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# Dislocation-based approaches for simulating plastic deformation in BCC metals under high-rate loading

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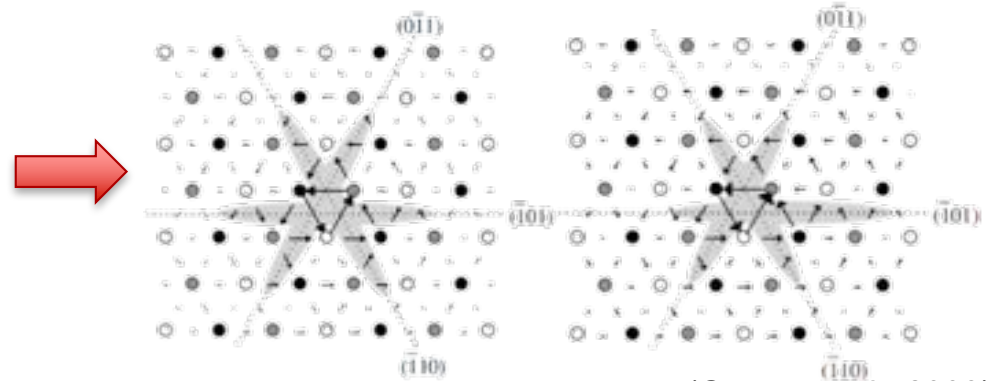
**International Symposium on Plasticity**  
January 3 2017



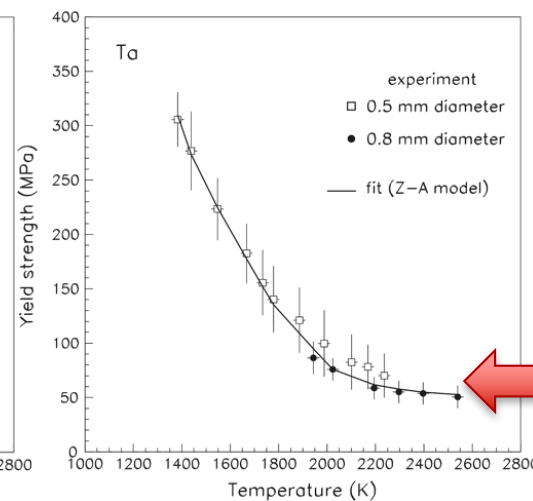
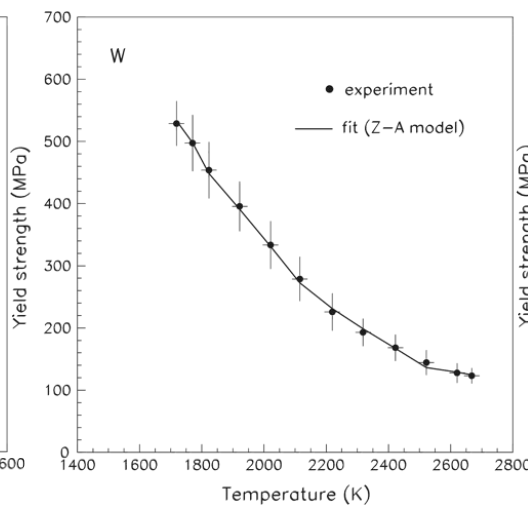
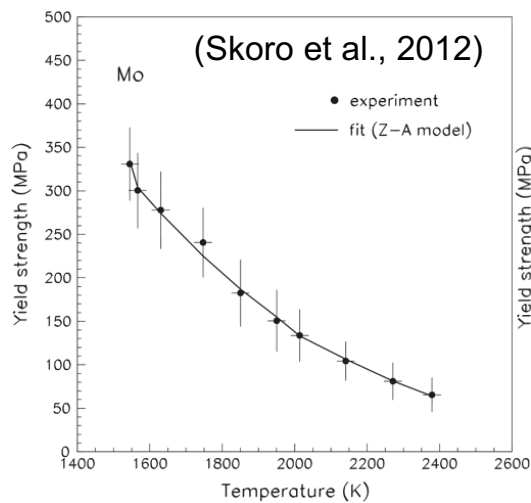
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# Dislocation Plasticity in BCC Metals

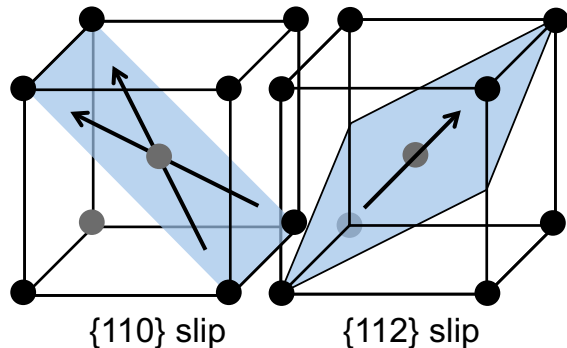
BCC metals deform via the propagation of screw dislocations, whose atomic structure is complex. Dislocation cores “spread” onto multiple crystallographic planes.



(Groger et al., 2008)



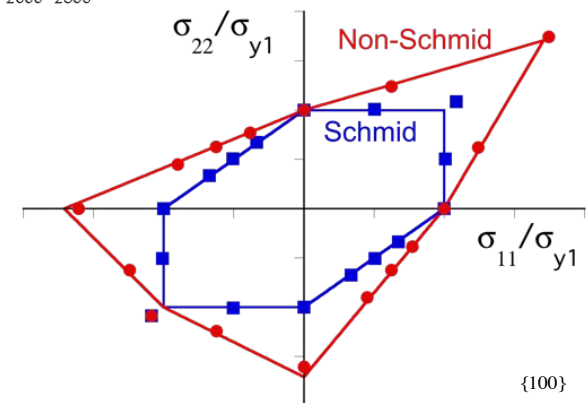
Therefore, BCC metals' strength is strongly dependent on  $T$  (and  $\dot{\epsilon}$ ).



$$\tau(T, \dot{\gamma}) = \underbrace{\tau^*(T, \dot{\gamma})}_{\text{Thermal}} + \underbrace{\tau_{obs}}_{\text{Athermal}}$$

In FCC metals,  $\tau^* \approx 0$

In BCC metals,  $\tau^* \gg 0$  ( $T \ll T_c$ )



# Multi-scale Modeling of BCC Metals

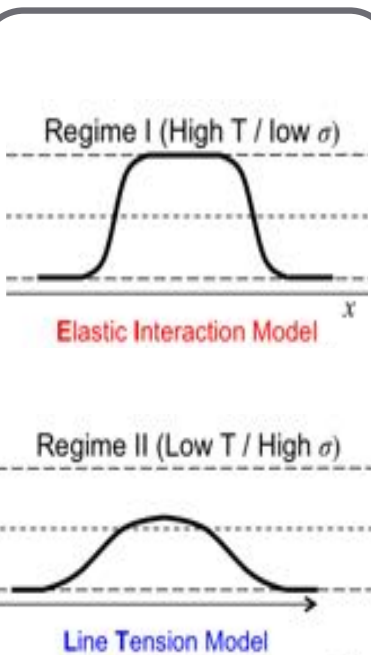
Atomic scale  
phenomena

Single crystal  
behavior

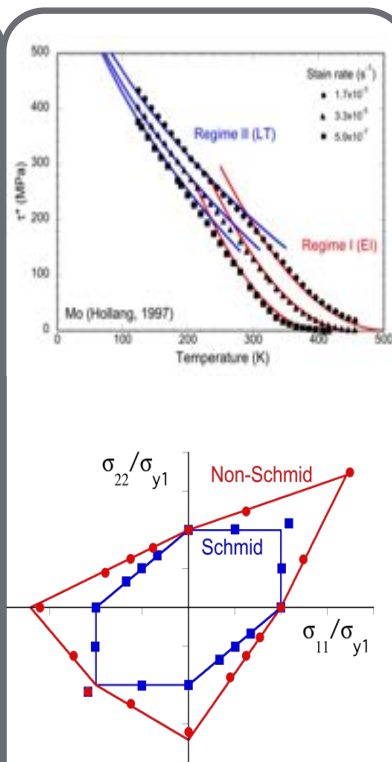
Validation

Polycrystal  
behavior

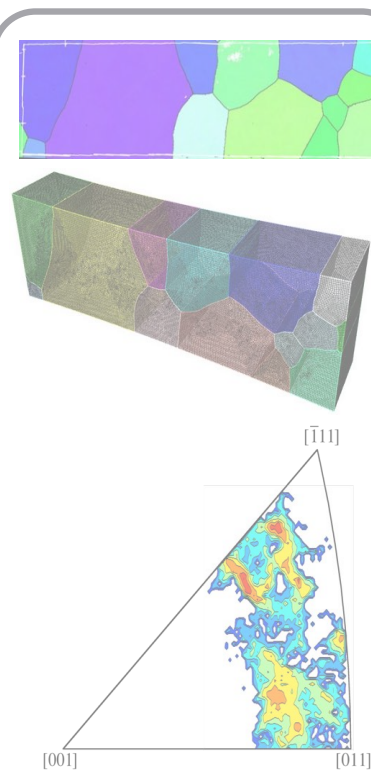
Continuum  
response



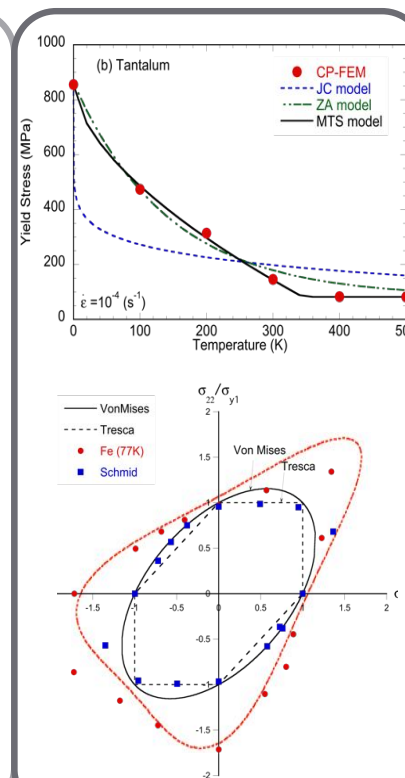
Dislocation  
kink-pair theory



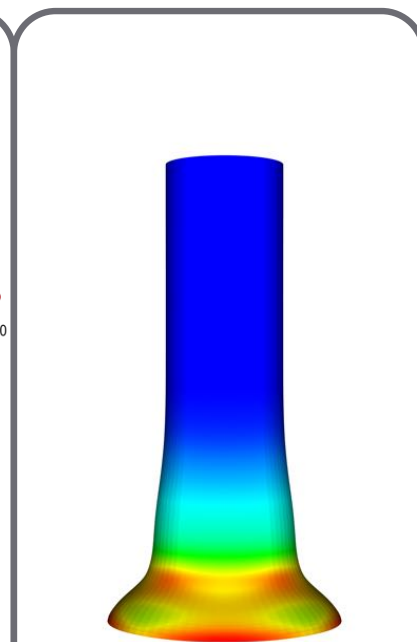
Single crystal  
constitutive model



Compare  
with experiments

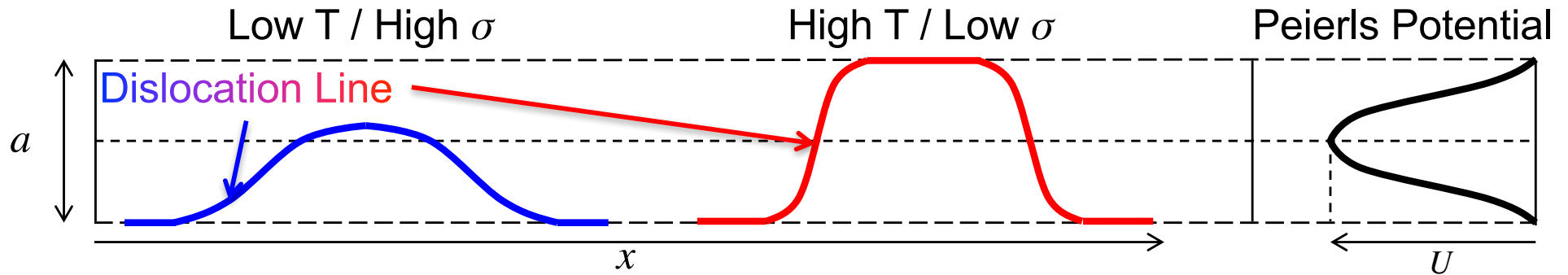


Bulk  
strength model



Simulations of  
impact tests

# Dislocation Kink-Pair Theory



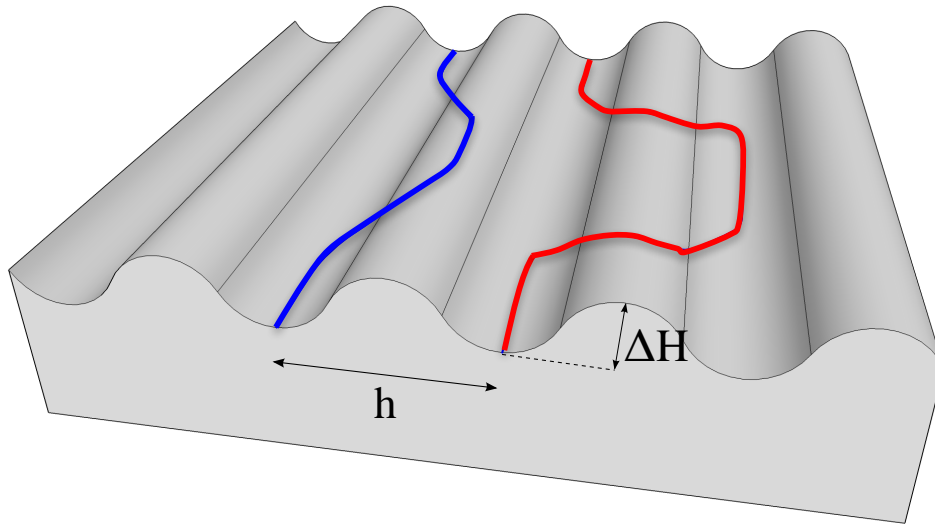
Line Tension Model

Elastic Interaction Model

$$\tau_{LT}^*(T, \dot{\gamma}) = \tau_{LT}^0 \left( 1 - \left( \frac{T}{T_c(\dot{\gamma})} \right)^{1/2} \right)$$

$$\tau_{EI}^*(T, \dot{\gamma}) = \tau_{EI}^0 \left( 1 - \frac{T}{T_c(\dot{\gamma})} \right)^2$$

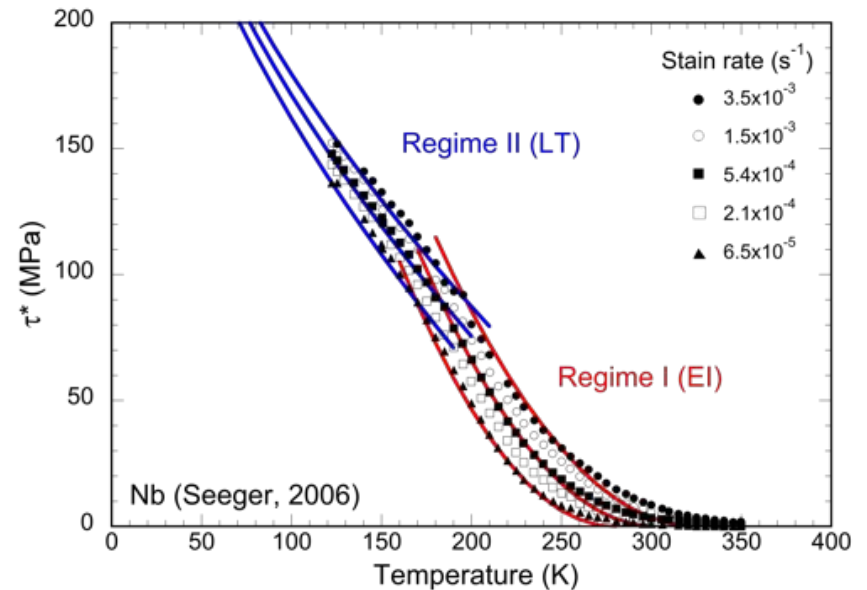
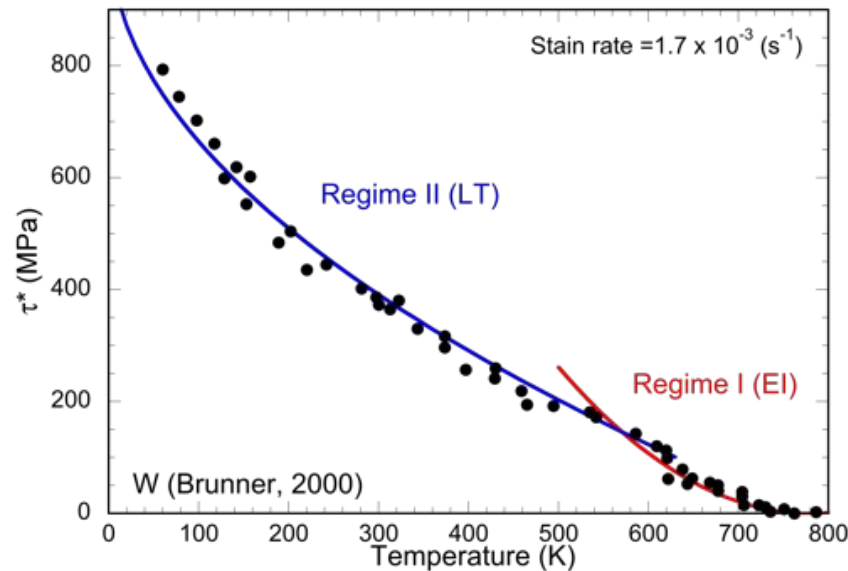
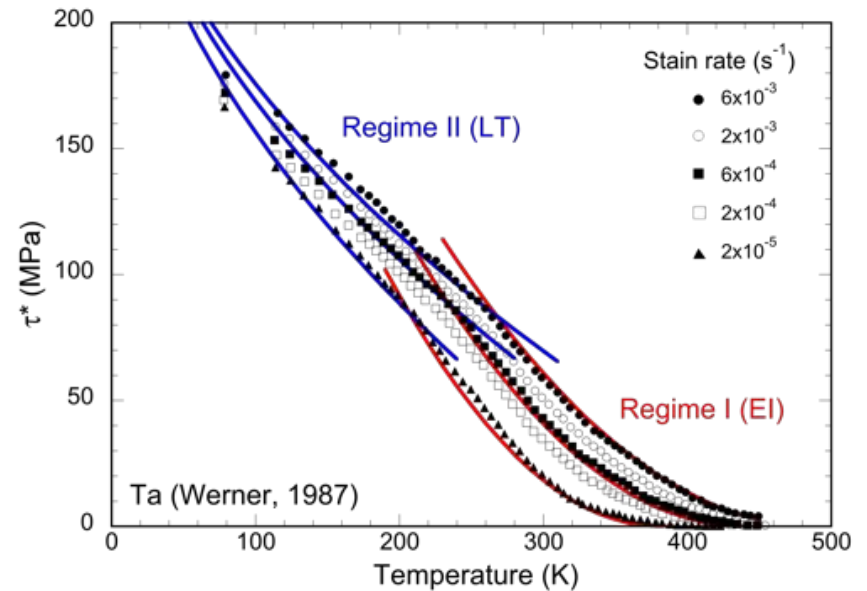
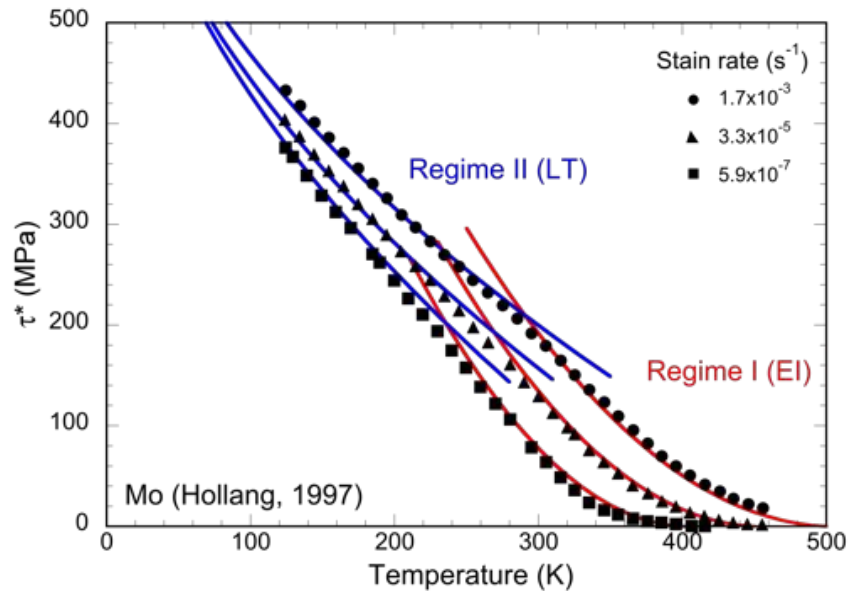
$$T_c(\dot{\gamma}) = \frac{2H_k}{k_B \ln(\dot{\gamma}_0 / \dot{\gamma})}$$



