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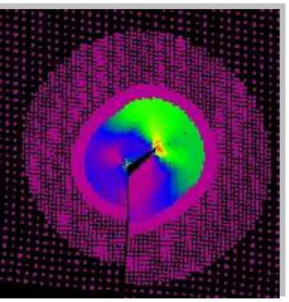

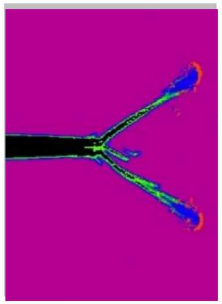

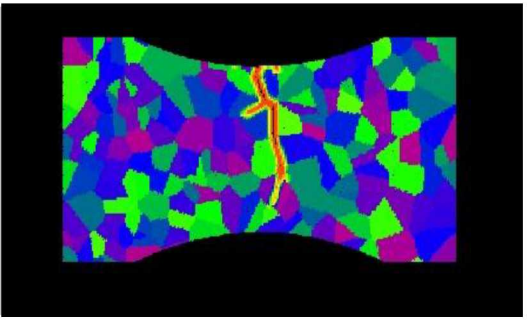

Peridynamic model for fatigue cracks

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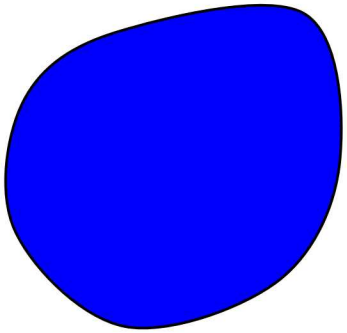
Outline

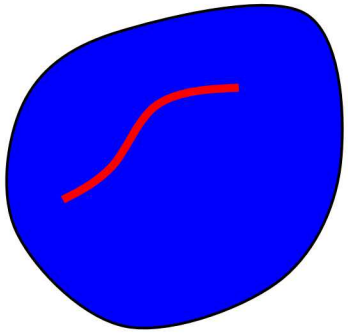
- Peridynamics: What it is
- Damage and fracture
- Fatigue model
- Calibration of parameters

Purpose of peridynamics*

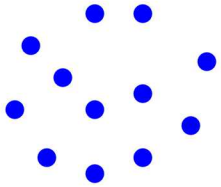
- To unify the mechanics of continuous and discontinuous media within a single, consistent set of equations.



Continuous body



Continuous body with a defect



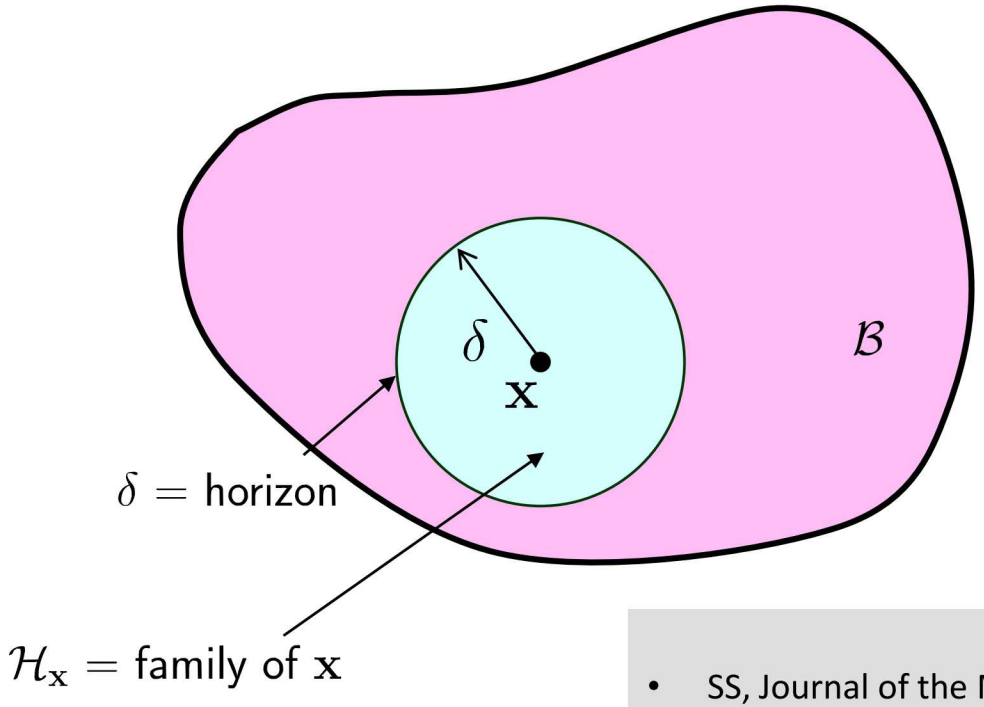
Discrete particles

- Why do this?
  - Avoid coupling dissimilar mathematical systems (A to C).
  - Model complex fracture patterns.
  - Communicate across length scales.

* Peri (near) + dyn (force)

Peridynamics basics: Horizon and family

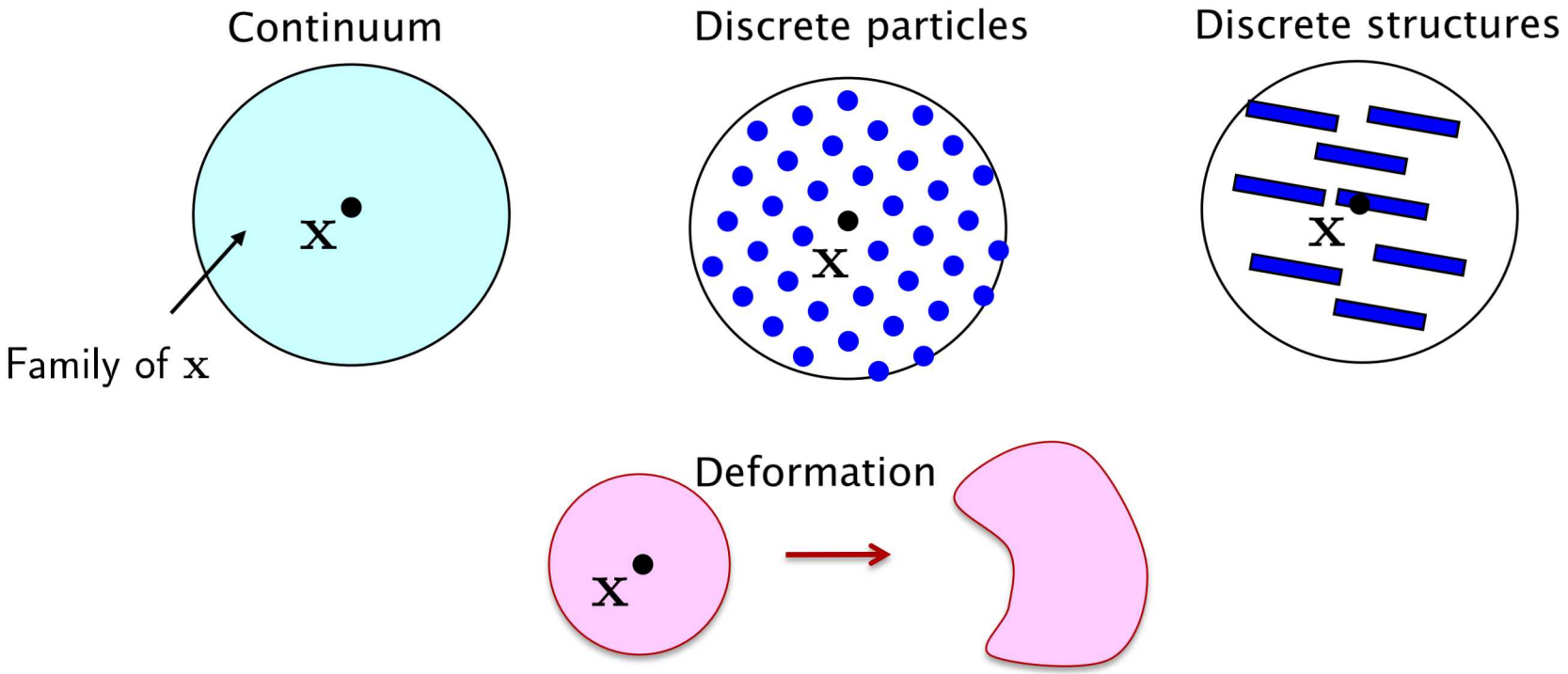
- Any point $\mathbf{x}$ interacts directly with other points within a distance δ called the "horizon."
- The material within a distance δ of $\mathbf{x}$ is called the "family" of $\mathbf{x}$, $\mathcal{H}_{\mathbf{x}}$.



