

Discussion of Electron Inertia implementation, and Nernst term affect on Maglif calculations.

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Regular MHD

(This approach currently taken our Lab ICF design codes)

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$$

$$\partial_t \mathbf{E} = c^2 (\nabla \times \mathbf{B} - \mu_0 \mathbf{J})$$

Usually drop this term and close with a simple ohms law

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J}$$

Advection

$$\partial_t \mathbf{B} = \nabla \times \mathbf{u} \times \mathbf{B}$$

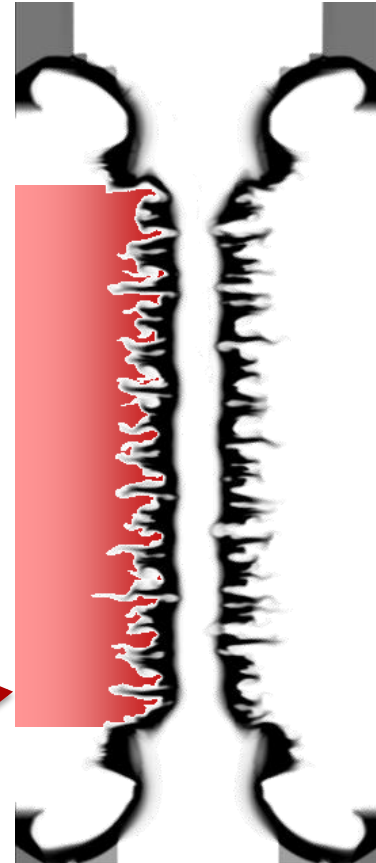
Diffusion

$$\partial_t \mathbf{B} = -\nabla \times \left(\frac{\eta}{\mu_0} \nabla \times \mathbf{B} \right)$$

Can't propagate field through vacuum so its assigned a high resistivity and a choice is made on the density this switches to plasma.

Density can't be arbitrarily low as MHD breaks down and $\mathbf{E} = \eta \mathbf{J}$ could support unrealistic currents

Sort of OK for Z targets – an unknown for Z-Next



Incorporate Electron Inertia

Electron inertia:
(Allows physical transition to low densities)

$$\partial_t \mathbf{J} = \varepsilon_0 \omega_{pe}^2 (\mathbf{E} - \eta \mathbf{J})$$

$$\mathbf{J} = \frac{\mathbf{E}}{\eta}$$

$$\mathbf{J}^{n+1} = \mathbf{J}^n - \Delta t \varepsilon_0 \omega_{pe}^2 \mathbf{E}$$

$$\omega_{pe}^2 = \frac{n_e e^2}{m_e \varepsilon_0}$$

Rate at which current can ramp up is now tied to electron mass – can't go super-luminal

Advocated for many years by
Matt Martin / C. Seyler

I follow the formalism developed for Gorgon by **Mark Sherlock** cast in terms of the electron equation of motion

$$m_e \frac{d\mathbf{v}}{dt} = -e \underbrace{(\mathbf{E} + \mathbf{v} \times \mathbf{B})} - m_e \gamma \mathbf{v} \underbrace{(\mathbf{v} - \mathbf{v}_i)}$$

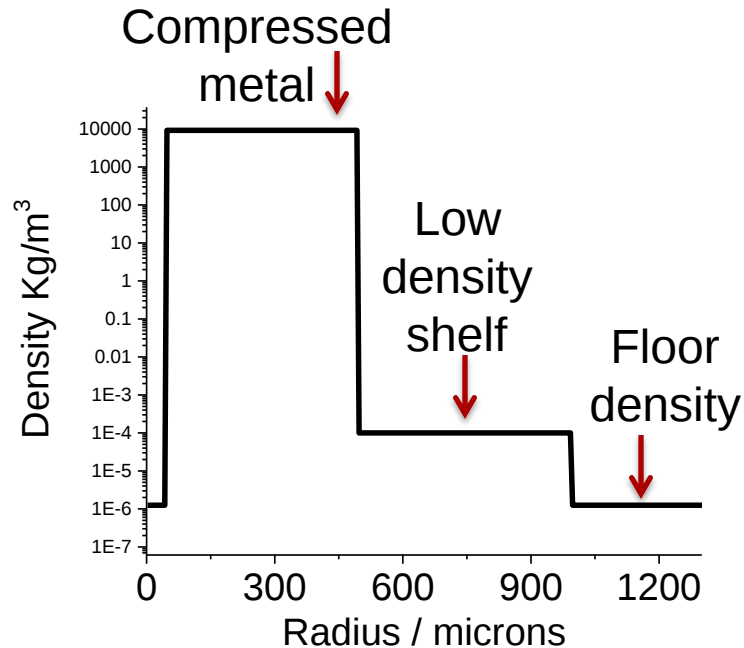
These terms equate to the form on the left, with ion motion assumed small and $\mathbf{v} \times \mathbf{B}$ handled elsewhere.

$$\frac{d\mathbf{p}}{dt} = -e \left(\mathbf{E} + \frac{\mathbf{p}}{\gamma m_e} \times \mathbf{B} \right) - m_e \gamma \mathbf{v} \left(\frac{\mathbf{p}}{\gamma m_e} - \mathbf{v}_i \right)$$

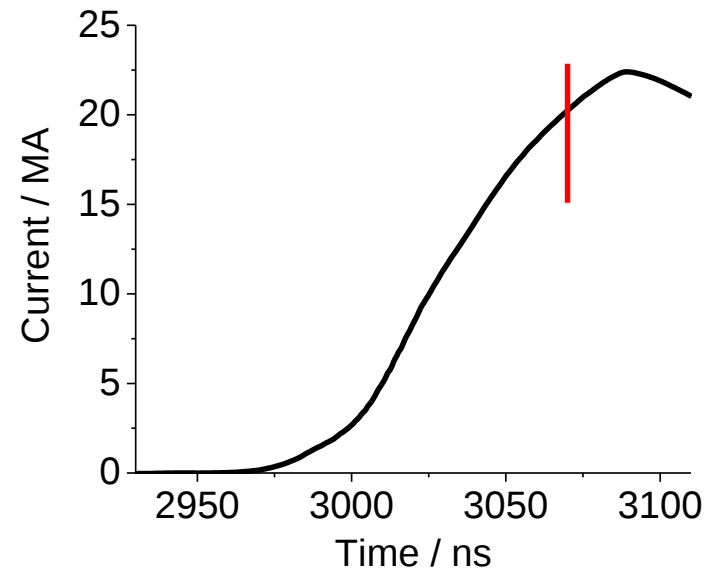
Cast in terms of electron momentum we can cap electron velocity

Simple test case:

Field solver on static plasma



Static beryllium density profile
“inspired by Maglif”
Uniformly 10eV (no heating)
Calculate conductivities from
Lee-Moore-Desjarlais tables.