Best-fit material parameters

| Material | $\dot{\gamma}_0$ ( $s^{-1}$ ) | $2H_k$ (eV) | $\tau_{EI}^0$ (MPa) | $\tau_{LT}^0$ (MPa) |
|----------|-------------------------------|-------------|---------------------|---------------------|
| Mo       | $3.75 \times 10^9$            | 1.27        | 1156                | 835                 |
| Ta       | $2.99 \times 10^6$            | 0.85        | 406                 | 320                 |
| W        | $3.71 \times 10^{10}$         | 2.06        | 2035                | 1038                |
| Nb       | $1.14 \times 10^8$            | 0.68        | 576                 | 402                 |

# Calibration / Validation of KP Theory



Measured  $\tau^*$  are accurately reproduced by the EI and LT models

- FEM code developed at Sandia National Laboratories (JAS-3D)
- 24 {110}<111> slip systems

- Slip rate:  $\dot{\gamma}^\alpha = \dot{\gamma}_0^\alpha \left( \frac{\tau^\alpha}{g^\alpha} \right)^{1/m}$  (Hutchinson, 1976)

- Slip resistance:  $g^\alpha = \min(\tau_{EI}^{*\alpha}, \tau_{LT}^{*\alpha}) + \tau_{obs}^\alpha$ 

$\tau_{EI}^{*\alpha}$   
 $\downarrow$   
 Lattice friction

$\tau_{LT}^{*\alpha}$   
 $\downarrow$   
 Obstacle stress

$\tau_{EI}^*(T, \dot{\gamma}) = \tau_{EI}^0 \left( 1 - \frac{T}{T_c(\dot{\gamma})} \right)^2$   
  
 $\tau_{LT}^*(T, \dot{\gamma}) = \tau_{LT}^0 \left( 1 - \left( \frac{T}{T_c(\dot{\gamma})} \right)^{1/2} \right)$

- Obstacle stress:  $\tau_{obs}^\alpha = A\mu b \sqrt{\sum_{\beta=1}^{NS} \rho^\beta}$  (Taylor, 1934)

$$\dot{\rho}^\alpha = \left( \kappa_1 \sqrt{\sum_{\beta=1}^{NS} \rho^\beta} - \kappa_2 \rho^\alpha \right) \cdot |\dot{\gamma}^\alpha| \quad (\text{Kocks, 1976})$$

# More Details on Crystal Plasticity FEA

Cauchy stress resolved on each of 24 slip systems:  $\tau = \mathbf{P}_s : \boldsymbol{\sigma}$        $\mathbf{P}_s = \frac{1}{2}(\mathbf{m} \otimes \mathbf{s} + \mathbf{s} \otimes \mathbf{m})$

Non-Schmid stress:  $\tau_{ns} = \mathbf{P}_{ns} : \boldsymbol{\sigma}$        $\mathbf{P}_{ns} = c_1 \mathbf{m} \otimes \mathbf{t} + c_2 \mathbf{s} \otimes \mathbf{t} + c_3 \mathbf{s} \otimes \mathbf{s} + c_4 \mathbf{t} \otimes \mathbf{t} + c_5 \mathbf{m} \otimes \mathbf{m}$

Slip resistance ("hardness"):  $g = \max(\tau_{cr} - \tau_{ns}, 0) + \tau_{obs}$

Slip rate on each system:  $\dot{\gamma} = \dot{\gamma}_0 \left| \frac{\tau}{g} \right|^{\frac{1}{m}} \text{sign}(\tau)$

Plastic velocity gradient:  $\hat{\mathbf{L}}_p = \dot{\gamma}(\hat{\mathbf{s}} \otimes \hat{\mathbf{m}})$

Plastic deformation gradient:  $\dot{\mathbf{F}}_p = \hat{\mathbf{L}}_p \cdot \mathbf{F}_p \Rightarrow \begin{cases} \mathbf{F}_p = \exp(\hat{\mathbf{L}}_p \Delta t) \mathbf{F}_p \\ \exp(\hat{\mathbf{L}}_p \Delta t) = \mathbf{I} + \frac{\sin \phi}{\phi} \hat{\mathbf{L}}_p \Delta t + \frac{1 - \cos \phi}{\phi^2} (\hat{\mathbf{L}}_p \cdot \hat{\mathbf{L}}_p) \Delta t^2 \\ \phi = \Delta t \sqrt{\frac{1}{2} (\hat{\mathbf{L}}_p : \hat{\mathbf{L}}_p)} \end{cases}$   
(Cayley-Hamilton theorem)

Elastic deformation gradient and strain:  $\mathbf{F}_e = \mathbf{F} \cdot (\mathbf{F}_p)^{-1} \Rightarrow \hat{\mathbf{E}}_e = \frac{1}{2} [(\mathbf{F}_e)^T \cdot \mathbf{F}_e - \mathbf{I}]$

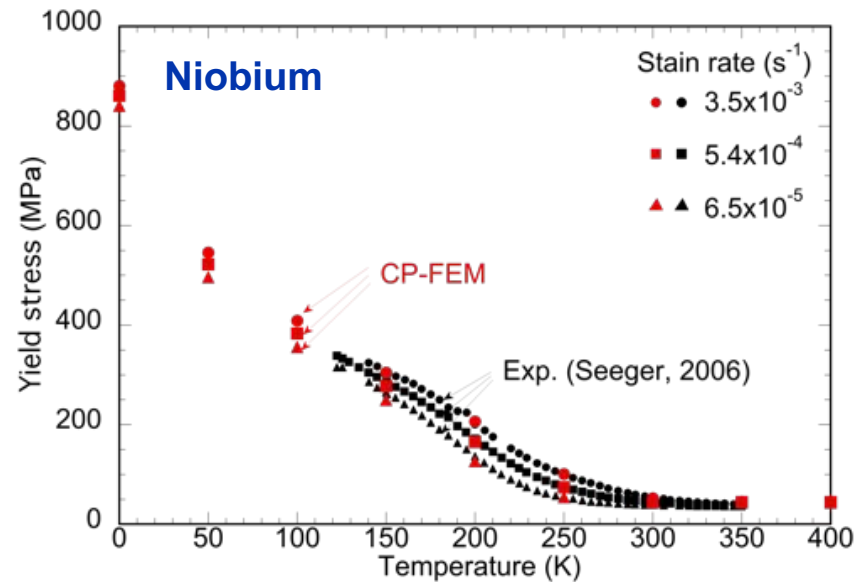
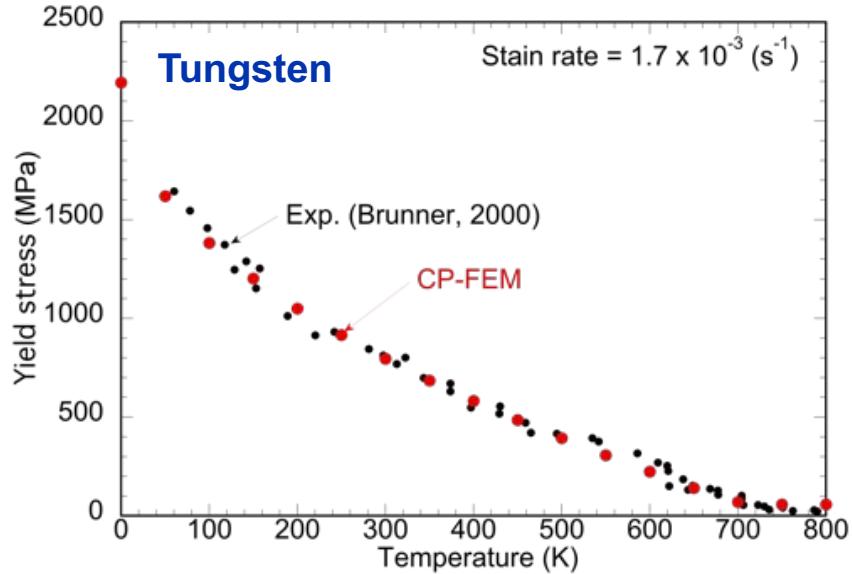
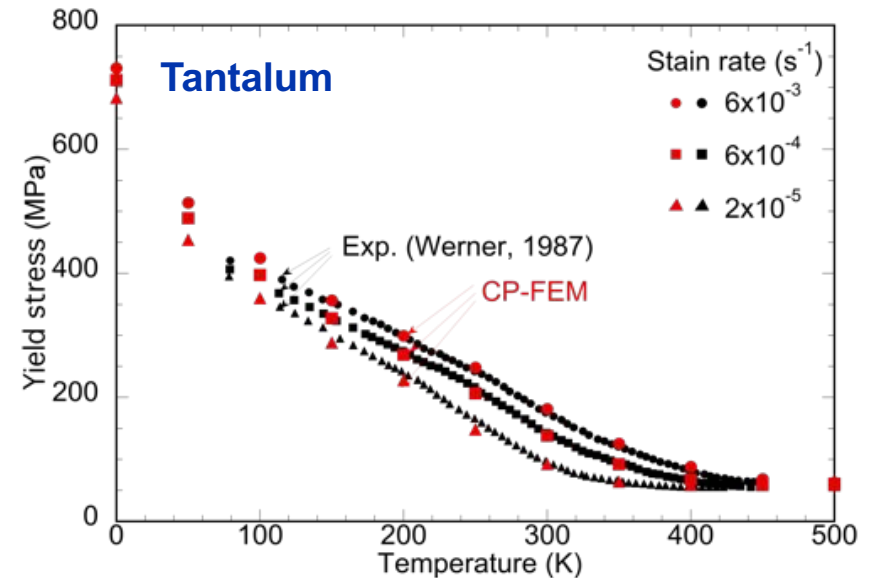
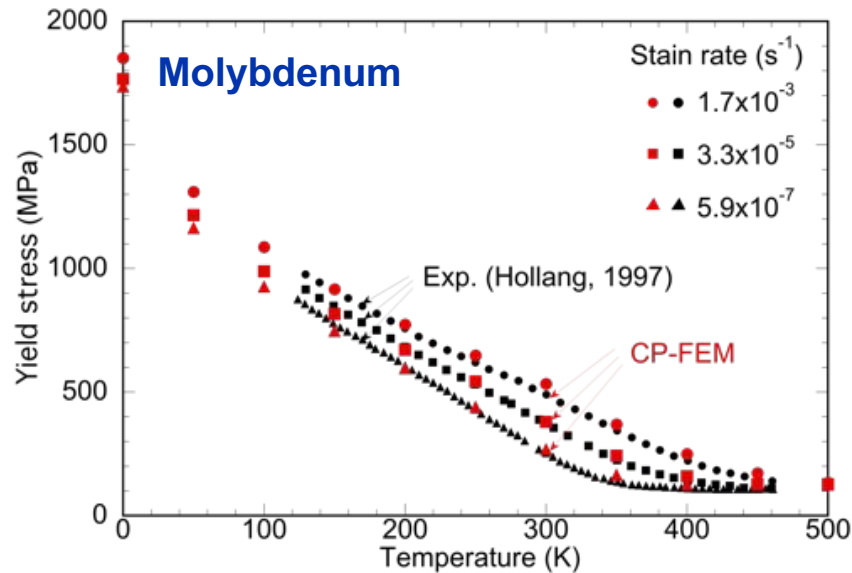
2nd Piola-Kirchhoff stress (hyper-elasticity):  $\hat{\boldsymbol{\sigma}}_{PK2} = \mathbf{C} : \hat{\mathbf{E}}_e$

Updated Cauchy stress:  $\boldsymbol{\sigma} = \frac{1}{J_{\mathbf{F}_e}} [\mathbf{F}_e \cdot \hat{\boldsymbol{\sigma}}_{PK2} \cdot (\mathbf{F}_e)^T]$

Updated crystallographic orientation:  $\mathbf{F}_e = \mathbf{U}_e \cdot \mathbf{R}_e \Rightarrow \mathbf{R}_{lattice} = \mathbf{R}_e \cdot \mathbf{R}_h$

Updated hardness:  $\tau_{obs} = \alpha \mu b \sqrt{\sum_{\beta=1}^{NS} \rho^\beta}$        $d\rho = \left( \kappa_1 \sqrt{\sum_{\beta=1}^{NS} \rho^\beta} - \kappa_2 \rho \right) \cdot |d\gamma|$

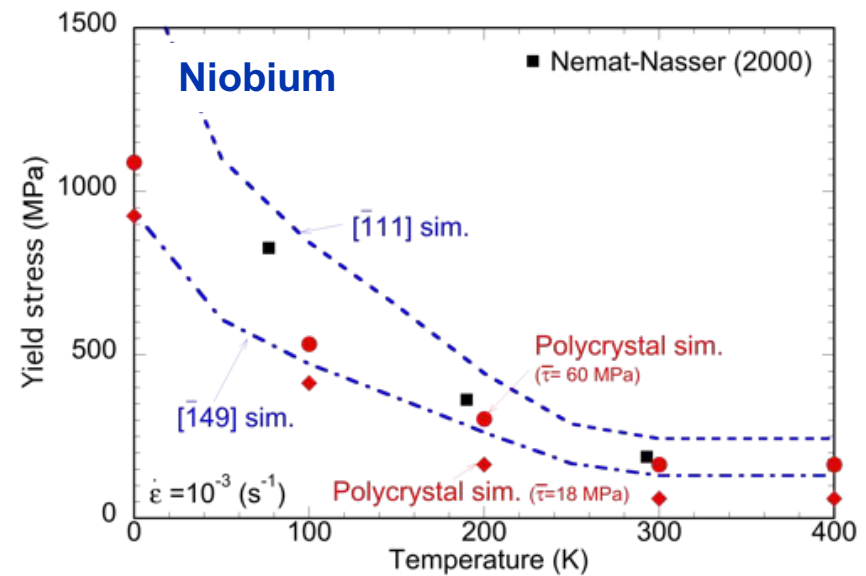
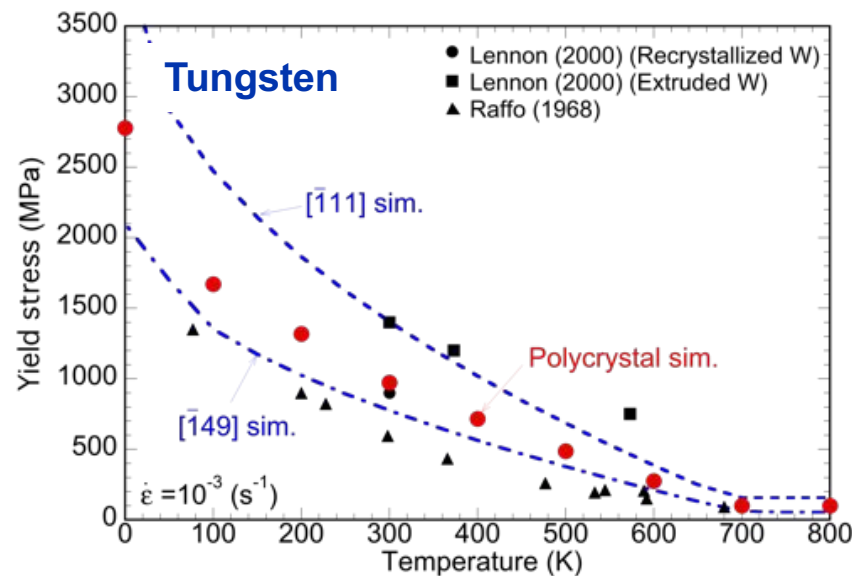
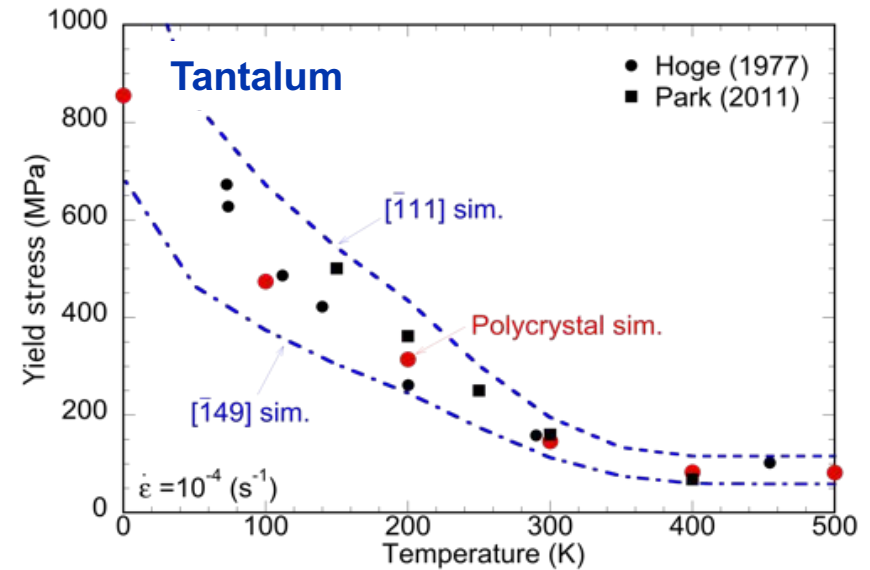
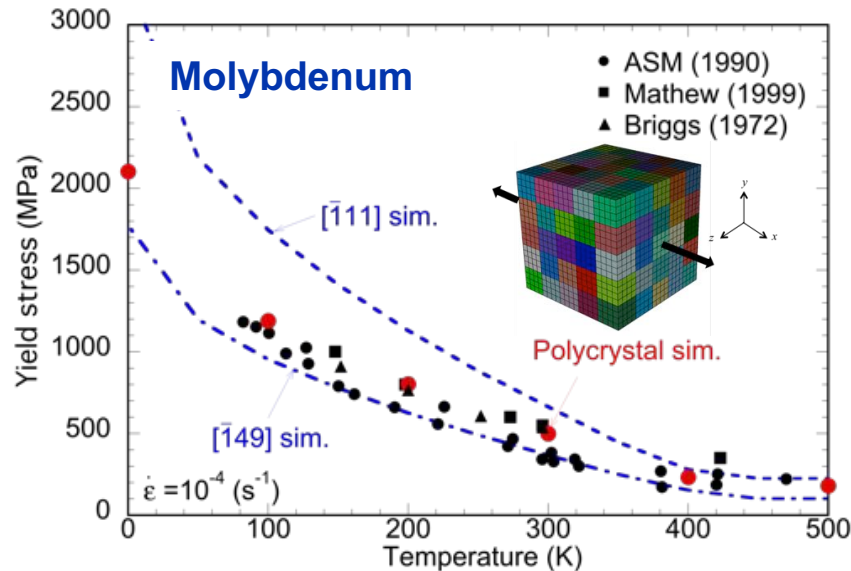
# Single Crystal CP-FEM Predictions