General references

- SS, Journal of the Mechanics and Physics of Solids (2000)
- SS and R. Lehoucq, Advances in Applied Mechanics (2010)
- Madenci & Oterkus , *Peridynamic Theory & Its Applications* (2014)

Peridynamic nonlocality: Strain energy at a point



- Key assumption: the strain energy density at $W(\mathbf{x})$ is determined by the deformation of its family.

Potential energy minimization yields the peridynamic equilibrium equation

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- Potential energy:

$$\Phi = \int_{\mathcal{B}} (W - \mathbf{b} \cdot \mathbf{y}) \, dV_{\mathbf{x}}$$

where W is the strain energy density, $\mathbf{y}$ is the deformation map, $\mathbf{b}$ is the applied external force density, and $\mathcal{B}$ is the body.

- Euler-Lagrange equation is the equilibrium equation:

$$\int_{\mathcal{H}_{\mathbf{x}}} \mathbf{f}(\mathbf{q}, \mathbf{x}) \, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x}) = 0$$

for all $\mathbf{x}$. $\mathbf{f}$ is the *pairwise bond force density*.

Peridynamics basics: States

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- A *peridynamic state* is a mapping on bonds in a family.
- We write:

$$\mathbf{u} = \underline{\mathbf{A}}\langle \boldsymbol{\xi} \rangle$$

  where $\boldsymbol{\xi}$ is a bond, $\underline{\mathbf{A}}$ is a state, and $\mathbf{u}$ is some vector.

- States play a role in peridynamics similar to that of second order tensors in the local theory.

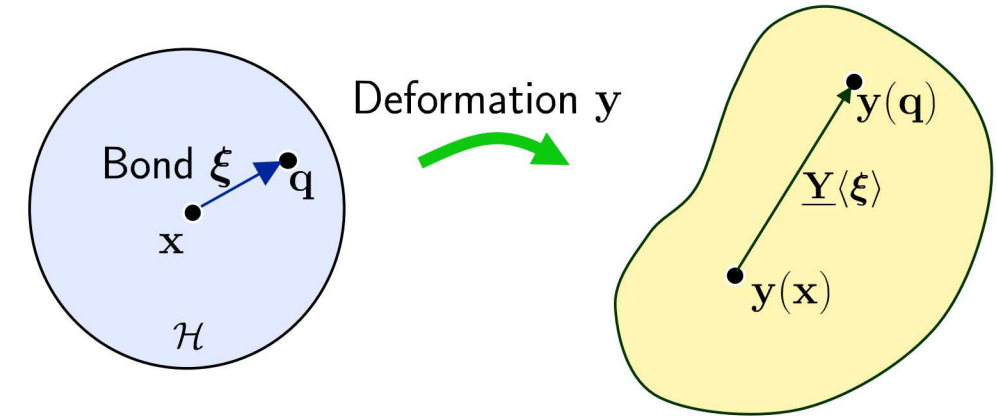
Peridynamics basics: Kinematics

- The *deformation state* is the function that maps each bond $\boldsymbol{\xi}$ into its deformed image:

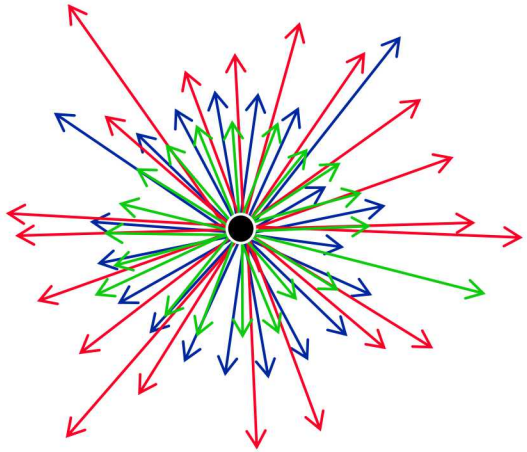
$$\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x})$$

where $\mathbf{y}$ is the deformation and

$$\boldsymbol{\xi} = \mathbf{q} - \mathbf{x}.$$



Undeformed family of $\mathbf{x}$ Deformed family of $\mathbf{x}$



Deformed images of bonds:
State description allows complexity

Peridynamics basics: Force state

- $\mathbf{f}(\mathbf{x}, \mathbf{q})$ has contributions from the material models at both $\mathbf{x}$ and $\mathbf{q}$.

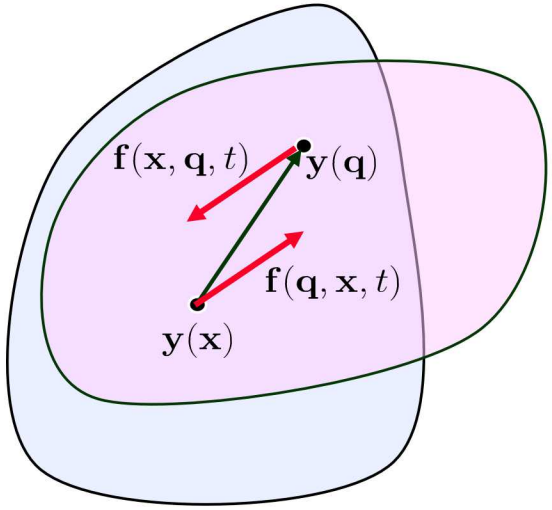
$$\mathbf{f}(\mathbf{x}, \mathbf{q}) = \mathbf{t}(\mathbf{x}, \mathbf{q}) - \mathbf{t}(\mathbf{q}, \mathbf{x})$$

$$\mathbf{t}(\mathbf{x}, \mathbf{q}) = \underline{\mathbf{T}}[\mathbf{x}]\langle \mathbf{q} - \mathbf{x} \rangle, \qquad \mathbf{t}(\mathbf{x}, \mathbf{q}) = \underline{\mathbf{T}}[\mathbf{q}]\langle \mathbf{x} - \mathbf{q} \rangle$$

- $\underline{\mathbf{T}}[\mathbf{x}]$ is the *force state*: maps bonds onto bond force densities. It is found from the constitutive model:

$$\underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}})$$

where $\hat{\underline{\mathbf{T}}}$ maps the deformation state to the force state.



Peridynamics basics: Bonds and bond force density

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- The vector from $\mathbf{x}$ to any point $\mathbf{q}$ in its family in the reference configuration is called a *bond*.

