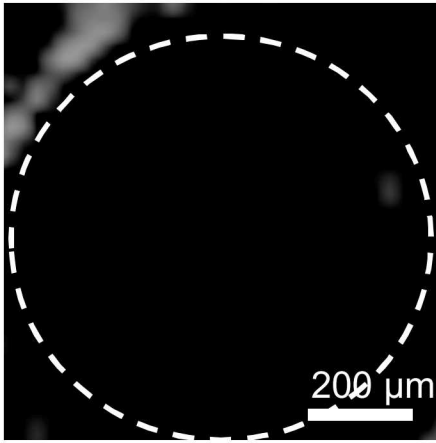


Videography of NiV/Al impact at 440 m/s

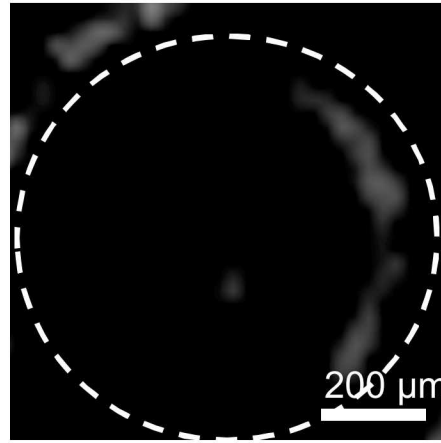
(a) $t=0 \mu\text{s}$



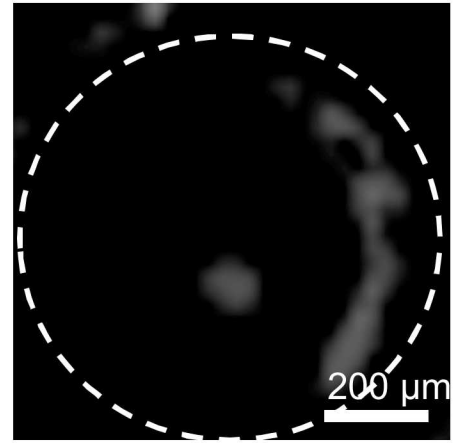
