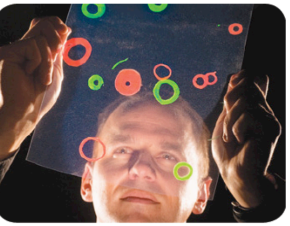
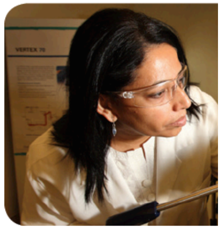
SAND2017-7257C

Solving the Mixed Integer Non-linear Programming Problem of Unit Commitment on AC Power Systems





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Overview

- Today's Practice
- Contributions
  - Overview
  - Local Solution Method
  - Global Solution Method
  - UC+ACOPF Results
- Ongoing Work

TODAY'S PRACTICE

Issues in Day-Ahead Markets

- Operational Challenges
  - Committing Least Cost + Maintaining Reliability
  - Out-of-Merit Reliability Commitments
  - Improving convergence between day-ahead and real-time prices
- Algorithmic Challenges
  - Accounting for reliability needs in dispatch and pricing optimization
  - Better physical representation of the generating units and underlying network

Unit Commitment in the Day-Ahead Market

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Current Practices	Proposed Approach
UC/Security-Constrained UC • Copper-plate (no network/single node) • Ignores congestion; requires cutsets to proxy capacity limits on network • Most tractable **SCUC DCOPF** • Real power flows only (proportional to current) • BΘ (full) or PTDF (compact) approach **Extensions:** • Accounts for losses • Nomograms/cutsets to proxy reliability requirements	**SCUC ACOPF** • Co-optimizes real and reactive power dispatch • Accounts for commitments needed for blackstart service, reactive support, voltage support, and interface control • Nonlinear, nonconvex on meshed networks

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The link between physics and prices

- Locational marginal pricing (LMP) is the spot price of electricity
- Dual variable/Lagrange multiplier (λ_n) to real power balancing at all buses

$$p_n - p_n^d + p_n^g = 0 \qquad (\lambda_n)$$

ACOPF $$p_n = |v_n| \sum_{m \in \mathcal{N}} |v_m| \left(G_{nm} \cos \theta_{nm} + B_{nm} \sin \theta_{nm}\right)$$

DCOPF $$p_n = \sum_{m \in \mathcal{N}} \left(B_{nm} \theta_{nm}\right) \approx |\tilde{v}_n| \sum_{m \in \mathcal{N}} |\tilde{v}_m| \left(B_{nm} \theta_{nm}\right)$$

DCOPF with losses $$p_n = \sum_{m \in \mathcal{N}} \left(G_{nm} \left(\theta_{nm}\right)^2 \Big/ 2 + B_{nm} \theta_{nm}\right)$$

The LMP incorporates the marginal cost of supplying the next MW of load for a given location in time; includes

1. marginal unit cost,
2. cost of network congestion (due to thermal line limits), and
3. cost of real power losses on the network

CONTRIBUTIONS

CONTRIBUTIONS OVERVIEW

UC+ACOPF: MINLP

Min Production Costs + Startup Costs + No-Load Costs

subject to

AC Network Limits

Real power balancing

Reactive power balancing

Voltage magnitude bounds

Thermal line limits

Spinning reserves

Apparent Power Production Limits §

Max/min real/reactive power generation

Ramp up/down rates on real power

Minimum up/down time

System Data

Nodal voltage limits

Reserve requirements

Real/reactive power load

Transformer tap ratio and phase-shifters

Thermal line limits and line R/X/B

Shunts

Generator Data

Synchronous condensers

T0 state and startup lags

Minimum up/down time

Ramp up/down limits

Startup/shutdown ramp limits

Min/max real/reactive power limits

§ Extends Morales-España, Latorre, and Ramos, "Tight and compact MILP formulation for the thermal unit commitment problem," *IEEE Trans. on Power Syst.*, vol. 28, no. 4, pp. 4897–4908, 2013.

Nodal Power Balancing is Nonconvex

- Polar Power-Voltage Power Flow Formulation (**PSV**)

$$|v_{n,t}| \sum_{m \in \mathcal{N}} |v_{m,t}| \left(G_{nm} \cos \theta_{nm,t} + B_{nm} \sin \theta_{nm,t}\right) - p^+_{n,t} + p^-_{n,t} = 0, \qquad \forall n \in \mathcal{N}$$

$$|v_{n,t}| \sum_{m \in \mathcal{N}} |v_{m,t}| \left(G_{nm} \sin \theta_{nm,t} - B_{nm} \cos \theta_{nm,t}\right) - q^+_{n,t} + q^-_{n,t} = 0, \qquad \forall n \in \mathcal{N}$$