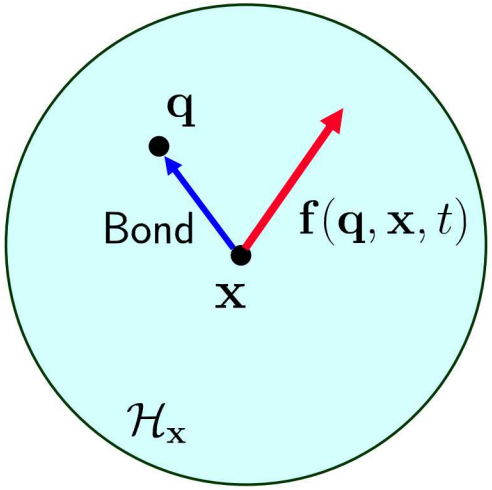
$$\boldsymbol{\xi} = \mathbf{q} - \mathbf{x}$$

- Each bond has a *pairwise force density* vector that is applied at both points:

$$\mathbf{f}(\mathbf{q}, \mathbf{x}, t).$$

- Equation of motion is an integro-differential equation, not a PDE:

$$\rho(\mathbf{x})\ddot{\mathbf{y}}(\mathbf{x}, t) = \int_{\mathcal{H}} \mathbf{f}(\mathbf{q}, \mathbf{x}, t)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x}, t).$$



Peridynamics basics: The nature of internal forces

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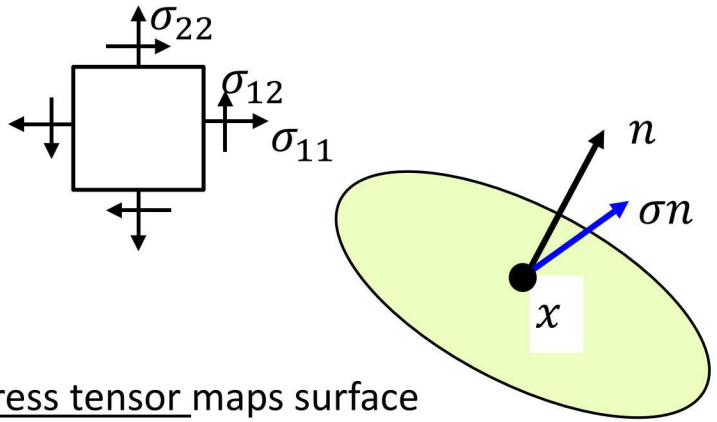
Standard theory

Stress tensor field
(assumes continuity of forces)



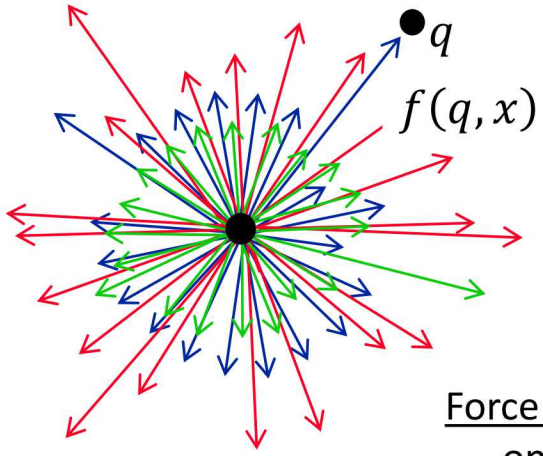
Stress tensor maps surface normal vectors onto surface forces

$$\rho\ddot{u}(x,t) = \nabla \cdot \sigma(x,t) + b(x,t)$$

Differentiation of surface forces

Peridynamics

Bond forces between neighboring points
(allowing discontinuity)



Force state maps bonds onto bond forces

$$\rho\ddot{u}(x,t) = \int_{H_x} f(q,x)dV_q + b(x,t)$$

Summation over bond forces

Peridynamics basics: Elastic materials

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- A peridynamic elastic material has strain energy density given by

$$W(\underline{\mathbf{Y}}).$$

- The force state is given by

$$\underline{\hat{\mathbf{T}}}(\underline{\mathbf{Y}}) = W_{\underline{\mathbf{Y}}}(\underline{\mathbf{Y}})$$

where $W_{\underline{\mathbf{Y}}}$ is the Frechet derivative of the strain energy density.

Peridynamic vs. local equations

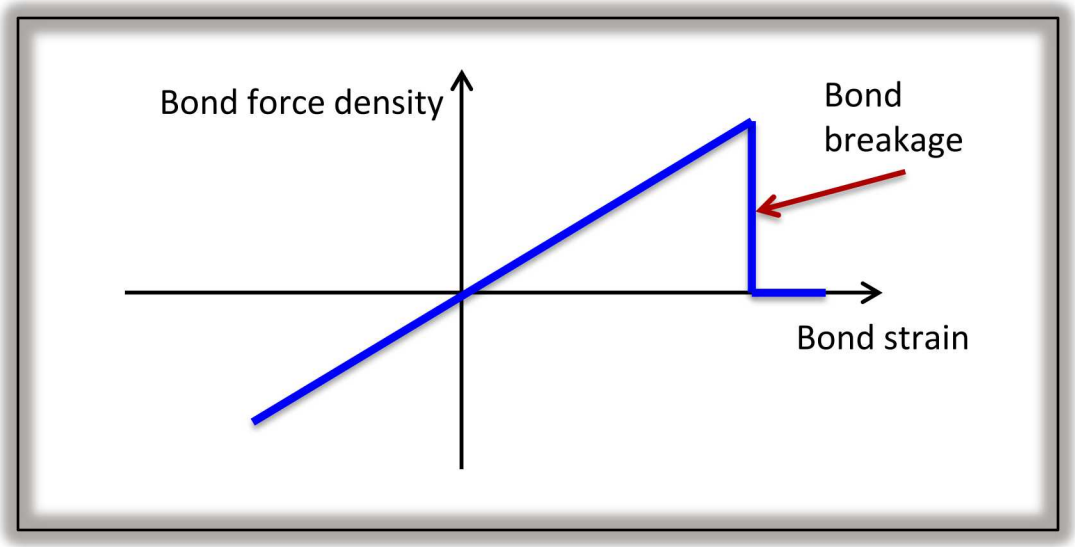
- The structures of the theories are similar, but peridynamics uses nonlocal operators.

Relation	*Peridynamic theory*	*Standard theory*
Kinematics	$\underline{\mathbf{Y}}\langle \mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q})-\mathbf{y}(\mathbf{x})$	$\mathbf{F}(\mathbf{x}) = \dfrac{\partial \mathbf{y}}{\partial \mathbf{x}}(\mathbf{x})$
Linear momentum balance	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \displaystyle\int_{\mathcal{H}} \Big(\mathbf{t}(\mathbf{q},\mathbf{x}) - \mathbf{t}(\mathbf{x},\mathbf{q})\Big)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x})$	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b}(\mathbf{x})$
Constitutive model	$\mathbf{t}(\mathbf{q},\mathbf{x}) = \underline{\mathbf{T}}\langle \mathbf{q}-\mathbf{x}\rangle, \qquad \underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}})$	$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\mathbf{F})$
Angular momentum balance	$\displaystyle\int_{\mathcal{H}} \underline{\mathbf{Y}}\langle \mathbf{q}-\mathbf{x}\rangle \times \underline{\mathbf{T}}\langle \mathbf{q}-\mathbf{x}\rangle\, dV_{\mathbf{q}} = \mathbf{0}$	$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$
Elasticity	$\underline{\mathbf{T}} = W_{\underline{\mathbf{Y}}}$ (Fréchet derivative)	$\boldsymbol{\sigma} = W_{\mathbf{F}}$ (tensor gradient)
First law	$\dot{\varepsilon} = \underline{\mathbf{T}} \bullet \underline{\dot{\mathbf{Y}}} + q + r$	$\dot{\varepsilon} = \boldsymbol{\sigma}\cdot\dot{\mathbf{F}} + q + r$

$$\underline{\mathbf{T}} \bullet \underline{\dot{\mathbf{Y}}} := \int_{\mathcal{H}} \underline{\mathbf{T}}\langle \boldsymbol{\xi}\rangle \cdot \underline{\dot{\mathbf{Y}}}\langle \boldsymbol{\xi}\rangle\, dV_{\boldsymbol{\xi}}$$