CP-FEM model accurately reproduced yield stresses of  $[\bar{1}49]$  single crystals

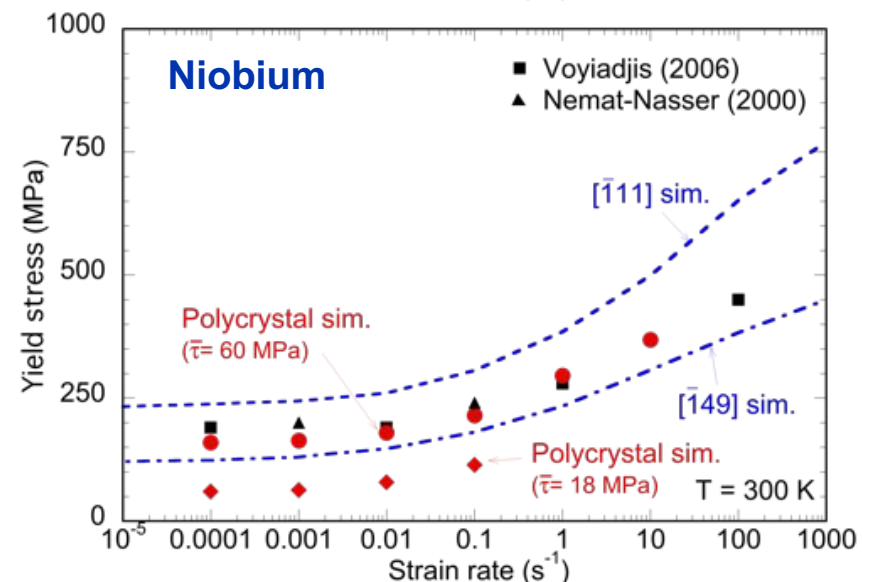
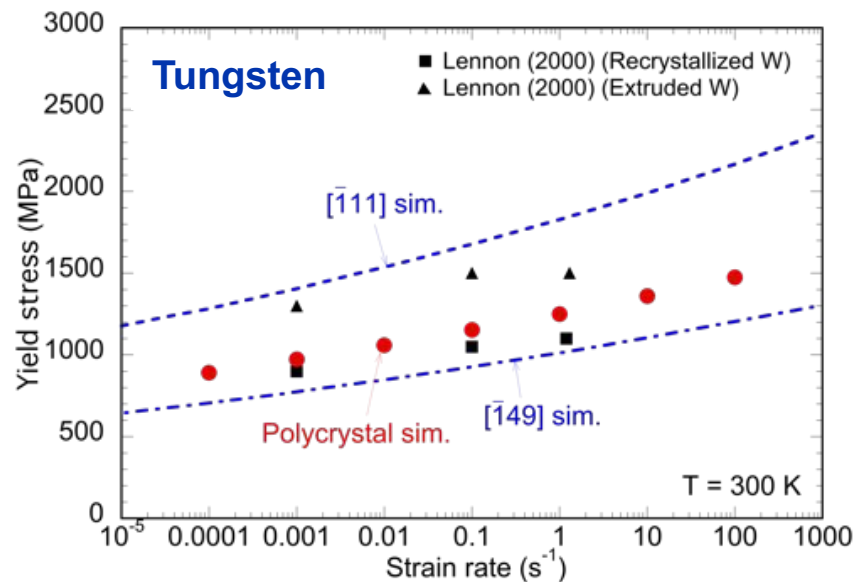
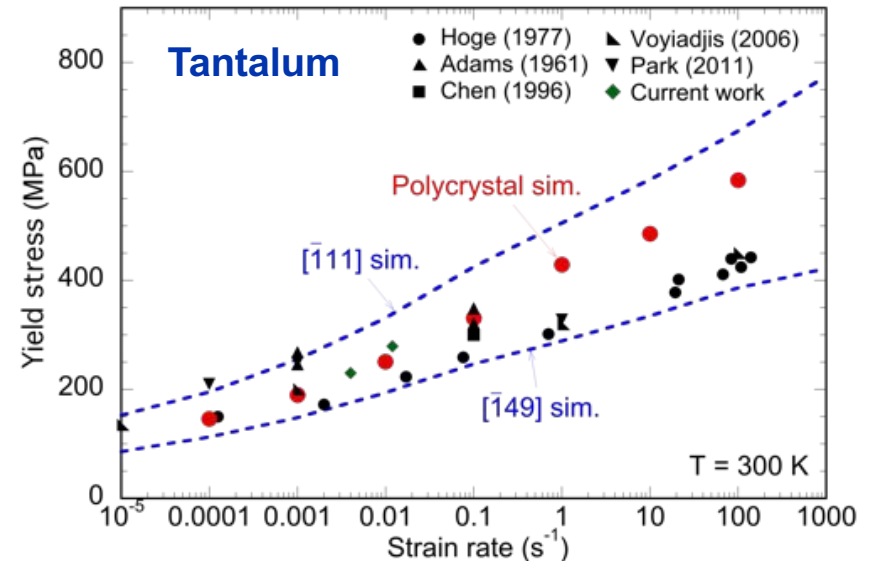
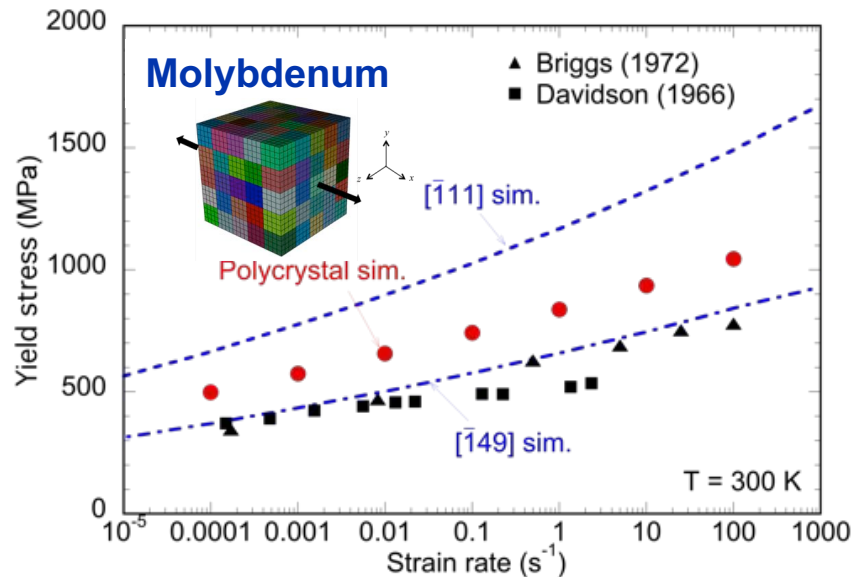


# Polycrystal CP-FEM Predictions



Measured yield stresses of BCC polycrystals lie between the bounds predicted by CP-FEM models on extreme single crystal orientation.

# Polycrystal CP-FEM Predictions



Measured yield stresses of BCC polycrystals lie between the bounds predicted by CP-FEM models on extreme single crystal orientation.

- Johnson and Cook (JC) model (Johnson and Cook, 1983, 1985)

$$\sigma_y^{JC} = A(1 + C \ln \dot{\epsilon})(1 - T^{*m})$$

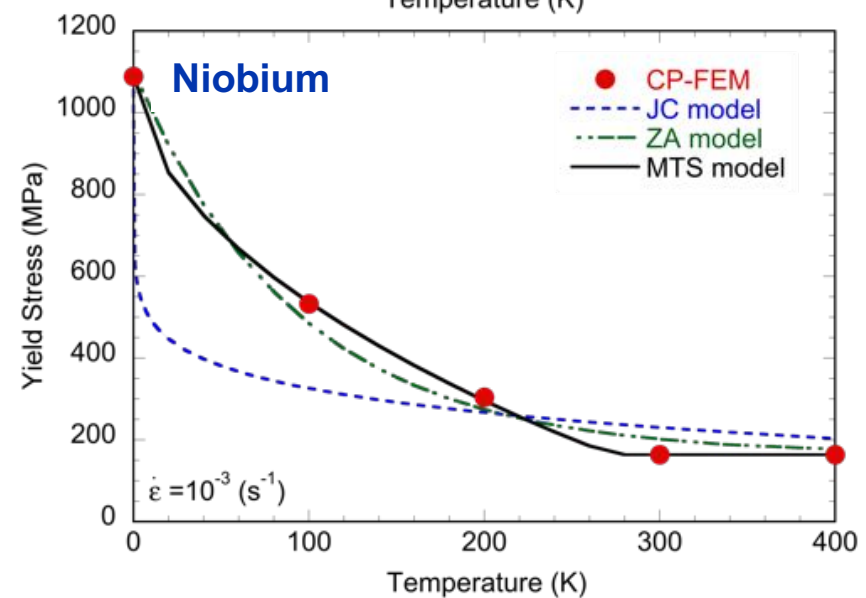
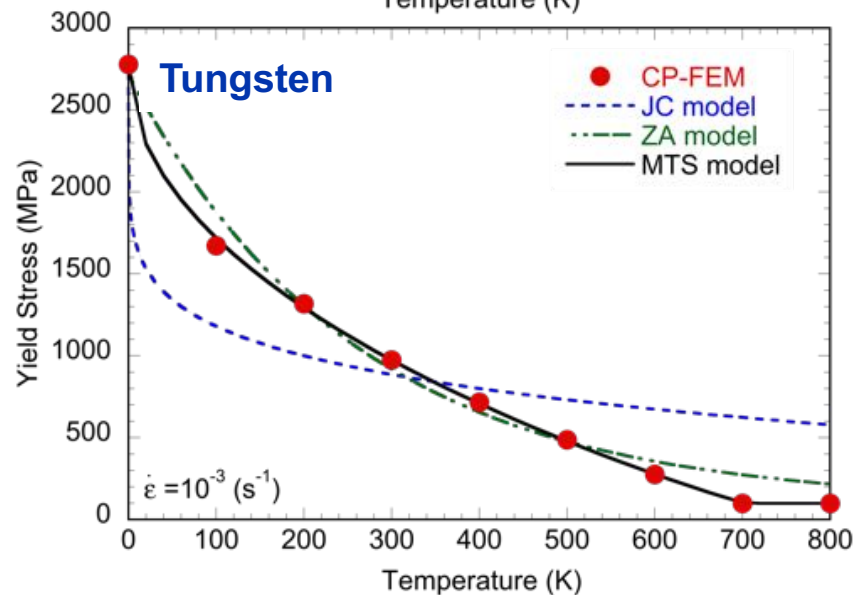
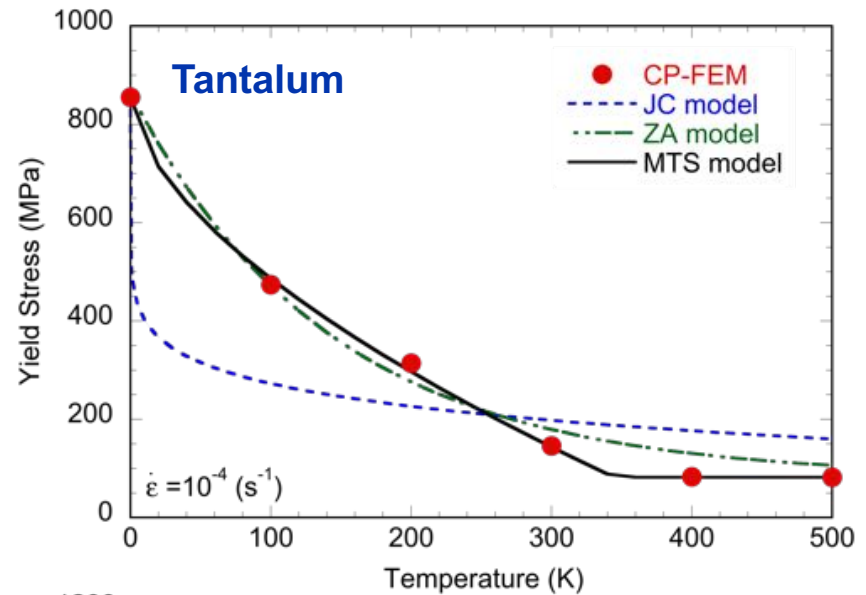
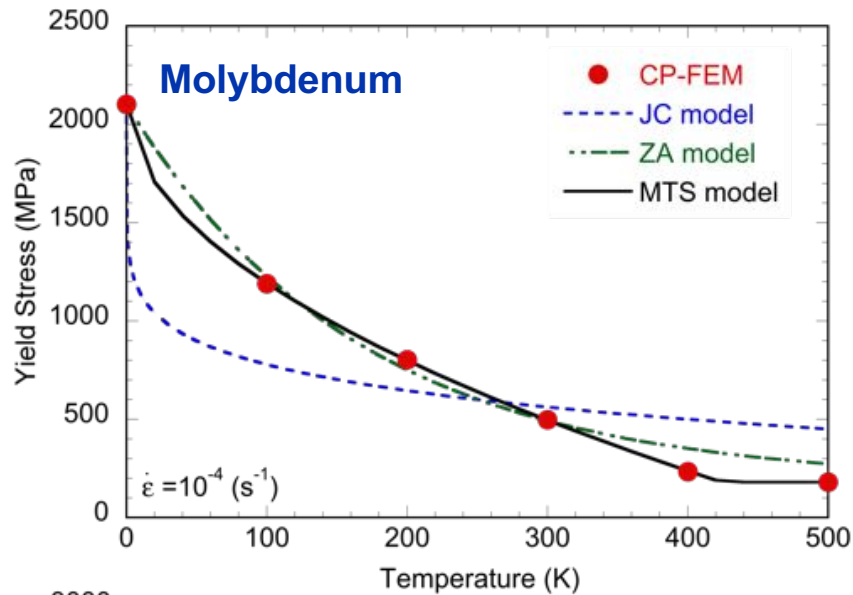
- Zerilli-Armstrong (ZA) model (Zerilli and Armstrong, 1987)