- Rectangular Power-Voltage Power Flow Formulation (**RSV**)

$$v^r_{n,t} \sum_{m \in \mathcal{N}} \left(G_{nm} v^r_{m,t} - B_{nm} v^j_{m,t}\right) + v^j_{n,t} \sum_{m \in \mathcal{N}} \left(G_{nm} v^j_{m,t} + B_{nm} v^r_{m,t}\right) - p^+_{n,t} + p^-_{n,t} = 0, \quad \forall n \in \mathcal{N}$$

$$v^j_{n,t} \sum_{m \in \mathcal{N}} \left(G_{nm} v^r_{m,t} - B_{nm} v^j_{m,t}\right) - v^r_{n,t} \sum_{m \in \mathcal{N}} \left(G_{nm} v^j_{m,t} + B_{nm} v^r_{m,t}\right) - q^+_{n,t} + q^-_{n,t} = 0, \quad \forall n \in \mathcal{N}$$

- Rectangular Current Injection Formulation (**RIV**)

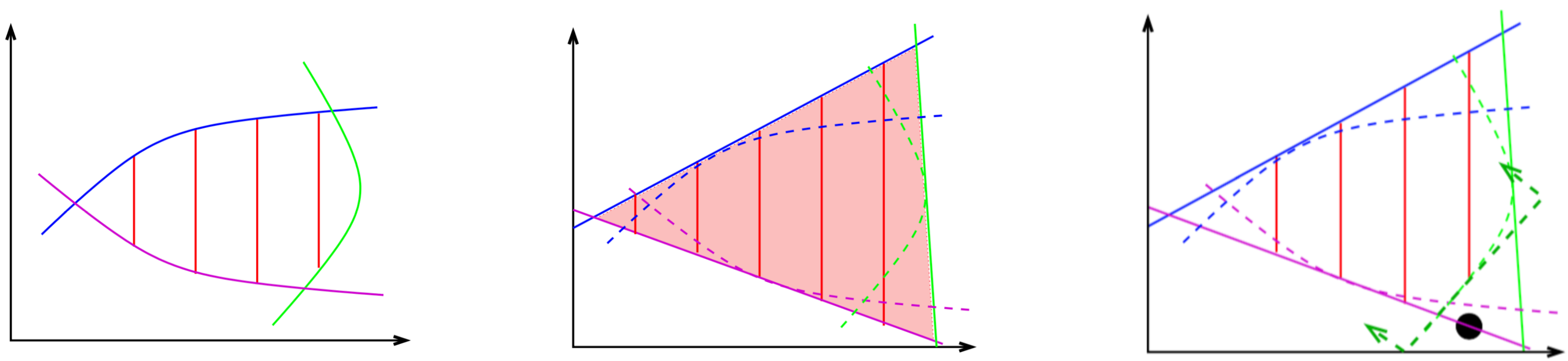
$$i^r_{n,t} - \left(\sum_{k(n,\cdot) \in \mathcal{F}} i^r_{k(n,m),t} + G^s_n v^r_{n,t} - B^s_n v^j_{n,t}\right) = 0, \left(v^r_{n,t} i^r_{n,t} + v^j_{n,t} i^j_{n,t}\right) - p^+_{n,t} + p^-_{n,t} = 0, \quad \forall n \in \mathcal{N}$$

$$i^j_{n,t} - \left(\sum_{k(n,\cdot) \in \mathcal{F}} i^j_{k(n,m),t} + G^s_n v^j_{n,t} + B^s_n v^r_{n,t}\right) = 0, \left(v^j_{n,t} i^r_{n,t} - v^r_{n,t} i^j_{n,t}\right) - q^+_{n,t} + q^-_{n,t} = 0, \quad \forall n \in \mathcal{N}$$

MINLP solved by Outer Approximation § (OA)

$$\begin{cases} minimize_x \; f(x), \\ subject\ to \; g(x) \leq 0, \\ \qquad\qquad x \in X, \\ \qquad\qquad x_i \in \mathbb{Z}, \forall i \in I \end{cases}$$

$f: \mathbb{R}^n \to \mathbb{R}, g: \mathbb{R}^n \to \mathbb{R}^m$ are twice continuously differentiable functions,
$X \subset \mathbb{R}^n$ is a bounded polyhedral set, and
$I \subseteq \{1, \dots, n\}$ is the index set of integer variables



§ Outer Approximation Algorithm (Duran and Grossman, 1986); Graphics (Belotti et al., 2013)

CONTRIBUTIONS

LOCAL SOLUTION METHOD

Successive Linear Programming (SLP) [R1]

MIN Piecewise linear cost function with penalty factors

s.t.