(b) $t=2.13 \mu\text{s}$



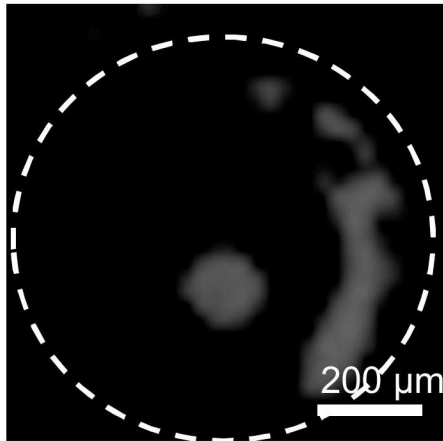
(c) $t=4.26 \mu\text{s}$



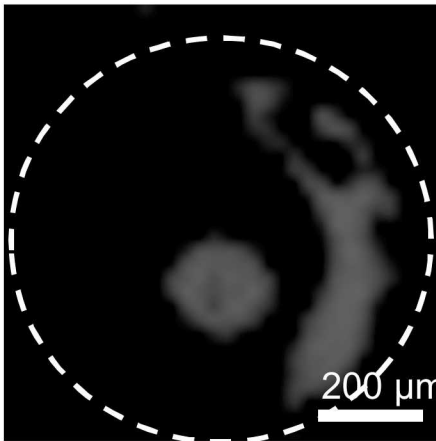
(d) $t=6.39 \mu\text{s}$



