SAND2017-7263C

Design of Continuously Graded Elastic Materials for Acoustic Cloaking

Clay Sanders [a], Wilkins Aquino [a], Timothy Walsh [b]

[a] *Duke University, Department of Civil & Environmental Engineering*

[b] *Sandia National Laboratories, Albuquerque NM*

US National Congress on Computational Mechanics, Montreal, Canada

July17-20, 2017

Duke UNIVERSITY

Sandia National Laboratories, Albuquerque, NM. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration. With main facilities in Albuquerque, N.M., and Livermore, C.A., Sandia has major R\&D responsibilities in national security, energy and environmental technologies, and economic competitiveness.

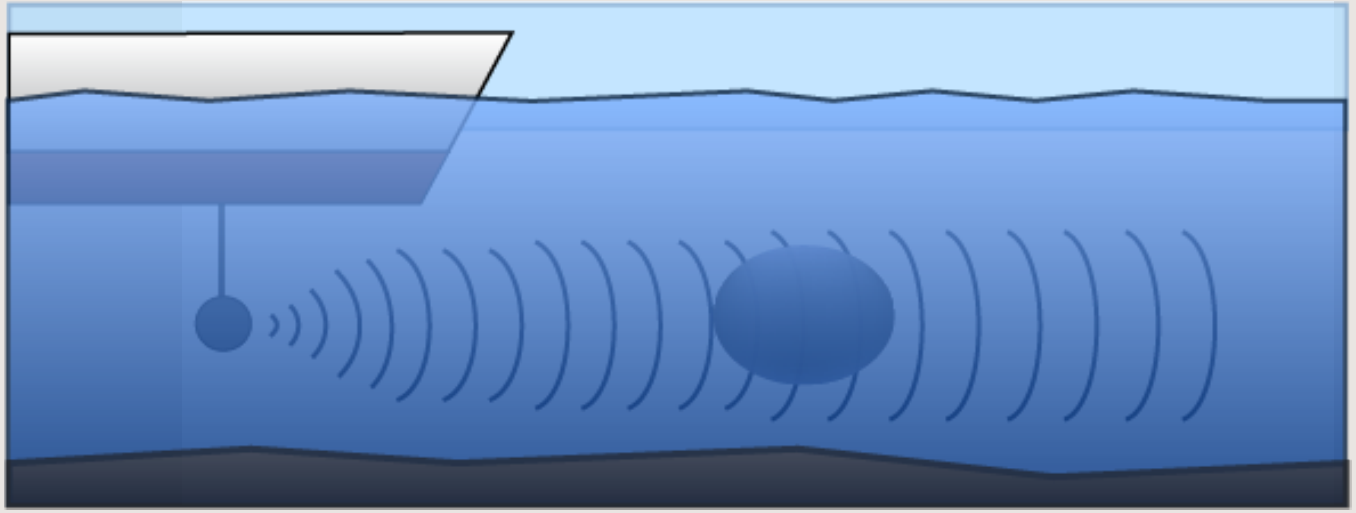
Sandia National Laboratories

Outline

- Research Motivation
- Problem Formulation
  - Governing acoustic-structural interaction
  - Abstract Optimization Formulation
- Numerical Results
  - Frequency-specific design
  - Extended frequency range support
- Conclusions & Questions

Research Motivation

- Interest in cloaking structures and materials
  - Electromagnetic cloaks use coordinate transformation methods
  - Difficulty in translating these solutions to acoustic wave energy management
- Proposed media for acoustic cloaks require extraordinary properties
  - Radially varying anisotropic mass density and bulk modulus
- Elastic acoustic metamaterials provide candidate for more-easily realized acoustic cloaking material