Run a Z style current through
it, and look at current
distributions at time marked
~20MA

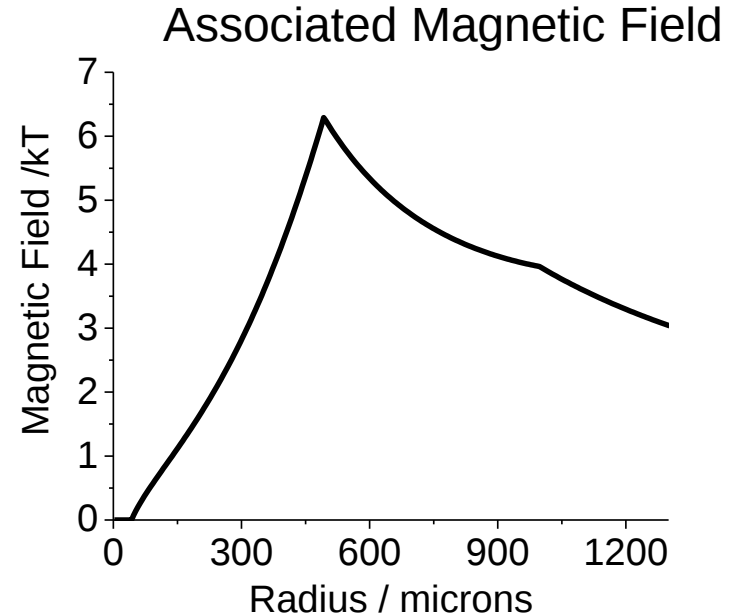
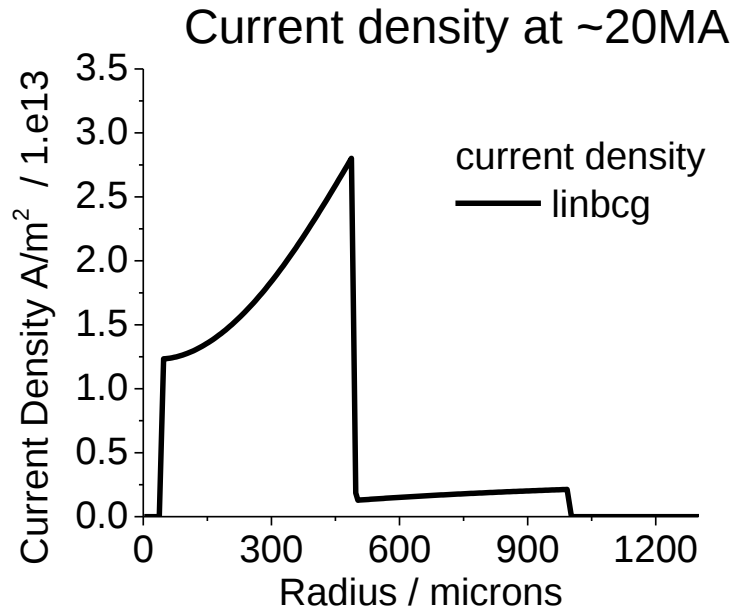
Iterative Matrix Diffusion Solve

$$\partial_t \mathbf{B} = -\nabla \times \frac{\eta}{\mu_0} \nabla \times \mathbf{B}$$

Integrate diffusion equation via fully implicit iterative sparse matrix inversion (typical of many ICF MHD codes)

Assign a vacuum resistivity to transport field over vacuum. (100 ohm-m on anything at floor density)

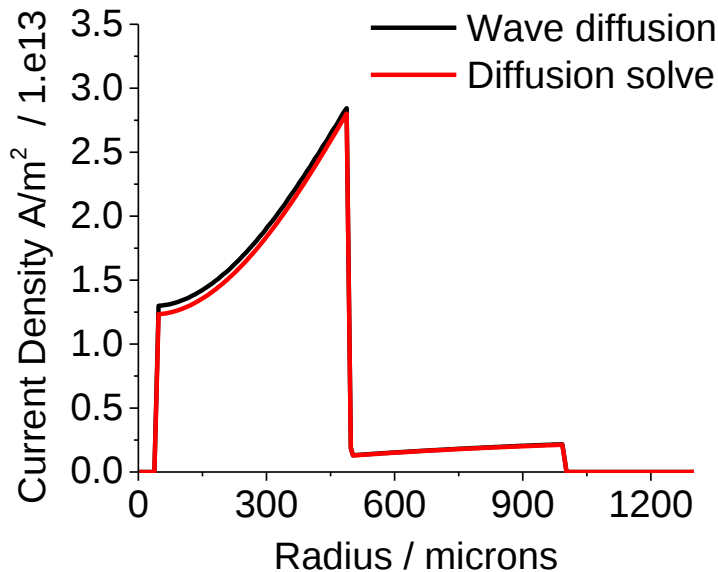
Use Numerical Recipes “linbcg” bi-conjugate gradient solver (itol=1,tol=1.e-5)



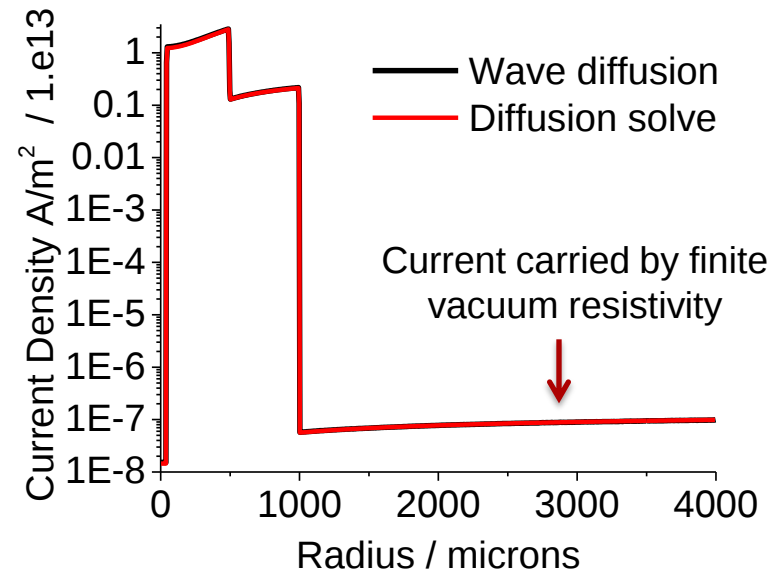
Contrast diffusion solve with Gorgon Wave-Diffusion solve

$$\partial_t \mathbf{B} = -\nabla \times \frac{\eta}{\mu_0} \nabla \times \mathbf{B}$$

Current Densities at ~20MA

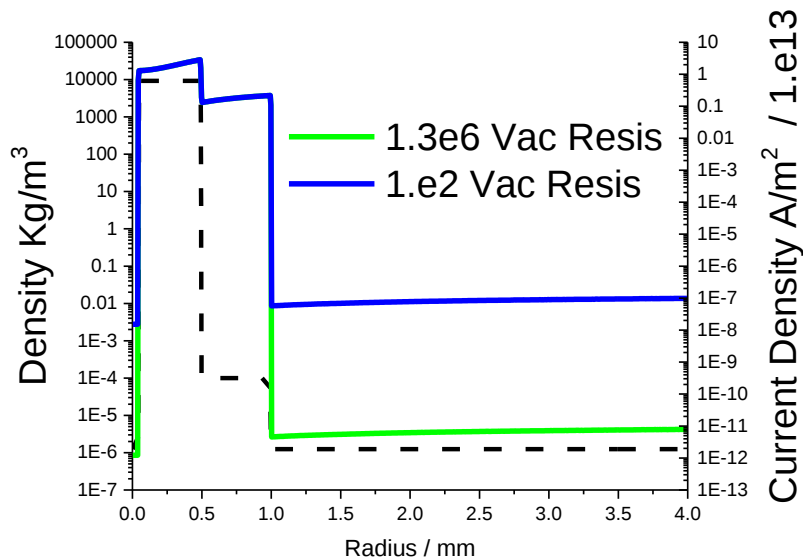


$$\begin{aligned} \partial_t \mathbf{B} &= -\nabla \times \mathbf{E} \\ \partial_t \mathbf{E} &= c^2 (\nabla \times \mathbf{B} - \mu_0 \mathbf{J}) \\ \mathbf{E} &= \eta \mathbf{J} \end{aligned}$$