$$\sigma_y^{ZA} = C_0 + C_1 \exp(-C_3 T + C_4 T \ln \dot{\epsilon})$$

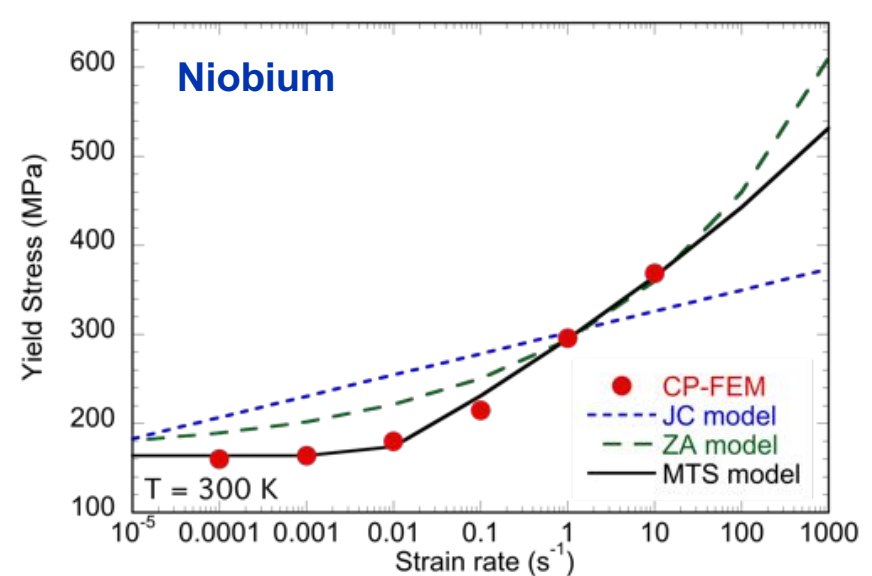
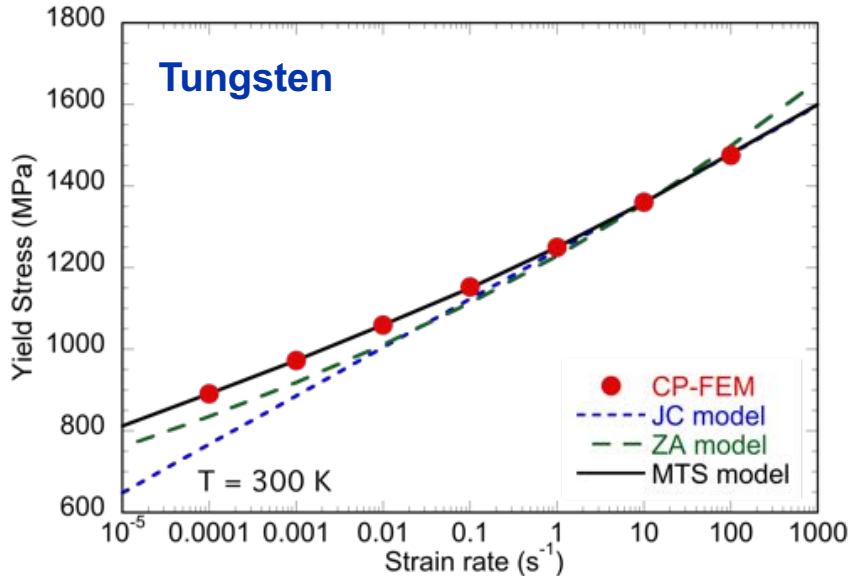
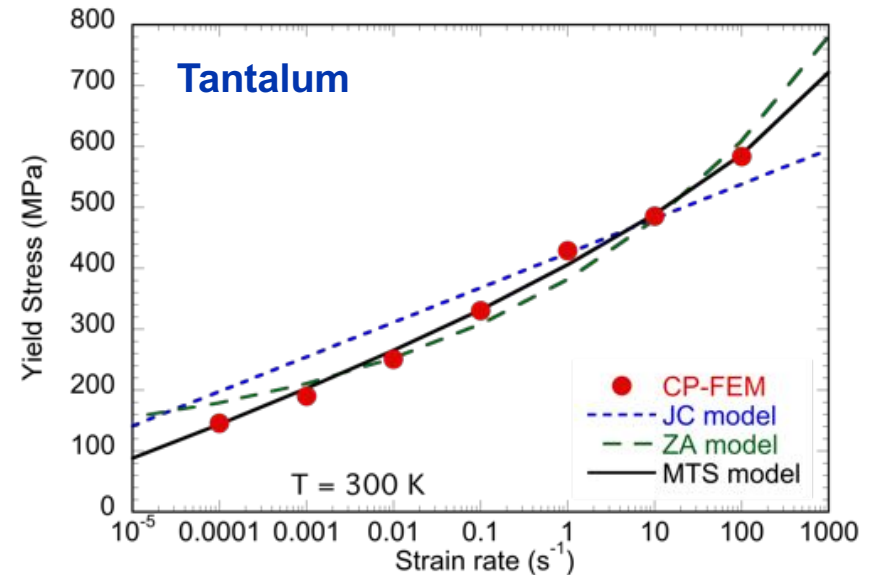
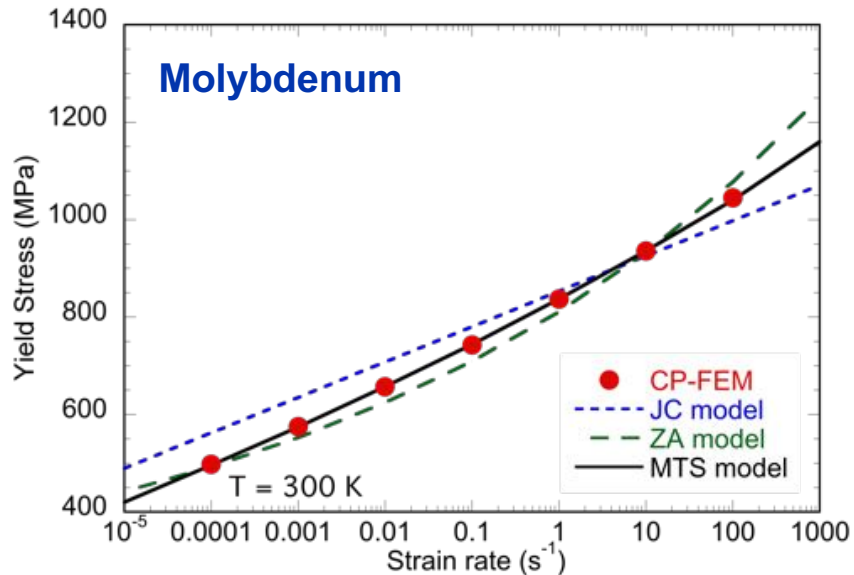
- Mechanical Threshold Stress (MTS) model (Follansbee, 1988)

$$\sigma_y^{MTS} = \sigma_0 + \hat{\sigma} \left( 1 - \left( \frac{k_B T}{G_0} \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{1/q} \right)^{1/p}$$

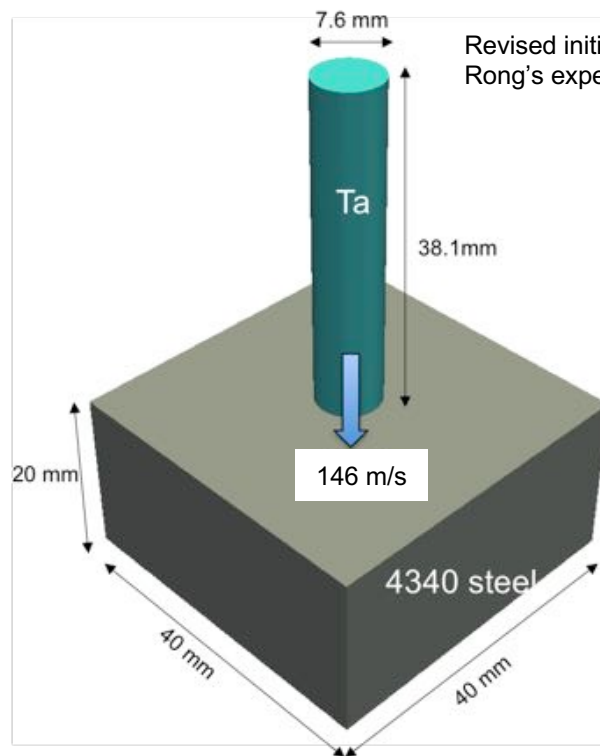
# Continuum Constitutive Model Calibration



# Continuum Constitutive Model Calibration

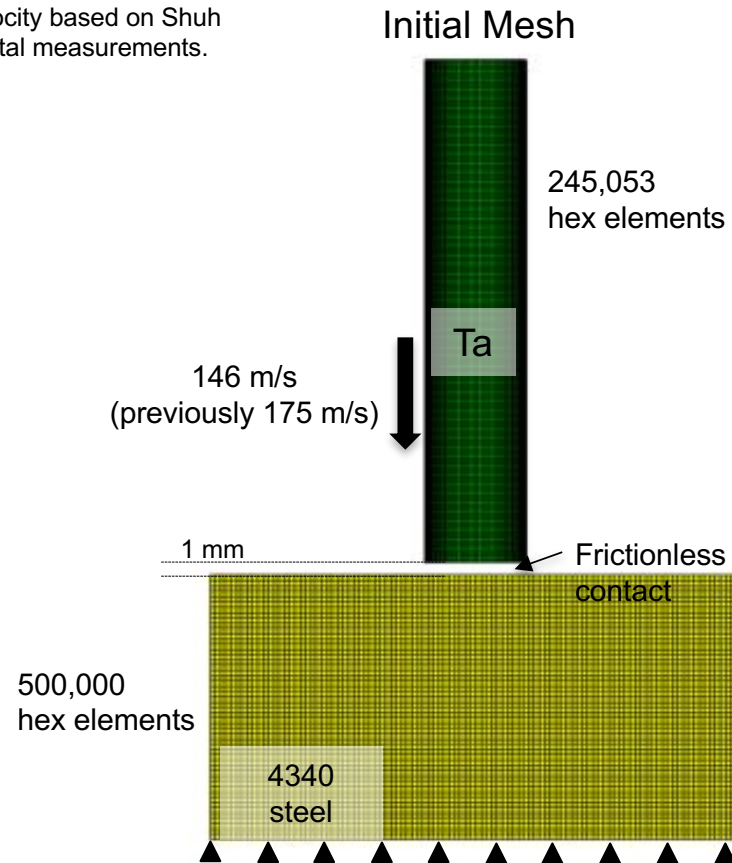


# Simulations of Taylor Cylinder Impact Tests



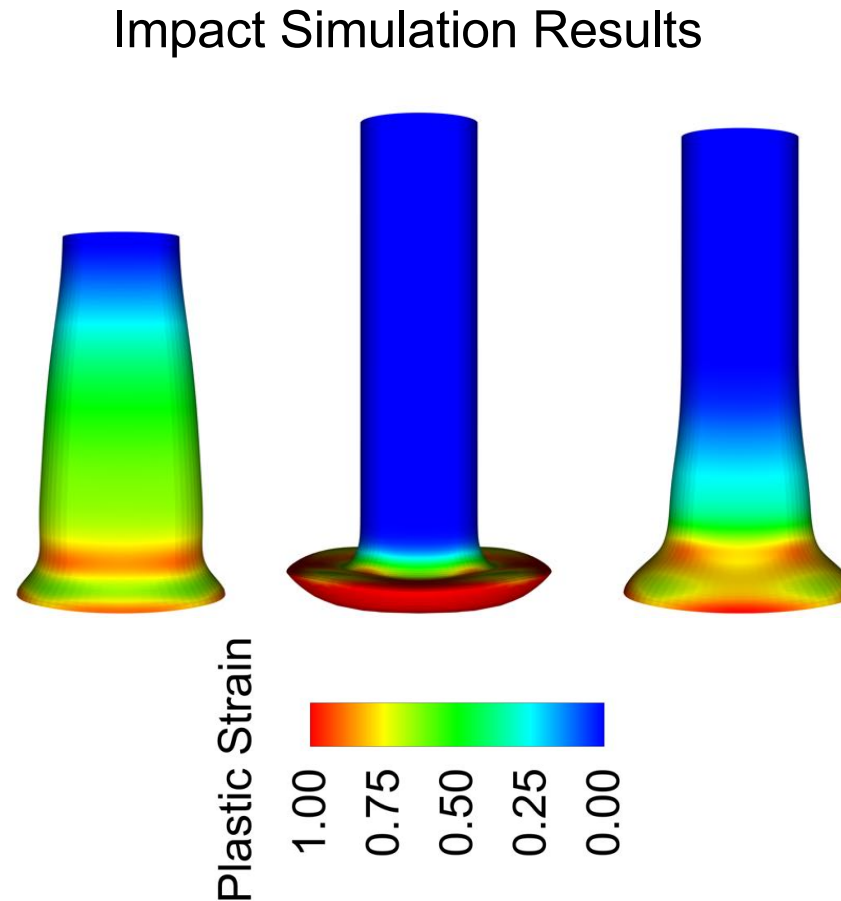
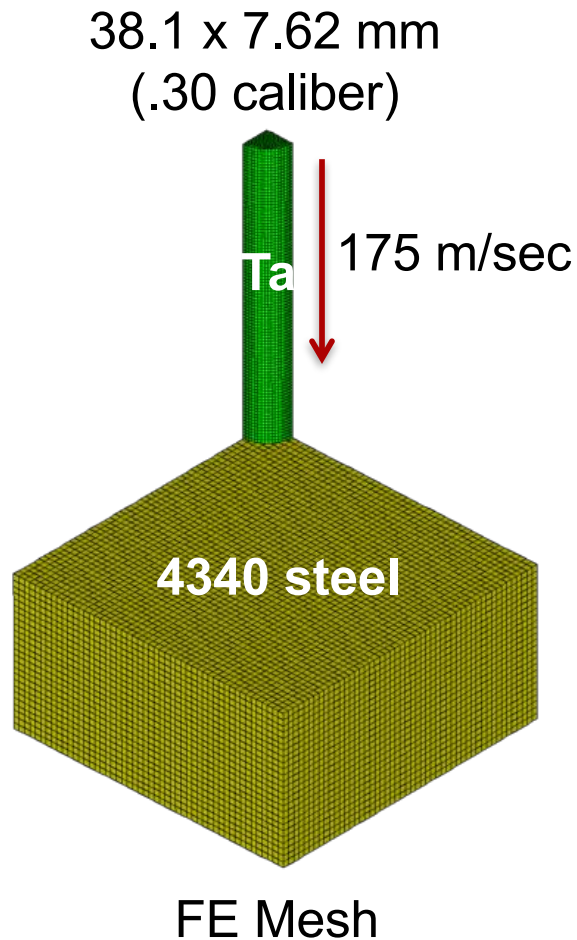
Revised initial velocity based on Shuh Rong's experimental measurements.

- Lagrangian
- Initial temperature = 298 K

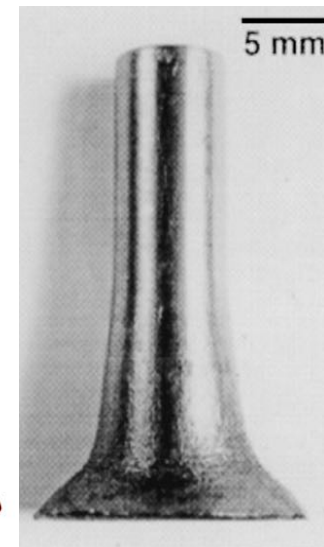




# Simulations of Ta Taylor Cylinder Impact



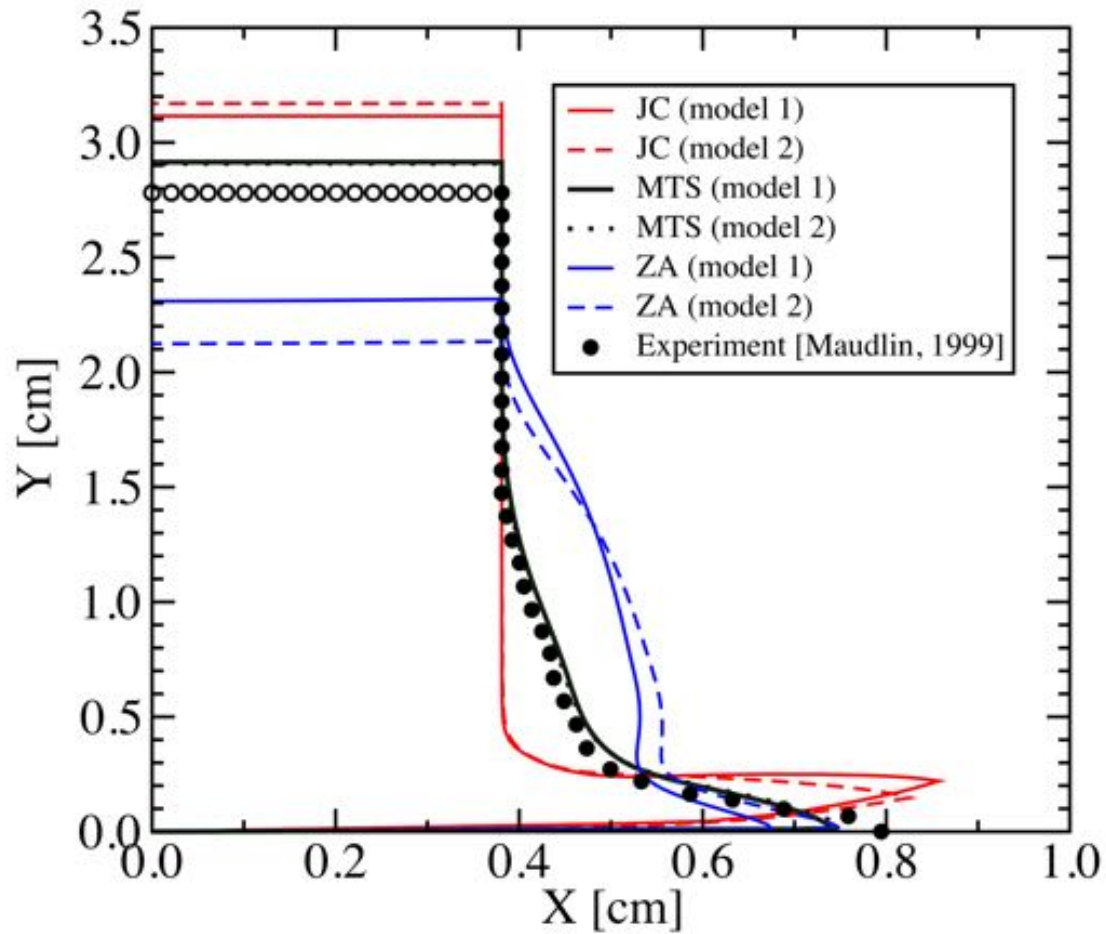
Experiment



(Maudlin 1999)

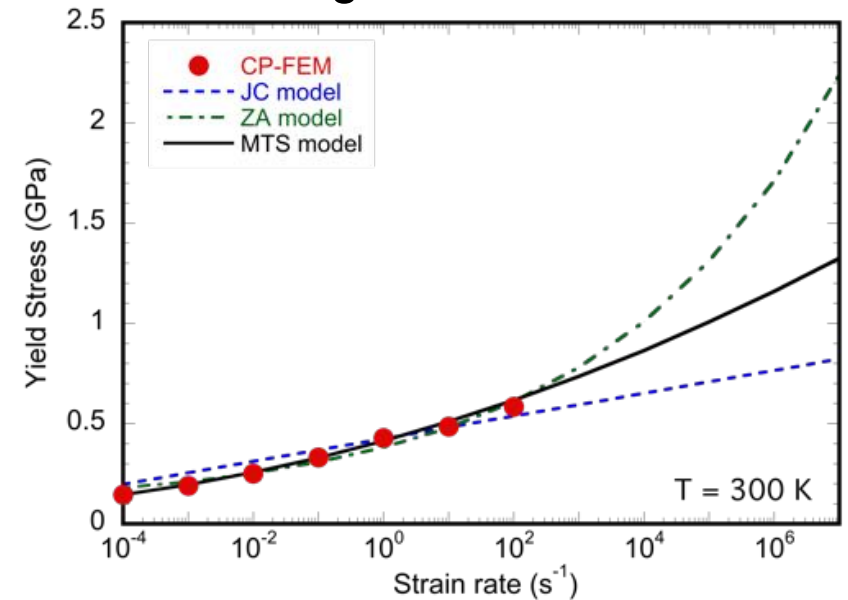


# Validation of Ta Taylor Cylinder Impact



Deformed Projectile Shape

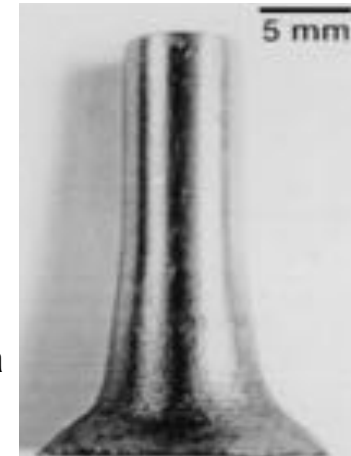
## Strength Model Predictions at Higher Strain Rates



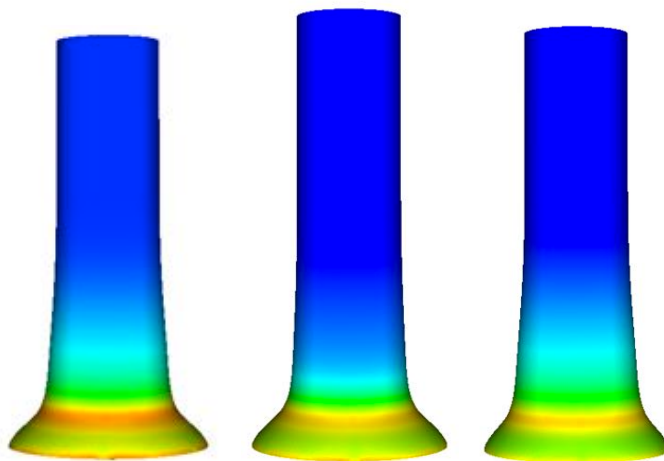
$V = 146 \text{ m/s}$  vs.  $175 \text{ m/s}$