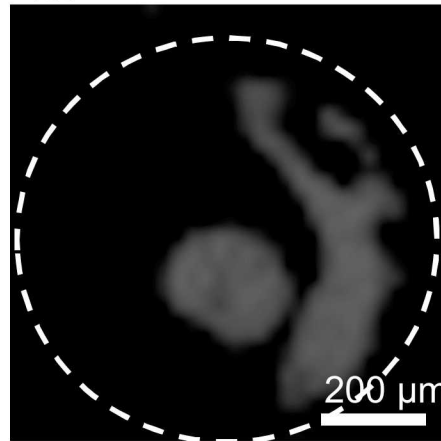
(e) $t=8.52 \mu\text{s}$



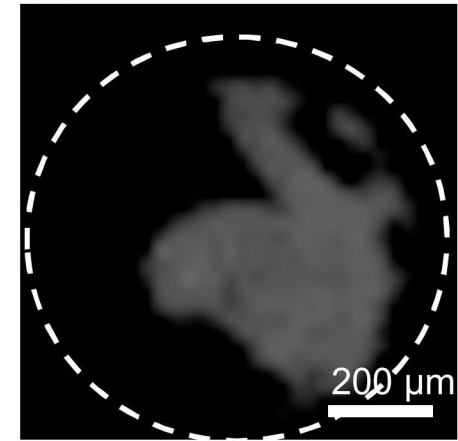
(f) $t=10.65 \mu\text{s}$



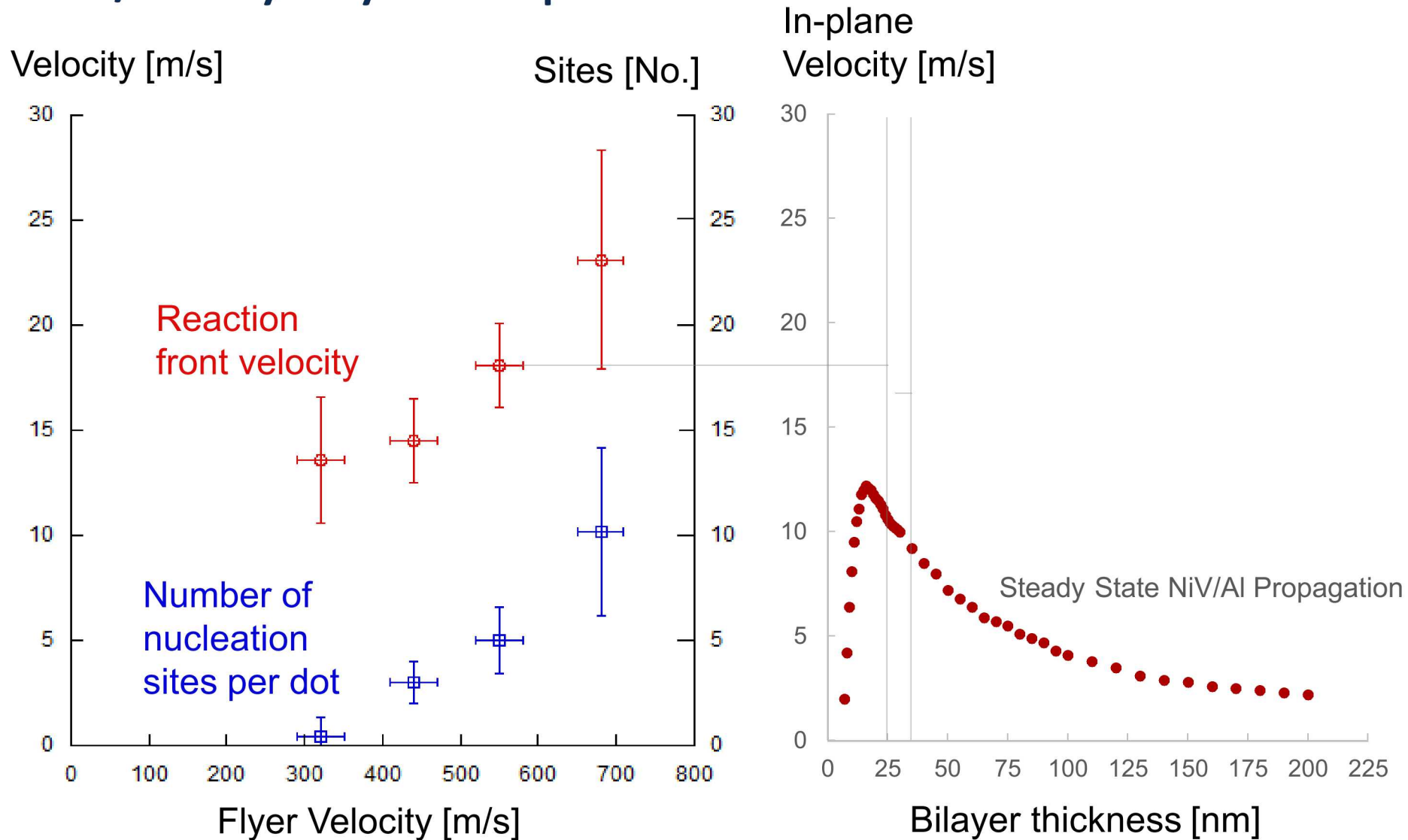
(g) $t=12.78 \mu\text{s}$