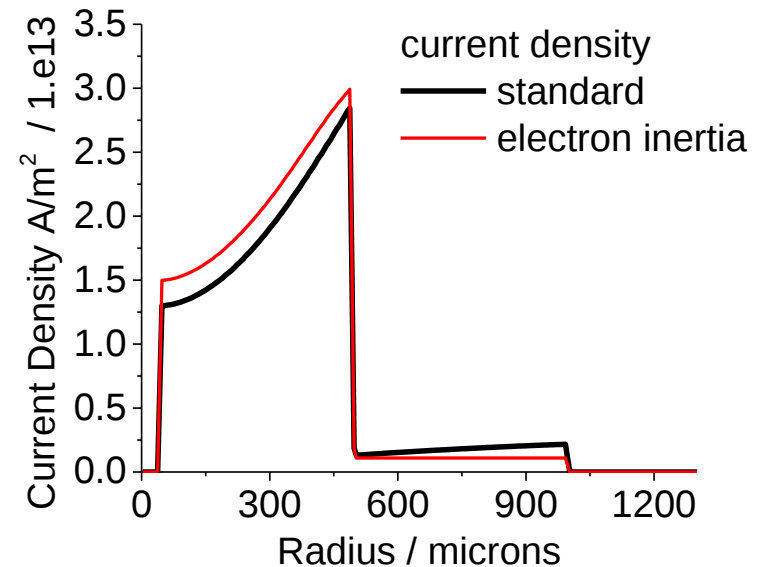
For this comparison vacuum resistivity set to be the same between the different solves

With wave-diffusion equation solve can drop that vacuum resistivity significantly further without any penalty



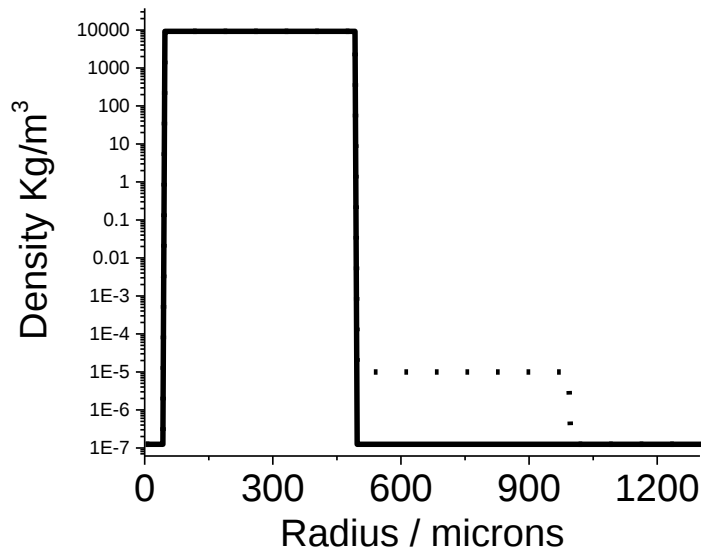
linbcg matrix solve won't run with that high a vacuum resistivity. Iteration doesn't converge and blows up the solution.

Wave-diffusion equation retains an electric field so relatively straight forward to couple to additional current density equations



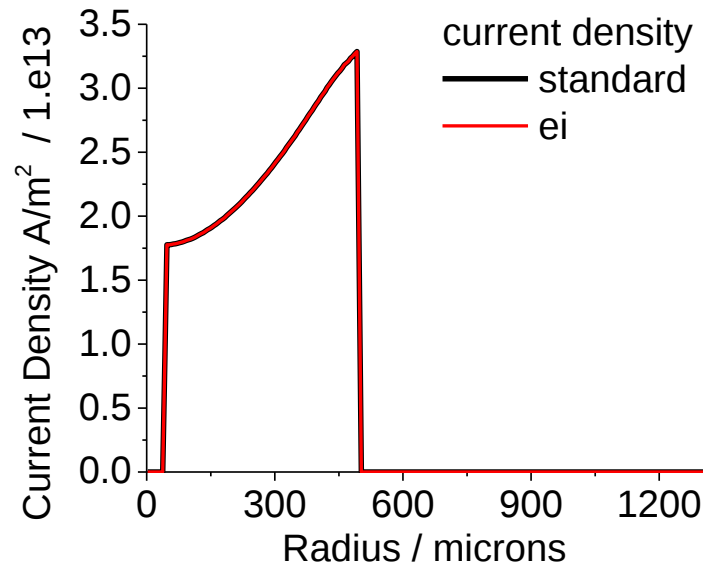
Current density saturates in low density material and ~speed of light drift velocity, and alters distribution of current

Contrast current distributions with and without low density shelf

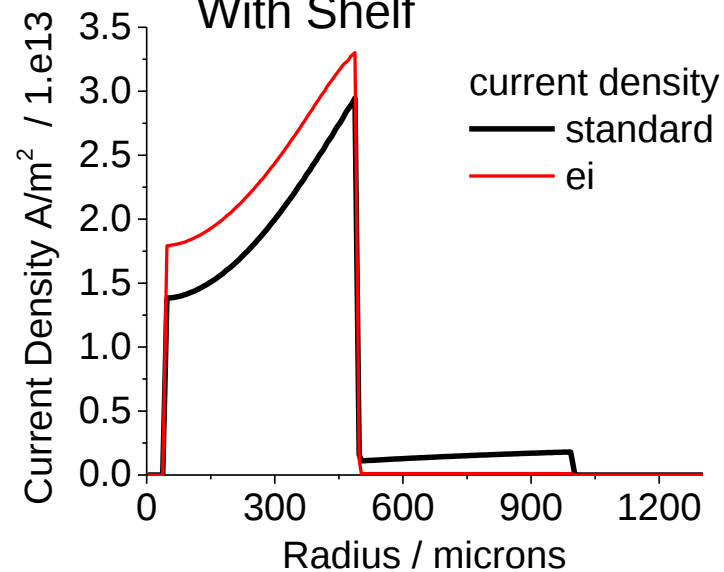


Differences in current distribution more evident for lower density halo $1.e-5 \text{ Kg/m}^3$