$V = 146 \text{ m/s}$   
HC Starck Ta



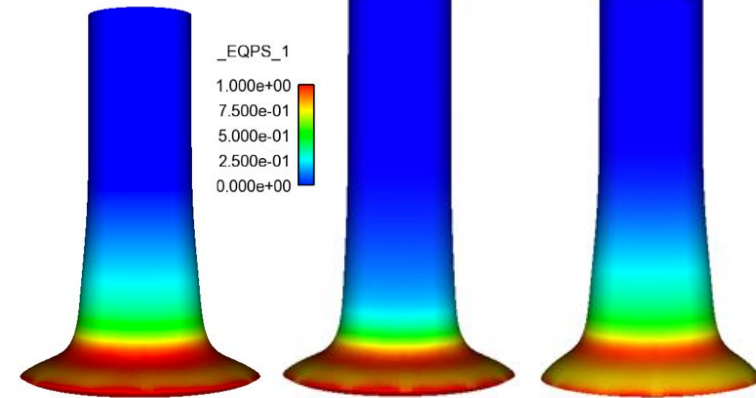
$V = 175 \text{ m/s}$   
DoD/DOE Ta



LANL's  
PTW model

Sandia's  
PTW model

Sandia's  
KP model



LANL's  
PTW model

Sandia's  
PTW model

Sandia's  
KP model

# Taking a “Step Back”: Simple Tension of Single Crystal Ta

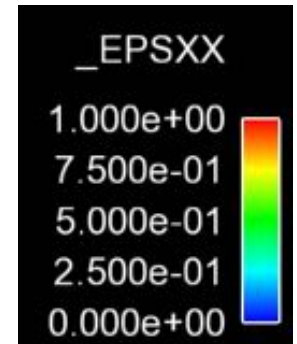
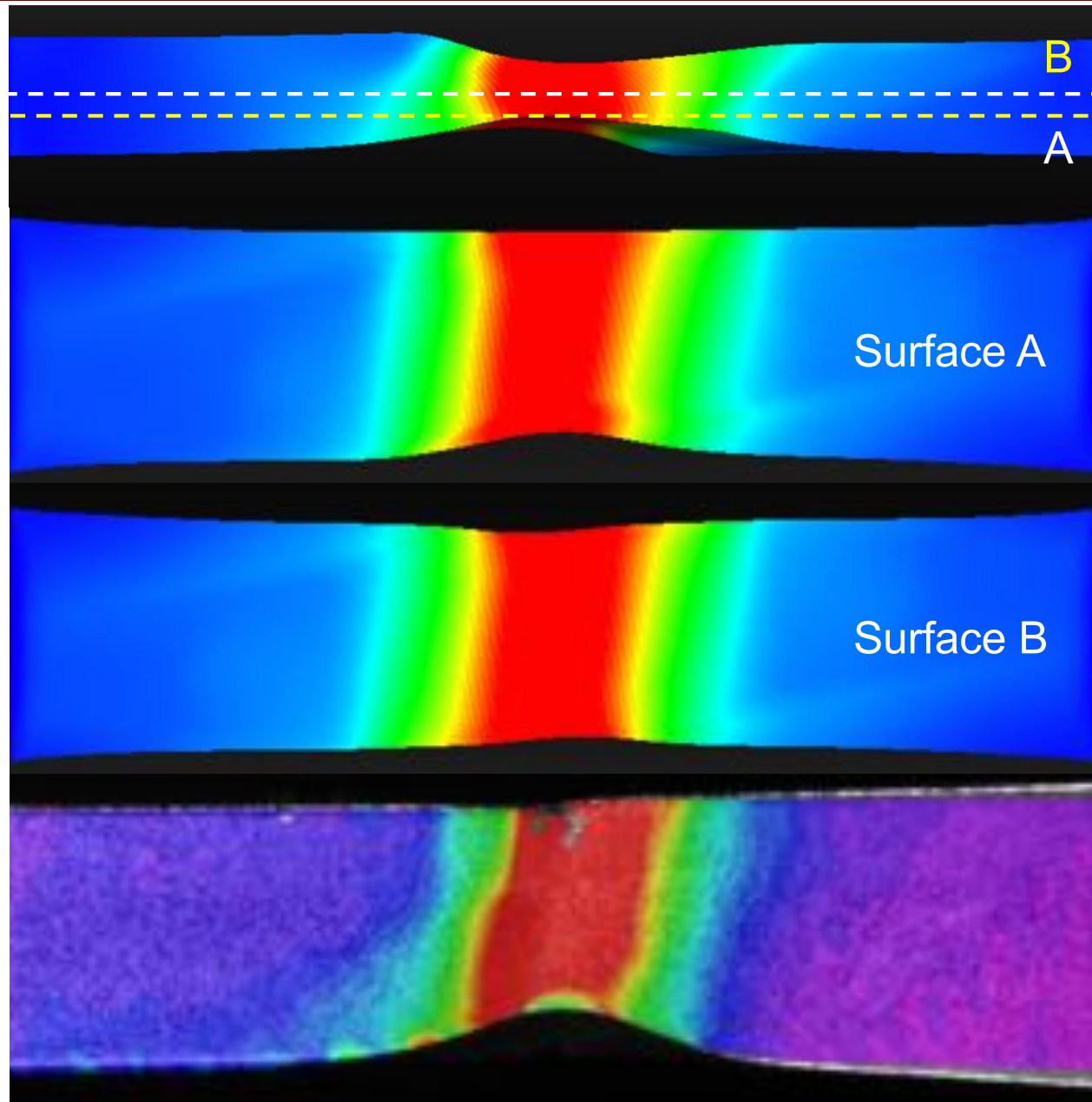
# Strain fields: HR-DIC vs. CP-FEM (fixed BC)

CP-FEM  
(Top view)

CP-FEM  
(Side view)

CP-FEM  
(Side view)

HR-DIC  
(Side view)



# Simulation of Ta single crystal

## Ta CP-FEM simulations

Uniaxial tension up to 30%

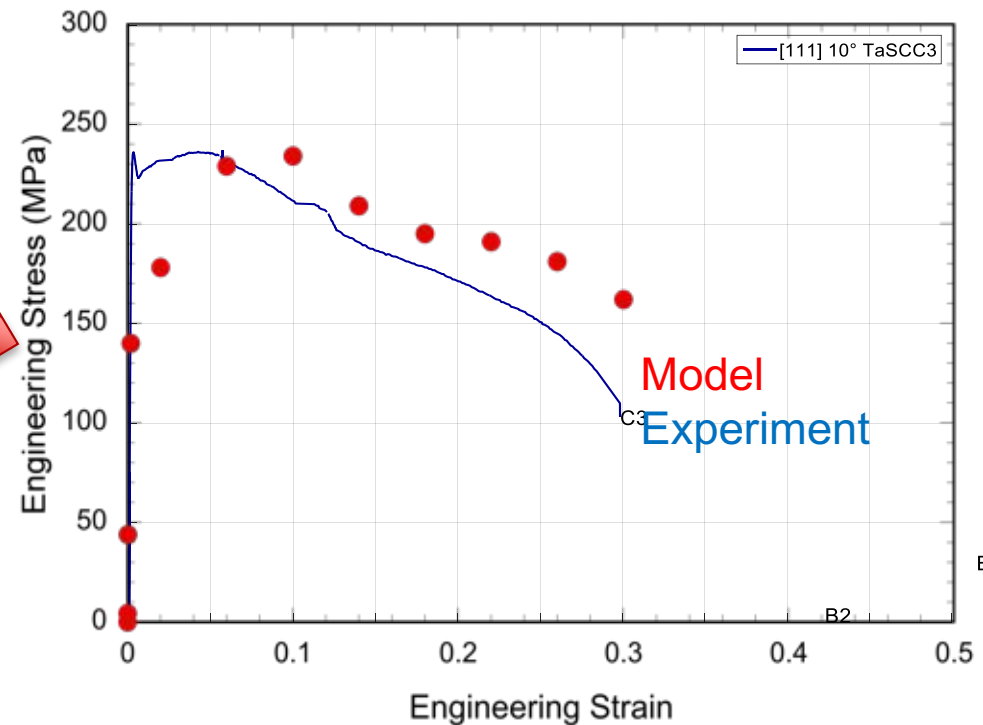
Strain rate =  $2 \times 10^{-4} \text{ s}^{-1}$

Euler angles = (94.272°, 35.24°, 308.81°)

Material parameters fit to Ta oligocrystals

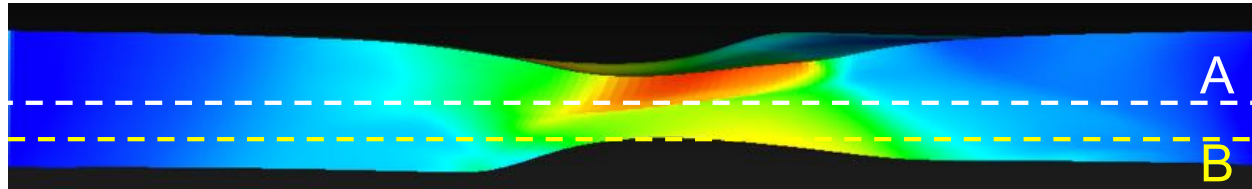
$1.3 \times 10^6$  hexahedral finite elements (tapered specimen)

No damage

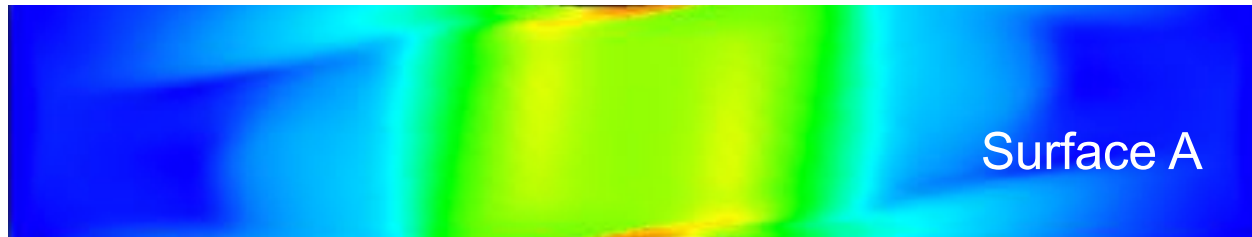


# Misorientations

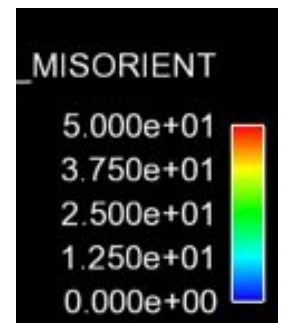
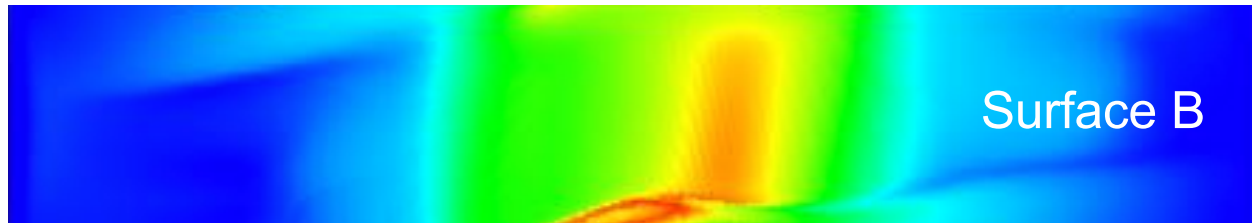
CP-FEM  
(Top view)



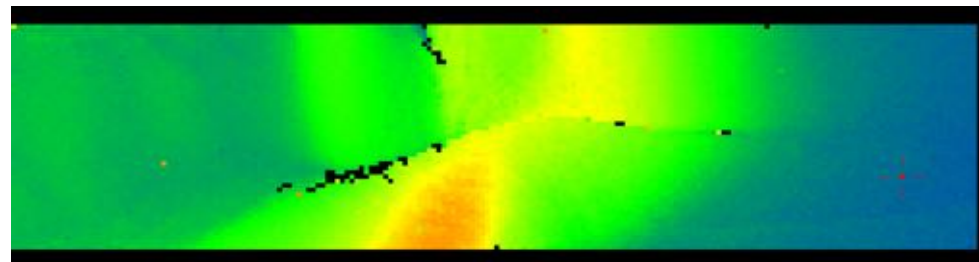
CP-FEM  
(Side view)



CP-FEM  
(Side view)

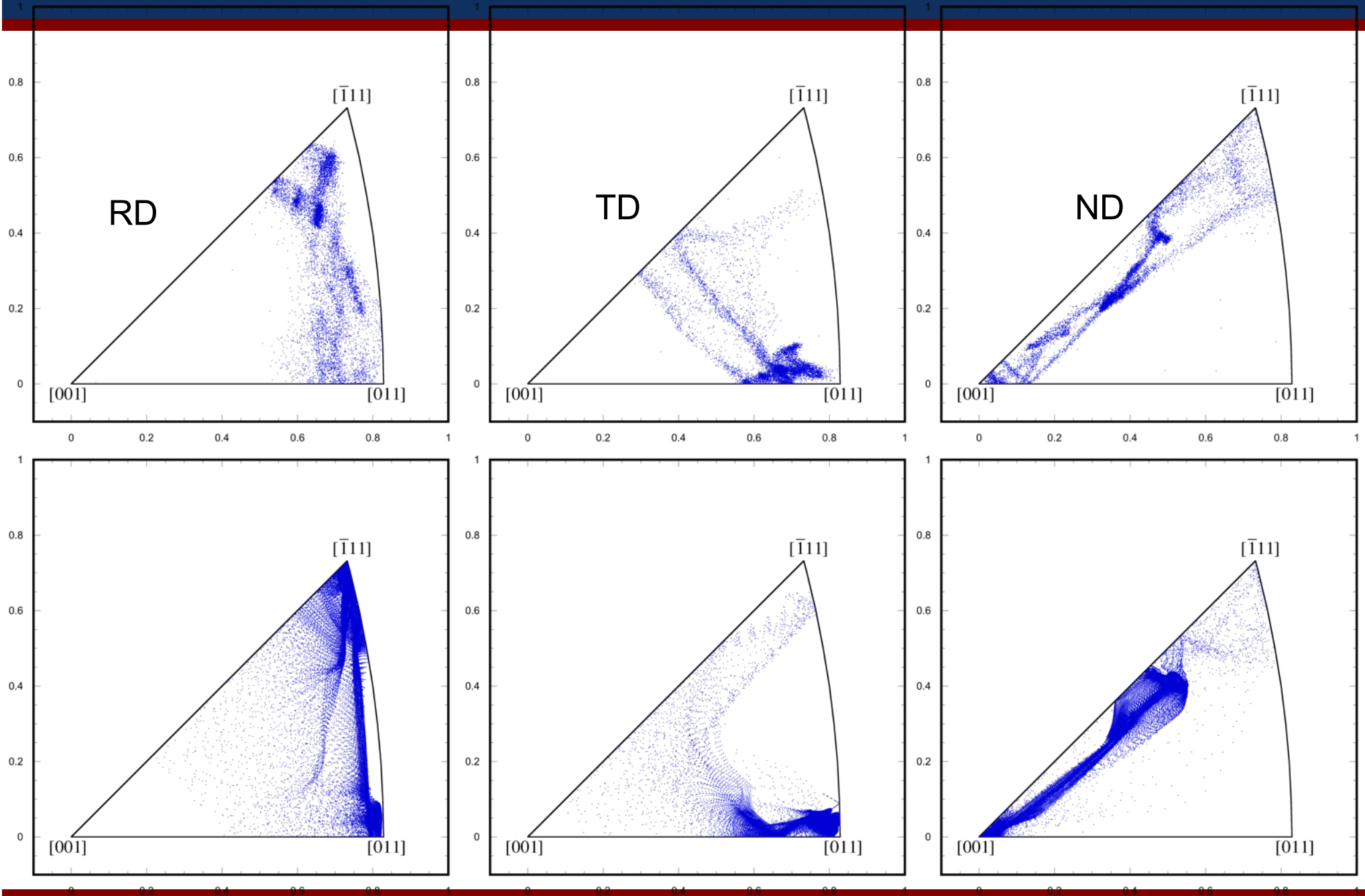


EBSD  
(Side view)