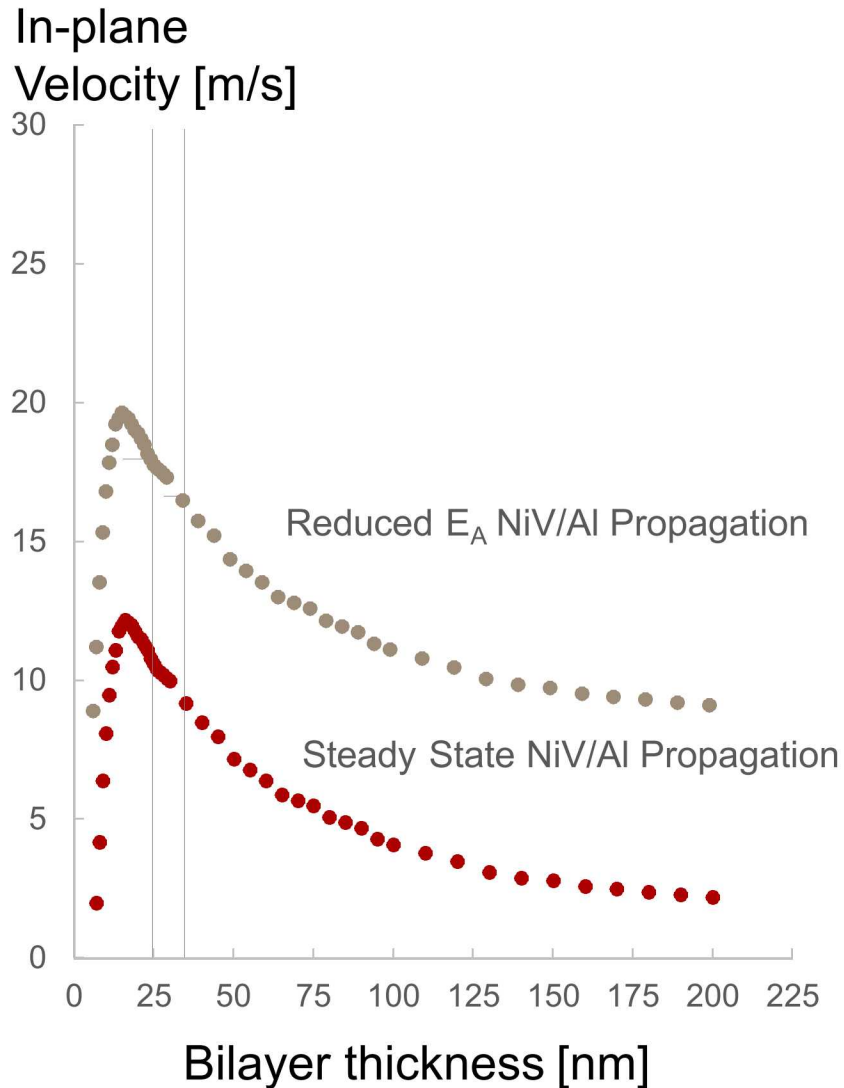
(h) $t=14.91 \mu\text{s}$



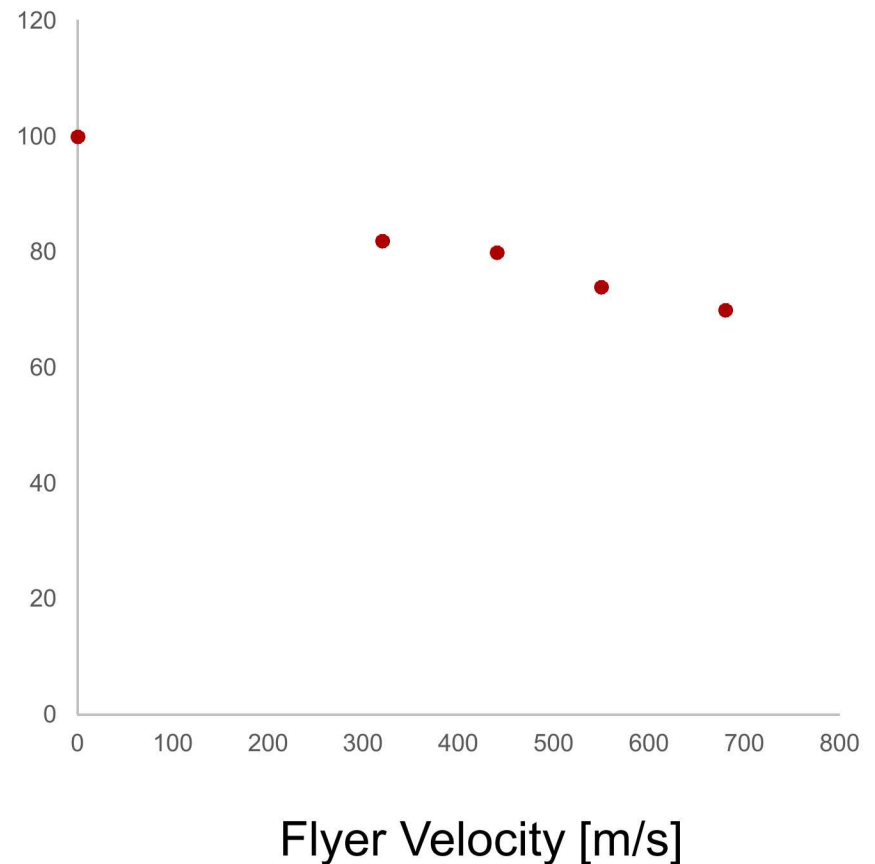
Reaction velocity of 25 nm Bilayer NiV/Al by Flyer Impact



