SAND2017-7214C

Exceptional service in the national interest

Sandia National Laboratories

Matrix Compression in the Method of Moments Code EIGER - Iterative Solver Accuracy and Parallel Efficiency

Joseph D. Kotulski

Sandia National Laboratories, Albuquerque ,NM, USA

jdkotul@sandia.gov

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Outline

- **EIGER Description**
- **Matrix compression**
- **Load- balancing (Matrix Fill)**
- **Results**
- **Conclusions / Future Work**

EIGER Description

- **Frequency-domain method of moments solution**
  - **Steady state solution**
  - **F90 (95, 2003) code – Object Oriented Design**

- **Boundary element formulation**
  - **Mesh surfaces of parts – interface between regions**

- **Normal formulation results in dense (fully populated) matrix**
  - **Galerkin testing - Rao, Wilton, and Glisson bases functions**

$$\bar{\bar{\mathrm{Z}}}\,\bar{\mathrm{I}} = \bar{\mathrm{V}}$$

  - **Simulations can be limited by available memory**

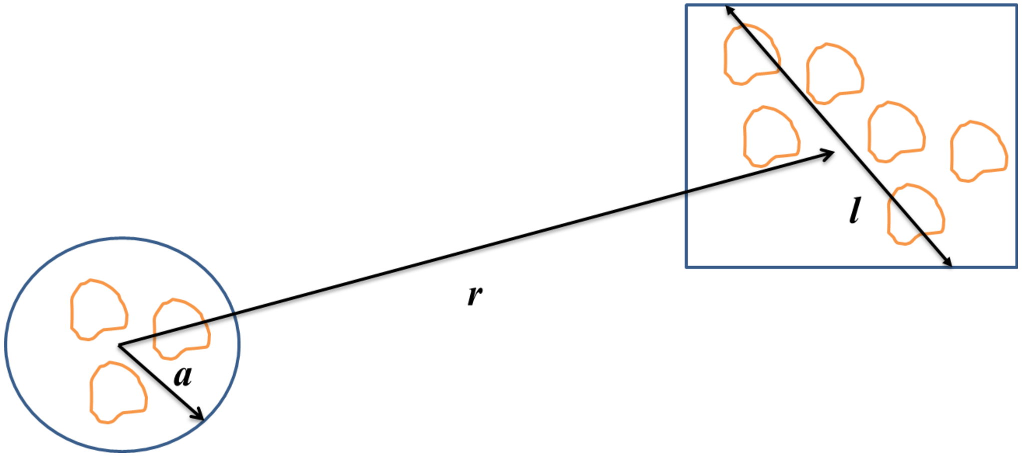
    (Entries are double precision complex)

Matrix Compression

- **These are techniques that no longer store the full matrix but a lower rank version of the matrix.**

- **Based on work by Bucci and Franceschetti**
  - **"On the Degrees of Freedom of Scattered Fields" IEEE AP, July 1989**

$$N_{dof} = \frac{4la}{r\lambda}$$



Compression Techniques

- Fast Multipole Method (FMM)
  - Compression achieved through Green's function simplification:
    - Factorization
    - Use of the addition theorem
    - Operator Diagonalization
    - Results in low-rank approximation of matrix blocks
    - N log (N) complexity

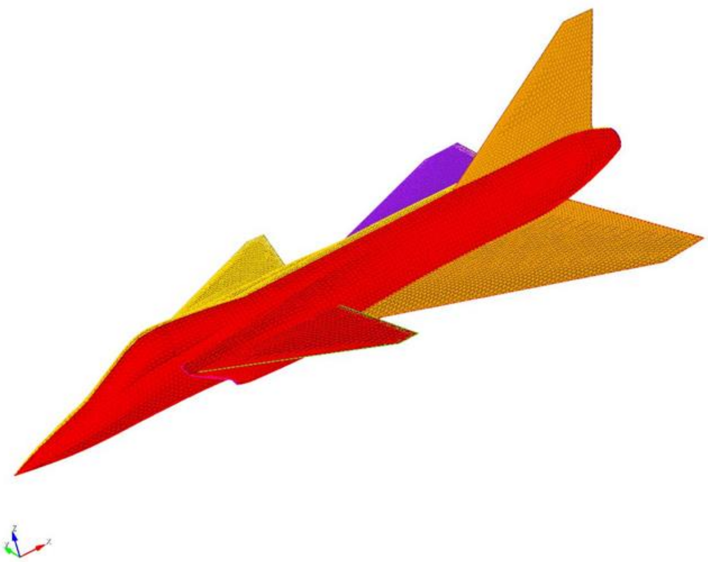
- Adaptive Cross Approximation (ACA)
  - Compression achieved:
    - Low-rank approximation of matrix blocks.
    - Done on the fly
      - Compressed matrix blocks never fully populated.
    - Since the process only operates on matrix blocks it is independent of Green's function simplification.
    - $N^{1.5}$ complexity
- Multilevel direct solve methods
  - University of Michigan, University of Colorado, and University of Texas at Austin