Bond based material models

- If each bond response is independent of the others, the resulting material model is called bond-based.
- The material model is then simply a graph of bond force density vs. bond strain.
- Damage can be modeled through bond breakage.
- Bond response is calibrated to:
  - Bulk elastic properties.
  - Critical energy release rate.



EMU numerical method

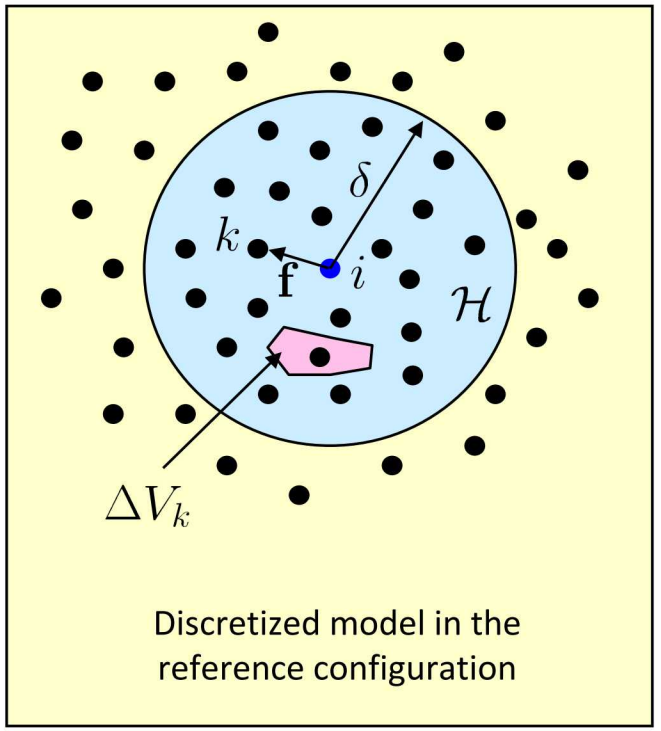
- Integral is replaced by a finite sum: resulting method is meshless and Lagrangian.

$$\rho\ddot{\mathbf{y}}(\mathbf{x},t) = \int_{\mathcal{H}} \mathbf{f}(\mathbf{q},\mathbf{x},t)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x},t)$$

$$\downarrow$$

$$\rho\ddot{\mathbf{y}}_i^n = \sum_{k\in\mathcal{H}} \mathbf{f}(\mathbf{x}_k,\mathbf{x}_i,t)\, \Delta V_k + \mathbf{b}_i^n$$

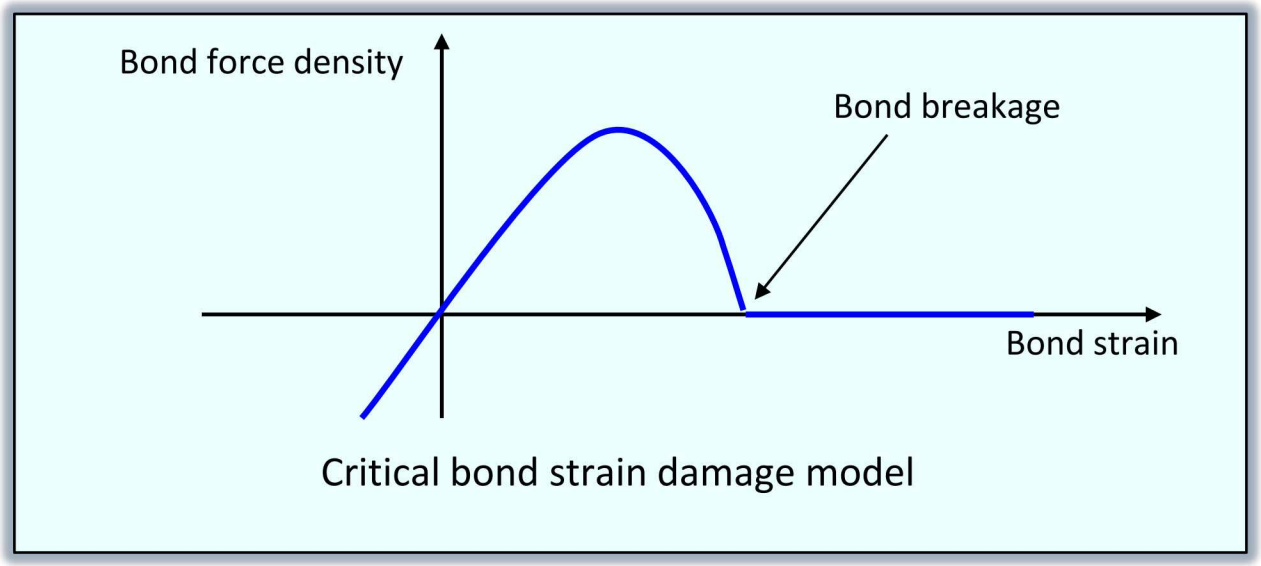
  - Looks a lot like MD.
  - Unrelated to Smoothed Particle Hydrodynamics
    - SPH solves the local equations by fitting spatial derivatives to the current node values.