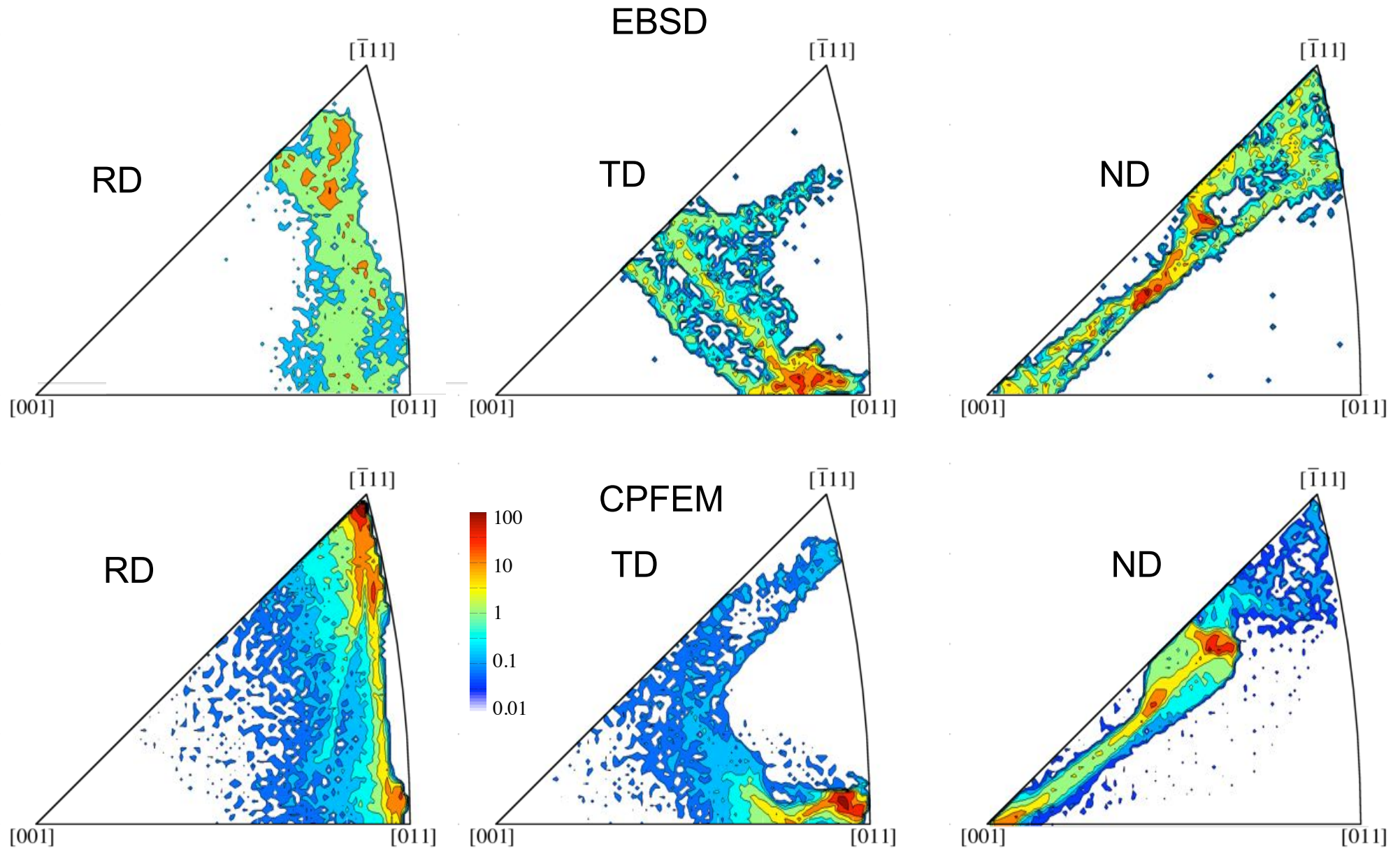


# Deformed texture (mid z, fixed BC)





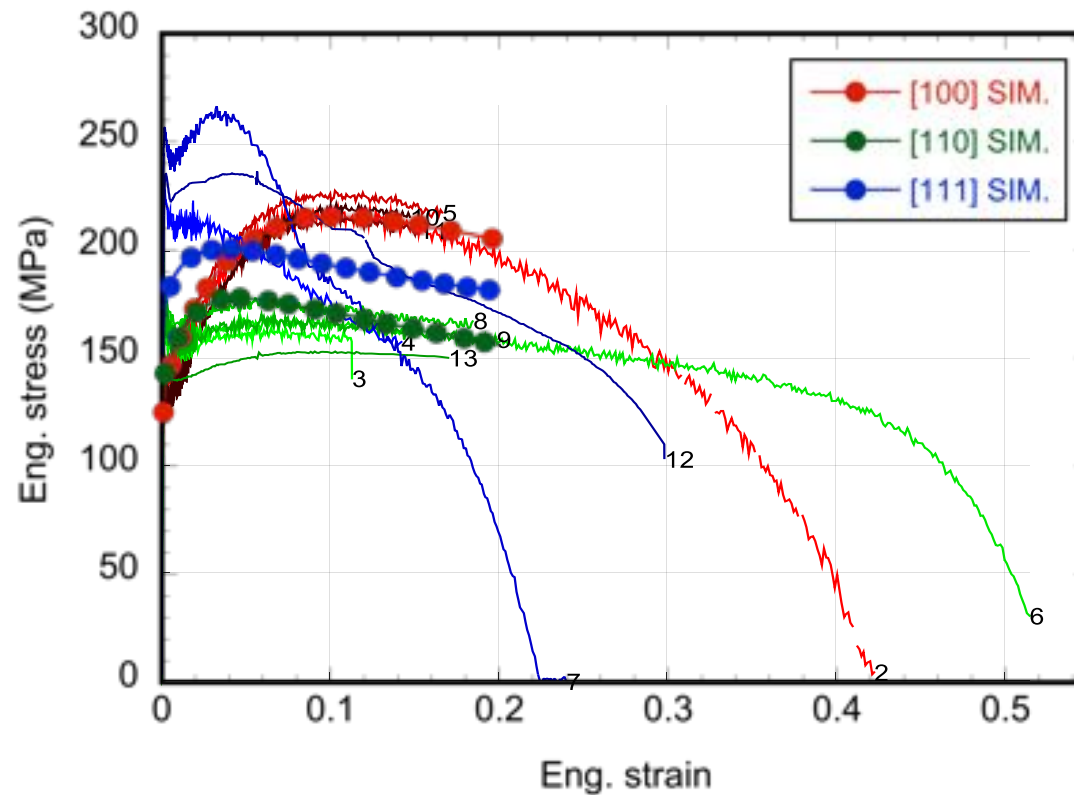
# Initial crystal orientations (mid-z plane)



# Tensile response

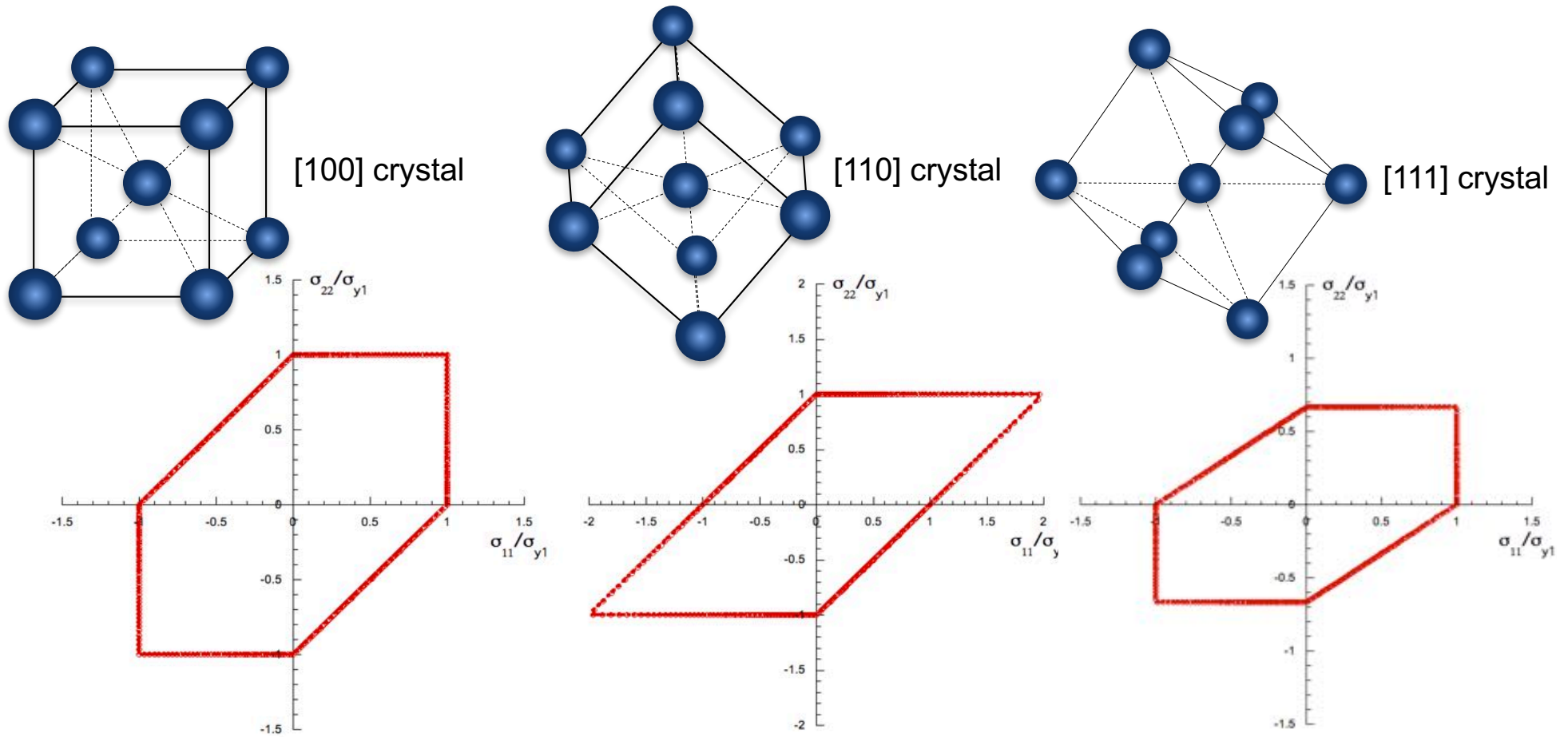
## ❖ Dislocation density-based hardening

$$g^\alpha = A\mu b \sqrt{\sum_{\beta=1}^{24} \mathbf{H}^{\alpha\beta} \rho^\beta} \quad \dot{\rho}^\alpha = \left( \kappa_1 \sqrt{\sum_{\beta=1}^{NS} \rho^\beta} - \kappa_2 \rho^\alpha \right) \cdot |\dot{\gamma}^\alpha|$$



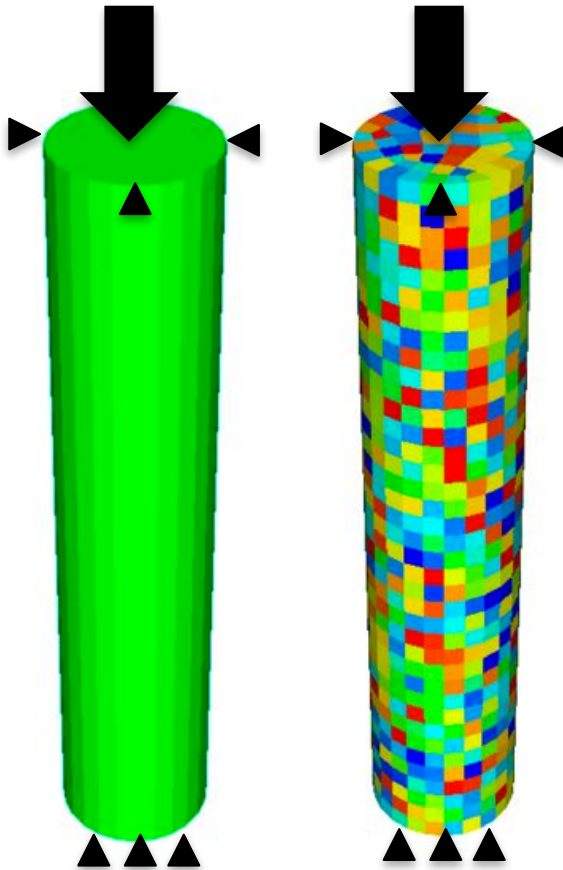
# Simulated Compression of Single Crystal Ta Cylinders

# Biaxial yield surfaces of single crystals

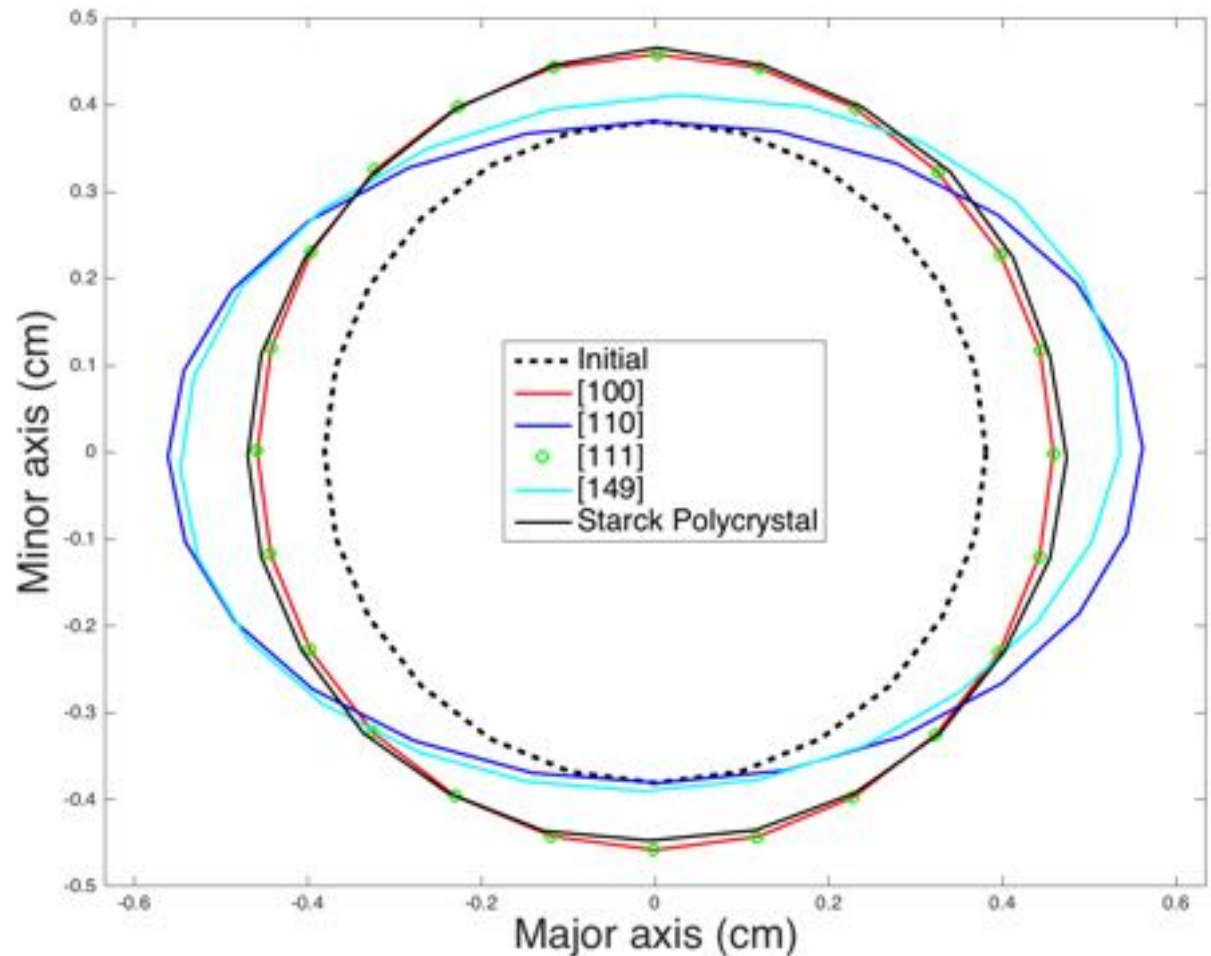


# Quasi-static compression of cylinder

30% compression  
Strain rate =  $10^{-3} \text{ s}^{-1}$

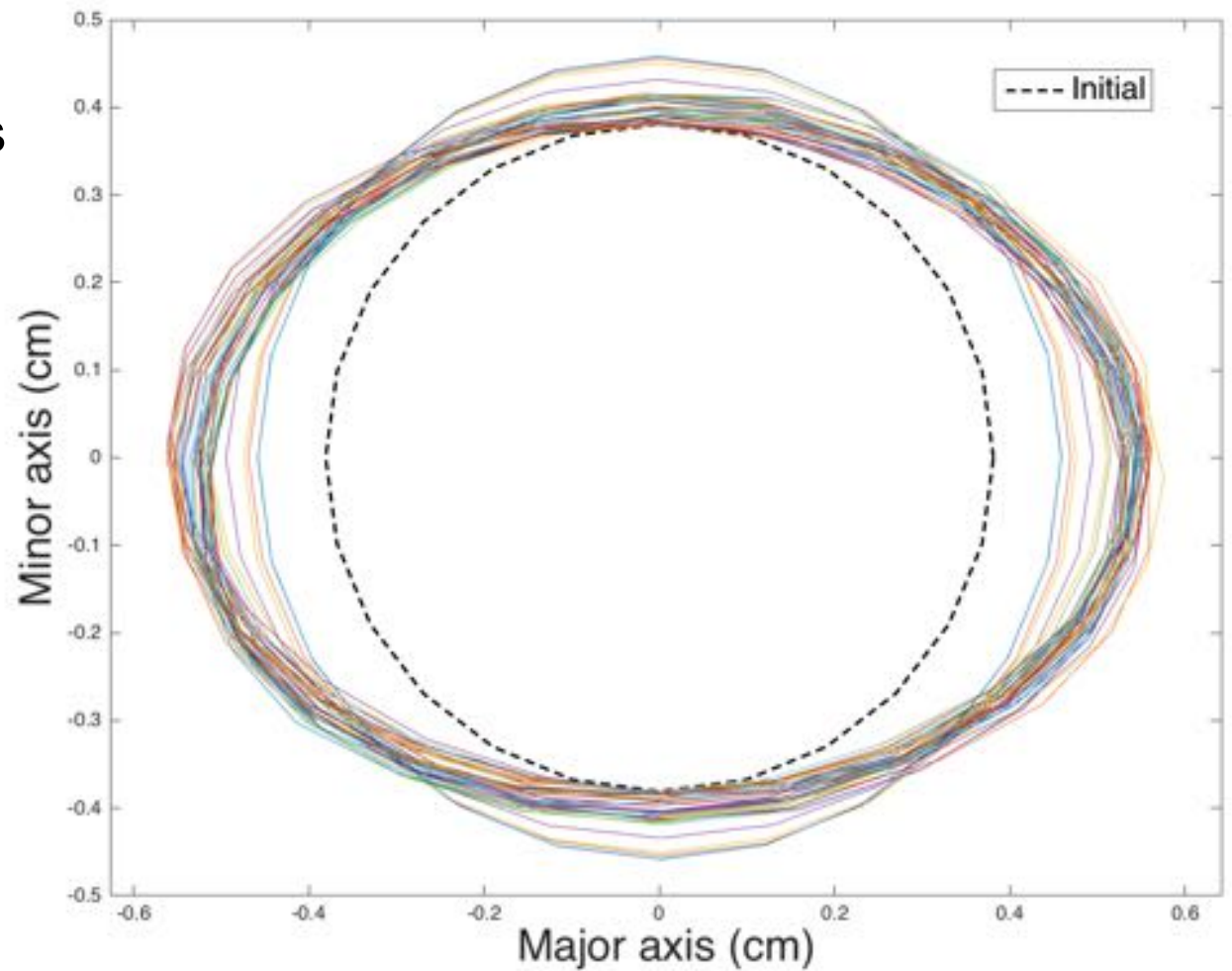
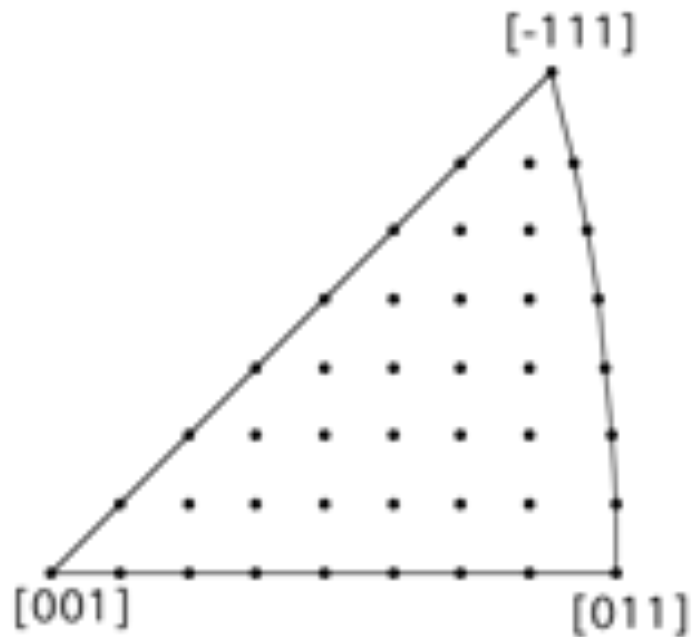