Matrix Compression Techniques

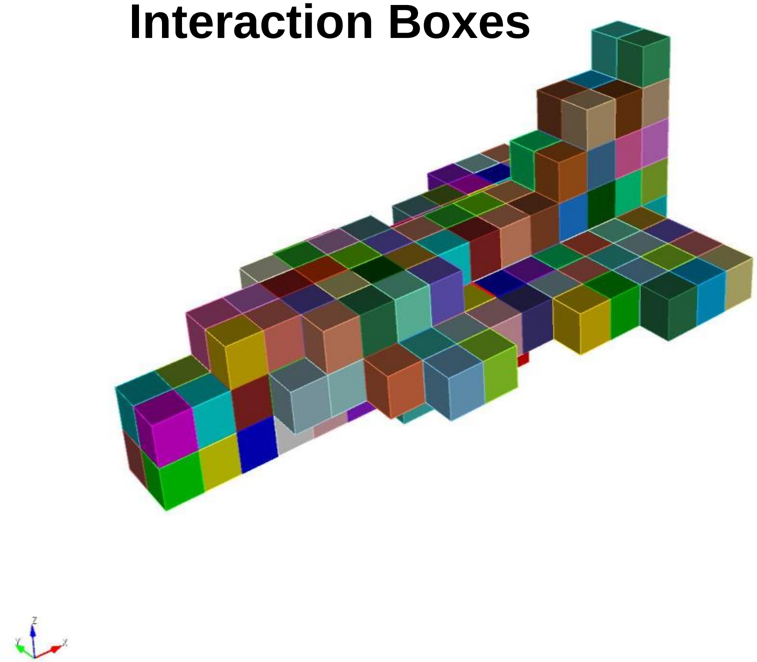
- **Identification of all matrix blocks**
  - **Discretized object (meshed) is encased in a oct-tree structure**

VFY 218

Meshed Object



Interaction Boxes



All compression techniques use this step in the solution process

ACA Matrix Compression

- **Each box contains elements with current unknowns on the elements.**
  - **Can be compared to a 1-level fast multipole algorithm**

- **2 boxes interact to form a matrix block.**

- **The distance between boxes, size of the boxes, and wavelength determine if a reduced or low-rank approximation can be used.**
  - **Not all blocks can be compressed.**
    - **Compression criterion :**
      - **Distance between the center of boxes > 2 * (box radius)**

ACA Matrix Compression

- **The matrix** $\bar{\bar{Z}}$ **is given by:**

$$\bar{\bar{Z}} = \sum_{j=1}^{MOM\_blocks} Z_j^{mom} + \sum_{i=1}^{COM\_blocks} \tilde{Z}_i^{com}$$

MOM_Blocks – Moment method matrix blocks (full matrix blocks)

COM_Blocks – Compressed matrix blocks (low-rank approximation)