Discretized model in the reference configuration

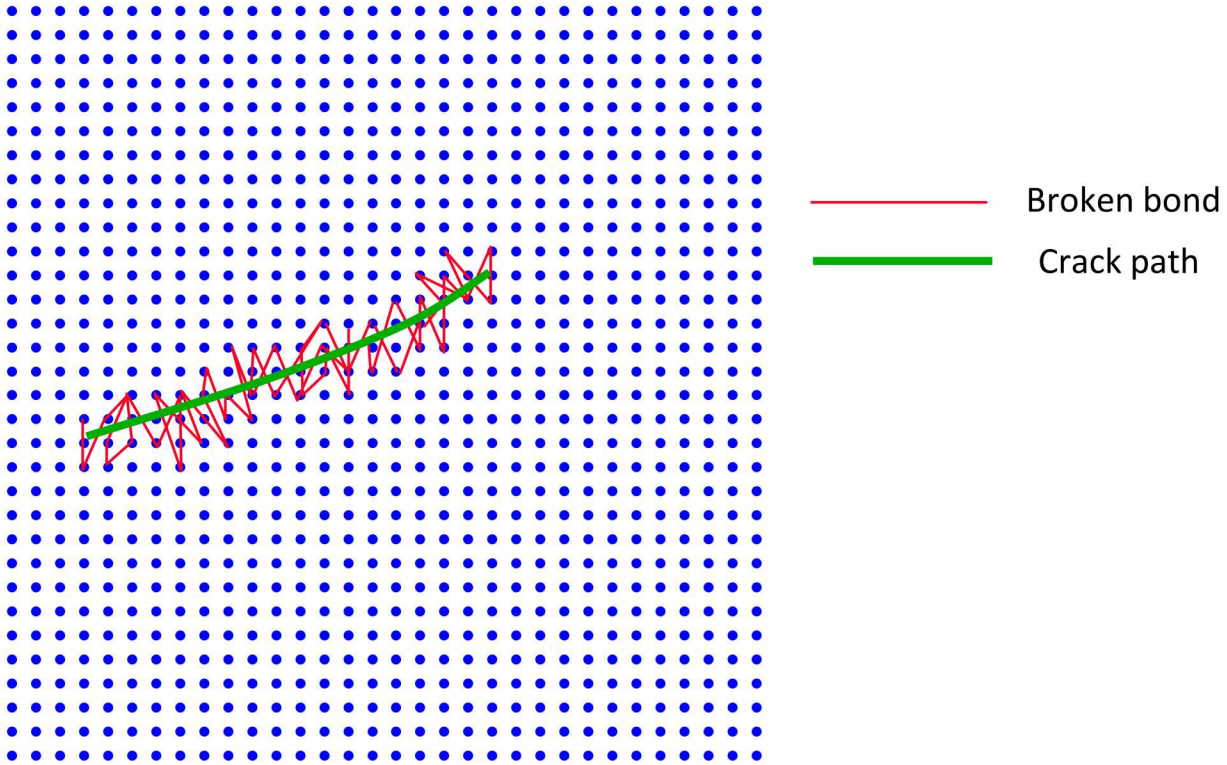
Damage due to bond breakage

- Recall: each bond carries a force.
- Damage is implemented at the bond level.
  - Bonds break irreversibly according to some criterion.
  - Broken bonds carry no force.
- Examples of criteria:
  - Critical bond strain (brittle).
  - Hashin failure criterion (composites).
  - Gurson (ductile metals).



Critical bond strain damage model

Autonomous crack growth

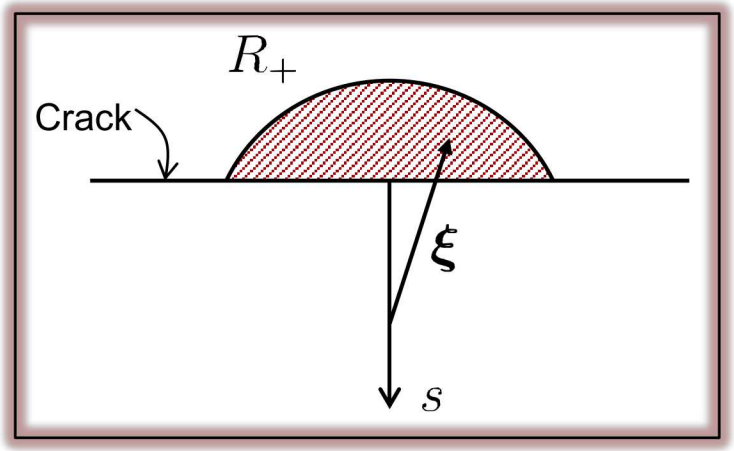


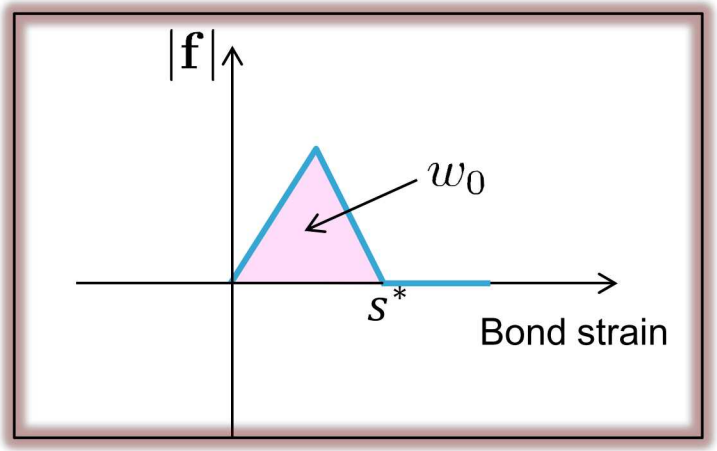
- When a bond breaks, its load is shifted to its neighbors, leading to progressive failure.

Critical bond strain: Relation to critical energy release rate

If the work required to break the bond $\boldsymbol{\xi}$ is $w_0(\boldsymbol{\xi})$, then the energy release rate is found by summing this work per unit crack area (J. Foster):

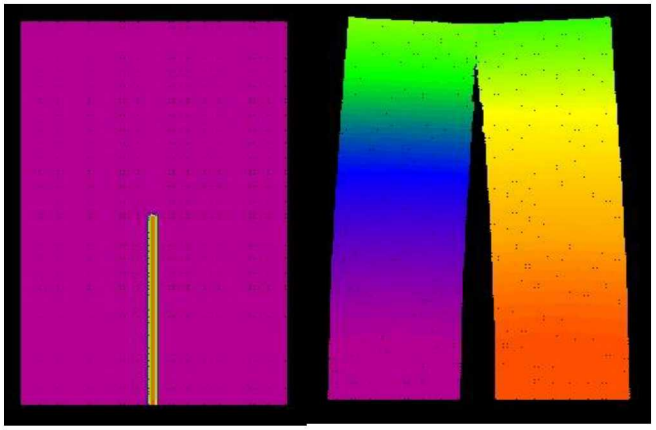
$$G = \int_0^{\delta} \int_{R_+} w_0(\boldsymbol{\xi})\, dV_{\boldsymbol{\xi}}\, ds$$

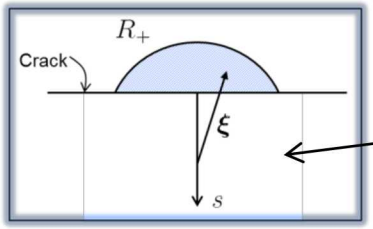




- Can then get the critical strain for bond breakage s^* in terms of G.
- Could also use the peridynamic J-integral as a bond breakage criterion.

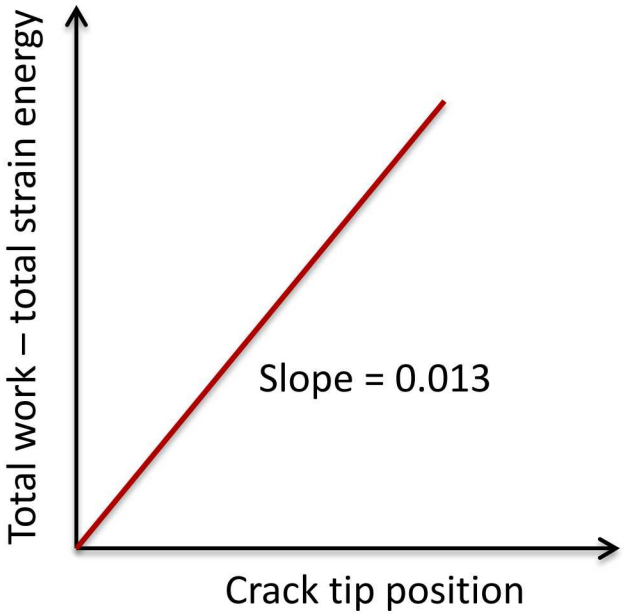
Constant bond failure strain reproduces the Griffith crack growth criterion





From bond properties, energy release rate should be

G = 0.013



- This confirms that the energy consumed per unit crack growth area equals the expected value from bond breakage properties.