Line Current Flows

$$i^r_{k(n,m)} = \mathrm{Re}\left(Y^k_{1,1} v_n + Y^k_{1,2} v_m\right),\ i^r_{k(m,n)} = \mathrm{Re}\left(Y^k_{2,1} v_n + Y^k_{2,2} v_m\right) \quad \forall k \in K$$

$$i^j_{k(n,m)} = \mathrm{Im}\left(Y^k_{1,1} v_n + Y^k_{1,2} v_m\right),\ i^j_{k(m,n)} = \mathrm{Im}\left(Y^k_{2,1} v_n + Y^k_{2,2} v_m\right) \quad \forall k \in K$$

Network Current Balancing

$$i^r_n - \left(\sum_{k(n,\cdot)} i^r_{k(n,m)} + G^{sh}_n v^r_n - B^{sh}_n v^j_n\right) = 0 \quad \forall n \in N$$

$$i^j_n - \left(\sum_{k(n,\cdot)} i^j_{k(n,m)} + G^{sh}_n v^j_n + B^{sh}_n v^r_n\right) = 0 \quad \forall n \in N$$

Nodal Power Injections

First-order Taylor series

Generator Limits

Inequality constraints with slack variables

Nodal Voltage Magnitude Limits

Outer approximation,
First-order Taylor series,
Step-size bounds,
Tangential cutting planes, &
Inequality constraints with slack variables

Thermal Line (Flowgate) Limits

Set reduction, Outer approximation,
First-order Taylor series,
Tangential cutting planes, &
Inequality constraints with slack variables

SLP Convergence Properties §

(1) A KKT point to the ACOPF is found

(2) The SLP optimal solution is ACOPF feasible but not optimal

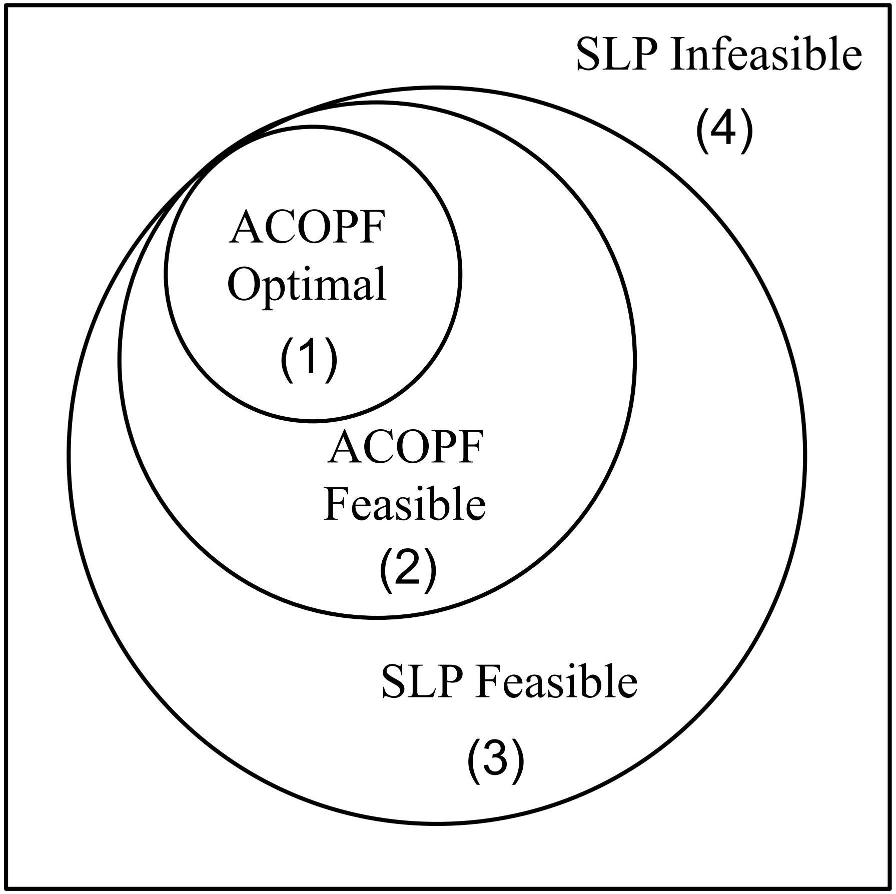
- Still a useful solution; may be better than a DCOPF with AC feasibility or decoupled OPF solution

(3) The SLP optimal solution is ACOPF infeasible

- Active penalties present
- Solution may be useful depending upon whether the violated limits are "soft" or "hard"