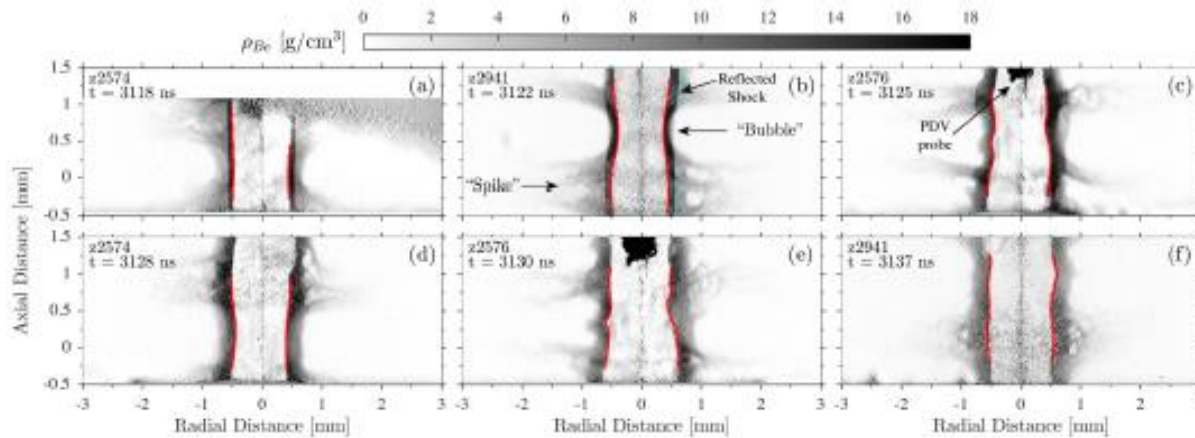
No Shelf



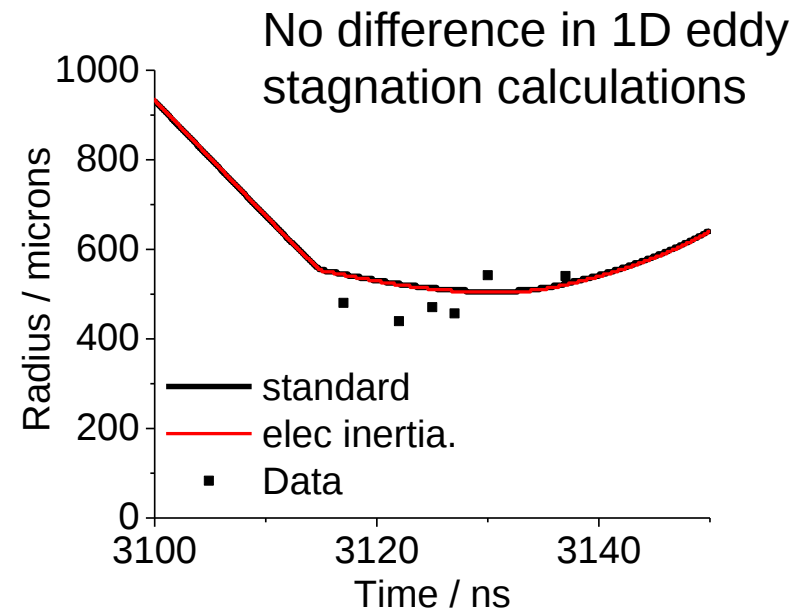
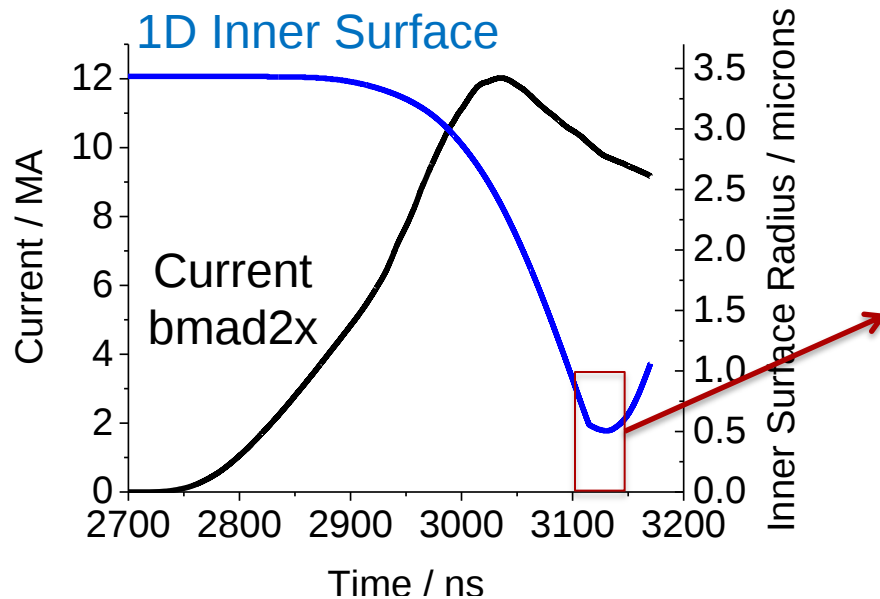
With Shelf



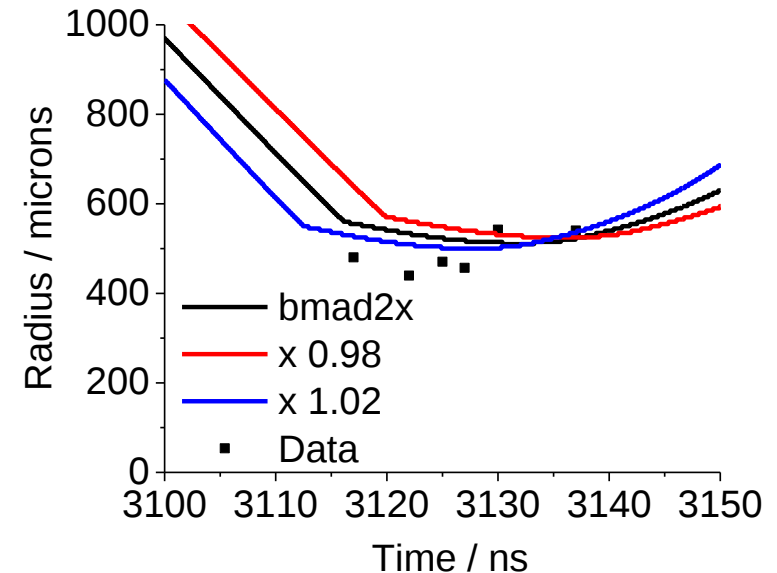
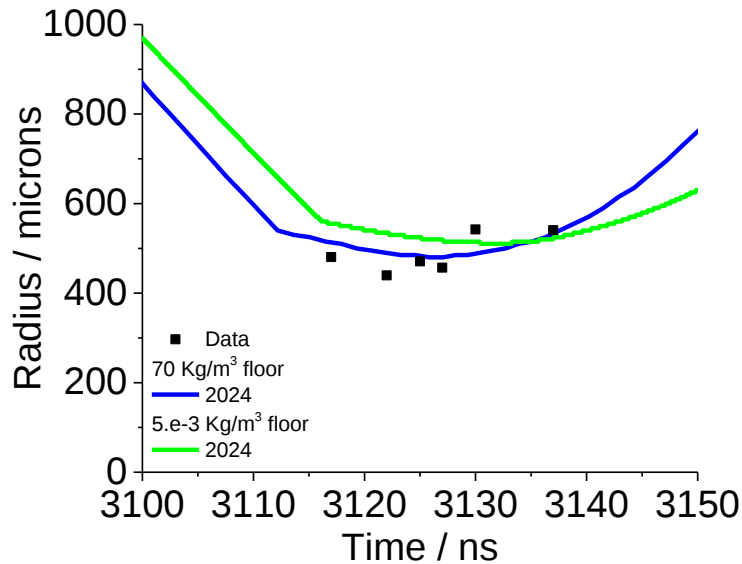
Contrast for Eddy Data – Be Liner compressing a liquid deuterium fill to enable radiography of stagnation



