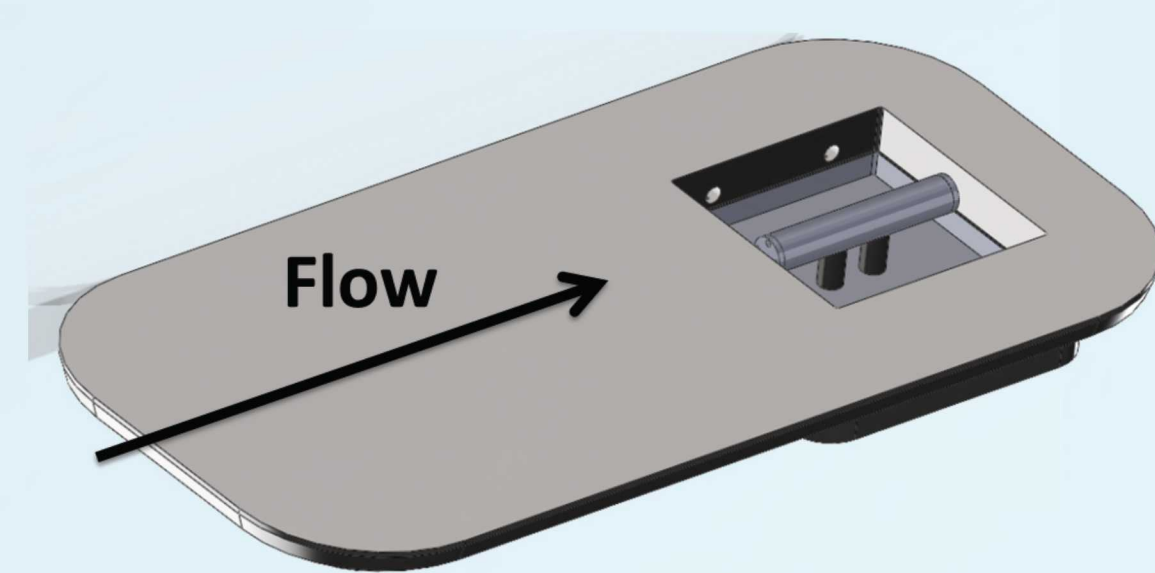


REDUCED-ORDER MODELING OF COMPRESSIBLE CAVITY FLOWS

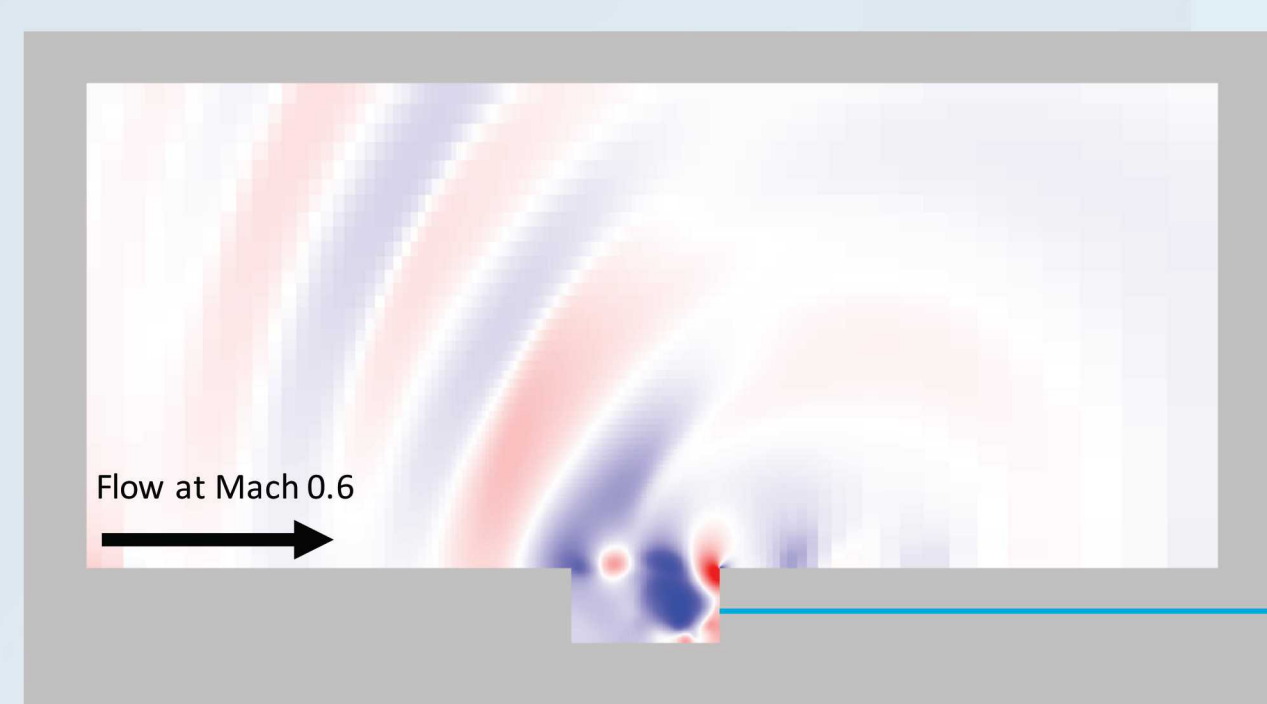
Jeffrey A. Fike, Irina K. Tezaur, Kevin T. Carlberg, and Matthew F. Barone

FLOW-INDUCED UNSTEADY PRESSURE LOADS

Flow past an open cavity, such as a bay on an aircraft, can create significant pressure fluctuations both inside and outside the cavity.