(4) The SLP is infeasible

- The ACOPF may have no solution
- The SLP requires a better initialization



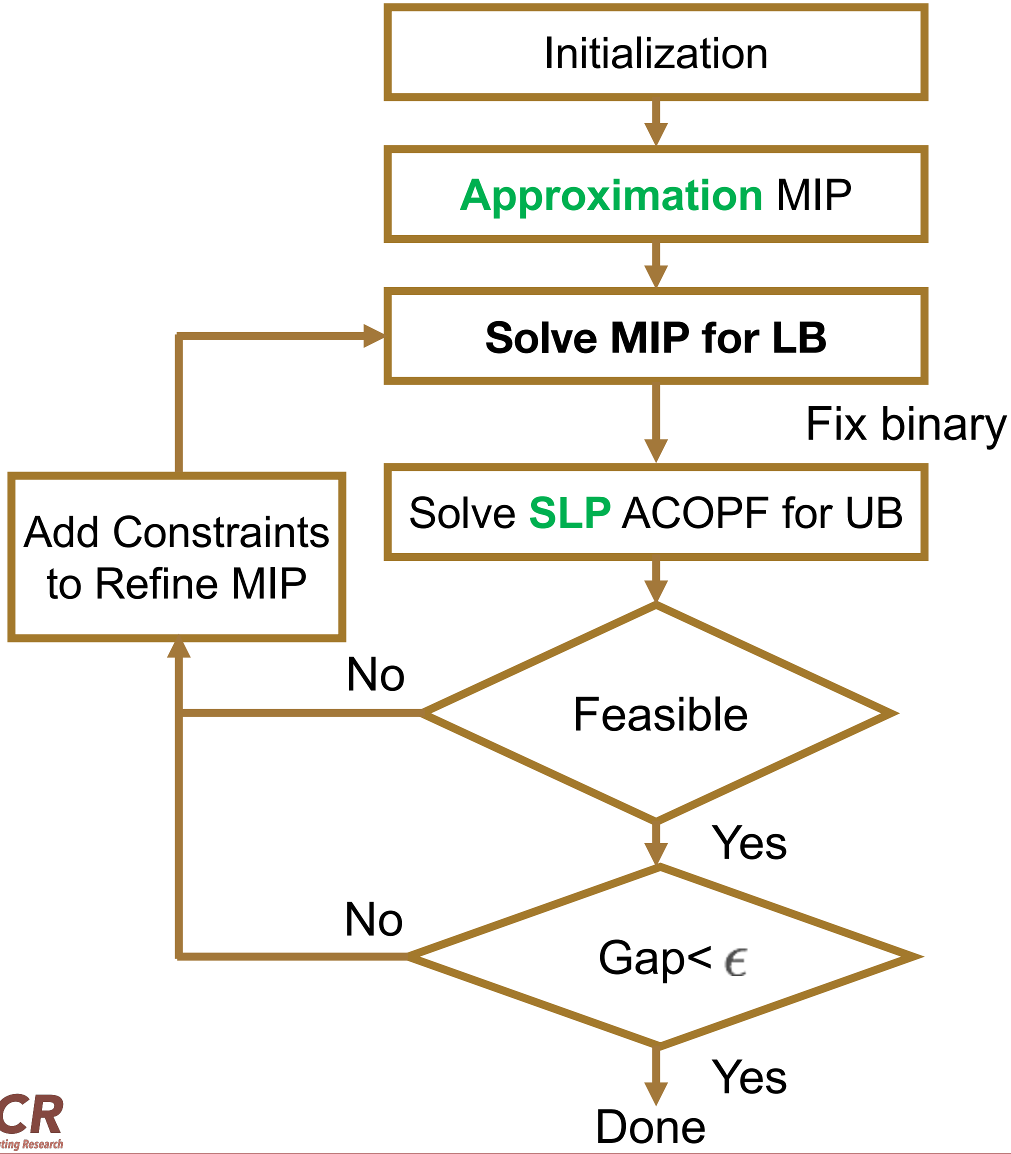
§ Extends Theorem 10.3.1 of Bazaraa et al. (2006)

Time Complexity Performance $\Theta\left(|\mathcal{N}|^p\right)$

	Best-Case Simulations			All Converged Simulations		
Baseline	p	R^2	RMSE (s)	p	R^2	RMSE (s)
NLP/KNITRO	1.42	0.83	1.46	1.47	0.82	1.40
NLP/Ipopt	1.13	0.95	0.60	1.34	0.97	0.50
SLP/CPLEX	0.97	0.99	0.20	1.01	0.98	0.33
SLP/Gurobi	1.01	0.99	0.21	1.03	0.98	0.33
Thermally Constrained						
NLP/KNITRO	1.39	0.88	1.13	1.39	0.89	1.08
NLP/Ipopt	1.11	0.98	0.36	1.22	0.97	0.50
SLP/CPLEX	0.99	0.99	0.17	1.00	0.98	0.31
SLP/Gurobi	1.06	0.99	0.23	1.05	0.97	0.36

- Running time increases linearly with the network size (p=1 corresponds to a linear algorithmic scaling) for the SLP algorithm
- *Potentially applicable in the strict time frames of the real-time markets*