Single Ta    Polycrystalline Ta  
(Starck Ta texture)



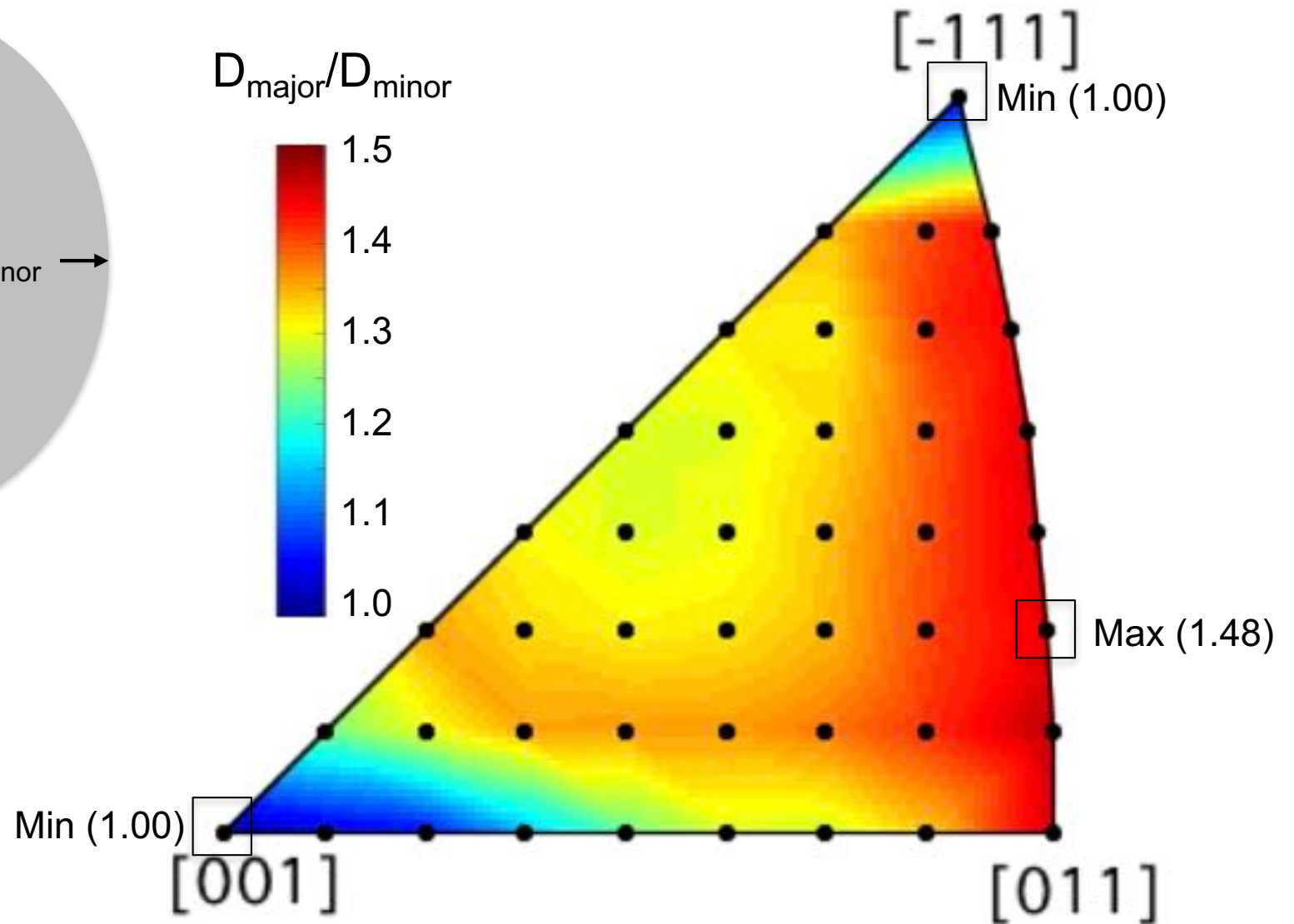
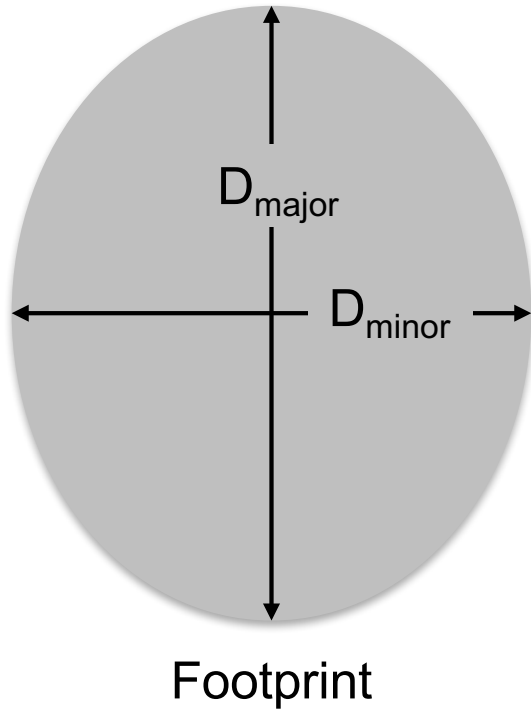
# Footprints of single crystals

43 single crystal orientations





# Aspect ratios ( $=D_{\text{major}}/D_{\text{minor}}$ )



\* STARCK polycrystalline Ta : 1.035

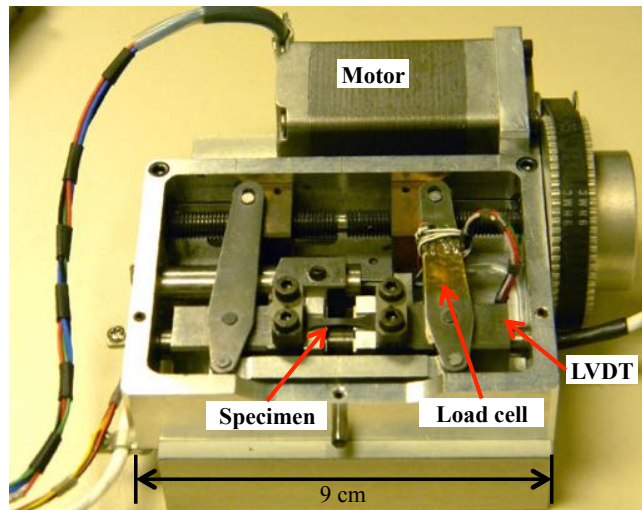


# Summary

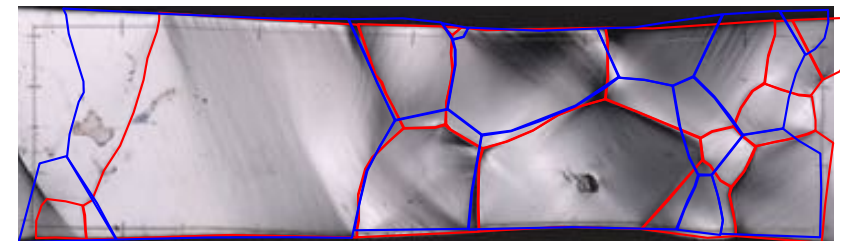
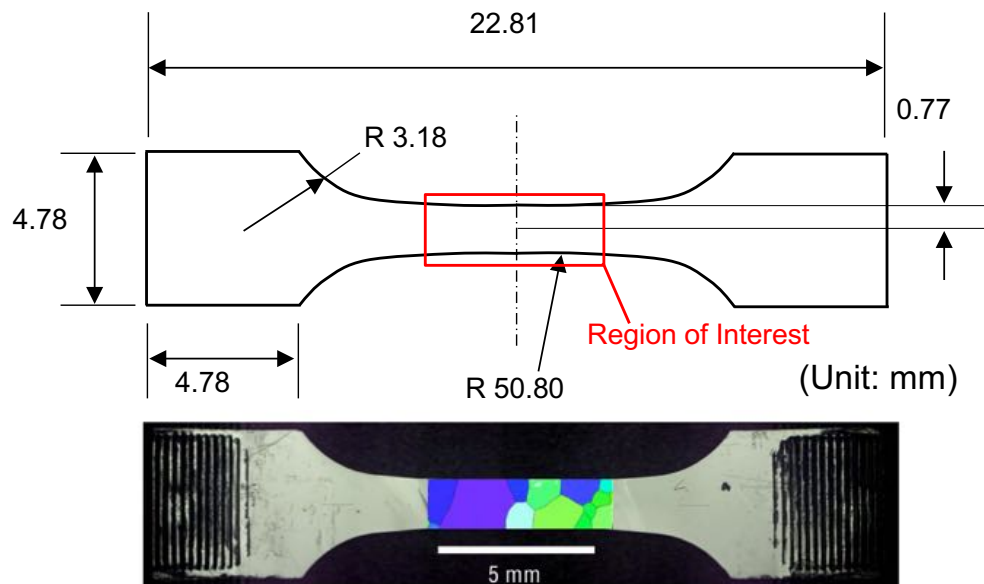
- Developed  $T$  and  $\dot{\epsilon}$  dependent flow rule based on dislocation kink-pair theory for Mo, Ta, W and Nb.
- Predicted  $T$  and  $\dot{\epsilon}$  dependent  $\sigma_y$  using CP-FEM model agree well with experimental data.
- MTS model accurately reproduced  $T$  and  $\dot{\epsilon}$  dependent  $\sigma_y$  of BCC polycrystals.
- CP-FEM predictions of Ta oligocrystals showed good agreement with measured surface strain fields (HR-DIC) and deformed textures (EBSD).
- Proposed computational method provides a convenient and direct link from the fundamental dislocation physics to the continuum-scale plastic deformation of BCC metals at the grain scale.

# Supplementary Slides: Validation on Ta Multicrystals

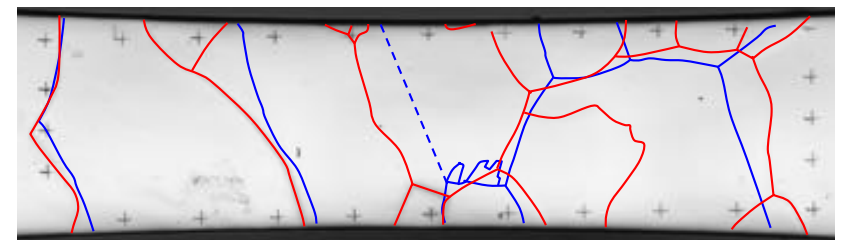
# Experimental Setup



- Tantalum oligocrystals with mostly columnar 2D grain structure eliminate unknown subsurface grain morphology.
- *In-situ* load frame developed at Sandia
- HR-DIC (surface strain fields) and EBSD (crystal orientations) measurements at load inside SEM



Specimen 1

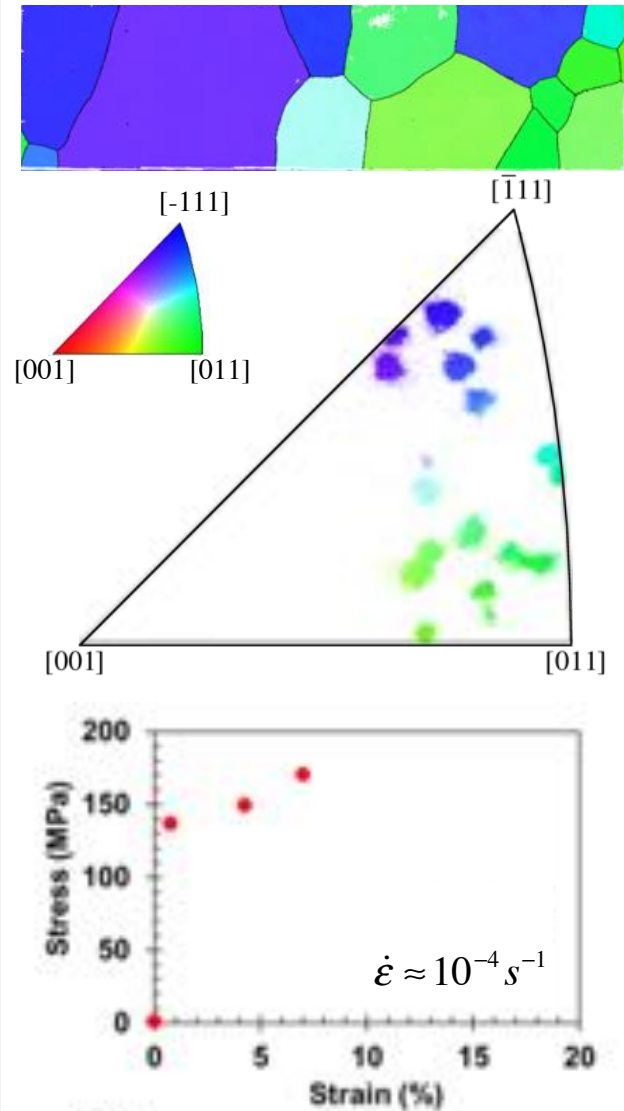


Specimen 2

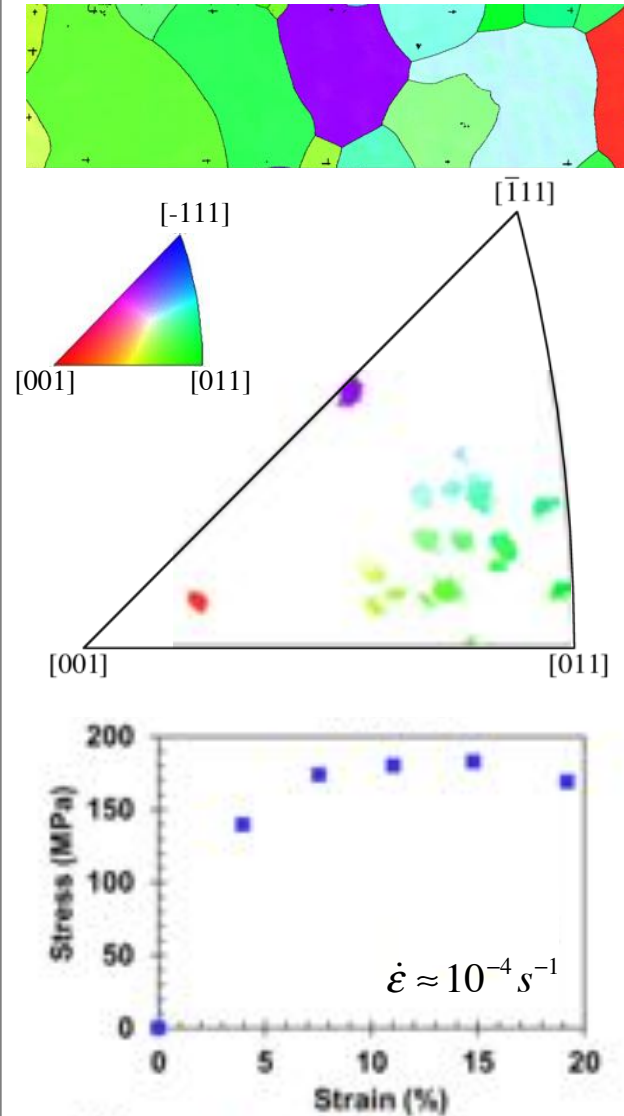
— Grain boundary (Front) — Grain boundary (Back)