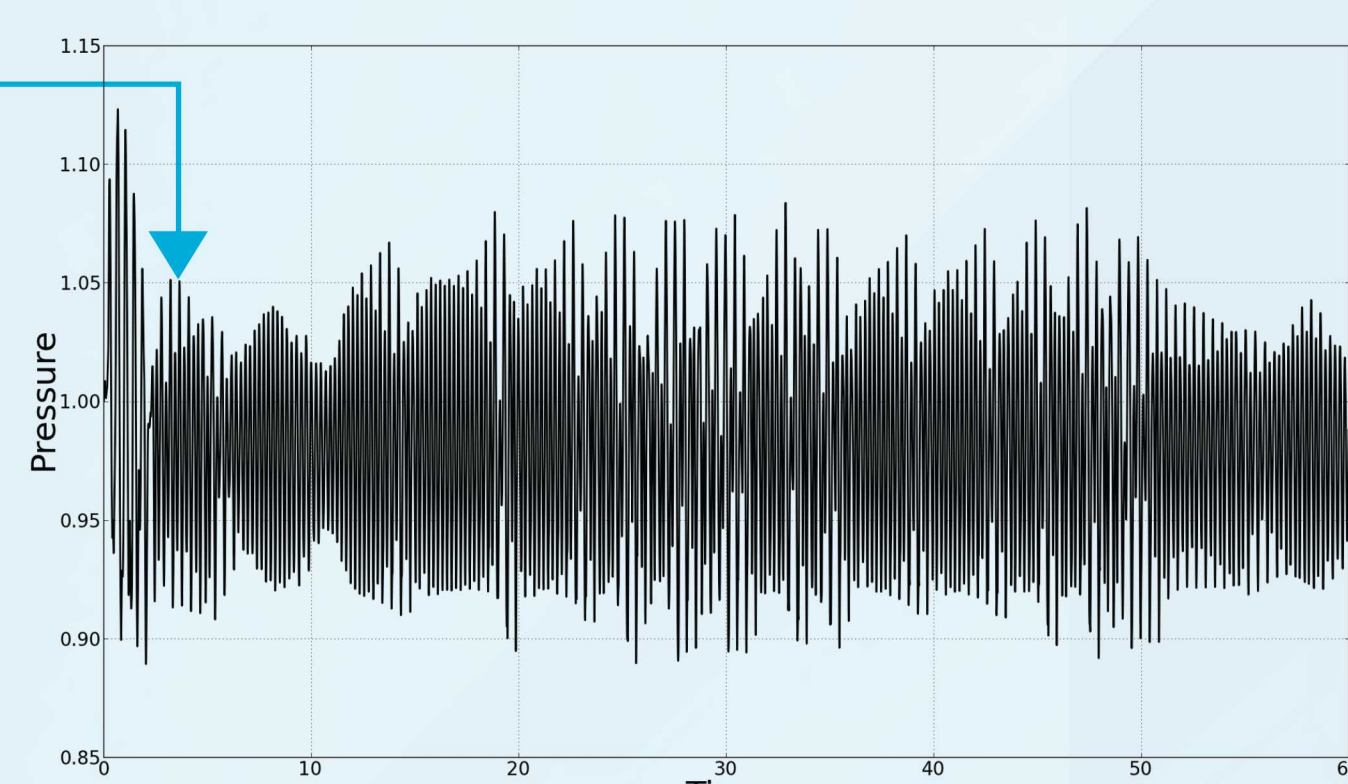
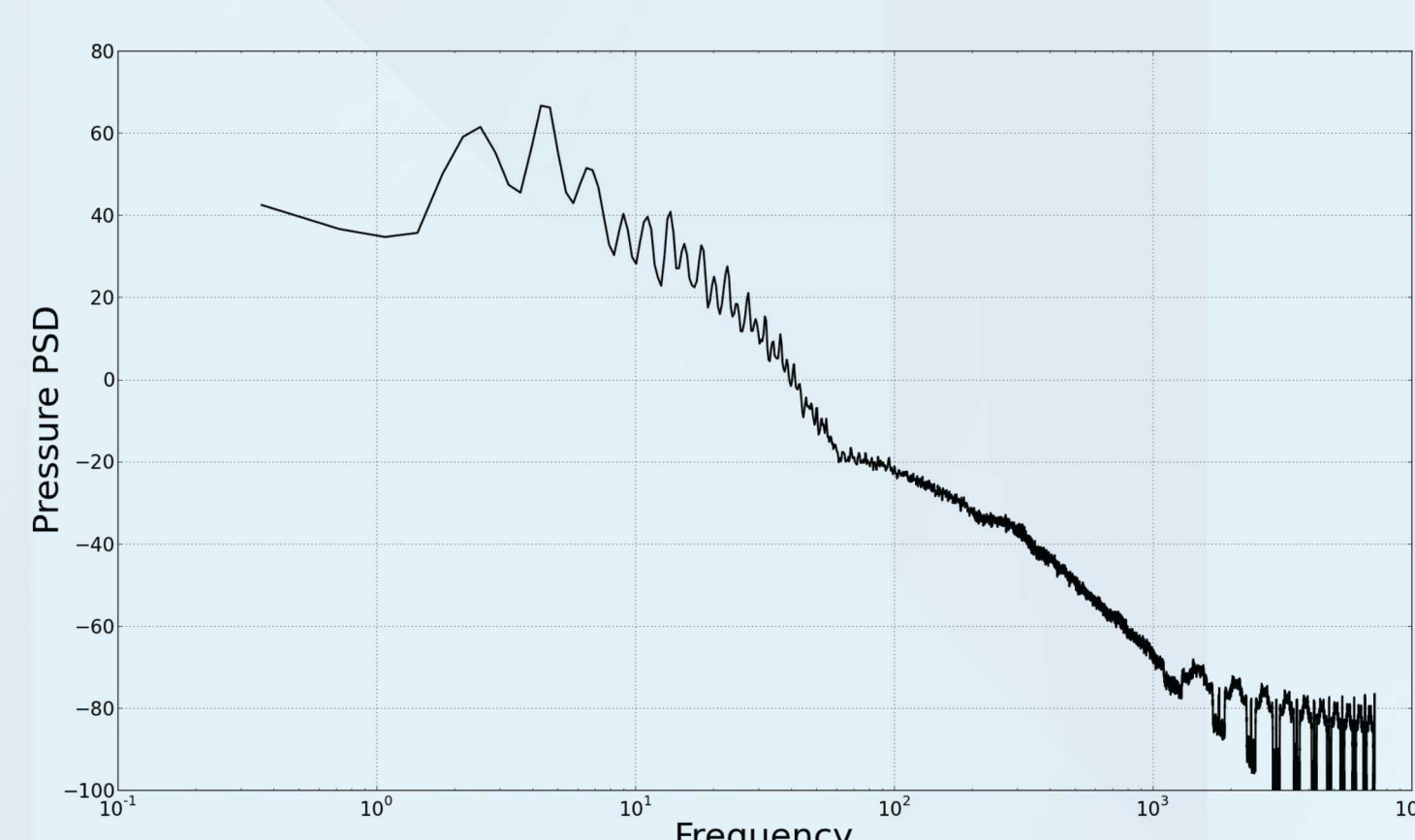