MINLP solved by Outer Approximation (OA)



Local Solution [R2]

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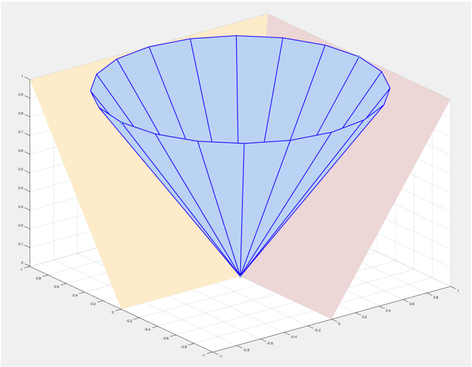
GLOBAL SOLUTION METHOD

ACOPF Second-Order Cone Relaxation § (SOCR)

$$c_{b,b} \equiv (v_b^r)^2 + (v_b^j)^2 = v_b^2$$

$$c_{b,k} \equiv v_b^r v_k^r + v_b^j v_k^j = |v_b||v_k| \cos\theta_{b,k}$$

$$s_{b,k} \equiv v_b^r v_k^j - v_k^r v_b^j = -|v_b||v_k| \sin\theta_{b,k}$$



$$\min \quad \sum_{g \in \mathcal{G}} [A_g^2 (p_g^G)^2 + A_g^1 p_g^G + A_g^0]$$

$$\text{s.t.} \quad \sum_{l \in \mathcal{L}_b^{in}} p_l^t + \sum_{l \in \mathcal{L}_b^{out}} p_l^f + G_b^{sh} c_{b,b} + P_b^D - \sum_{g \in \mathcal{G}_b} p_g^G = 0 \qquad \forall\, b$$

$$\sum_{l \in \mathcal{L}_b^{in}} q_l^t + \sum_{l \in \mathcal{L}_b^{out}} q_l^f - B_b^{sh} c_{b,b} + Q_b^D - \sum_{g \in \mathcal{G}_b} q_g^G = 0 \qquad \forall\, b$$

$$p_l^f = G_l^{ff} c_{b,b} + G_l^{ft} c_{b,k} - B_l^{ft} s_{b,k} \qquad \forall\, l$$

$$q_l^f = -B_l^{ff} c_{b,b} - B_l^{ft} c_{b,k} - G_l^{ft} s_{b,k} \qquad \forall\, l$$

$$p_l^t = G_l^{tt} c_{k,k} + G_l^{tf} c_{k,b} - B_l^{tf} s_{k,b} \qquad \forall\, l$$

$$q_l^t = -B_l^{tt} c_{k,k} - B_l^{tf} c_{k,b} - G_l^{tf} s_{k,b} \qquad \forall\, l$$

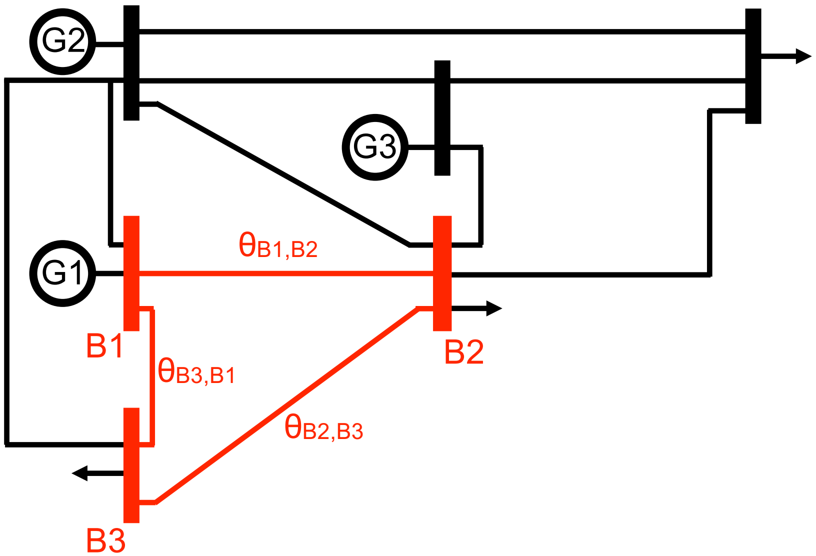
$$(p_l^f)^2 + (q_l^f)^2 \le (S_l^{max})^2, \ (p_l^t)^2 + (q_l^t)^2 \le (S_l^{max})^2 \qquad \forall\, l$$

$$(V_b^{min})^2 \le c_{b,b} \le (V_b^{max})^2 \qquad \forall\, b$$

$$P_g^{G,min} \le p_g^G \le P_g^{G,max}, \ Q_g^{G,min} \le q_g^G \le Q_g^{G,max} \qquad \forall\, g$$

§ Second-Order Cone Relaxation (Jabr, 2006; Kocuk, 2015)