Duke UNIVERSITY

Sandia National Laboratories

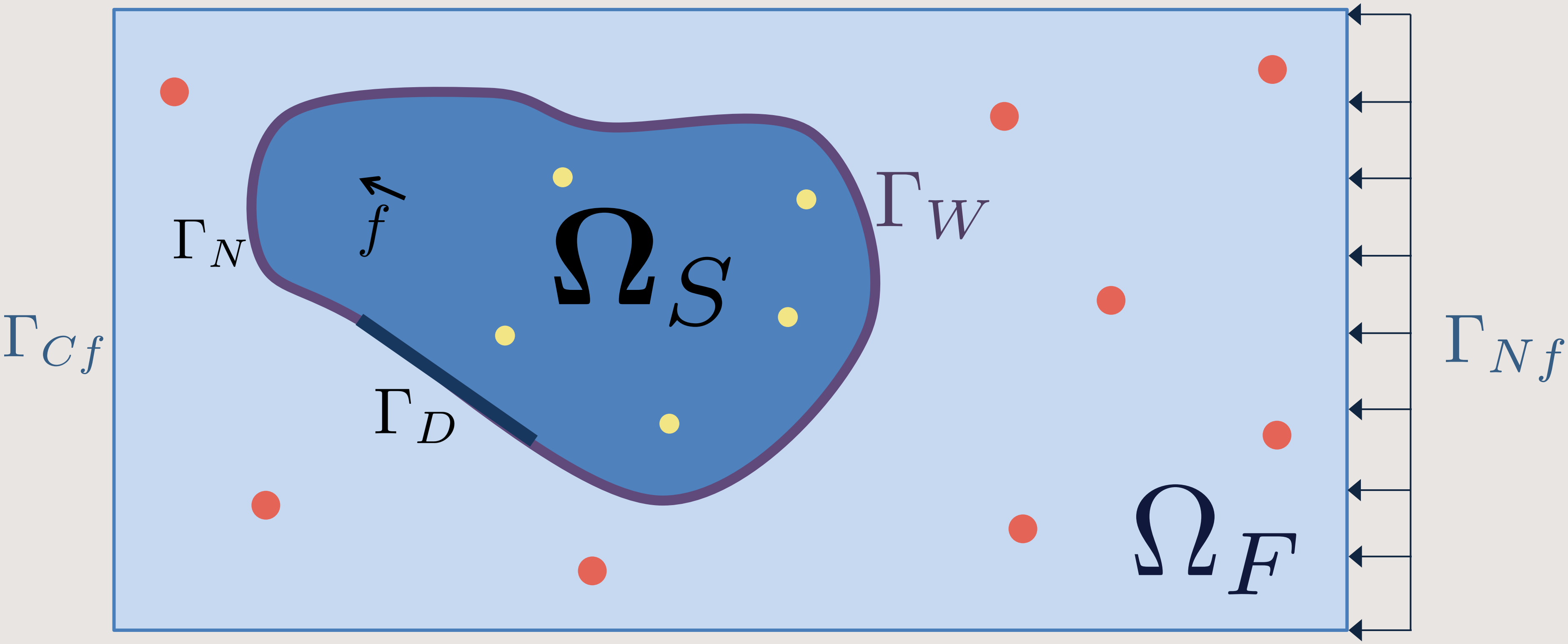
Research Motivation

- Select materials that provide optimal vibration control through large scale PDE constrained optimization
  - Many engineering systems use viscoelastic media to control vibration environments
  - Current engineering practice ‘ad hoc’ in the design of foam materials for damping

Duke UNIVERSITY

Sandia National Laboratories

Governing Acoustic-Structural Interaction



= Measurement locations (Red: microphones, Yellow: accelerometers)

Frequency-Domain Governing Equations for Coupled ASI Problem

Coupled PDE's for ASI govern system behavior and provide constraints for optimization

Elastodynamics

$$-\omega^2 \rho_s \boldsymbol{u} - \nabla \cdot \boldsymbol{\sigma} = \mathbf{0} \text{ in } \Omega_S$$

$$\boldsymbol{\sigma} = (b\boldsymbol{D}_b + G\boldsymbol{D}_G) : \boldsymbol{\epsilon}$$

Scalar Helmholtz Eqn. in Fluid

$$-k^2\psi - \Delta\psi = 0 \text{ in } \Omega_F$$

$$\nabla\psi \cdot \boldsymbol{n} = -\rho_f v_n \text{ on } \Gamma_{NF}$$

$$\nabla\psi \cdot \boldsymbol{n} = -ik\psi \quad \text{on } \Gamma_C$$

Coupled Boundary Conditions

$$\frac{\partial\psi}{\partial n} = -\rho_f v_n \text{ on } \Gamma_W$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = -\psi\boldsymbol{n} \text{ on } \Gamma_W$$

Duke UNIVERSITY

Sandia National Laboratories

Weak Form of Governing Equations

Find the mapping $(\boldsymbol{u}, \psi) \rightarrow V_s(\Omega_s) \times V_f(\Omega_f)$ such that:

$$-\omega^2 \int_{\Omega_S} \rho_s \boldsymbol{u}\boldsymbol{w} d\Omega + \int_{\Omega_S} \boldsymbol{\sigma} : \nabla^s \boldsymbol{w} d\Omega - \int_{\Gamma_W} \sigma_n \boldsymbol{w} ds = \int_{\Omega_S} f \boldsymbol{w} d\Omega$$

$$-k^2 \int_{\Omega_F} \psi \phi d\Omega + \int_{\Omega_F} \nabla \psi \cdot \nabla \phi d\Omega + \int_{\Gamma_W} \frac{\partial \psi}{\partial n} \phi ds = \int_{\Gamma_{NF}} \frac{\partial \psi}{\partial n} \phi ds$$

$$\forall \boldsymbol{w} \in V_s, \phi \in V_f$$

Discrete Coupled ASI Equations

- Introduce discretization of structural displacements & velocity potential:

$$\hat{\boldsymbol{u}} = \sum_{A}^{n_{sdof}} \boldsymbol{N}_A \boldsymbol{c}_A, \;\; \boldsymbol{N}_A \in \Omega_S, \boldsymbol{c}_A \in \mathbb{R}^{n_{sdof}}$$

$$\hat{\psi} = \sum_{B}^{n_{fdof}} N_B d_B, \;\; N_B \in \Omega_F, d_B \in \mathbb{R}^{n_{fdof}}$$