Some results about peridynamics

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- For any choice of horizon, we can fit material model parameters to match the bulk properties and energy release rate.
  - Using nonlocality, can obtain material model parameters from wave dispersion curves (Weckner).
- Coupled coarse scale and fine scale evolution equations can be derived for composites (Lipton and Alali).
- A set of discrete particles interacting through any multibody potential can be represented exactly as a peridynamic body.
- Well posedness has been established under certain conditions (Mangesha, Du, Gunzburger, Lehoucq).

Cyclic strain in a bond

- For a given bond $\boldsymbol{\xi}$, the *bond elongation* is the change in bond length:

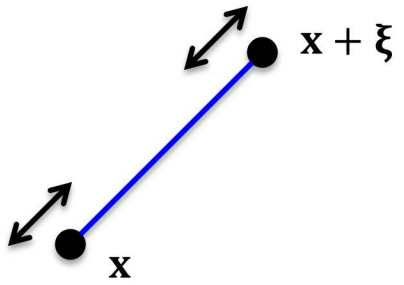
$$e = |\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle| - |\boldsymbol{\xi}| = |\mathbf{y}(\mathbf{x}+\boldsymbol{\xi}) - \mathbf{y}(\mathbf{x})|.$$

- The *bond strain* is the change in length over initial length:

$$s = \frac{e}{|\boldsymbol{\xi}|}.$$

- Let s^+ and s^- be the two extremes under cyclic loading of $\boldsymbol{\xi}$.
- The *cyclic bond strain* is defined by

$$\varepsilon = |s^+ - s^-|.$$

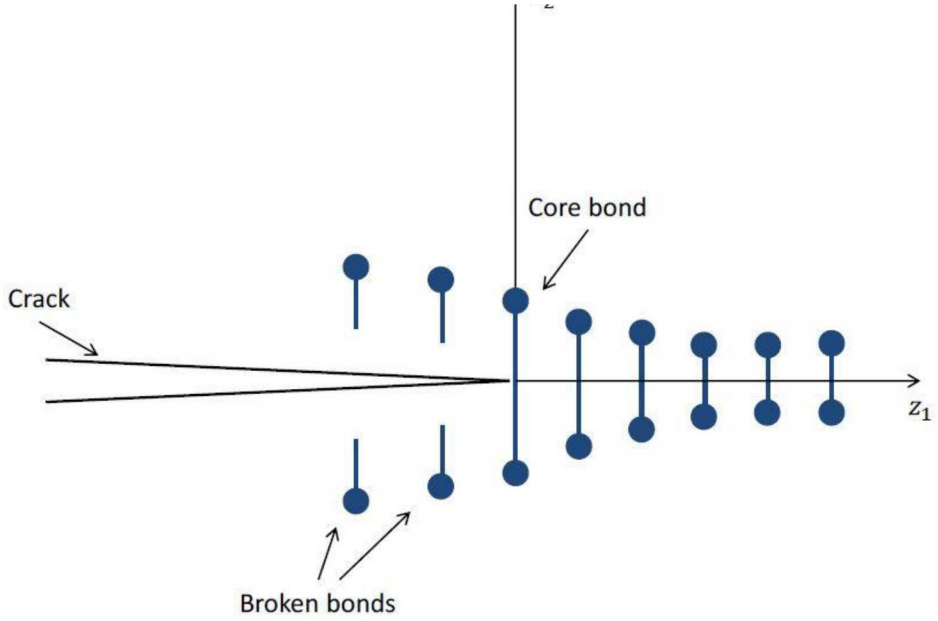


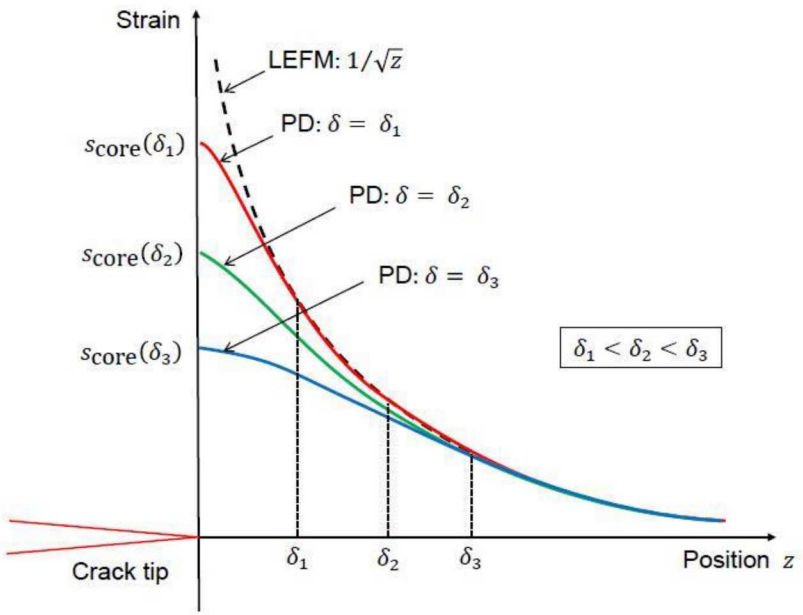
Structure of a crack tip field

- Let $\varepsilon_{\text{core}}(\delta)$ be the largest cyclic strain in any bond.
- Can show by a dimensional argument $\exists\ \hat{\varepsilon}_{\text{core}}$ such that

$$\varepsilon_{\text{core}}(\delta) = \hat{\varepsilon}_{\text{core}} \frac{\Delta K}{E\sqrt{\delta}}$$

where ΔK = cyclic stress intensity factor and E = modulus.





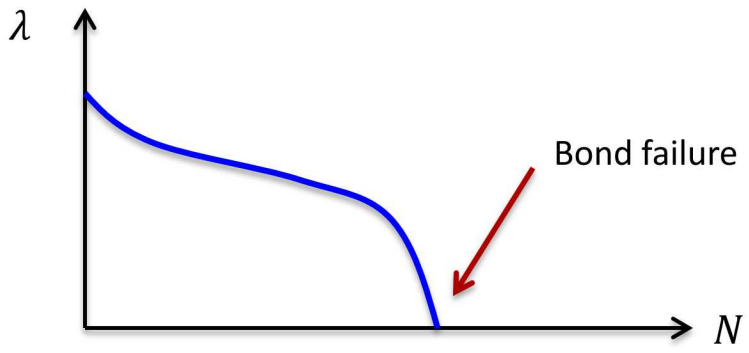
Remaining life of a bond

- Each bond in the body has a *remaining life* $\lambda(N)$ where N is the cycle number.
- The remaining life is monotonically decreasing over time.

$$\lambda(0) = 1, \qquad \dot{\lambda} \leq 0.$$

- The bond fails at the first cycle N when

$$\lambda(N) \leq 0.$$



Fatigue model

- The fatigue model specifies how the remaining life of each bond depends on the loading.

$$\frac{d\lambda}{dN}(N) = -A\varepsilon^m$$

  where A and m are constants and ε is the cyclic bond strain.

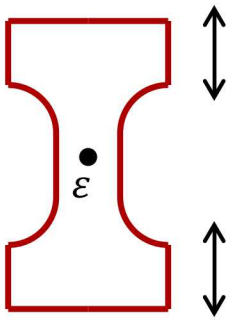
- The constants are calibrated separately for phases I and II (nucleation and growth).