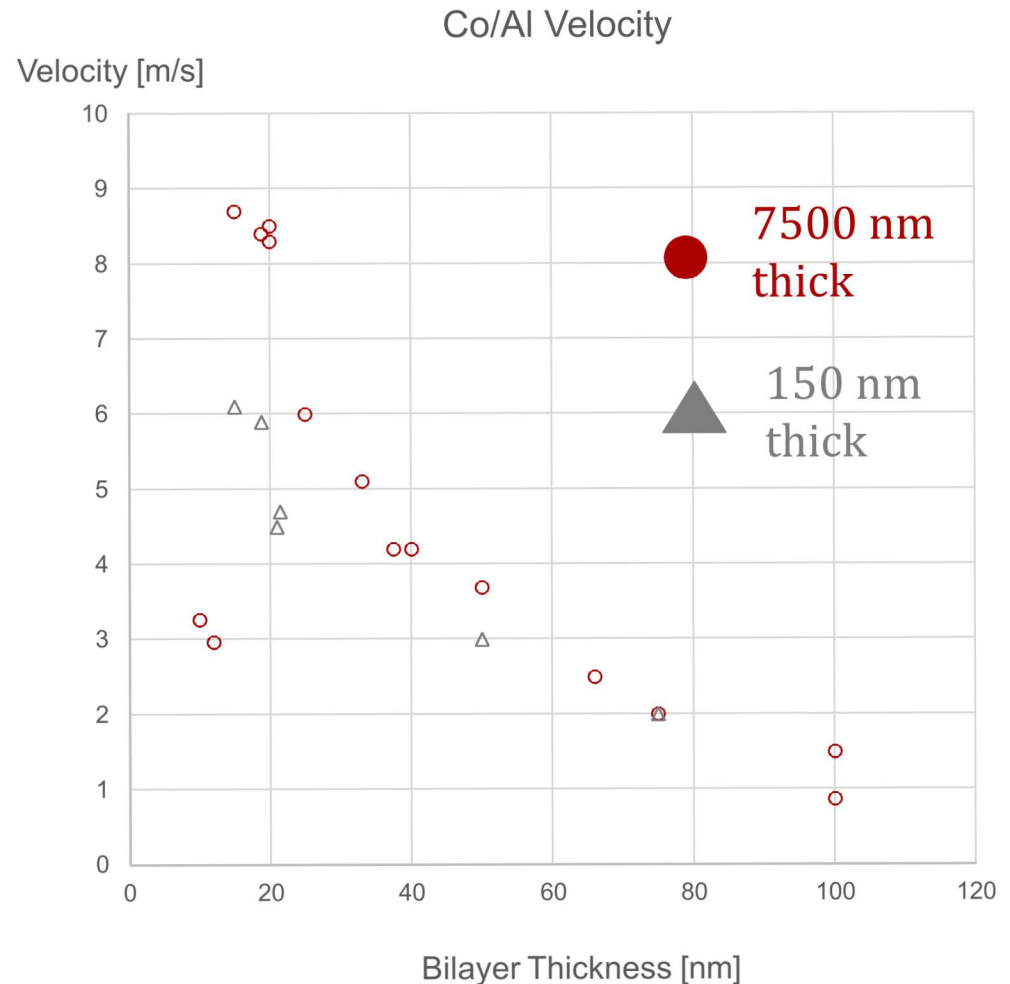
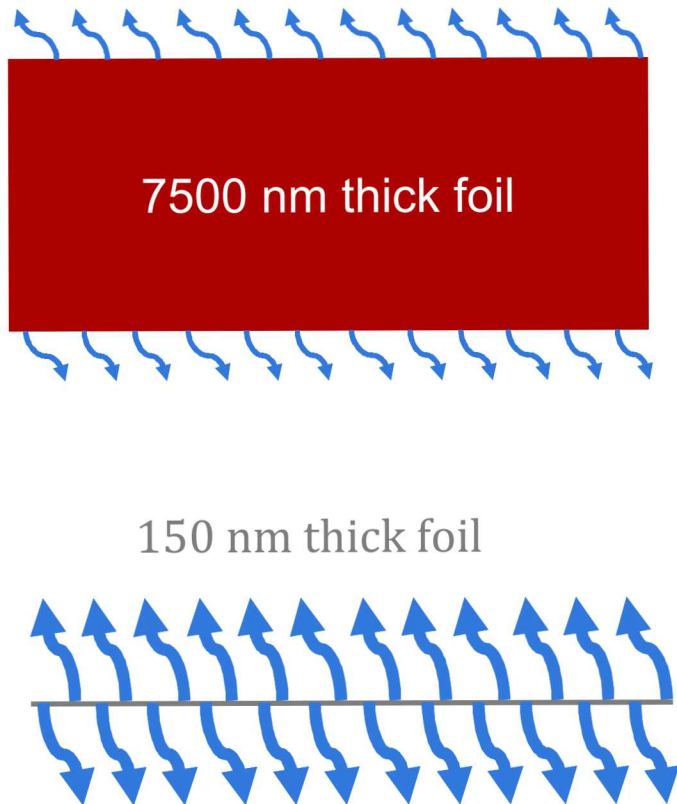
Reduction in Ni/Al Activation Energy after Impact



Activation Energy [J/mole-at.]