Objects in the cavity are subjected to these unsteady pressure loads, which can excite structural vibrations.



Long time histories are required to accurately compute statistics that characterize the unsteady environment.

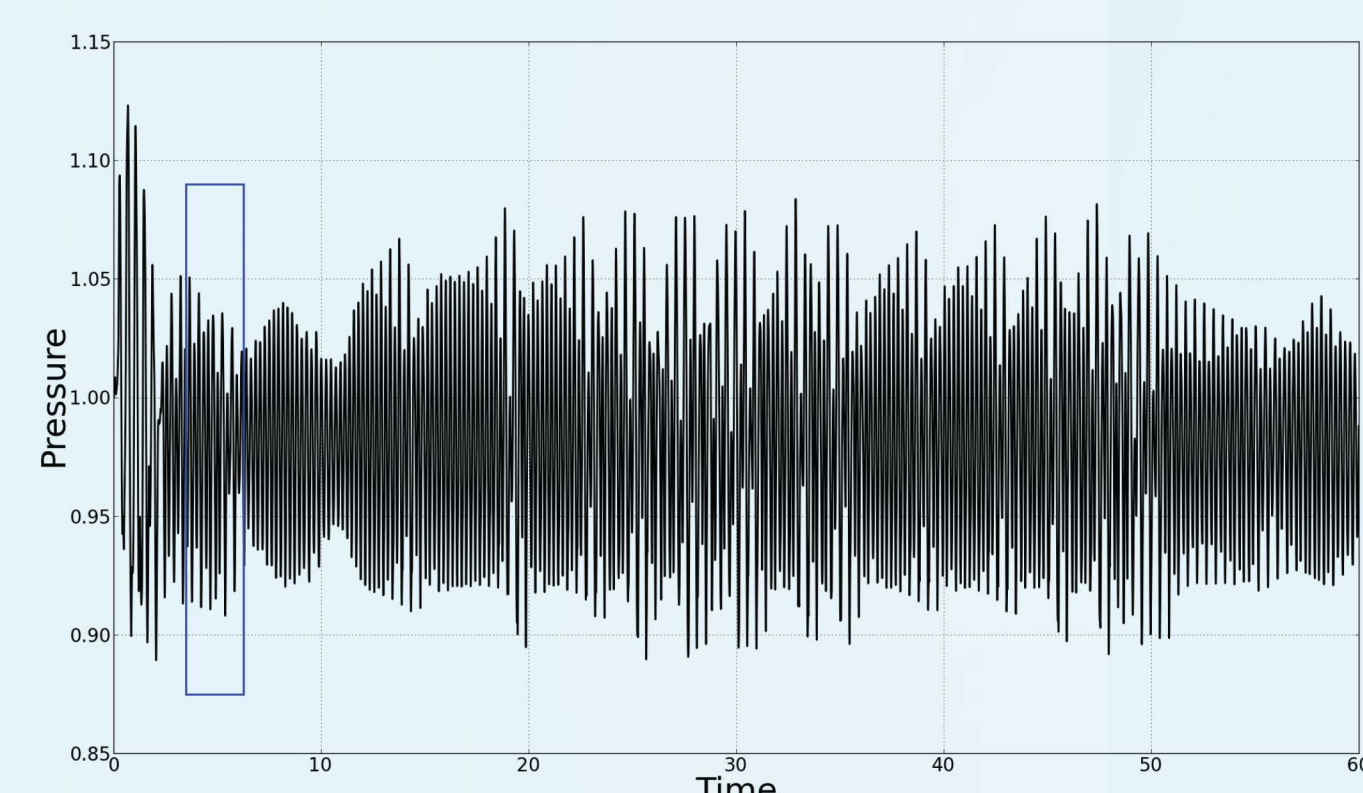
- Root Mean Square (RMS)
- Power Spectral Density (PSD)



Computationally expensive to run simulations for very many time steps.

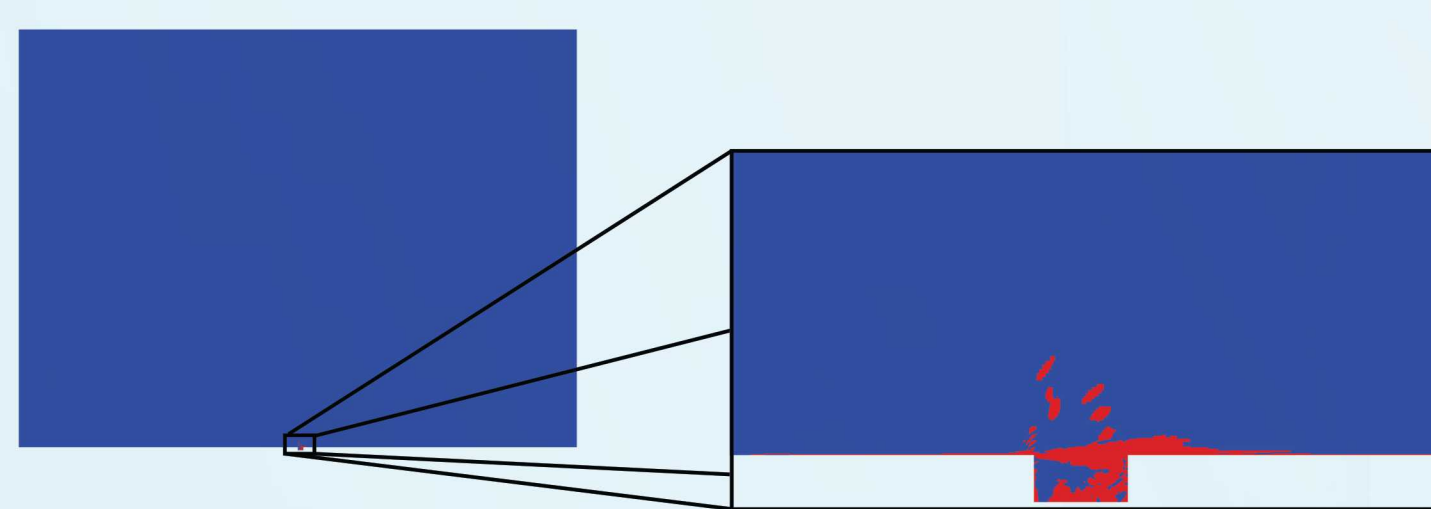
PROJECTION-BASED REDUCED-ORDER MODELS

Attempt to create a Reduced-Order Model (ROM) using data from a short time window that accurately reproduces the long term statistical behavior of the full model and has a substantially lower computational cost.



- Reduce dimensionality of problem by solving for a set of modal coefficients rather than the full degrees of freedom.
- Discrete-optimal Least-Squares Petrov-Galerkin (LSPG) ROMs have shown promise for non-linear fluid flow simulations.

- Cavity simulation using a different code demonstrated a 220x reduction in CPU core hours.
- This reduction in computational cost requires nonlinear operator approximation with a sample mesh.