- Discretized coupled weak form equation represented as:

$$\begin{aligned} &\boldsymbol{g}\left(\begin{bmatrix} \hat{\boldsymbol{u}} \\ \hat{\psi} \end{bmatrix}, \boldsymbol{p}\right) = \\ &= \left(\begin{bmatrix} \boldsymbol{K}_s(\boldsymbol{p}) & 0 \\ 0 & -\boldsymbol{K}_f/\rho_f \end{bmatrix} + i\omega \begin{bmatrix} \boldsymbol{C}_s & \boldsymbol{L} \\ \boldsymbol{L}^T & -\boldsymbol{C}_f/\rho_f \end{bmatrix} - \omega^2 \begin{bmatrix} \boldsymbol{M}_s & 0 \\ 0 & \boldsymbol{M}_f/\rho_f \end{bmatrix}\right) \begin{bmatrix} \hat{\boldsymbol{u}} \\ \hat{\psi} \end{bmatrix} - \begin{bmatrix} \boldsymbol{f}_s \\ \boldsymbol{f}_f \end{bmatrix} \\ &= \boldsymbol{0} \end{aligned}$$

- $\{\mathbf{K}_s, \mathbf{C}_s, \mathbf{M}_s\}$ = Structural Stiffness, Damping, and Mass Matrices
- $\{\mathbf{K}_f, \mathbf{C}_f, \mathbf{M}_f\}$ = Fluid Stiffness, Damping, and Mass Matrices
- $\mathbf{L}$ = Coupling Matrix

Duke UNIVERSITY

Sandia National Laboratories

Design Variables

- Represent spacial distribution of material moduli with design variable $\boldsymbol{p}$
- Generalize material as viscoelastic, with frequency-dependent complex bulk and shear moduli, $\{\boldsymbol{b}, \boldsymbol{G}\}$:

$$
\begin{aligned}
b(\omega, \boldsymbol{x}) &= b_R(\omega, \boldsymbol{x}) + b_I(\omega, \boldsymbol{x}) \\
G(\omega, \boldsymbol{x}) &= G_R(\omega, \boldsymbol{x}) + G_I(\omega, \boldsymbol{x}) \\
\boldsymbol{p} &= \{G_R, b_R, G_I, b_I\}
\end{aligned}
$$

- Structural stiffness matrices depend on material distribution:

$$
\begin{aligned}
\boldsymbol{K}_s &= \int_\Omega \boldsymbol{B}^T \boldsymbol{D} \boldsymbol{B} d\Omega \\
&= \int_{\Omega_S} \boldsymbol{B}^T (G_R \boldsymbol{D}_G + b_R D_b) d\Omega + i \int_{\Omega_S} (G_I \boldsymbol{D}_G + b_I D_b) d\Omega
\end{aligned}
$$

Duke UNIVERSITY

Sandia National Laboratories

Least-Squares Minimization Design Approach

Objective Function: Least-squares residual between computed and desired physical fields, with regularization term for design variable

$$J(\boldsymbol{v}, \boldsymbol{p}) := \frac{\kappa}{2} (\boldsymbol{v}^d - \boldsymbol{v})^T [\boldsymbol{Q}] (\boldsymbol{v}^d - \boldsymbol{v})$$

- $\boldsymbol{v} = \{\hat{\boldsymbol{u}}, \hat{\psi}\} \in \mathbb{C}^{sd+fd}$, State Variable
- $\boldsymbol{v}^d$ = Target data, $\in \mathbb{C}^{sd+fd}$
- $\boldsymbol{p}$ = Design variable, $\in \mathbb{C}^{dv}$
- $[Q]$ = Boolean measurement location matrix
- sd = Structural d.o.f.
- fd = Fluid acoustic d.o.f.
- dv = Discrete design variable dimension

Optimization Formulation

- Define the minimization problem:

$$\begin{aligned} \underset{\boldsymbol{v},\boldsymbol{p}}{\text{minimize}} \quad & J(\boldsymbol{v},\boldsymbol{p}) \\ \text{subject to} \quad & \boldsymbol{g}(\boldsymbol{v},\boldsymbol{p}) = \boldsymbol{0} \\ & G_l \leq \boldsymbol{p}_G \leq G_u \\ & b_l \leq \boldsymbol{p}_b \leq b_u \end{aligned}$$

Objective Function

with PDE constraint (structural-acoustic Helmholtz equation) + design variable bounds

- Bounds on placed on design variables to restrict to realistic values:
- Gradient-based optimization implementation in Rapid Optimization Library/Sierra SD
  - Numerical optimization using Newton-Krylov methods with Trust-Region Search

Duke UNIVERSITY

Sandia National Laboratories

Optimality Conditions

- Define a Lagrangian, where w is a vector of Lagrange multipliers:

$$\mathcal{L}(\boldsymbol{u}, \boldsymbol{p}, \omega) = \mathcal{J}(\boldsymbol{u}, \boldsymbol{p}) + \boldsymbol{w}^T \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{p}, \omega)$$