Improving the Lower Bound of SOCR [R3]

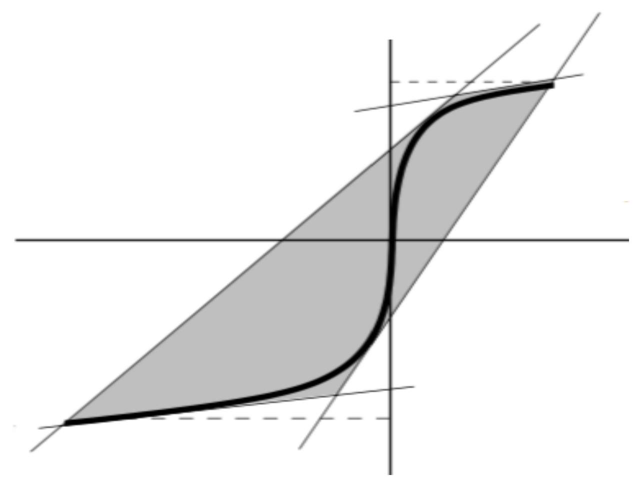


Cycle Constraints:
the sum of angle differences on each cycle equals to zero

$$\sum_{l \in \mathcal{L}_c} \theta_l = 0 \qquad \forall\, \mathcal{L}_c$$

$$\theta_l \equiv \theta_{b,k} = -\arctan\left(\frac{s_{b,k}}{c_{b,k}}\right) \qquad \forall\, l = (b,k)$$

Convex Relaxation of *arctan*:
Linear Over- and Under-Estimators
Optimality-Based Bound Tightening (OBBT)
Gradually Adding Cycle Constraints



Global ACOPF Performance

Case Name	Optimal Solution	Optimality Gap (%)	CPU Time (s)	Iteration Number
Case6ww	3126.36	0.008	0.26	4
Case14	8081.52	0.003	0.43	3
Case30	574.52	0.000	0.95	5
Case39	41864.18	0.005	1.21	3
Case57	41737.79	0.006	7.29	12
Case89	5817.60	0.009	46.2	44
Case118	129660.69	0.006	18.5	14
Case300	719725.10	0.009	82.7	49
NESTA Case6ww	3143.97	0.000	0.74	7
NESTA Case14	244.05	0.003	0.22	3
NESTA Case30	204.97	0.000	0.57	4
NESTA Case39	96505.52	0.009	3.00	8
NESTA Case57	1143.27	0.006	9.62	20
NESTA Case89	5819.81	0.009	55.8	57
NESTA Case118	3718.64	0.000	93.7	55
NESTA Case300	16891.28	0.000	138.2	26

MINLP solved by Outer Approximation (OA)

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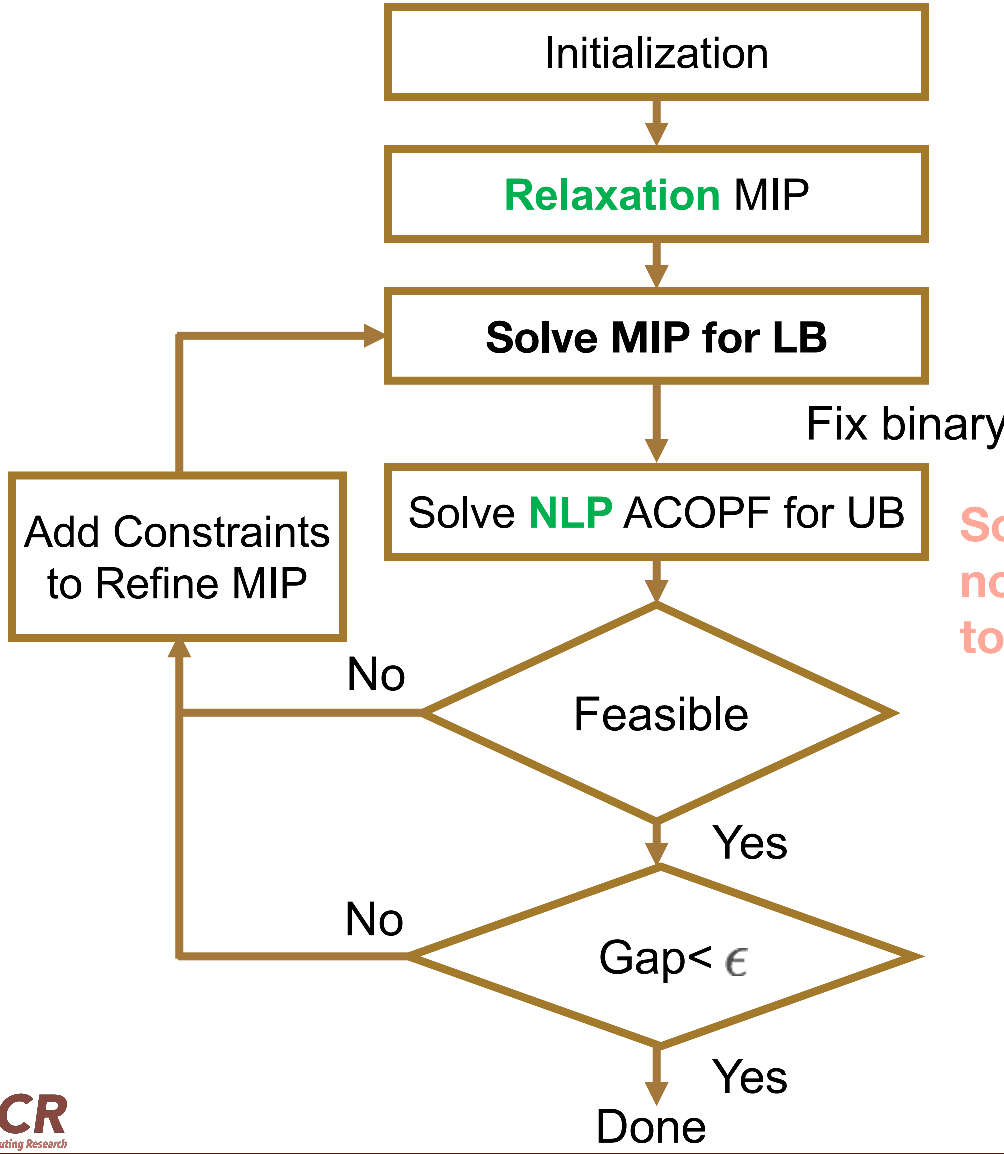