Offline Training Stage:

Collect Training Data: Run the Full-Order Model to generate snapshots of the flow state at several time instances.

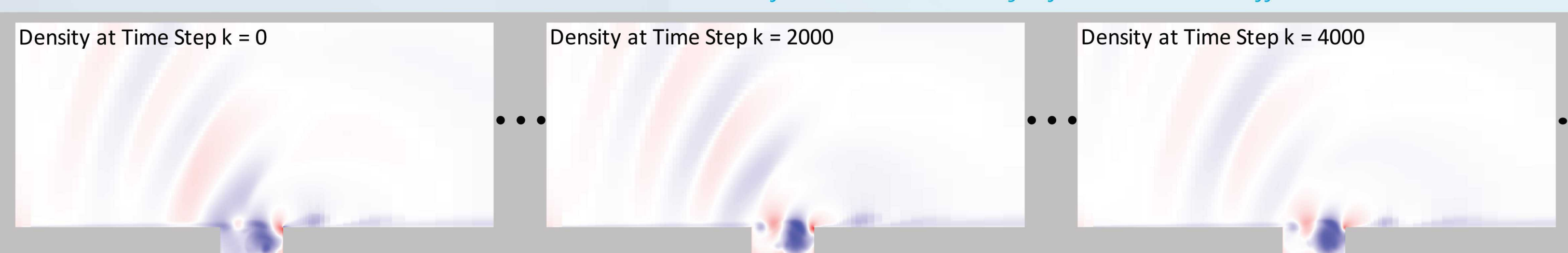


Machine Learning: Construct a modal basis Φ using Proper Orthogonal Decomposition (POD).



Online ROM Evaluation Stage:

Model Reduction: Run the Reduced-Order Model to solve for the time history of the modal coefficients.