ACA Matrix Compression

- **Approximate matrix description:**

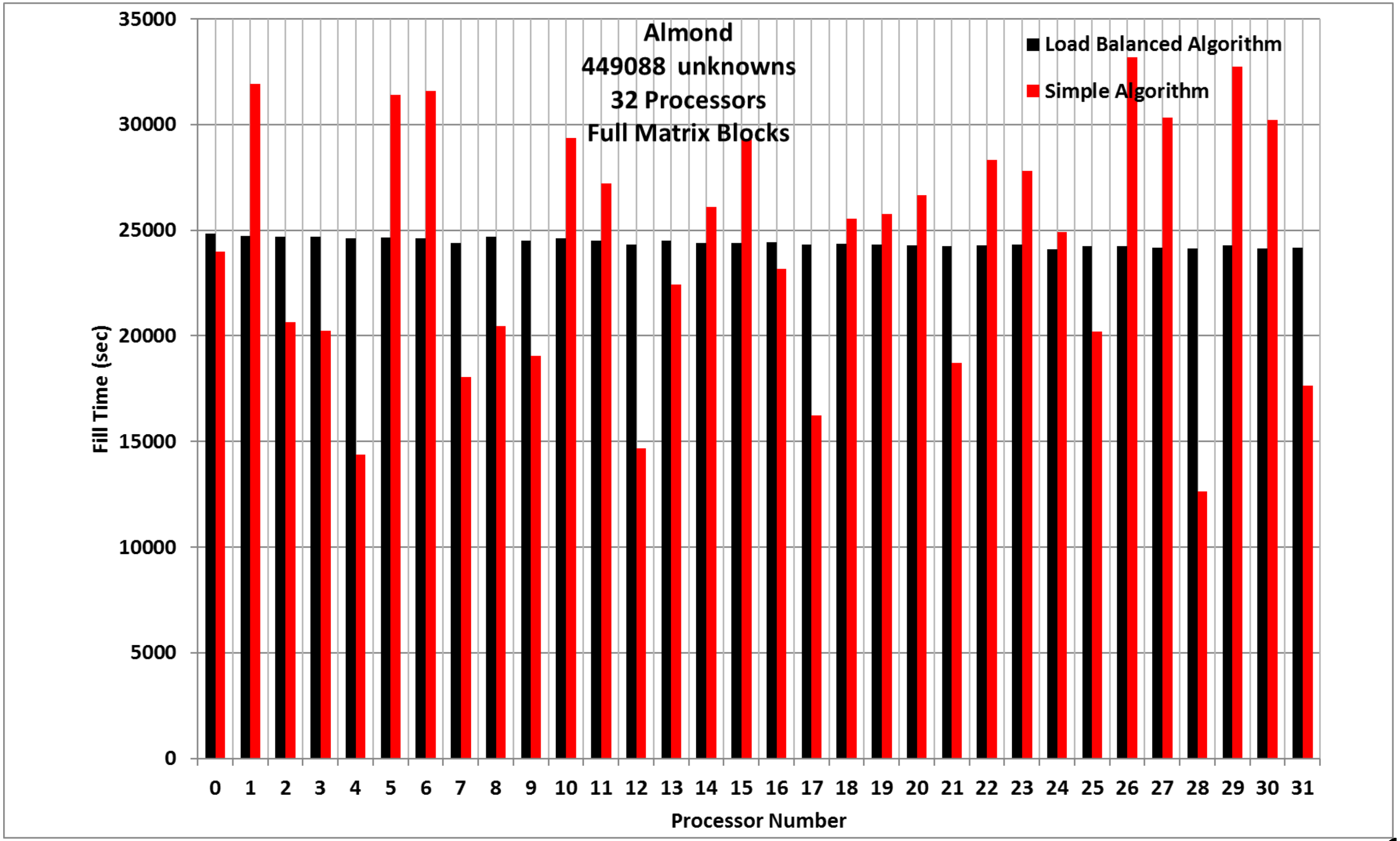
$$\tilde{Z}^{m\times n} = U^{m\times r} V^{r\times n} = \sum_{i=1}^{r} u_i^{m\times 1} v_i^{1\times n}$$