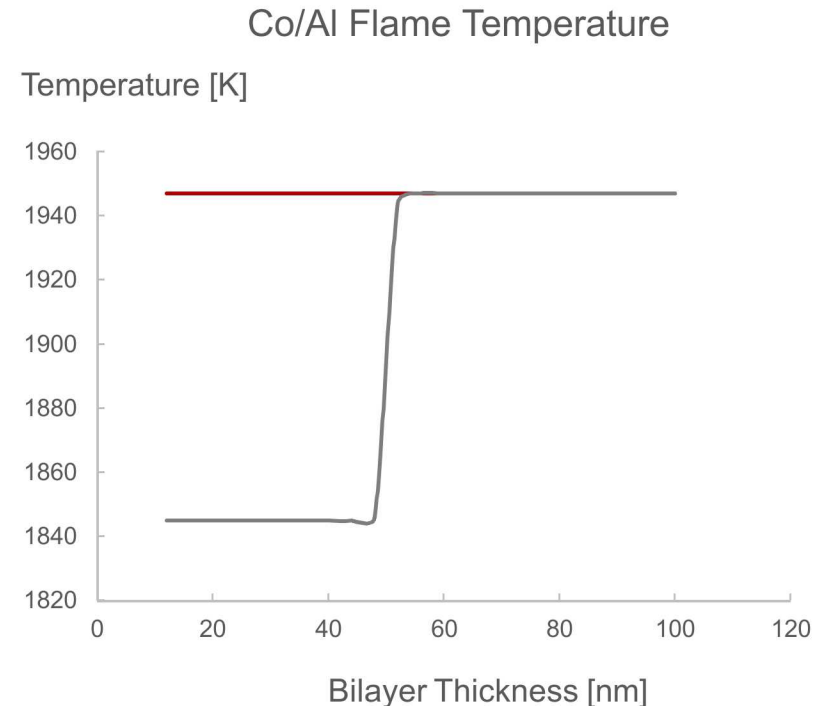
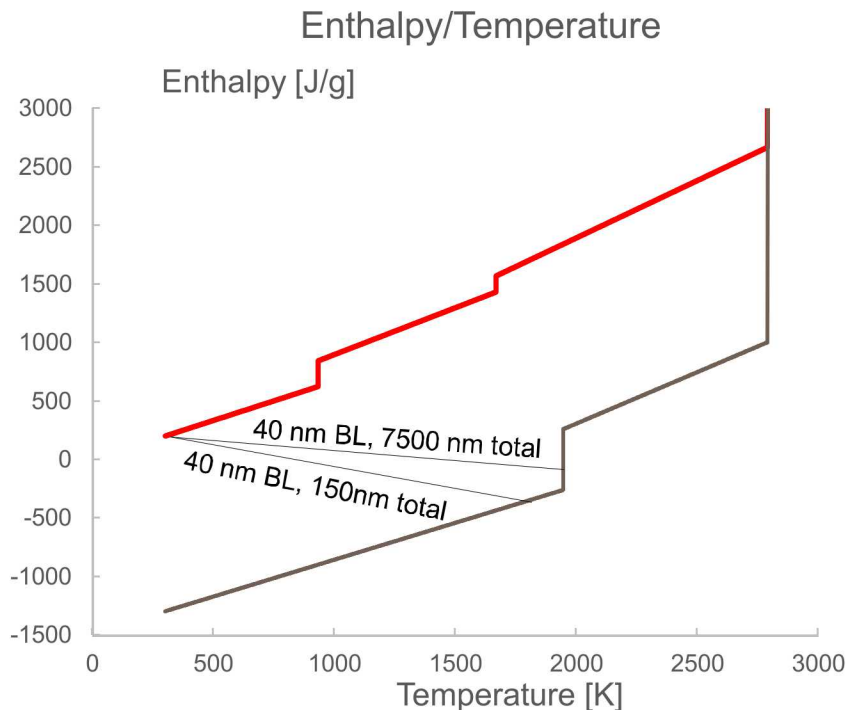
Radiation Loss in Electron Transparent Samples