Data: P. Knapp



Other sensitivities are evident in stagnation data

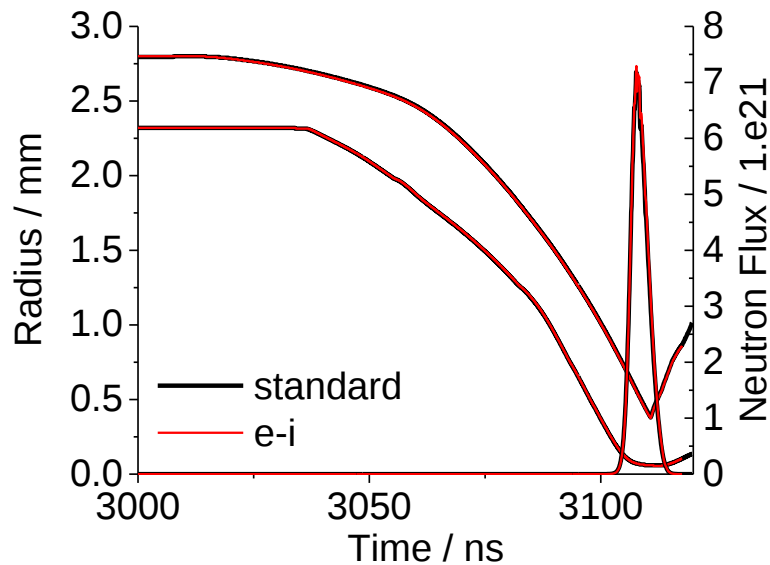


Raising floor density changes late time compression – aside from resistivity treatments, how confident are we in how we treat lower density material in general

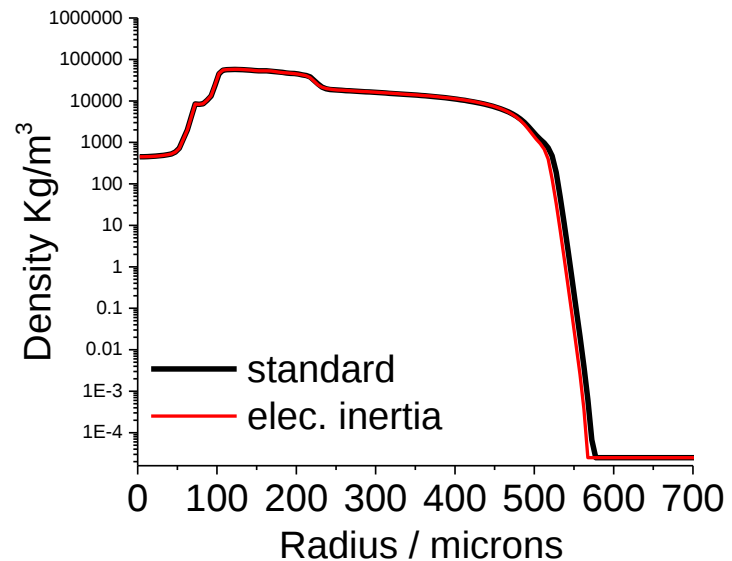
2% Uncertainty in current can alter compression at stagnation

For 1D Maglif Implosion – contrast electron inertial and standard resistivity treatment

No Difference in Implosion Trajectory



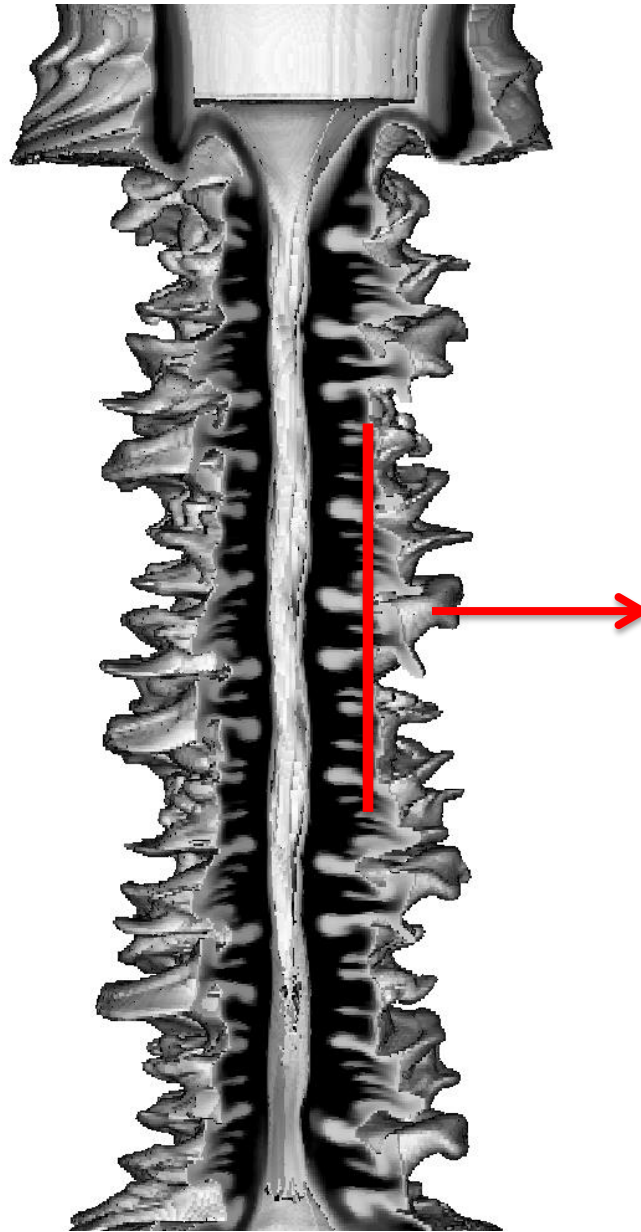
Steep density gradient on outer edge negates differences



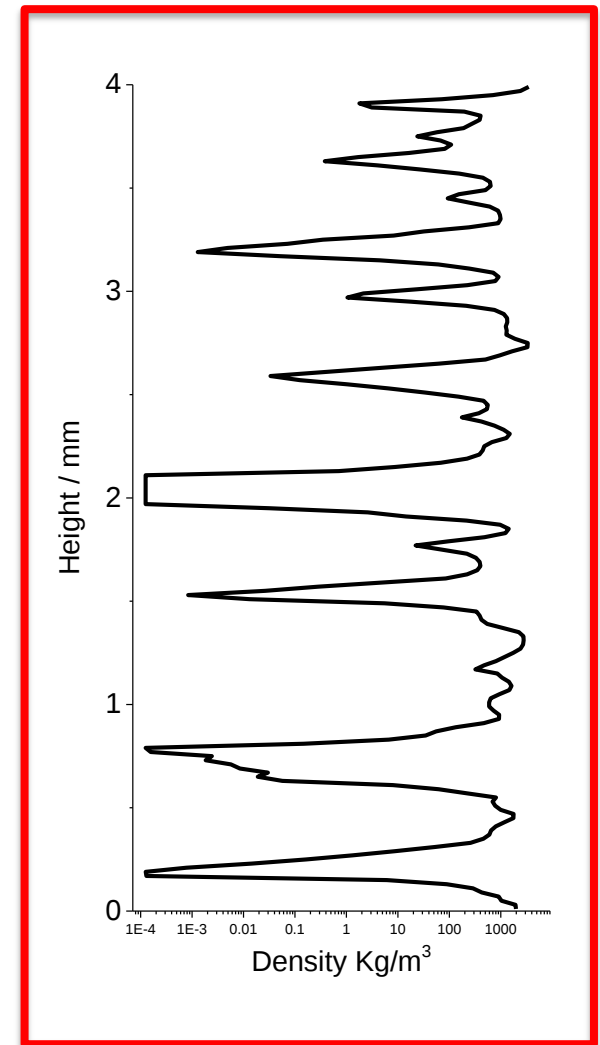
Maglif in reality is very unlikely to be a 1D mass profile.