- **The key step is the determination of the sub-matrices u and v.**

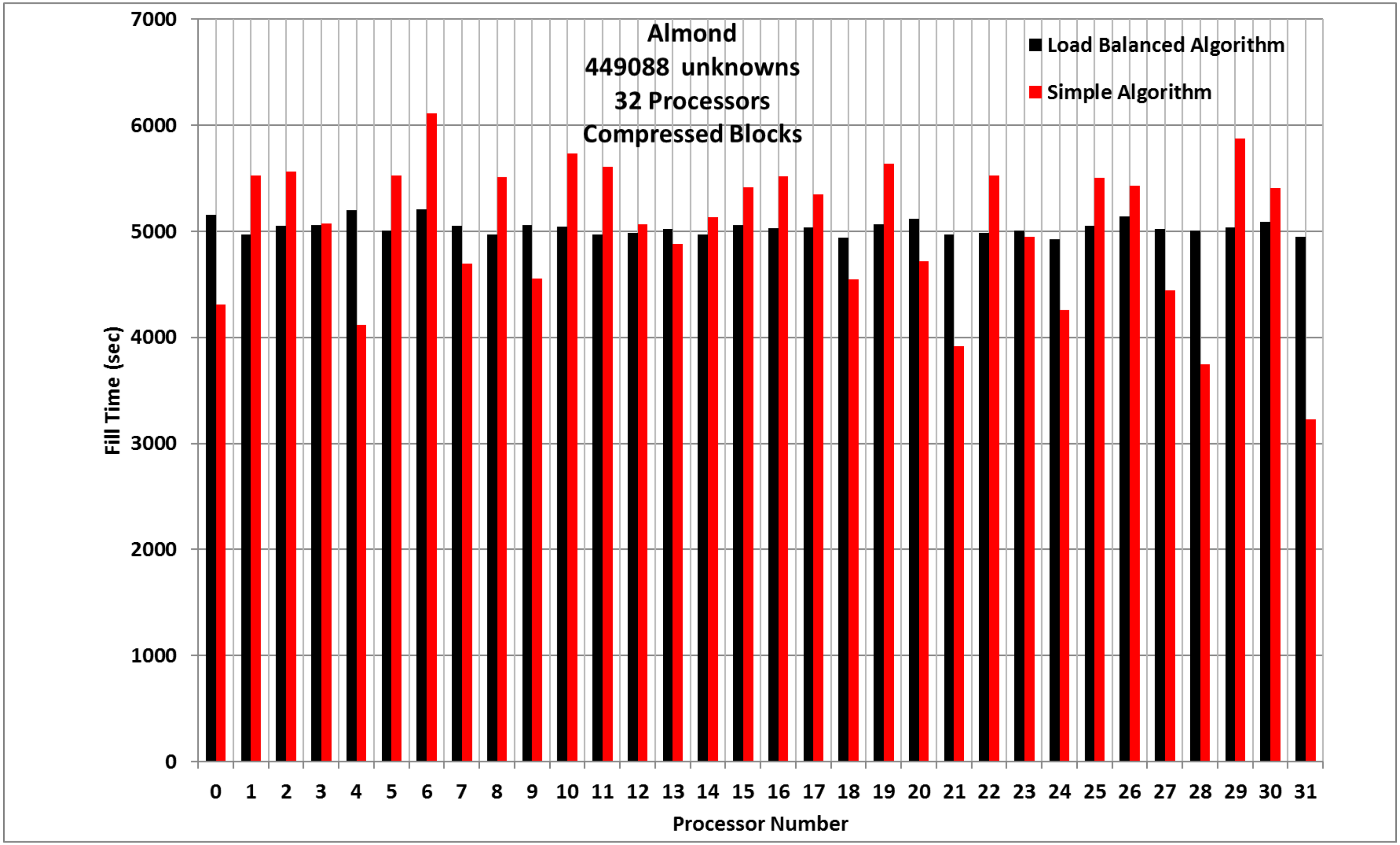
Parallel Computation of Matrix Blocks

- Each processor computes a subset of the total number of blocks
  - Both MOM_Blocks and COM_Blocks
- Initial decomposition of blocks done via block numbering
  - Neglects the work per block
- Improved decomposition of blocks performed
  - Sorted with regards to matrix block size
    - Known for full matrix blocks
    - Estimated using "Degrees of Freedom of the Scattered Field"

Parallel Computation of Matrix Blocks



Parallel Computation of Matrix Blocks



Solution of the Compressed System

- **The matrix equation to be solved is :**

$$\bar{\bar{Z}}\,\bar{I} = \bar{V}$$

- **The matrix is not completely available but is stored as:**

$$\bar{\bar{Z}} = \sum_{j=1}^{MOM\_blocks} Z_j^{mom} + \sum_{i=1}^{COM\_blocks} \tilde{Z}_i^{com}$$

- **Therefore a iterative solution approach needs to be used.**
  - **Generalized Minimum residual method(GMRES)**
    - **Saad and Schultz 1986**
  - **Transpose Free Quasi Minimum Residual (TFQMR)**
    - **Freund 1993**
  - **Both exercised and require matrix – vector products**