# Tantalum Oligocrystal Specimens

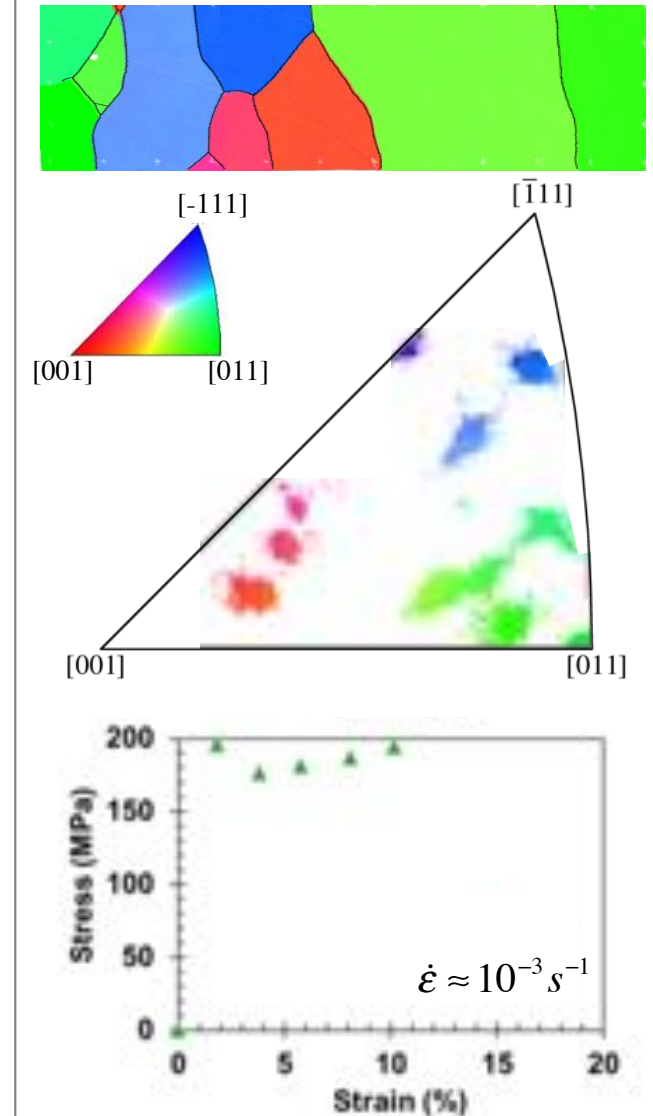
**Specimen 1**



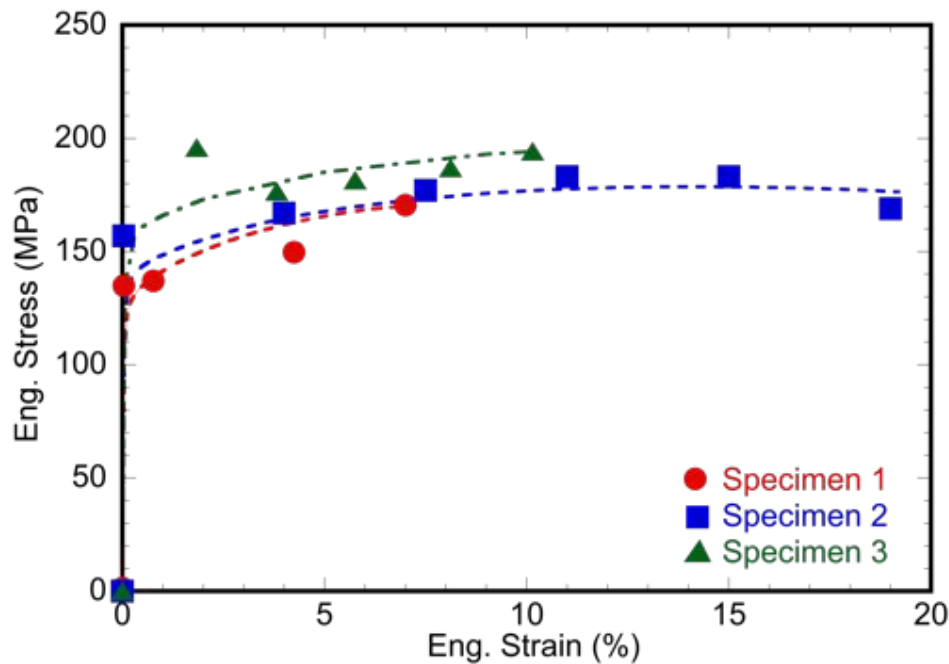
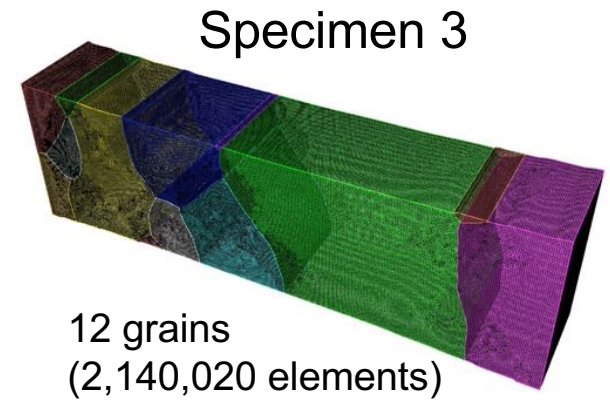
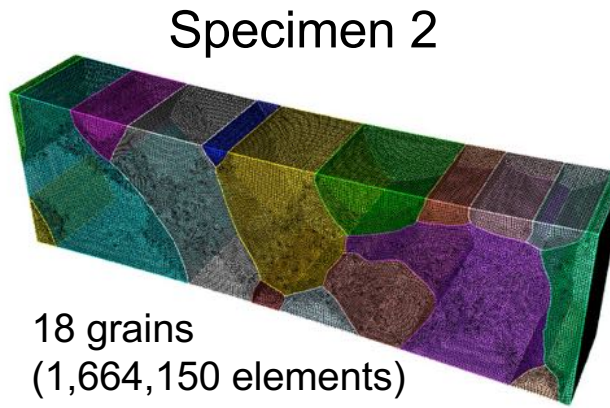
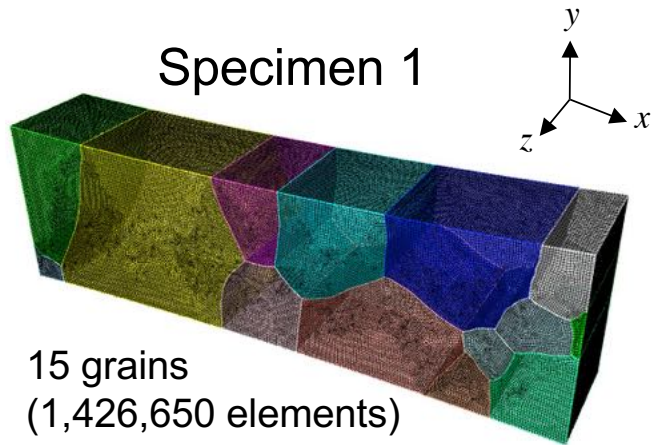
**Specimen 2**



**Specimen 3**



# Simulations of Ta Oligocrystals



Hardening parameters fit to measured stress-strain data.

$$\tau_{obs}^{\alpha} = A\mu b \sqrt{\sum_{\beta=1}^{NS} \rho^{\beta}} \quad \dot{\rho}^{\alpha} = \left( \kappa_1 \sqrt{\sum_{\beta=1}^{NS} \rho^{\beta}} - \kappa_2 \rho^{\alpha} \right) \cdot |\dot{\gamma}^{\alpha}|$$

$$\tau_{obs,0}(\text{Specimen 1}) = 27 \text{ MPa}$$

$$\tau_{obs,0}(\text{Specimen 2}) = 37 \text{ MPa}$$

$$\tau_{obs,0}(\text{Specimen 3}) = 27 \text{ MPa}$$

$$\kappa_1 = 1.4 \times 10^6 \text{ m}^{-1}$$

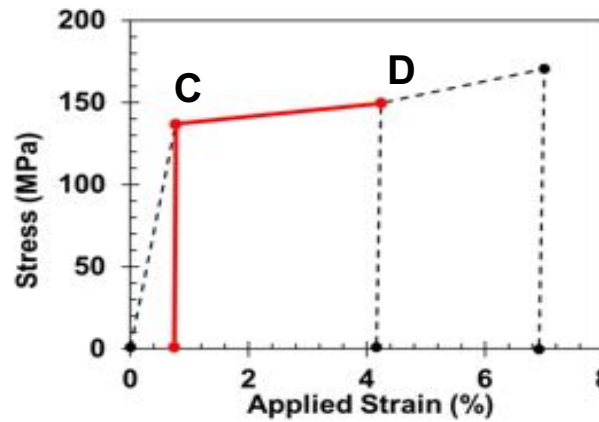
$$\kappa_2 = 14$$

“Initial yield”

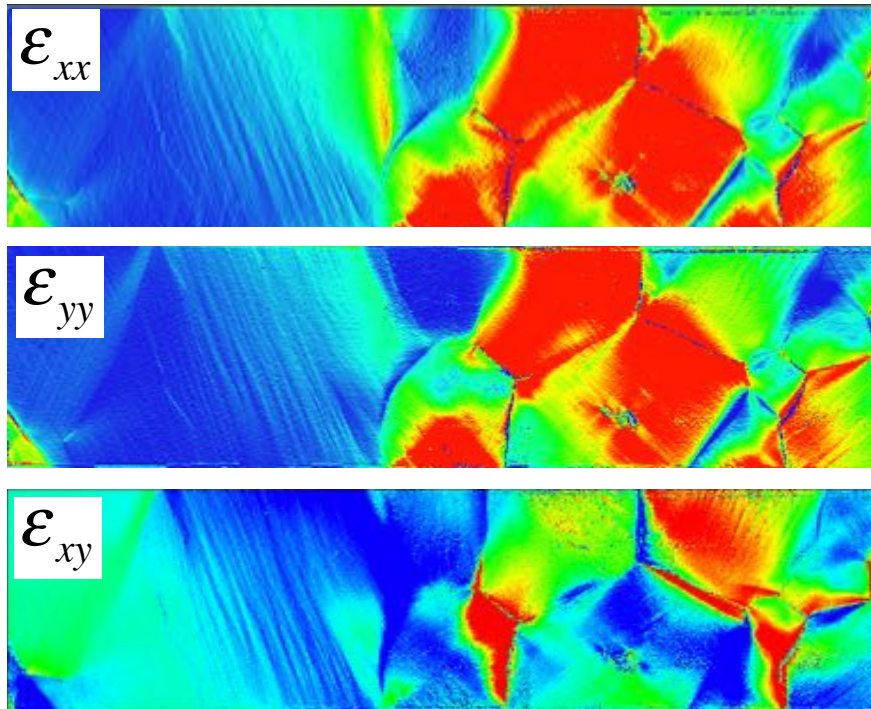
“Hardening shape”



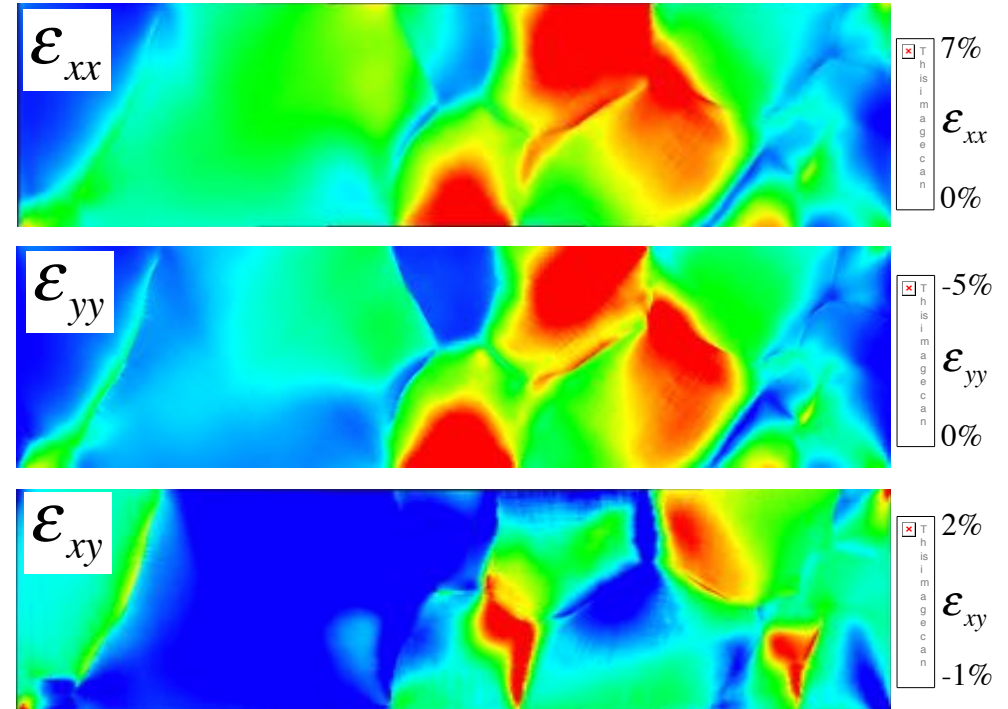
# Strain Field Analysis



$$\Delta\epsilon_{CD} = 3.4\%$$



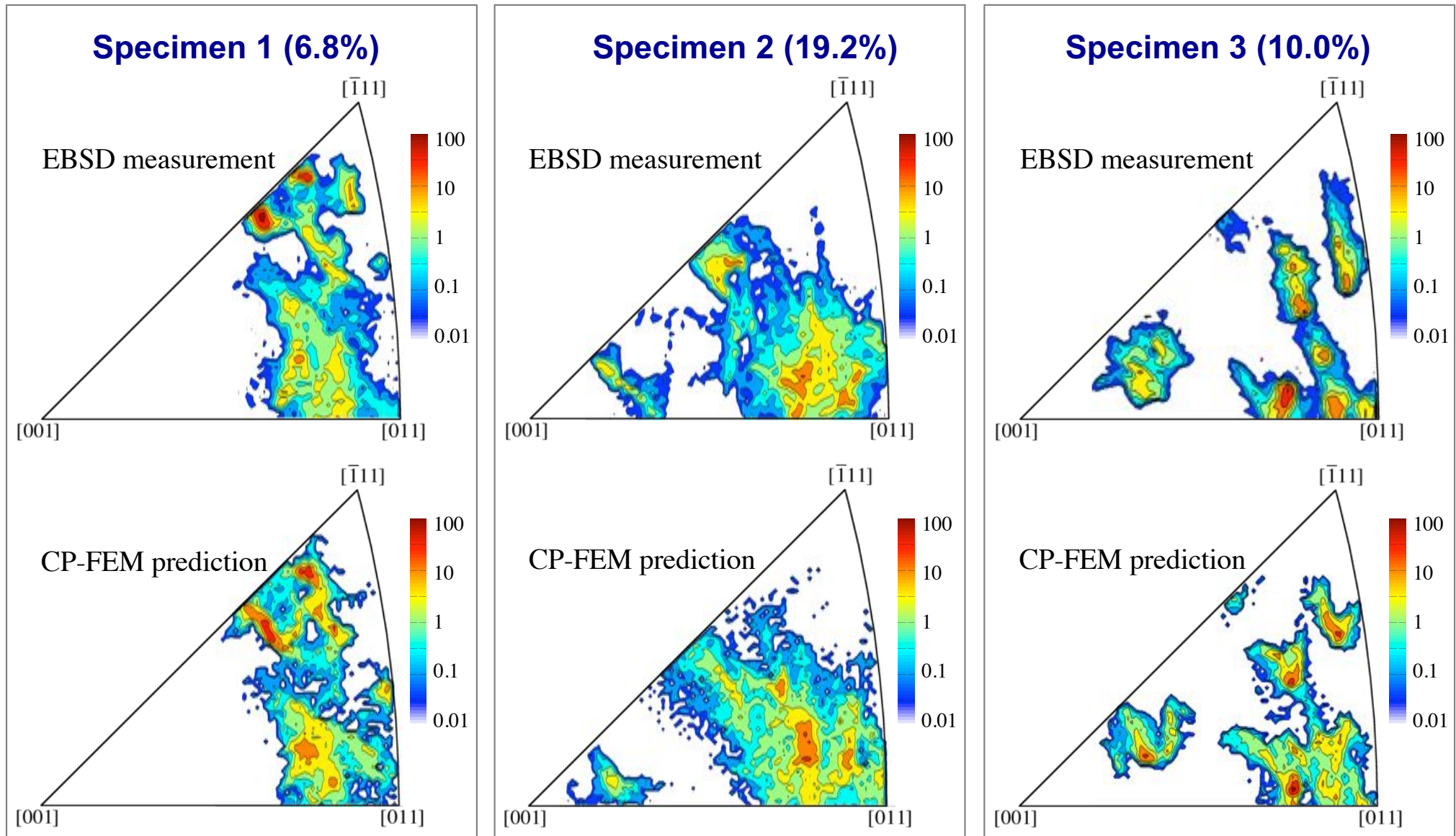
(a) HR-DIC measurements



(b) CP-FEM predictions

Measured and predicted strain fields agree well quantitatively.

# Texture Predictions

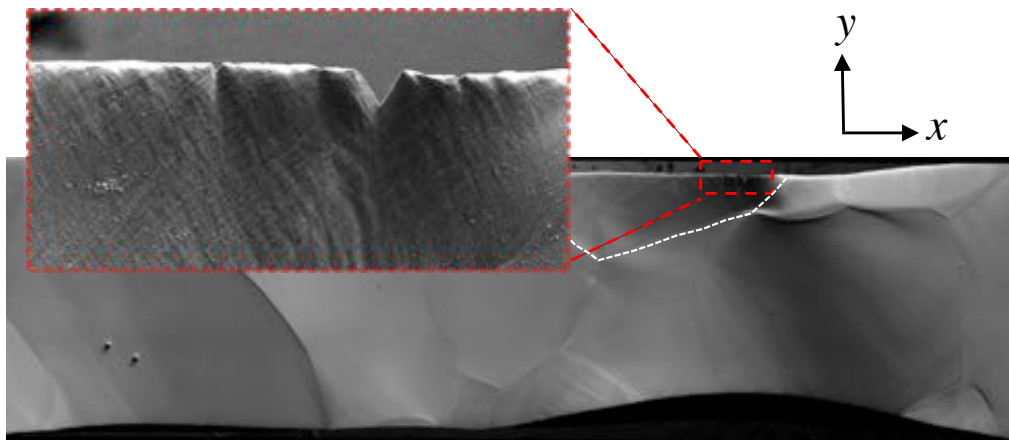
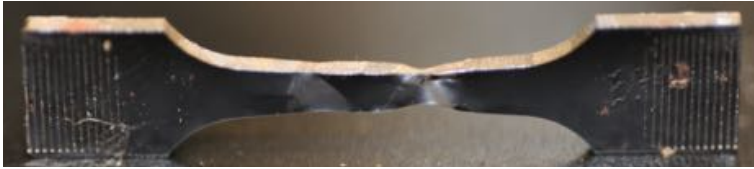


IPF contour plots indicate very good agreement between model and experiment.

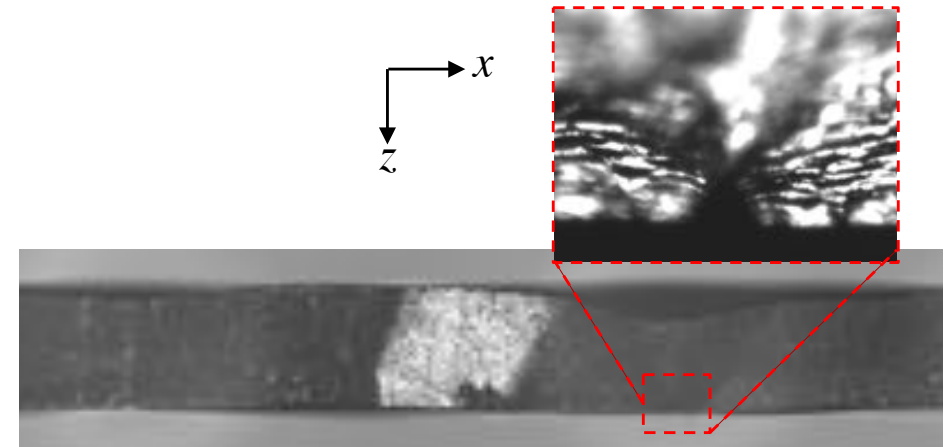


# Failure Predictions

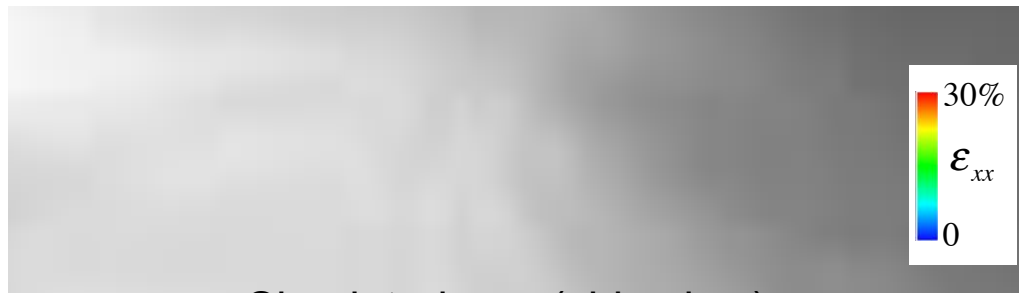
Ta oligocrystal specimen 2 at 19.2% deformation



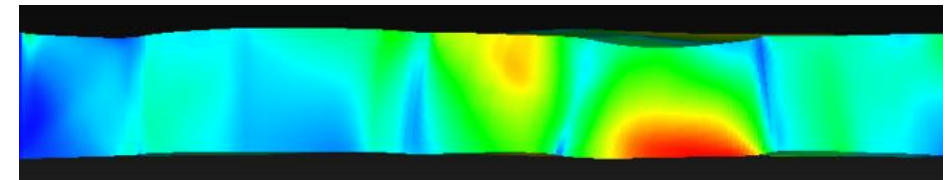
Surface image (side view)



Surface image (top view)



Simulated  $\epsilon_{xx}$  (side view)



Simulated  $\epsilon_{xx}$  (top view)

Failure location agrees with the location of the highest  $\epsilon_{xx}$  from the simulation