- KKT Conditions require:

$$\left\{ \begin{array}{c} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{array} \right\} = \left\{ \begin{array}{c} \mathcal{J}_u + \boldsymbol{g}_u^T \boldsymbol{w} \\ \mathcal{J}_p + \boldsymbol{g}_p^T \boldsymbol{w} \\ \boldsymbol{g} \end{array} \right\} = \vec{\mathbf{0}}$$

Duke UNIVERSITY

Sandia National Laboratories

Reduced Space Formulation

- Consider state variable ***v*** as a function of the design parameter ***p*** to define:

$$\hat{J}(\boldsymbol{p}) = J(\boldsymbol{v}(\boldsymbol{p}), \boldsymbol{p})$$

- Newton Step Optimization Process:

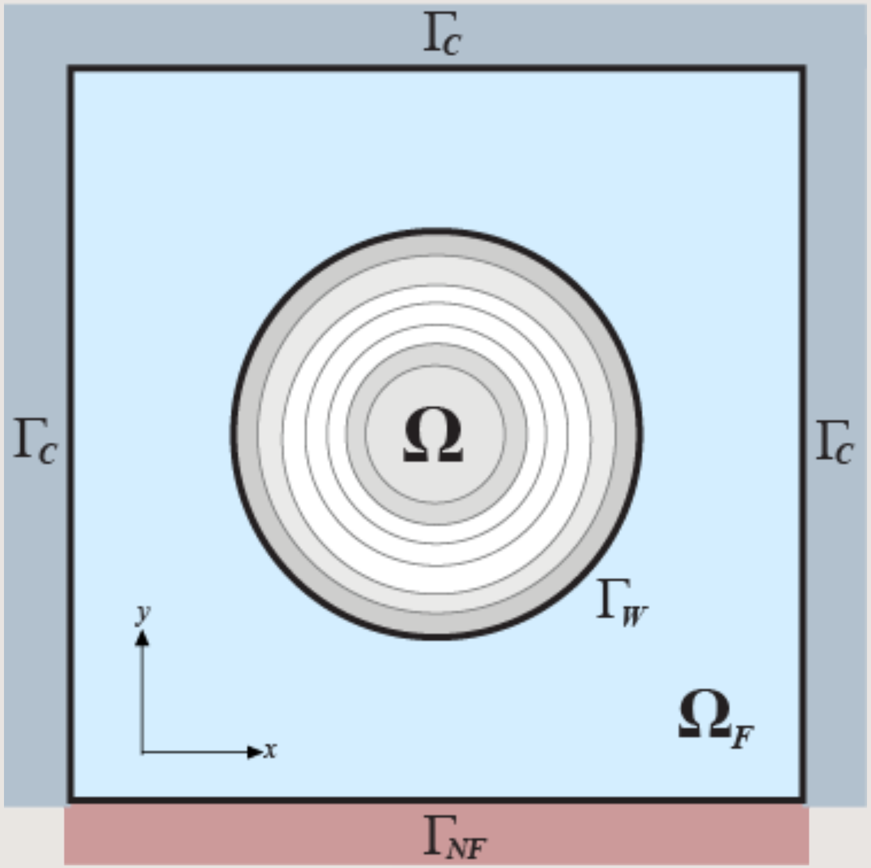
$\boldsymbol{g} = 0$	Compute state vector u
$\boldsymbol{g}_u^T \boldsymbol{w} = -\mathcal{J}_u$	Solve for adjoint vector w
$\mathcal{J}' = \mathcal{J}_p + \boldsymbol{g}^T \boldsymbol{w}$	Calculate reduced gradient
$\hat{\mathcal{J}}(\boldsymbol{p})$	Evaluate Reduced Objective
$\boldsymbol{H} \Delta \boldsymbol{p} = \mathcal{J}'$	Solve for Newton Step with Hessian H