Solution of the Compressed System

- **Low – rank approximation of the compressed sub matrices controlled by a parameter. (eps)**

- **Iterative solution controlled by chosen residual**
  - **For GMRES (number of vectors for restart)**

Test Object Almond

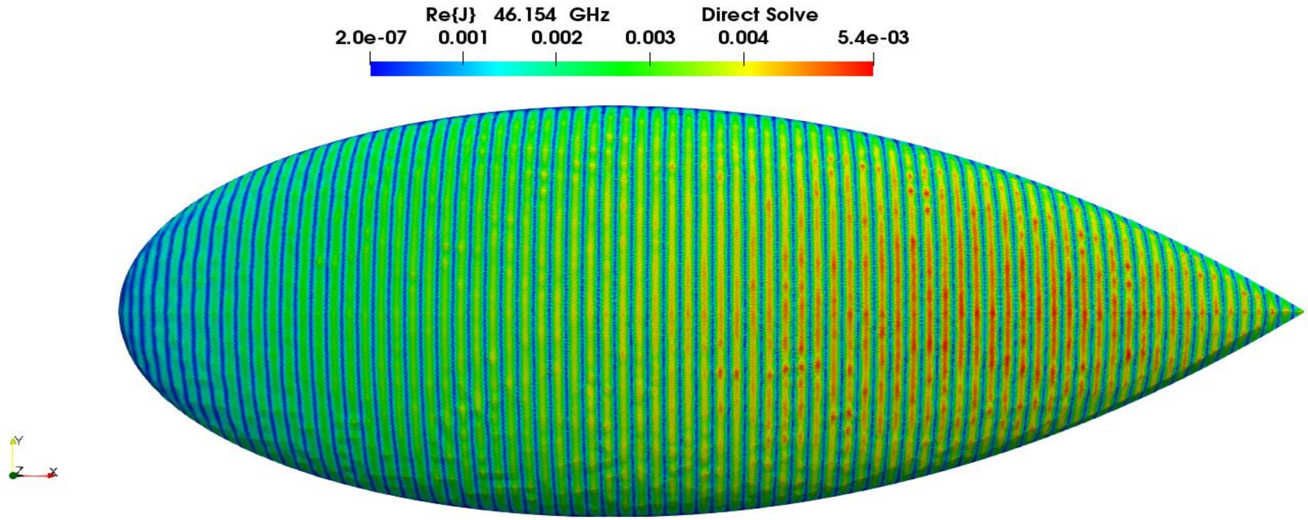
Sandia National Laboratories

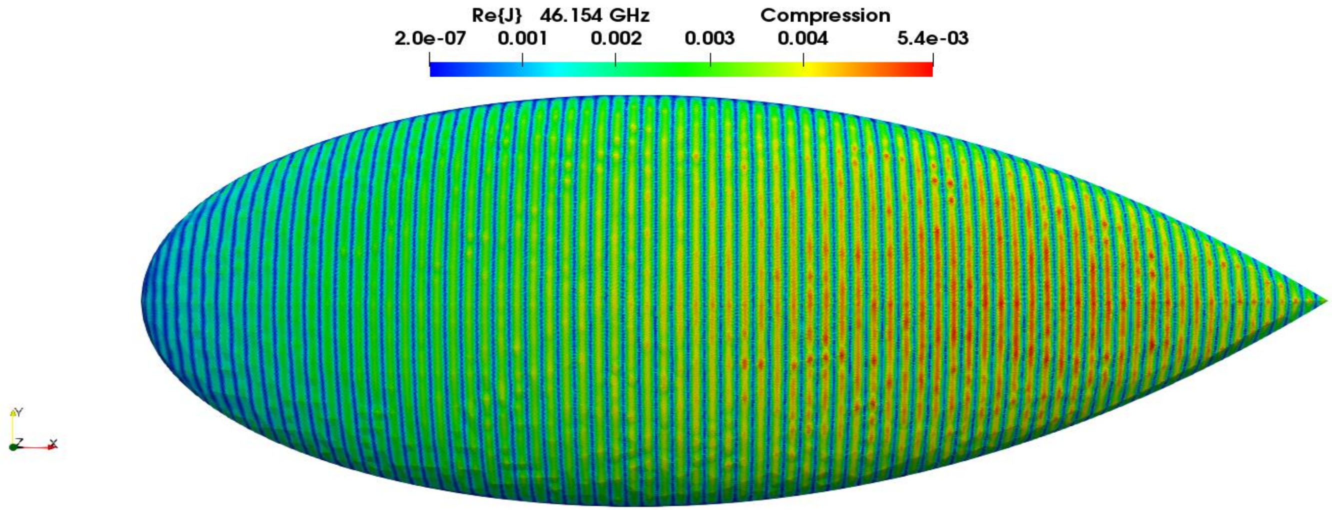
- **499088 Unknowns**
  - **Grid for 50 GHz**