NUMERICAL IMPLEMENTATION

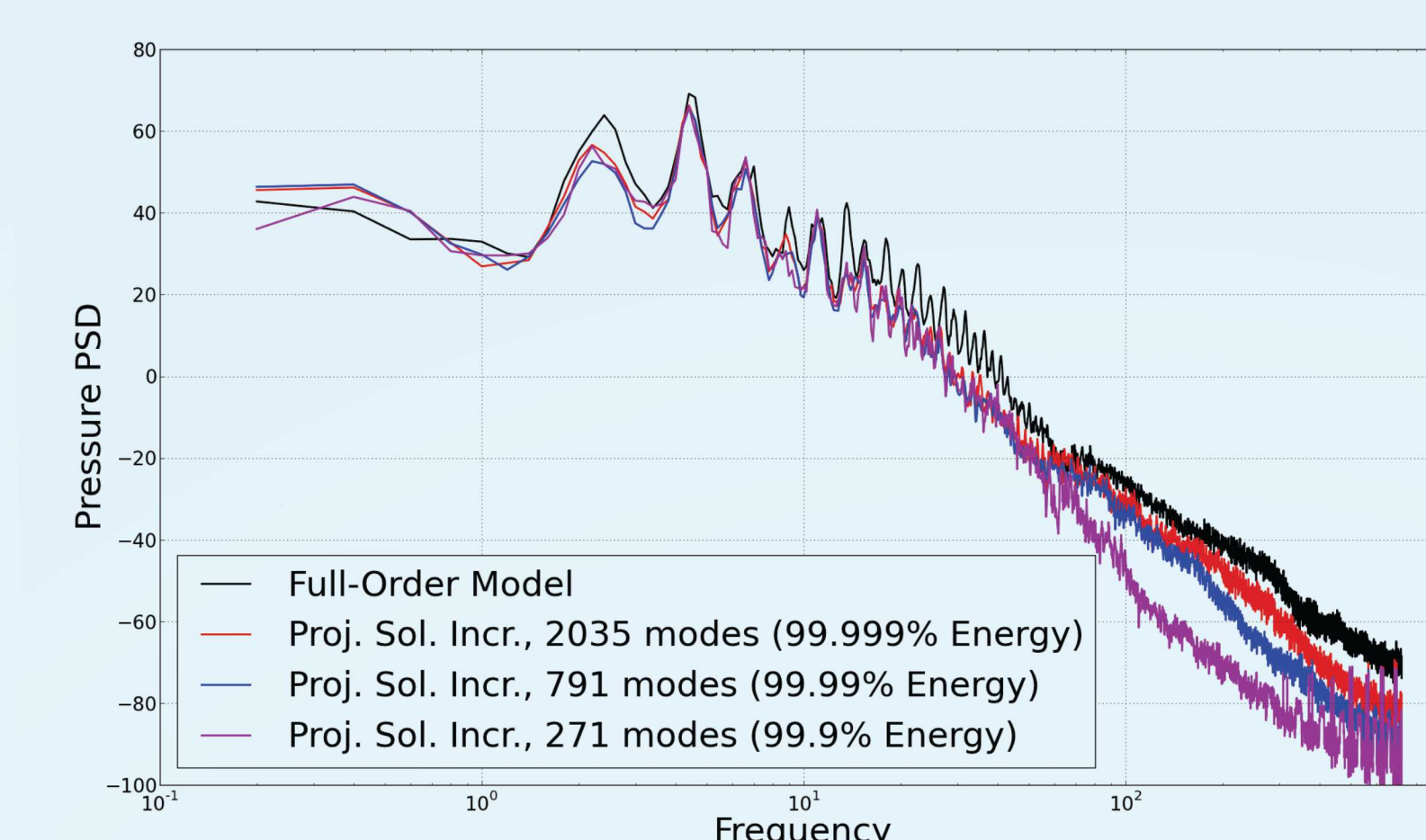
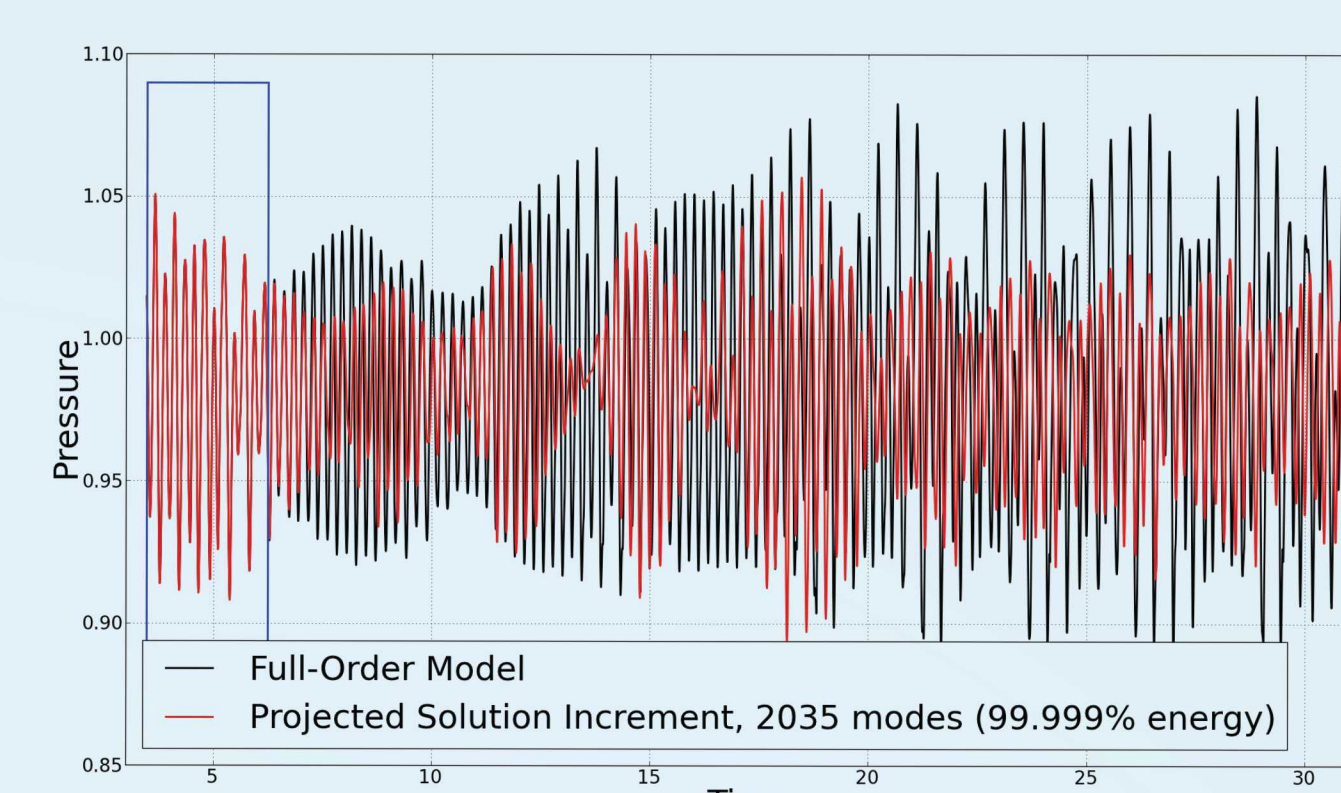
Reduced-Order Modeling capabilities have been added to SPARC, a computational fluid dynamics code currently under development at Sandia.

- Galerkin ROMs
- Least-Squares Petrov-Galerkin ROMs
- Preconditioned LSPG ROMs
- Projected Solution Increment or Ideally-Preconditioned LSPG ROMs
 - Provides an upper bound on the accuracy of the ROM implementations.