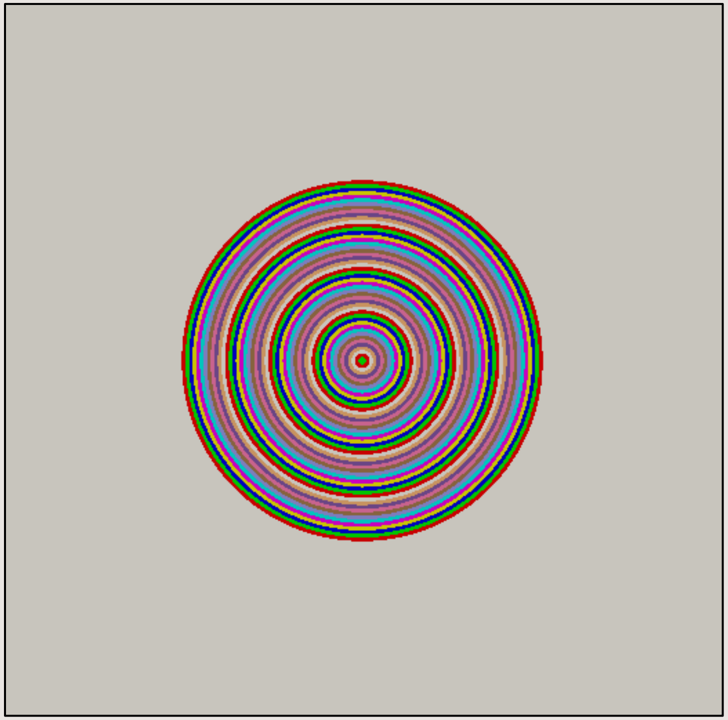
Duke UNIVERSITY

Sandia National Laboratories

Numerical Results

- 2-D fluid region with circular VE solid inclusion
- Inclusion consists of concentric rings w/ distinct material properties
- Harmonic acoustic pressure load applied to Γ_{NF}
- Match forward problem pressure distribution by adjusting VE material parameters





Left: Model Set up
Right: Finite element model mesh, with 50 layers, r = 0.25 m

Initial Guess and Optimization Bounds

- Initial guesses approximate a nearly-incompressible viscoelastic material, similar in magnitude to rubber
- Bounds selected to provide realistic damping/elasticity ratios

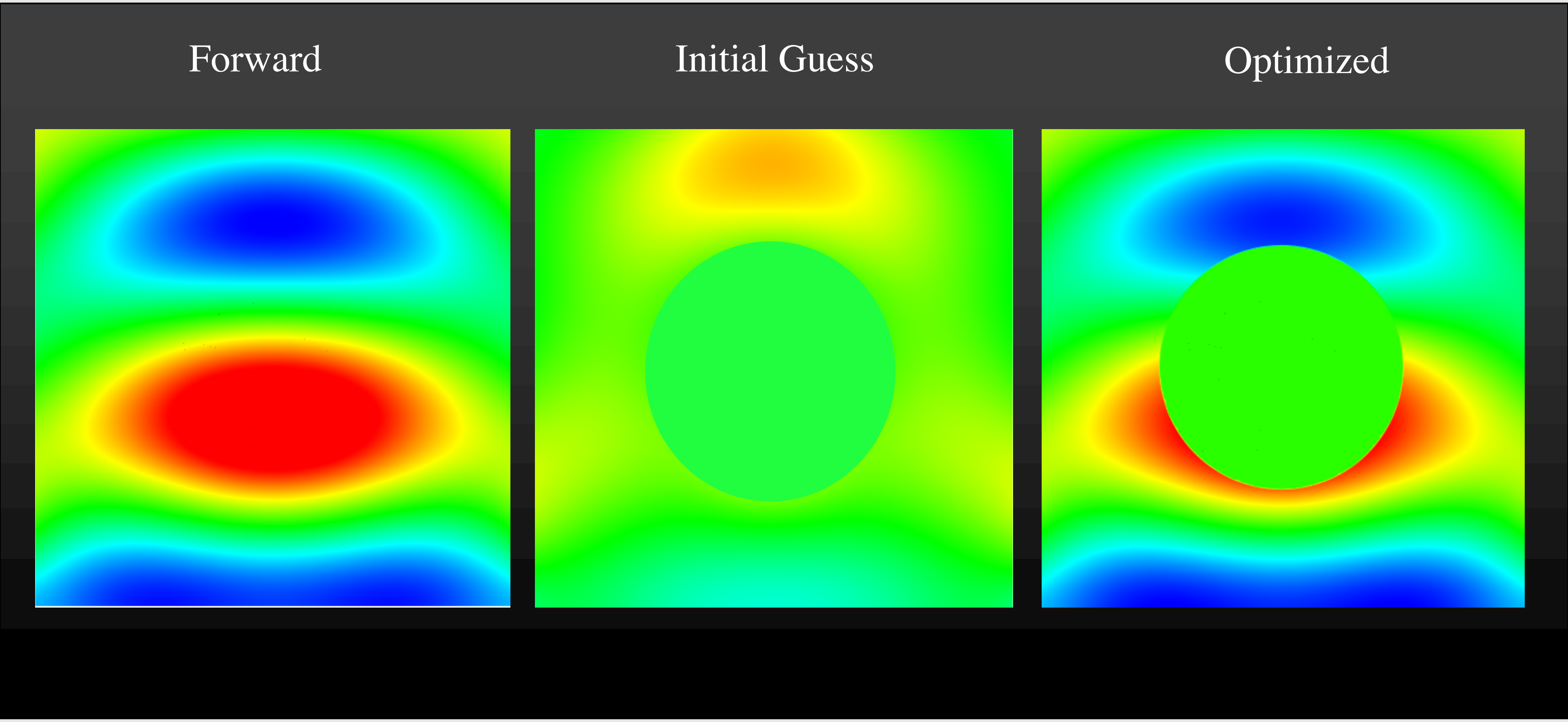
Modulus	Initial	Lower Bound	Upper Bound
b_R	5e+08	1e+08	1e+10
b_I	1e+03	1e+02	1e+05
G_R	1e+06	1e+05	1e+07
G_I	1e+03	1e+02	1e+05