Solve	# processors	Memory (Gbytes)	Solve Time (sec)
Direct	1980	3226	17992
Compression	48	322 (90%)	891

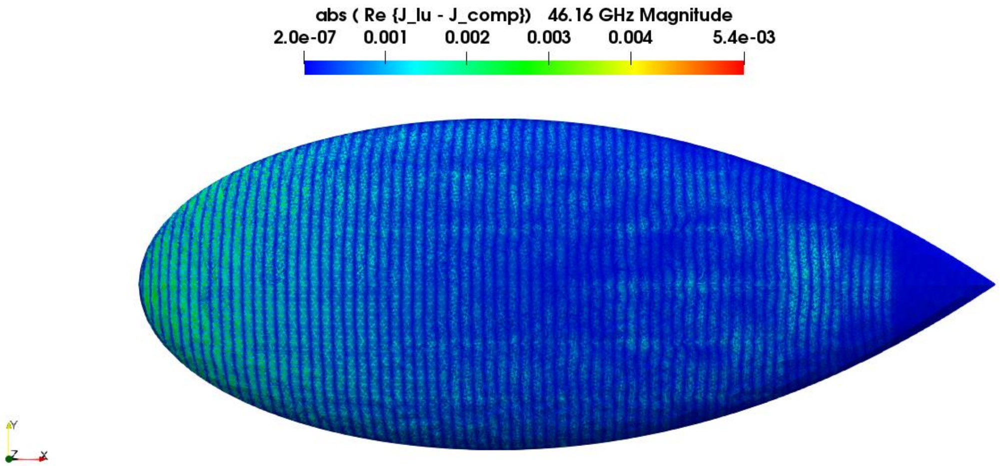
Eps 4e-03, Solver tolerance 1.e-06, GMRES (40)