Solving nonlinear, non-convex AC OPF to global optimality? ✔

Global Solution [R4]

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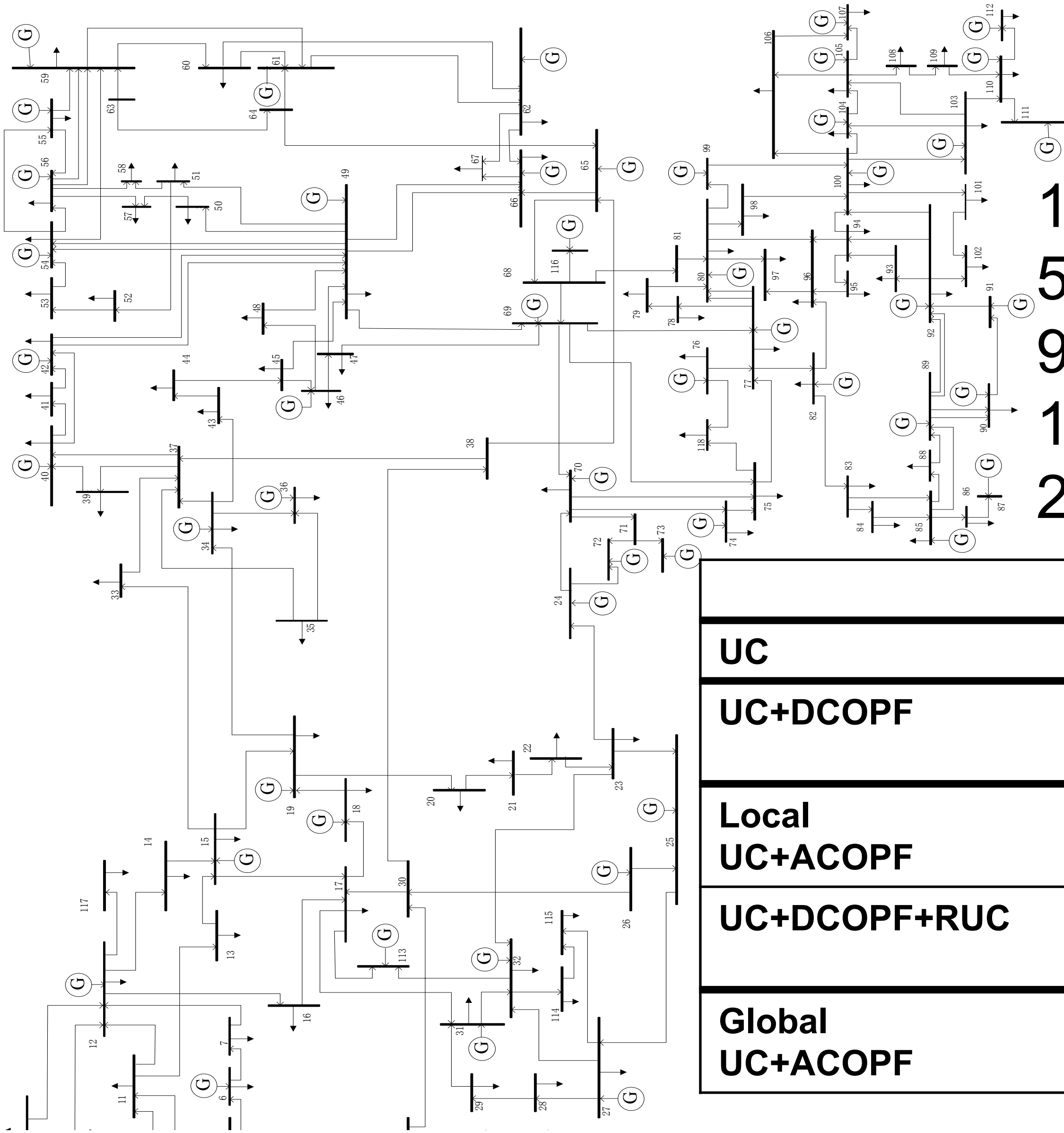
UC+ACOPF RESULTS