What we really care about is how current can be distributed through low density plasma distributed by instability growth.

Better treating current distribution in these profiles may directly affect how we grow instabilities of deliver current to small radius at stagnation

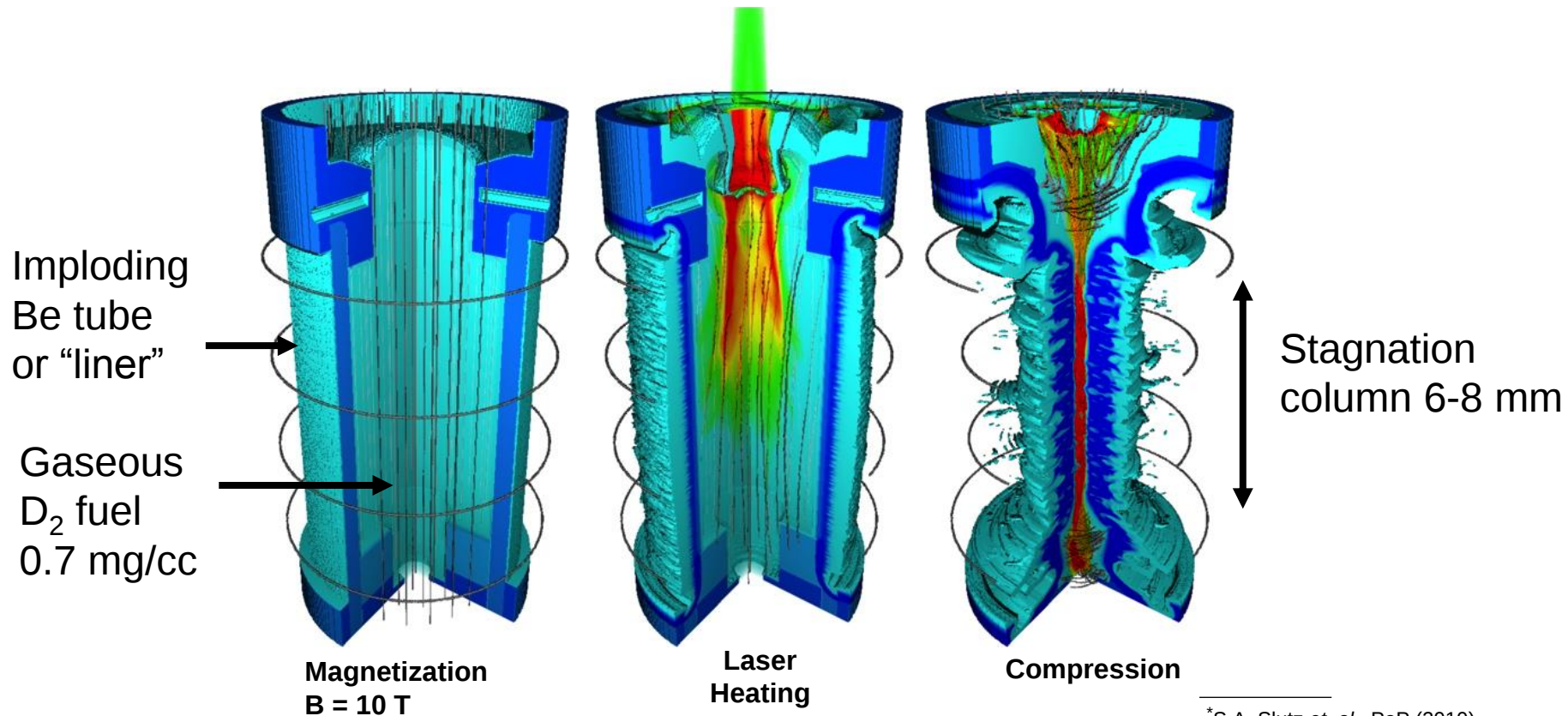


Lineout through trailing mass



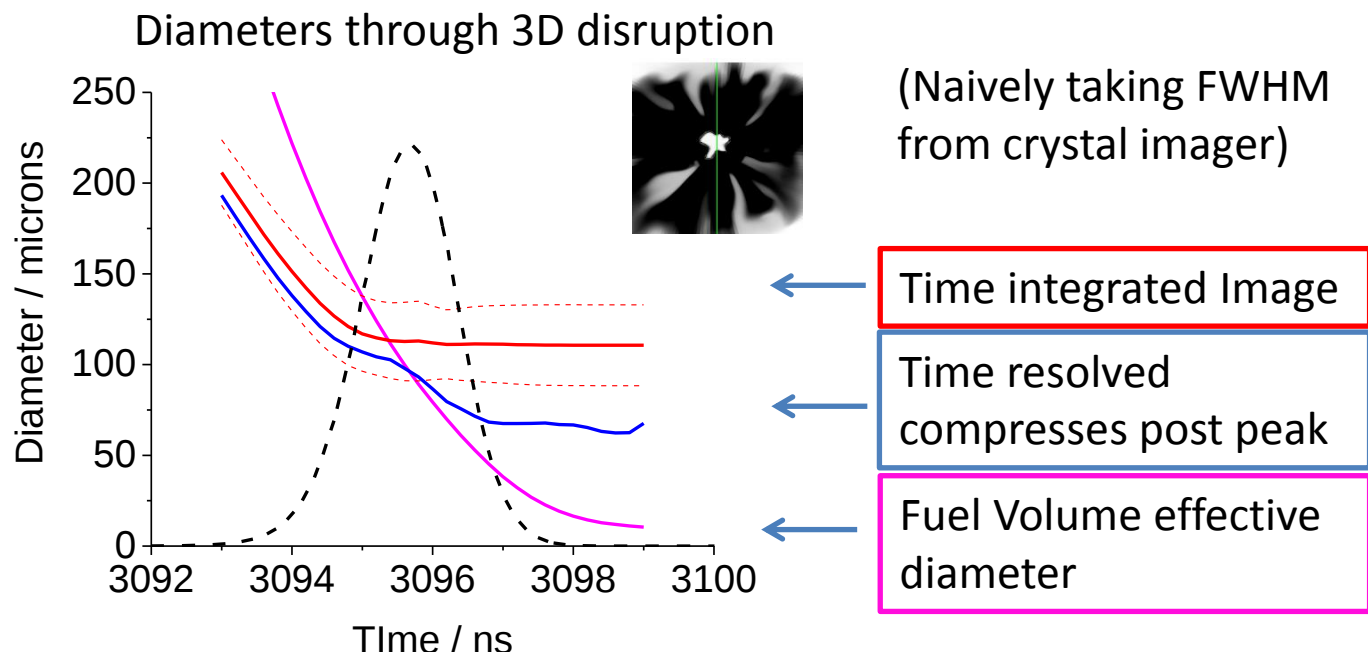
Affect of Nernst on Maglif stagnation

The Magnetized Liner Inertial Fusion (MagLIF) concept

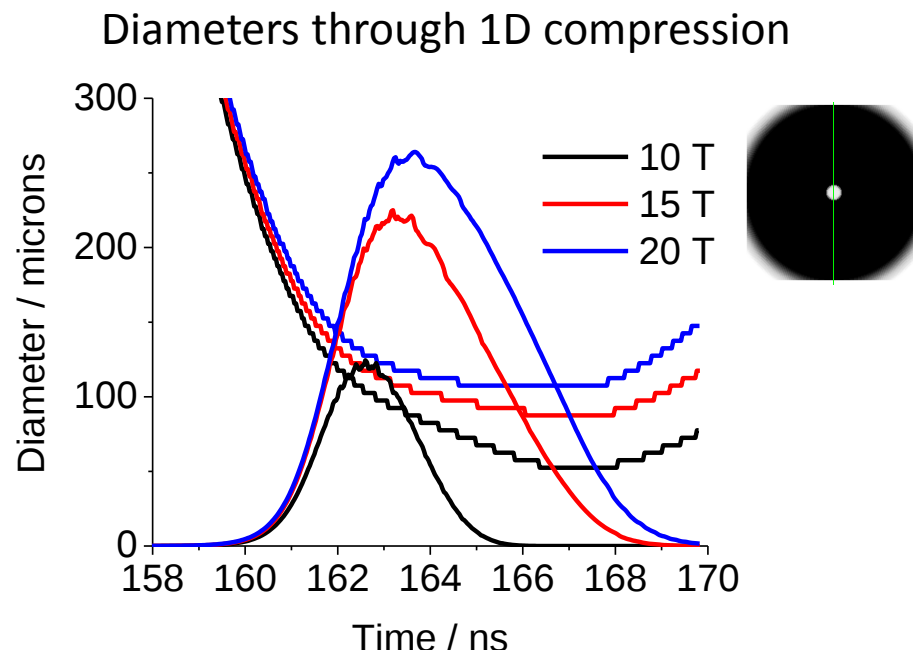


*S.A. Slutz *et. al.*, PoP (2010)