Duke UNIVERSITY

Sandia National Laboratories

Acoustic Cloaking

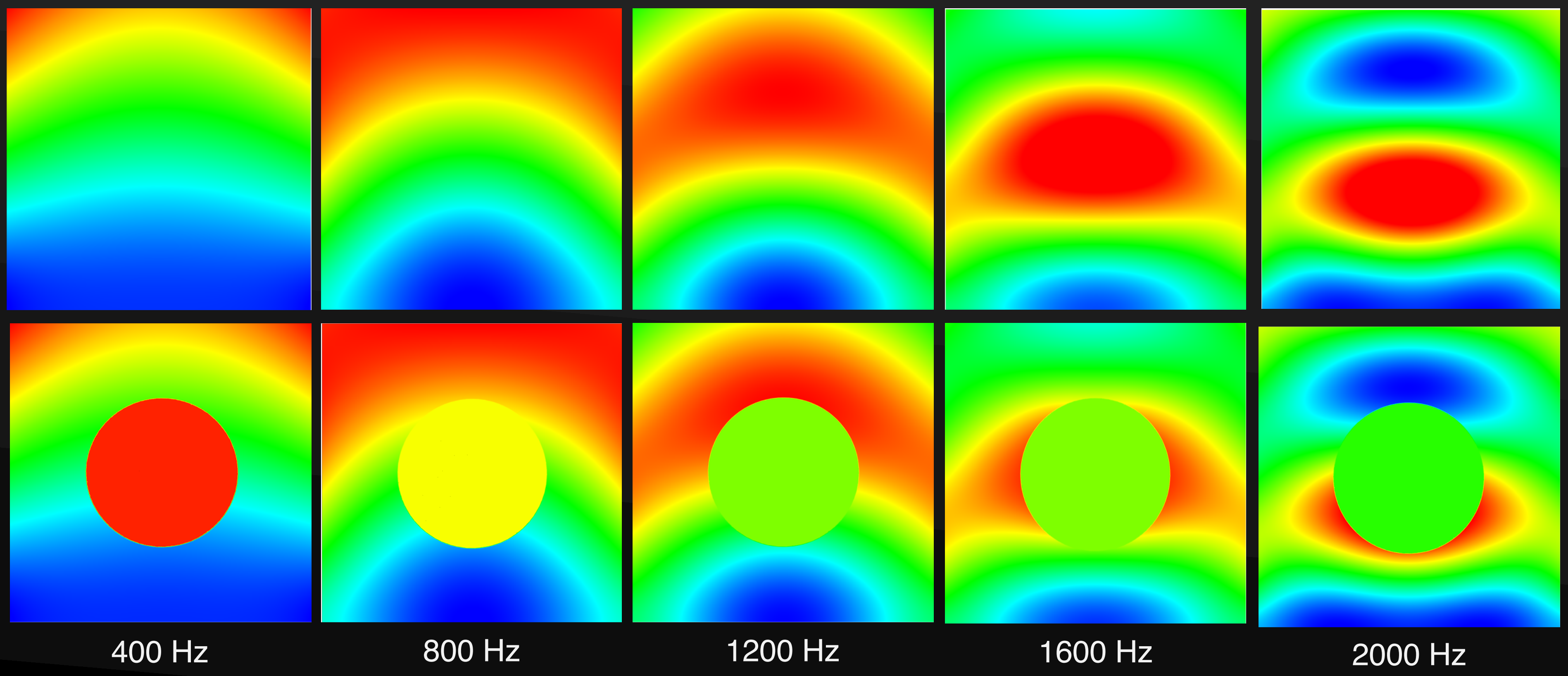
- Optimized VE foams allow recovery of desired pressure distribution



Left: Target acoustic pressure distribution, from forward problem
Center: Acoustic pressure distribution with initial material guess (2000 Hz Loading)
Right: Pressure distribution after convergence to optimized design