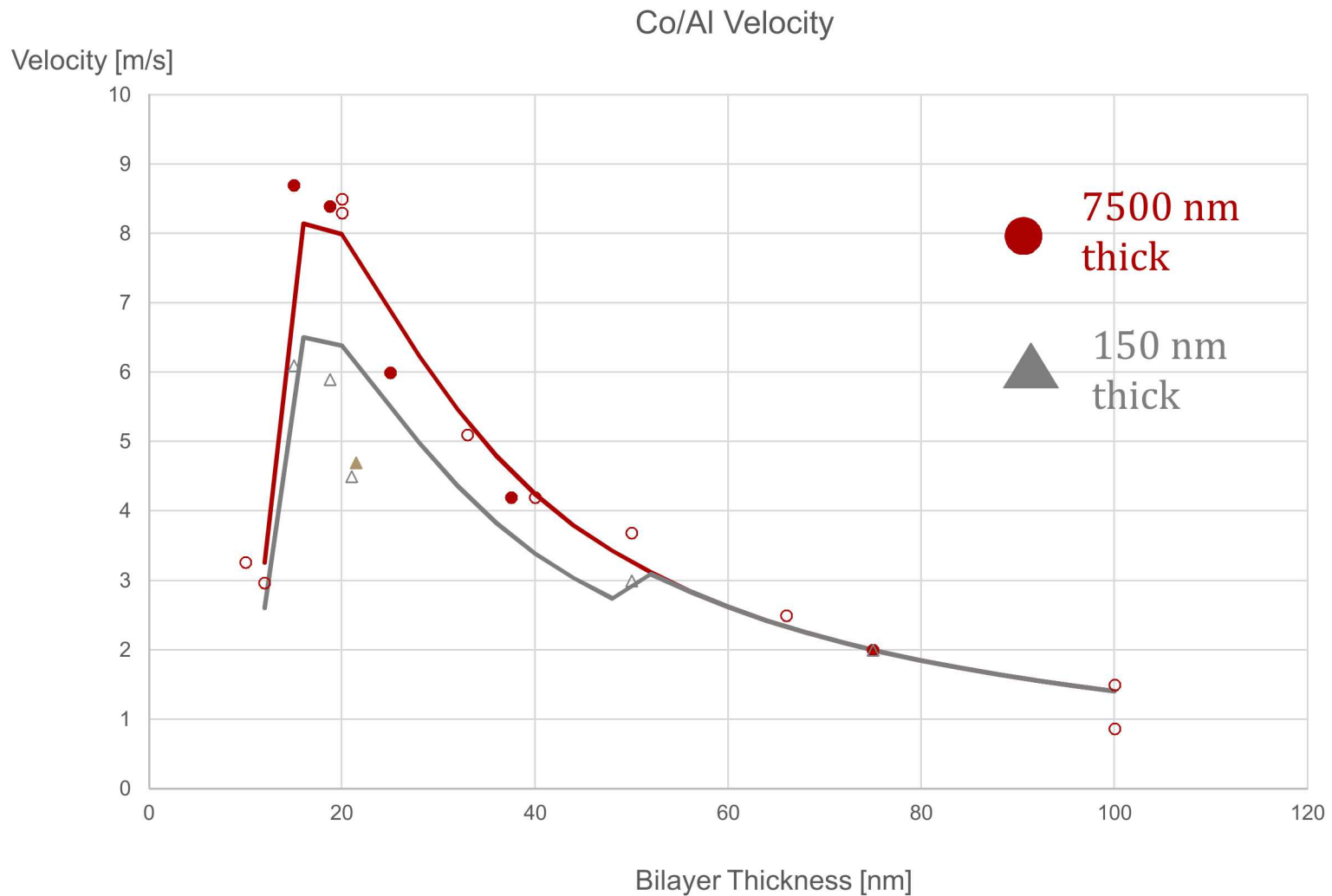
Accounting for Heat Loss in Model

Flame temperature is determined based on the enthalpy/temperature diagram from the heat of reaction minus heat lost to radiation by:

$$Q_{rad}(T) = -\varepsilon \frac{2\sigma(T^4 - T_0^4)}{b_{Total\ Thickness}}$$



Matching to Experimental Data



Summary

We have populated the Mann model based on experimental data in order to fit values for activation energy and diffusion coefficient

This framework can be applied to the propagation of shock impacted samples to calculate changes in activation energy

Energy loss to the environment can be incorporated into the model based on its effects on flame temperature for studies involving self-propagation of electron transparent reactive nanolaminates