 S.A. Slutz and R. A. Vesey, PRL (2012)
 M.R. Gomez *et. al.*, PRL (2014)
 P.F. Schmit *et. al.*, PRL (2014)
 A.B. Sefkow, *et. al.*, PoP (2014)
 M.R. Gomez, *et. al.*, PoP (2015)
 S.B. Hansen, *et. al.*, PoP (2015)
 R.D. McBride, *et. al.*, PoP (2016)

During 3D disruption, time resolved imaging would enable tracking of emission to lower radius (lack of bounce)

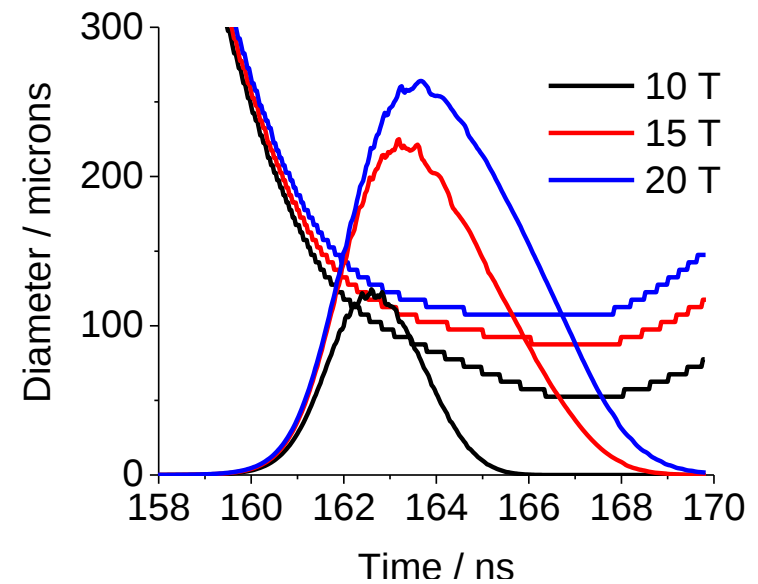
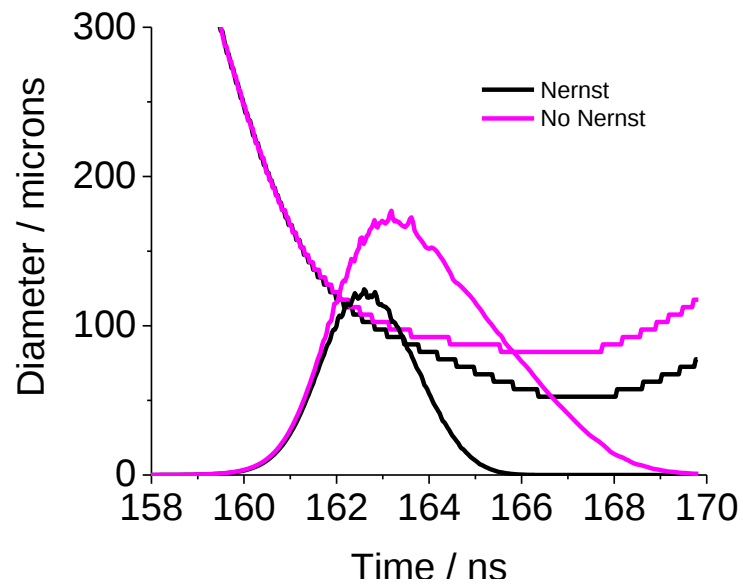


However “lack of bounce” is not a unique indication of disruption. It can be conflated with poor thermal transport inhibition.