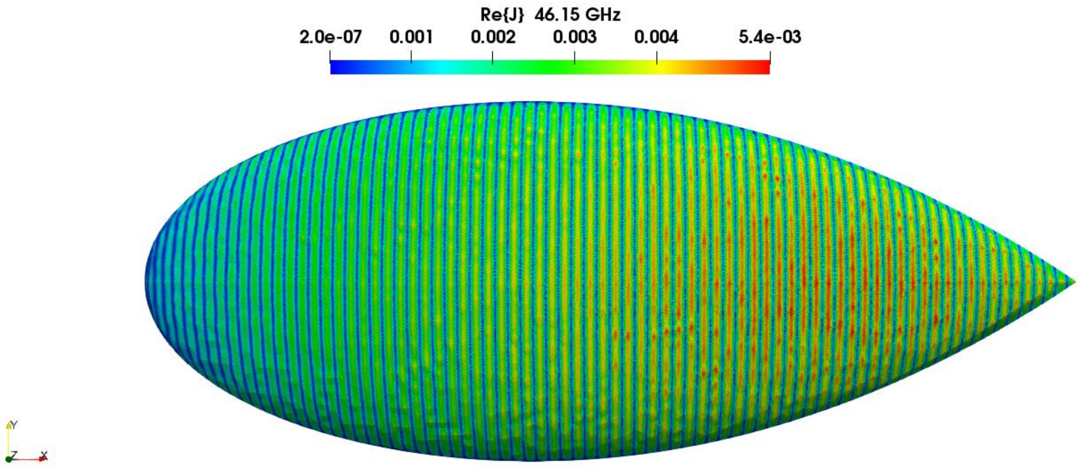
Surface Currents Comparison



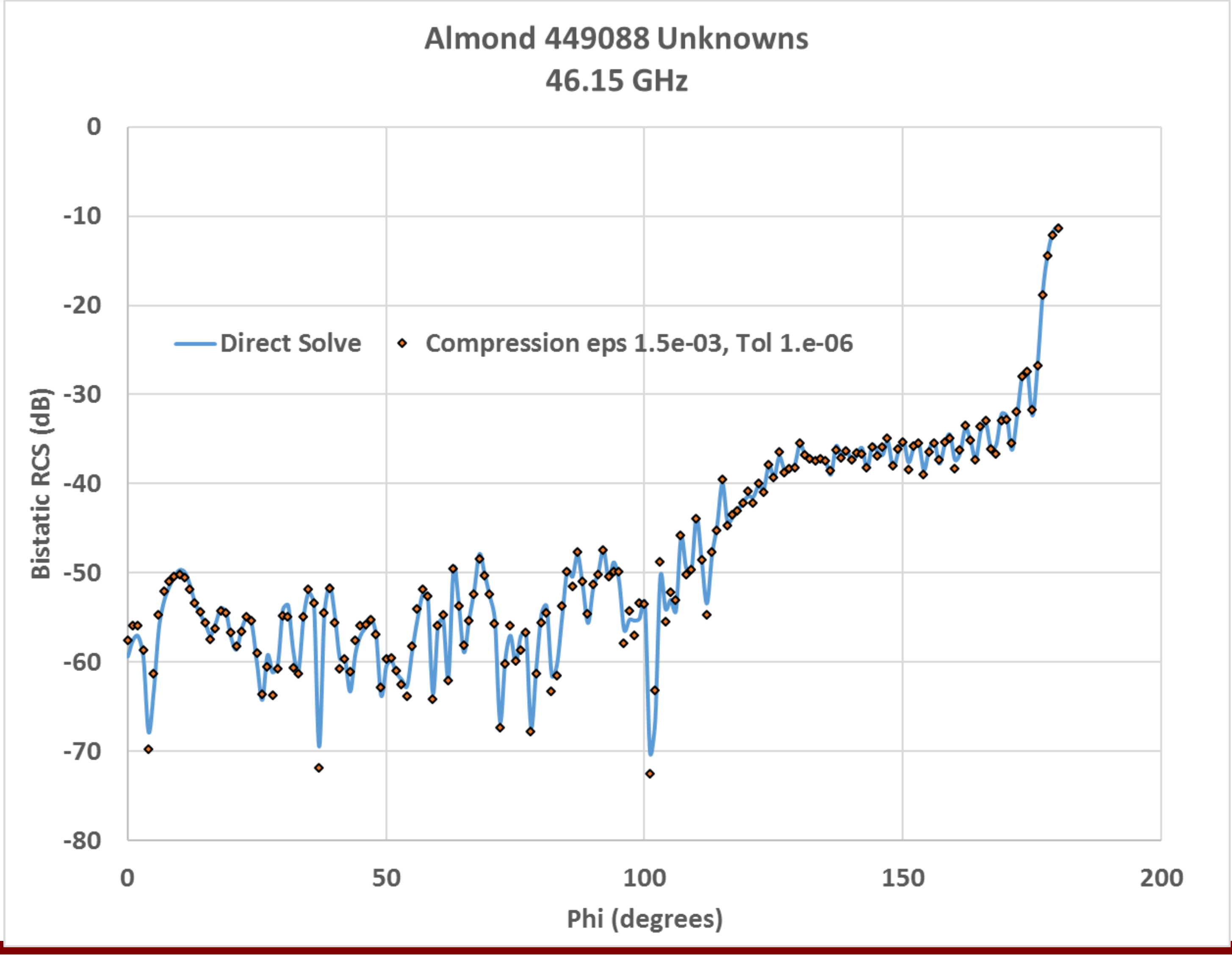


Surface Currents Comparison

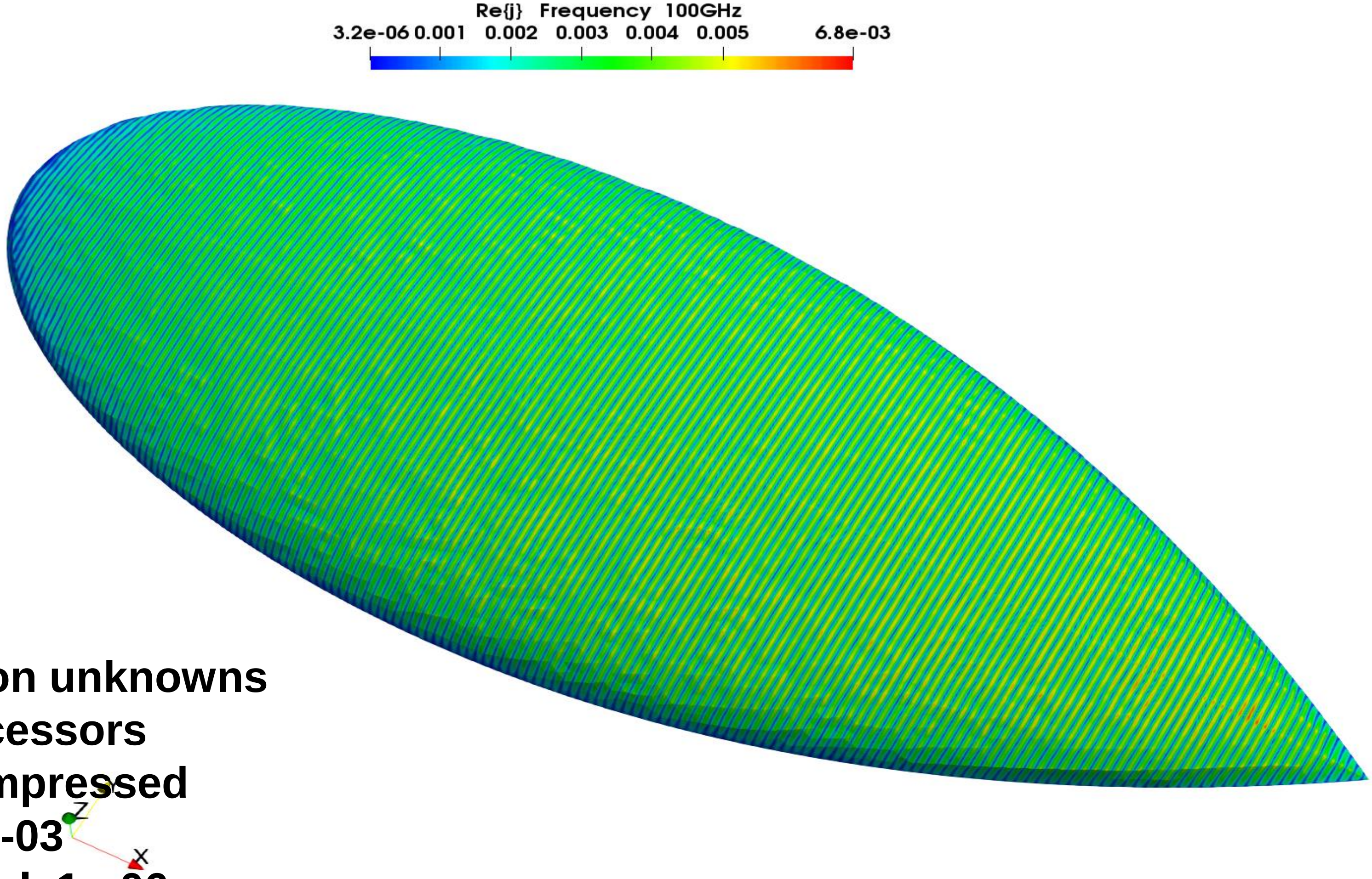




Far Field Comparison



Surface Currents – Problem 2



1.8 million unknowns
150 Processors
98% Compressed
Eps 1.5e-03
Solver Tol 1e-06
Solution Time 685 Sec

Conclusions / Future Work

- **Presented work using matrix compression (via ACA).**

- **Increase efficiency and robustness**
  - **Memory usage**
  - **Fill time**
  - **Matrix-vector product**

- **Collaboration with Solver Team (Trilinos)**
  - **Preconditioning – necessary for the EXTERNAL / INTERNAL problem**
  - **Block methods (Multiple excitation / directions)**
  - **MPI + threading**

- **Continue to extend solution frequency applicability**
  - **Alternative methods of compression**