IEEE-118 §

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- 118 nodes
- 54 generators
- 91 loads
- 186 network elements/lines
- 24-hour hourly commitment

	Cost ($)	AC Feasible?
UC	811,658 (base)	NO
UC+DCOPF	814,715 (+0.4%)	NO
Local UC+ACOPF	843,591 (+3.9%)	YES
UC+DCOPF+RUC	844,922 (+4.1%)	YES
Global UC+ACOPF	835,926 (+3.0%)	YES

- Key Takeaway: Results indicate considerable divergence between the market settlements and stability/reliability requirements

§ Data from Fu et al. (2006)

Computational Results (Local Method)

	UC	UC+DCOPF	UC+ACOPF		UC+DCOPF+RUC	
	MILP	MILP	MILP	SLP	MILP	SLP
Solution Time (s)						
6-Bus	0.13	0.21	0.88(3)	0.07(50)	1.02(1, 1)	0.06(33)
RTS-79	1.86	6.76	88.71(3)	0.75(36)	10.37(1, 2)	0.45(26)
IEEE-118	5.04	21.42	110.17(2)	5.06(46)	57.2(1, 1)	3.71(33)
Cost ($)						
6-Bus	101, 270	106, 987	101, 763		102, 523	
RTS-79	823, 145	823, 894	895, 281		896, 169	
IEEE-118	811, 658	814, 715	843, 591		844, 922	

- Most of the OA algorithm time spent in the MILP (MIP gap tolerance 0.1%)
- UC+ACOPF: 5x-15x slower than the UC+DCOPF
- UC+DCOPF+RUC: 1.5x-5x slower than the UC+DCOPF

Local v. Global UC+ACOPF Method

Case	Problem Formulation	Upper Bound	Lower Bound	Relative Gap (%)	CPU Time (s)
6-Bus	Global Local	101,763 101,763	101,655 -	0.11% *0.11%*	3.6 0.95
RTS-79	Global Local	895,096 895,281	893,967 -	0.13% *0.15%*	266.4 89.46
IEEE-118	Global Local	835,926 843,591	833,057 -	0.34% *1.25%*	8480 115.23

- **Note**: Thermal limits different in global solution method (apparent power thermal limit) and local solution method (current thermal limit) so a direct comparison (above) is *inexact*
- On the largest test case, the approximation method is over 70x faster, at the cost of 0.91% in relative optimality gap change

ONGOING WORK

Ongoing Work

- Study of global solution techniques applied to the PSV, RSV and RIV ACOPF formulations
- Implications on market settlements for including AC network constraints in the day-ahead
- Improving the performance of the MIP solution time in the OA algorithm (e.g., hybrid OA + branch-and-bound)
- Comparing the fidelity and computational performance to current market practices on larger scale, more realistic networks (GRIDDATA)

References

- [R1] A. Castillo, P. Lipka, J.-P. Watson, S.S. Oren, and R.P. O'Neill. "A Successive Linear Programming Approach to Solving the IV-ACOPF." Transactions on Power Systems (2015).
- [R2] A. Castillo, C. Laird, C. A. Silva-Monroy, J.-P. Watson, and R.P. O'Neill. "The Unit Commitment Problem with AC Optimal Power Flow Constraints." Transactions on Power Systems (2016).
- [R3] J. Liu, M. Bynum, A. Castillo, J.-P. Watson and C. Laird. "Global Solution of ACOPF Problems Using a Piecewise Outer-Approximation Approach Based on SOCP Relaxations." (2017) submitted.
- [R4] J. Liu, A. Castillo, J.-P. Watson, and C. Laird. "Global Solution Strategies for the Network-Constrained Unit Commitment (NCUC) Problem with Nonlinear AC Transmission Models." (2017) submitted.