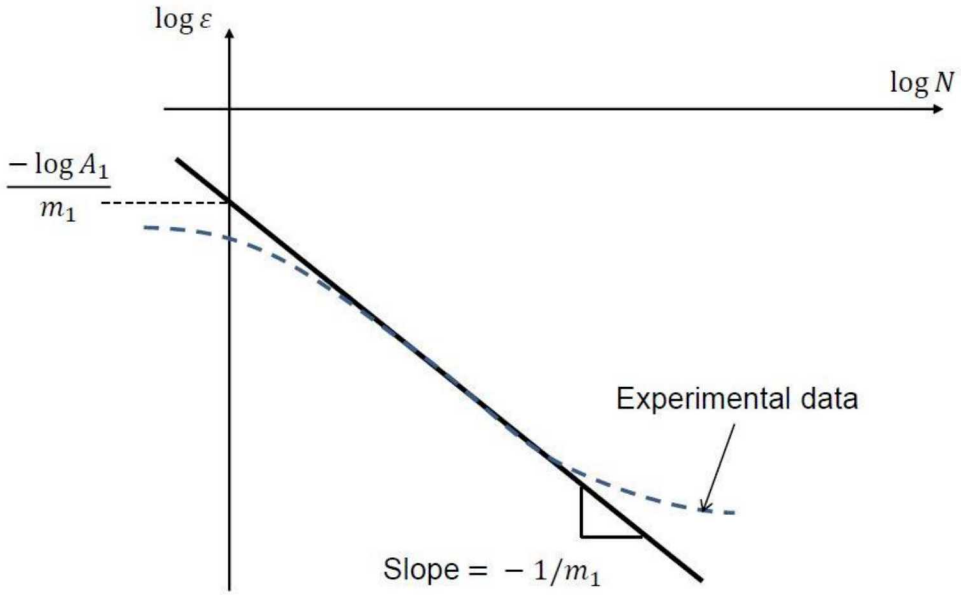
Phase I calibration from S-N data

- Run many cyclic loading tests at different values of ε (constant for each test).
- For each test, compute when damage starts:

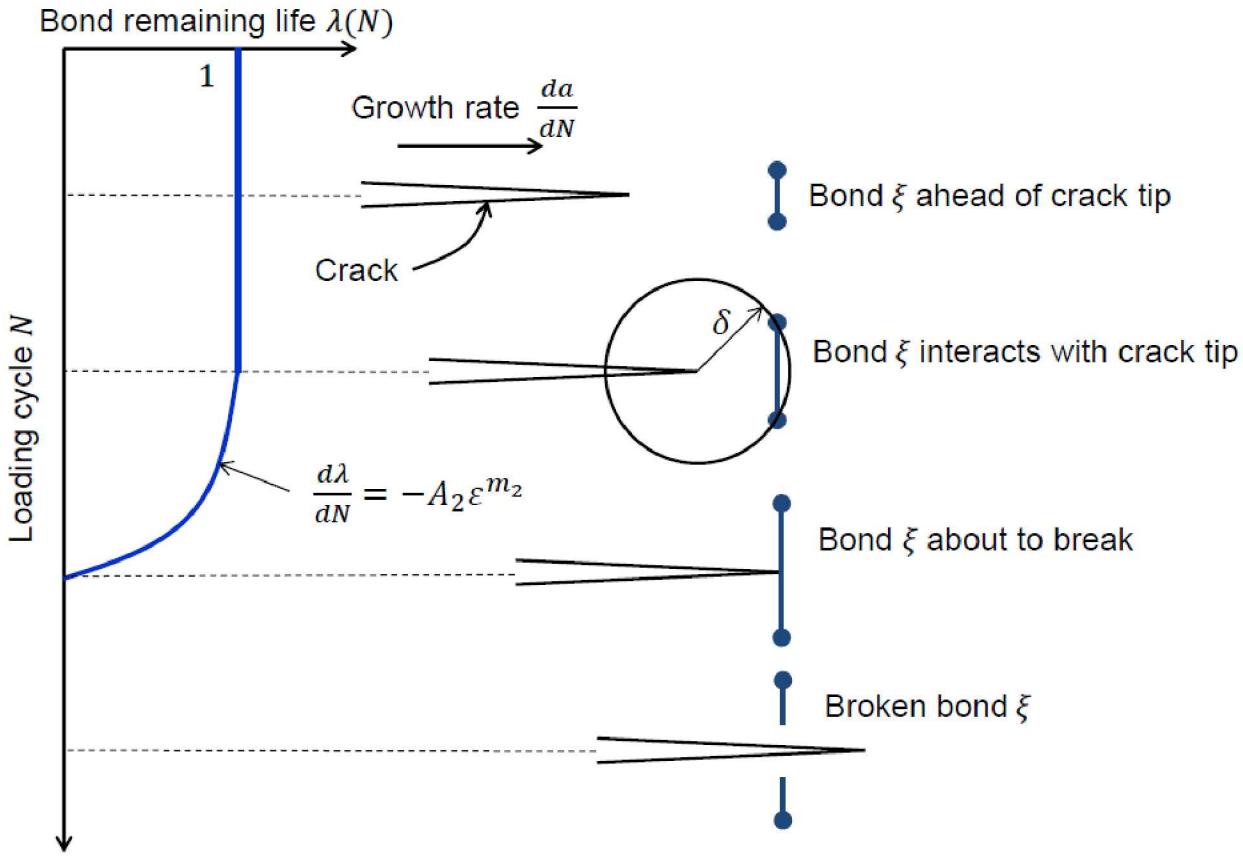
$$\frac{d\lambda}{dN}(N) = -A\varepsilon^m \qquad \Longrightarrow \qquad N = \frac{1}{A\varepsilon^m}.$$

- Compare this to data on an ε-N plot, fit A and m.





Growth: Bonds interact with the strain field near an approaching crack



Relate crack growth to remaining life

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- Evolution of remaining life:

$$\lambda(\delta) - \lambda(0) = \int_0^\delta \frac{d\lambda}{dz}\, dz = \int_0^\delta \frac{d\lambda}{dN}\frac{dN}{dz}\, dz.$$

- Recall

$$\frac{d\lambda}{dN} = -A\varepsilon^m.$$

- Denote by da/dN the crack growth rate.

$$1 - 0 = \frac{A}{da/dN}\int_0^\delta \varepsilon^m(z)\, dz$$

- Cyclic strain ahead of a crack:

$$\varepsilon(z) = \varepsilon_{\text{core}} f\left(\frac{z}{\delta}\right) = \frac{\Delta K}{E\sqrt{\delta}} f\left(\frac{z}{\delta}\right).$$

- Thus, for some c,

$$\frac{da}{dN} = cA\Delta K^m$$

Phase II calibration from Paris Law data

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- Now have

$$\frac{da}{dN} = cA\Delta K^m$$

  where c and m are as yet unknown.

- Assume the Paris Law holds:

$$\frac{da}{dN} = C\Delta K^M$$

  where C and M are constants that can be found from test data.

- Conclude

$$m = M.$$

- Need to do one computational simulation with an assumed value $C = 1$ to evaluate A.

Summary so far

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- Each bond has a remaining life $\lambda(N)$:

$$\lambda(0) = 1, \qquad \frac{d\lambda}{dN}(N) = -A\varepsilon^m, \qquad \lambda \leq 0 \text{ means failure.}$$

- In Phase I, use A and m from S-N data.
- In Phase II, use a different calibration from Paris law data.