ASSESSMENT OF ACCURACY

The accuracy of the ROM predictions can be assessed in several ways:

- Flow field visualizations
- Pointwise difference in time history
- Error in RMS of pressure fluctuations
- Error in PSD over a frequency range of interest



For a given training set, we can examine the effect of:

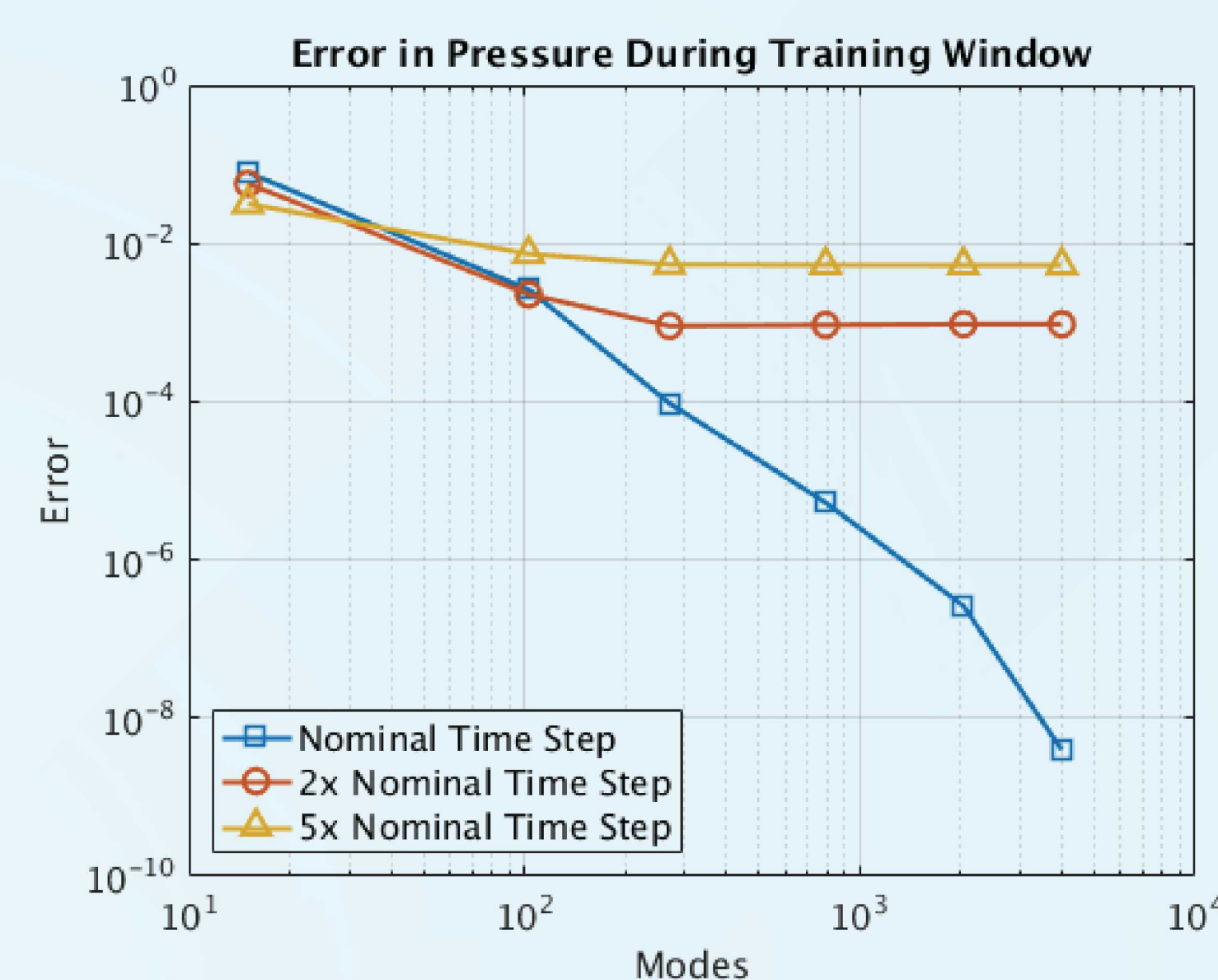
- Number of modes used in ROM
- Timestep used for ROM
- Projection type
- Preconditioner

Error in RMS of pressure fluctuation

Modes	% Energy	1xdt	2xdt	5xdt
4000	100.0	0.059	0.066	0.050
2035	99.999	0.056	0.073	0.051
791	99.99	0.069	0.062	0.057
271	99.9	0.068	0.060	0.043
103	99.0	0.097	0.095	0.095
15	90.0	0.152	0.120	0.086