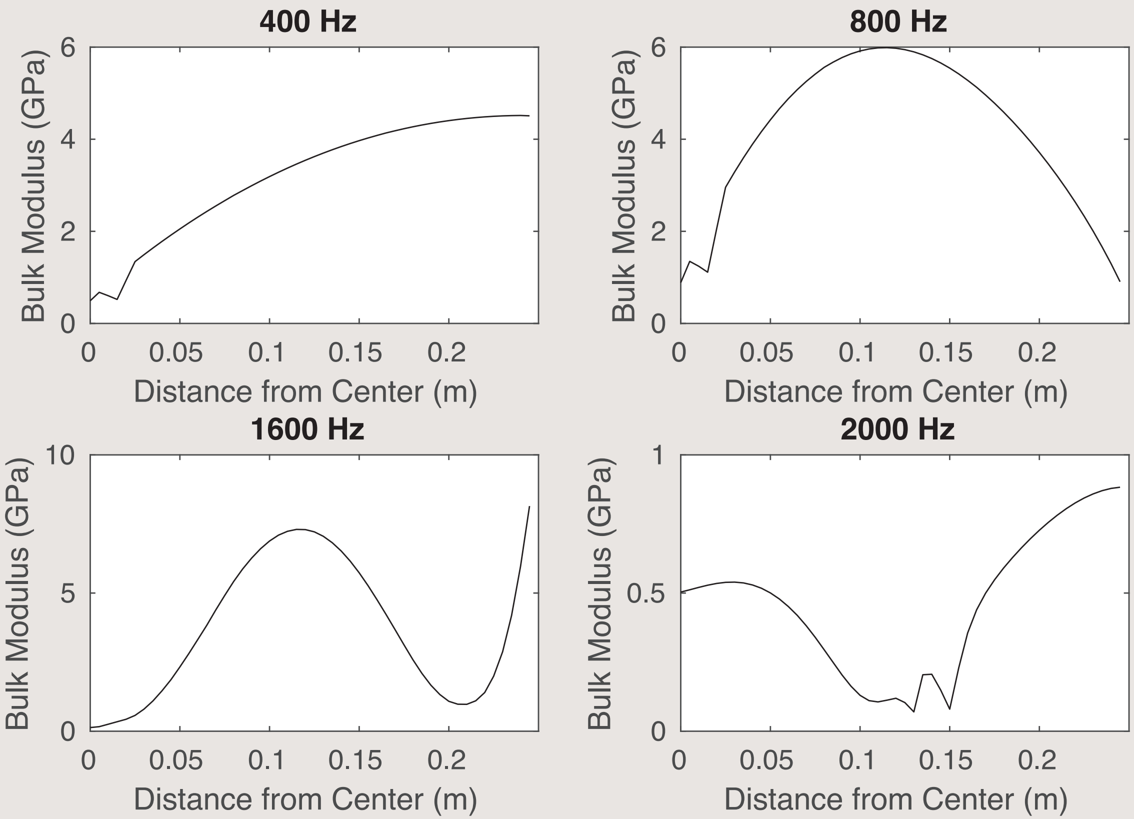
Solutions at Different Frequencies

- Optimized VE foams allow recovery of desired forward pressure distribution at variety of frequencies
  - Top : Acoustic pressure from forward analysis
  - Bottom : Acoustic pressure around optimized solid inclusion



Bulk Modulus Variation

- Bulk modulus sensitive to frequency, and varies nontrivially along disk radius
- Results suggest radially graded dispersive, elastic material can serve as acoustic cloak



Figures: Real component of bulk modulus along radius, for various frequencies

Duke
UNIVERSITY

Sandia National Laboratories

Shear Modulus & Damping Components

- A constant shear modulus shown to be effective in achieving cloaking behavior
- Minimized lossy components shown to be important for cloaking behavior
  - Imaginary components remained at initial values in optimized solution
  - Eliminating lossy components (Setting $G_I = b_I = 0$) can improve objective

Table 1: Comparison of Objective Function Values for Post-Processed Modulus Distributions (1200 Hz Example)

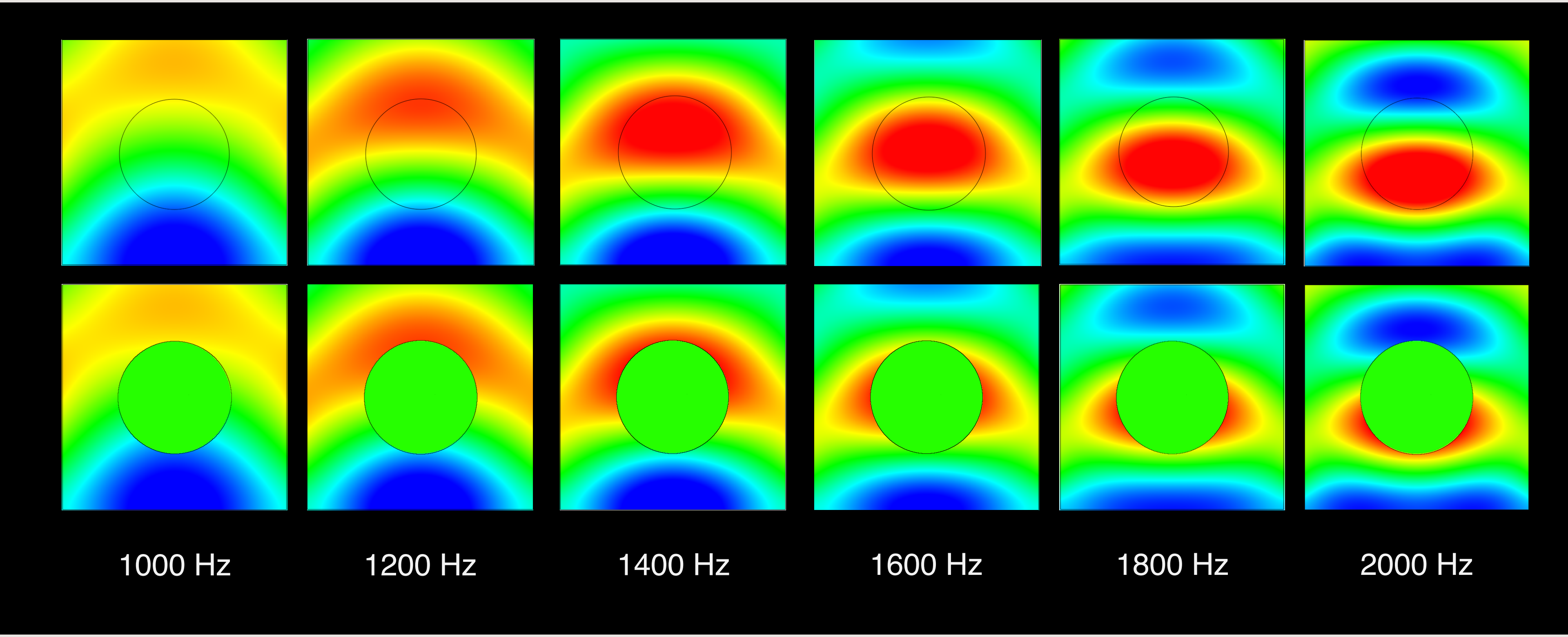
Case	**Objective Value**
Initial Guess	6.1041e-01
Optimized Solution	8.6274e-05
Averaged G_R	8.9483e-05
Constant G_R/b_R = 1e-3	8.9732e-05
$G_I = b_I = 0$	8.5905e-05