Time mapping permits very large N

- We can avoid modeling each cycle explicitly.
- Define the *loading ratio* by

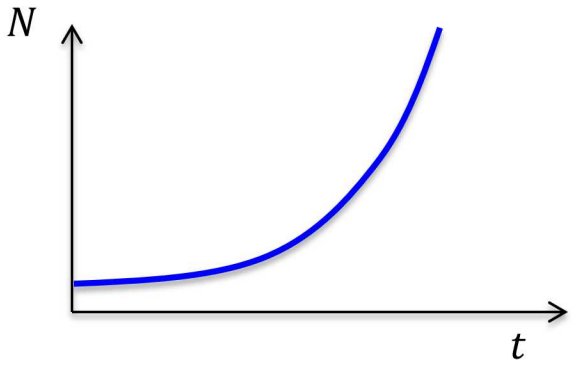
$$R = \frac{s^-}{s^+} \qquad \Longrightarrow \qquad \varepsilon = |s^+ - s^-| = |(1-R)s^+|.$$

- Map t to N:

$$N = e^{t/\tau}$$

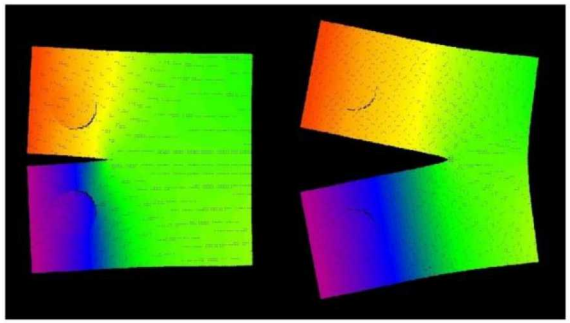
  where τ is a constant chosen according to convenience.

- Fatigue model in terms of t instead of N:

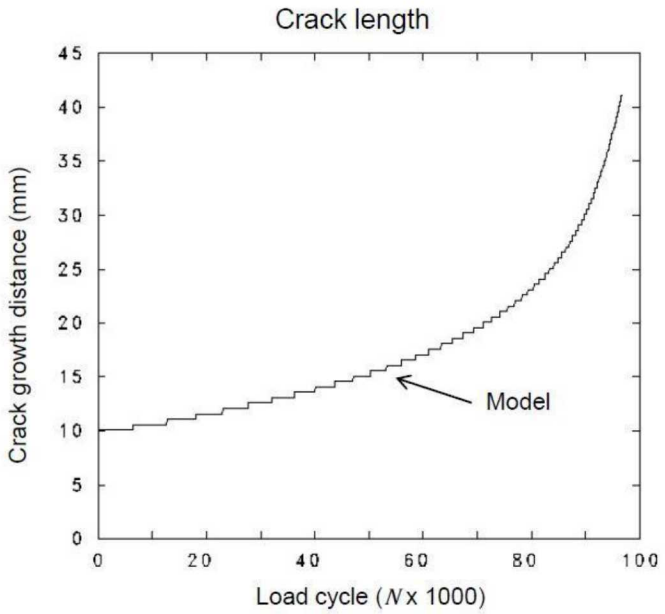


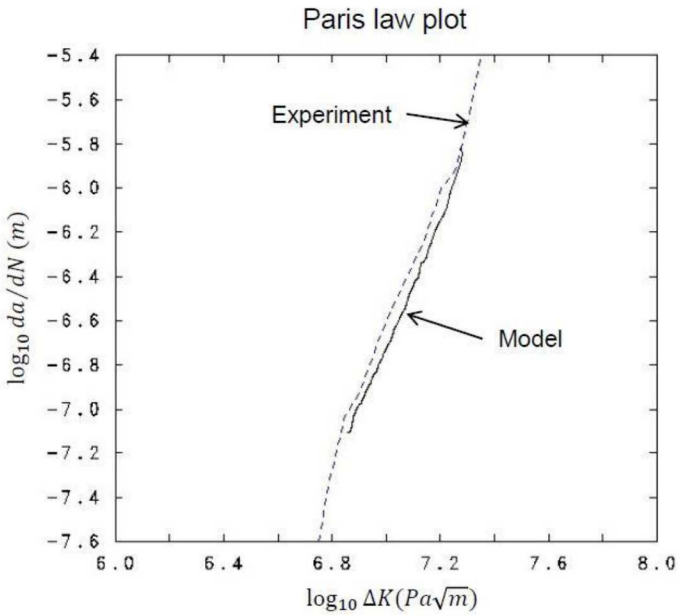
$$\frac{d\lambda}{dt} = \frac{d\lambda}{dN}\frac{dN}{dt} = \frac{-|1-R|AN}{\tau}|s^+|^m.$$

Aluminum compact test specimen

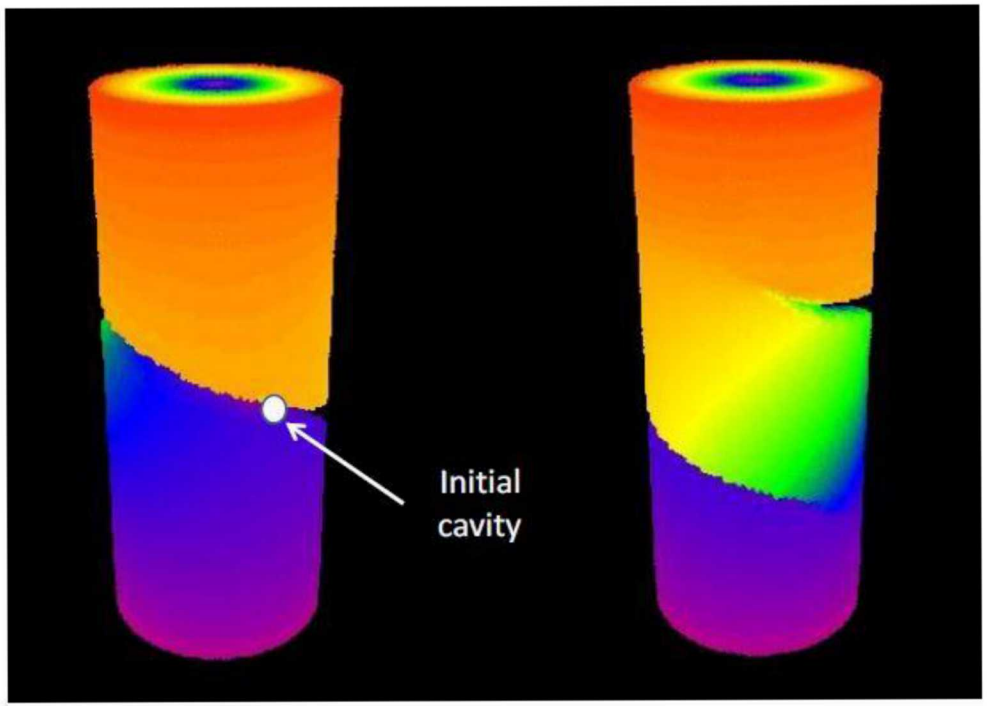


Data: T. Zhao and Y. Jiang. Fatigue of 7075-T651 aluminum alloy. International Journal of Fatigue, 30:834{849, 2008.





Spiral crack in a rod under torsion



Discussion

Sandia National Laboratories

- Method retains the main advantages of peridynamics.
  - Autonomous crack growth
  - Includes both nucleation and growth phases
  - Permits interaction between multiple cracks
  - Arbitrary crack path in 3D.
- A simple enhancement allows a spectrum of loading frequencies
  - This is a peridynamic version of Miner's rule.