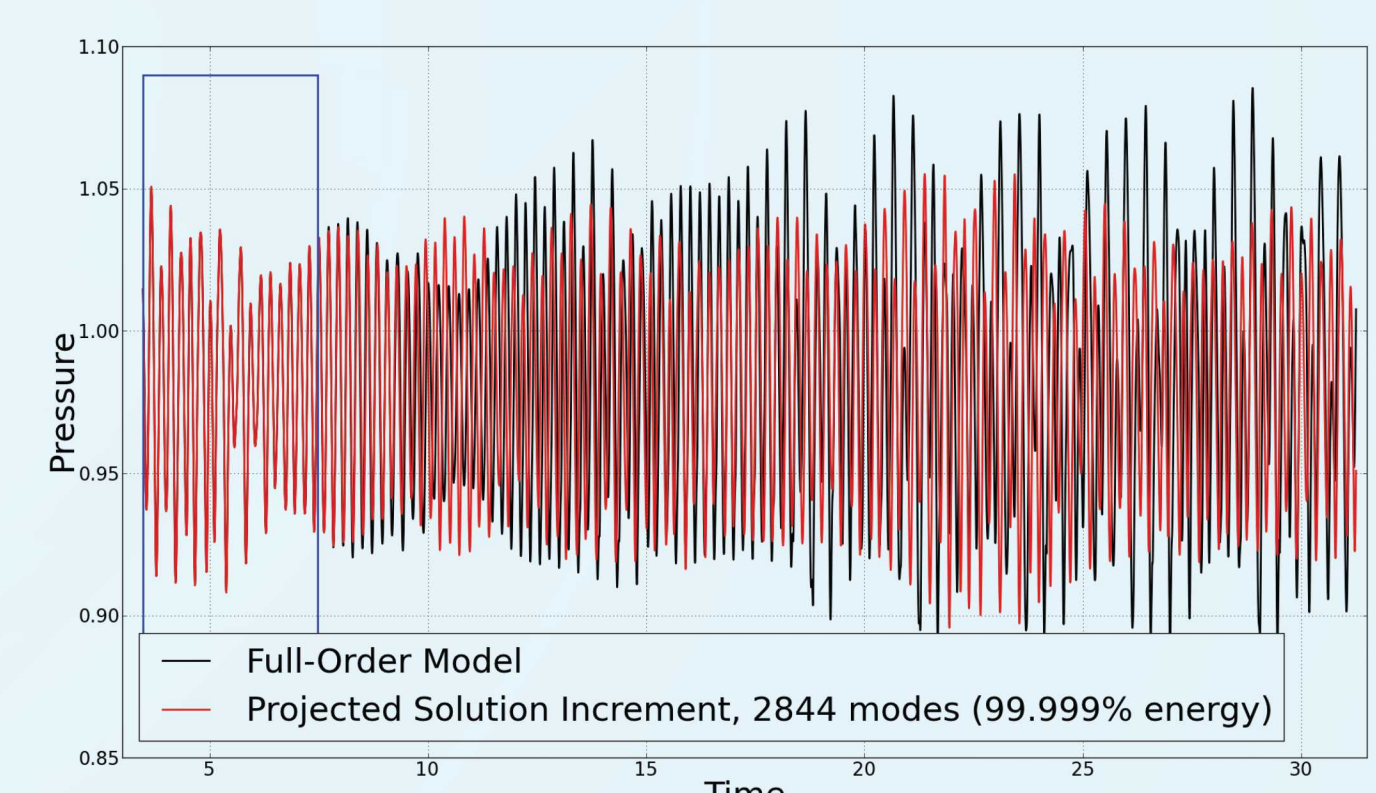
Error in PSD of pressure, normalized by RMS values

Modes	% Energy	1xdt	2xdt	5xdt
4000	100.0	0.106	0.119	0.199
2035	99.999	0.103	0.114	0.207
791	99.99	0.102	0.141	0.197
271	99.9	0.116	0.169	0.222
103	99.0	0.289	0.313	0.431
15	90.0	0.410	0.460	0.627



The training set is expected to have a significant impact on the accuracy and efficiency of the ROM.

- Training window length
- Training window position
- Stride between snapshots
- Disjoint training windows



CONCLUSIONS AND FUTURE WORK

- Demonstrated that Reduced-Order Models trained on a fairly short time interval can be used to recreate long term behavior of full-order model.
- Performed initial assessment on the impact of various parameters on the accuracy of the ROMs.
 - Continuing to explore variations in the training set and improved error metrics.
- Exploring techniques to improve ROM accuracy.
 - Structure preserving constraints to allow ROMs to better satisfy the physics of the FOM.
 - Basis modification for improved ROM accuracy and stability (with M. Balajewicz at UIUC).
- Need to implement sample mesh to reduce computational cost of the ROMs.

References

1. K. Carlberg, C. Farhat, J. Cortial, and D. Amsallam, "The GNAT method for nonlinear model reduction: Effective implementation and application to computational fluid dynamics and turbulent flows," *Journal of Computational Physics*, Vol. 242, p. 623–647 (2013).
 2. K. Carlberg, M. Barone, and H. Antil, "Galerkin v. least-squares Petrov-Galerkin projection in nonlinear model reduction," *Journal of Computational Physics*, Vol. 330, p. 693–734 (2017).
 3. M. Balajewicz, I. Tezaur, and E. Dowell, "Minimal subspace rotation on the Stiefel manifold for stabilization and enhancement of projection-based reduced order models for the compressible Navier-Stokes equations," *Journal of Computational Physics*, Vol. 321, p. 224–241 (2016).
- This work is funded through ASC P&EM.