Duke UNIVERSITY

Sandia National Laboratories

Multi-frequency Elastic Solution

- Single elastic distribution can minimize reflection for extended bandwidth of frequencies
- Solution for 1200 Hz evaluated over extended frequency range



Top: Target acoustic pressure distribution, from forward problem
Bottom: Acoustic pressure field for each frequency from single elastic distribution solution

Conclusions

- Abstract formulation for viscoelastic material design via numerical optimization
- Variation of purely elastic properties allows material to cloak itself from incident acoustic pressure for individual frequencies and frequency bandwidths
  - Bulk modulus is sensitive to radial variation for different frequencies
    - Generally observe decaying modulus values towards center of disk
    - For certain frequencies, appear to match impedance of surrounding fluid
  - Shear modulus generally insensitive; averaged shear modulus sufficient for cloaking behavior
  - Imaginary components need to be minimized for cloaking behavior
    - Decrease of imaginary components has no effect on objective; increase of imaginary components worsens performance

Duke UNIVERSITY

Sandia National Laboratories

Acknowledgments

- Support through:
  - DOE CSGF, provided under grant number DE-FG02-97ER25308
  - Sandia National Laboratories
- Prof. Wilkins Aquino, Ph.D.
- Timothy Walsh, Ph.D
- Software: Sierra SD & Rapid Optimization Library, produced by Sandia National Labs

References

- [1] Chen, Y., Barnhart, M., Chen, J., Hu, G., Sun, C., and Huang, G., Composite Structures 136, 358 (2016).
- [2] Cheng, Y., Yang, F., Xu, J. Y., and Liu, X. J., Applied Physics Letters 92, 151913 (2008).
- [3] Cummer, S. A., Popa, B. I., Schurig, D., Smith, D. R., Pendry, J., Rahm, M., and Starr, A., Physical review letters 100, 024301 (2008).
- [4] Everstine, G. C., Computers and Structures 65, 307 (1997).
- [5] Guild, M. D., Alu, A., and Haberman, M. R., The Journal of the Acoustical Society of America 129, 1355 (2011).
- [6] hoorickx, C. V., Sigmund, O., Schevenels, M., Lazarov, B., and Lombaert, G., Journal of Sound and Vibration 376, 95 (2016).
- [7] Kadic, M., Bu ̈ckmann, T., Schittny, R., Gumbsch, P., and Wegener, M., Physical Review Applied 2, 054007 (2014).
- [8] Kouri, D., Ridzal, D., Van Bloemen Waanders, B., and Von Winckel, G., "Rapid optimization library," Tech. Rep. (Sandia National Laboratories (SNL-NM), Albuquerque, NM (United States), 2014). Nakshatrala, P. and Tortorelli, D., Computer Methods in Applied Mechanics and Engineering 295, 305 (2015).
- [9] Nocedal, J. and Wright, S., Numerical optimization (Springer Science & Business Media, 2006). Norris, A. N., in Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, Vol. 464 (The Royal Society, 2008) pp. 2411–2434.
- [10] Reese, G., Bhardwaj, M. K., and Walsh, T., Sandia National Laboratories, Albuquerque, New Mexico SAND2011-8272 (2011).
- [11] Shen, H., Wen, J., Yu, D., Cai, L., Wen, X., et al., Modelling and Simulation in Materials Science and Engineering 21, 065011 (2013).

Duke UNIVERSITY