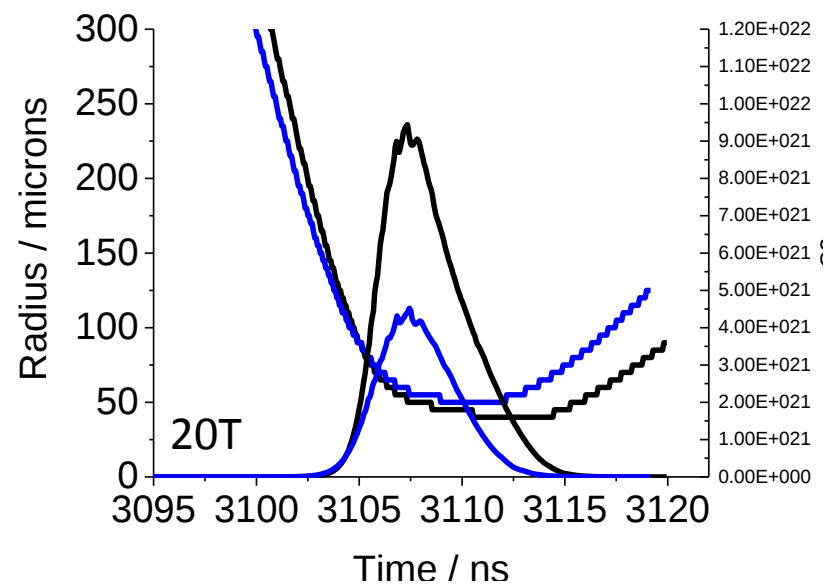
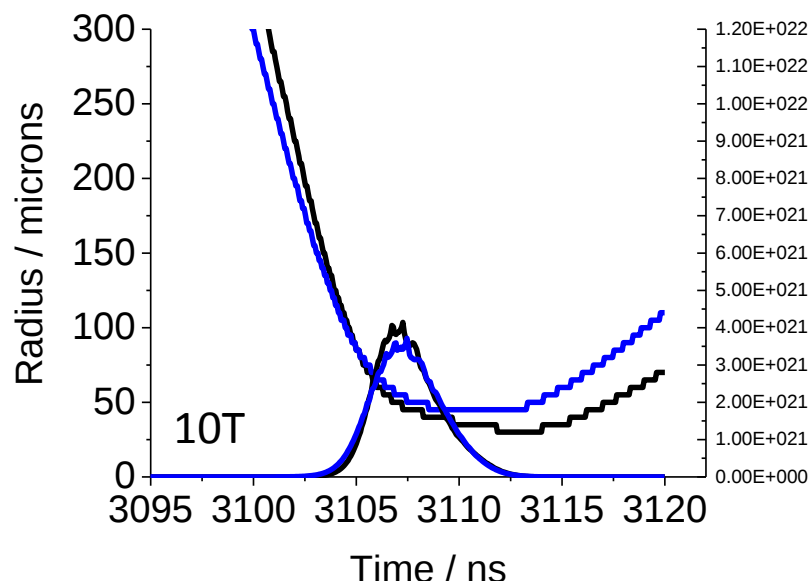
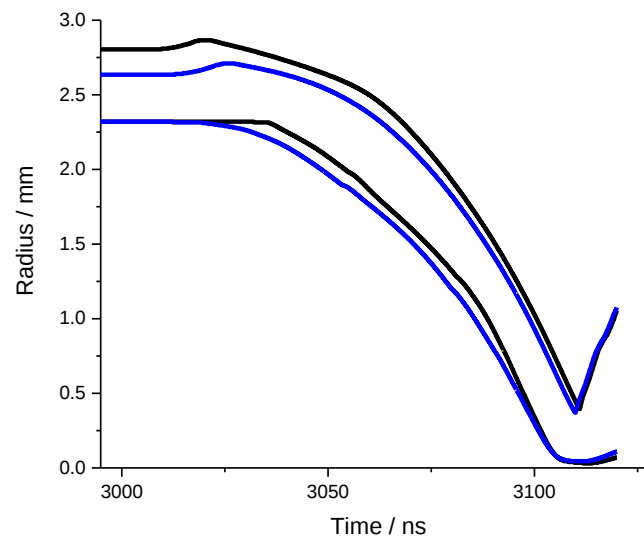
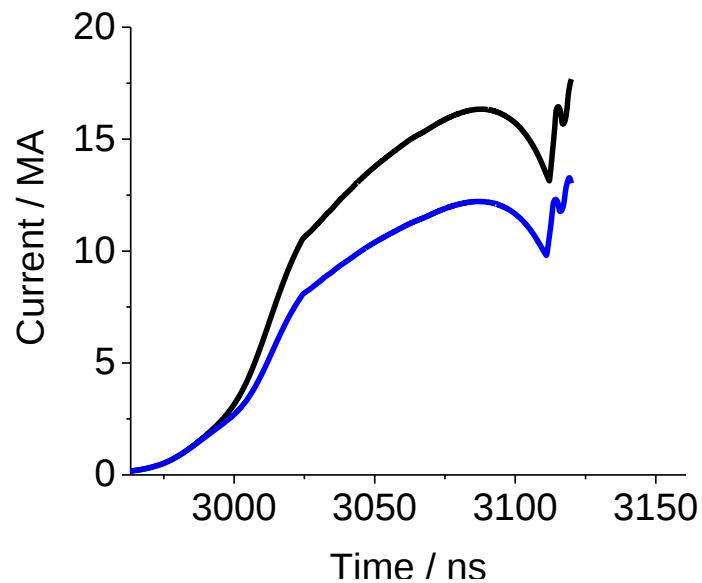
Nernst has similar effect to reducing magnetization and limiting burn duration in a Maglif stagnation

1D profiles of inner surface trajectory through neutron pulse



Implications for Current Scaling

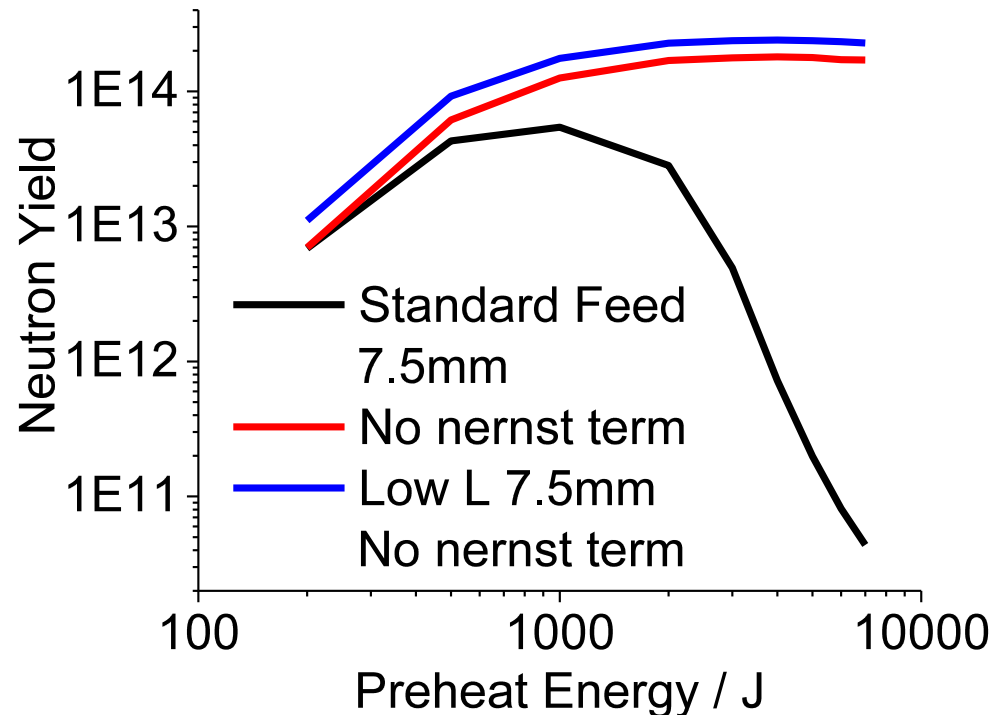
Reduce charge and shift to AR 9 liner



Nernst Magnetic field advection is leading to roll over in yield

2.79mm outer radius, 10mm tall standard Maglif target
10T Applied B_z
0.7 Kg/m³ Gas Density

Yield vs Preheat Energy



Implications for scaling of performance with increased preheat energy (if you don't push up fuel density and B_z field with that preheat energy)