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Sandia National Laboratories

# Flexible Optimization and Uncertainty-Enabled Design of Helical Compression Springs in Nonlinear Spring-Mass-Damper Systems


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Albuquerque, NM


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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration


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# Sandia National Laboratories

Sandia National Laboratories

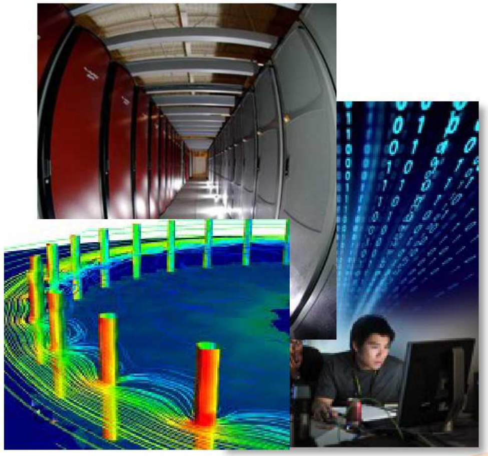
- Core Purpose: **help our nation secure a peaceful and free world through technology**.
- Provide objective, multidisciplinary technical assessments for complex problems.
- Focus on solutions with large science and technology content.
- Create prototypes for subsequent production and operation by industry.


U.S. DEPARTMENT OF ENERGY

# Scope & Complexity of National Security

**SNL Applies both BREADTH & DEPTH to solving our nation's most challenging problems.**

# Research Disciplines Drive Capabilities



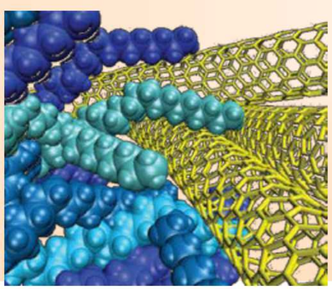
**High Performance Computing**

**Science & Technology Products**

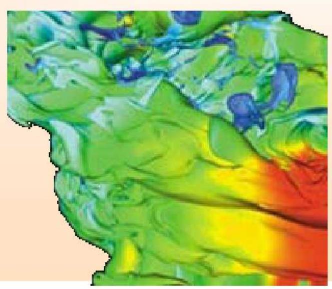
**Renewable Systems & Energy Infrastructure**

**Computer Sciences**

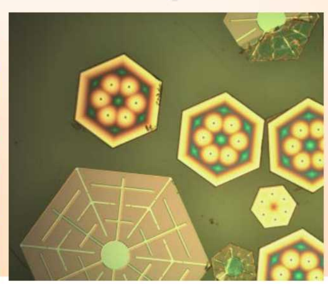
**Materials**



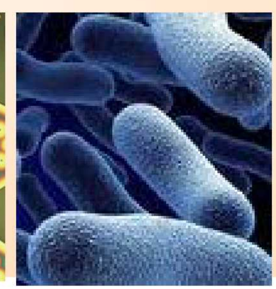
**Engineering Sciences**



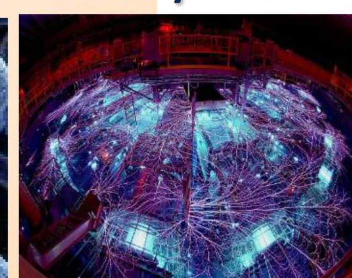
**Nanodevices & Microsystems**



**Bioscience**



**High Energy Density Science**



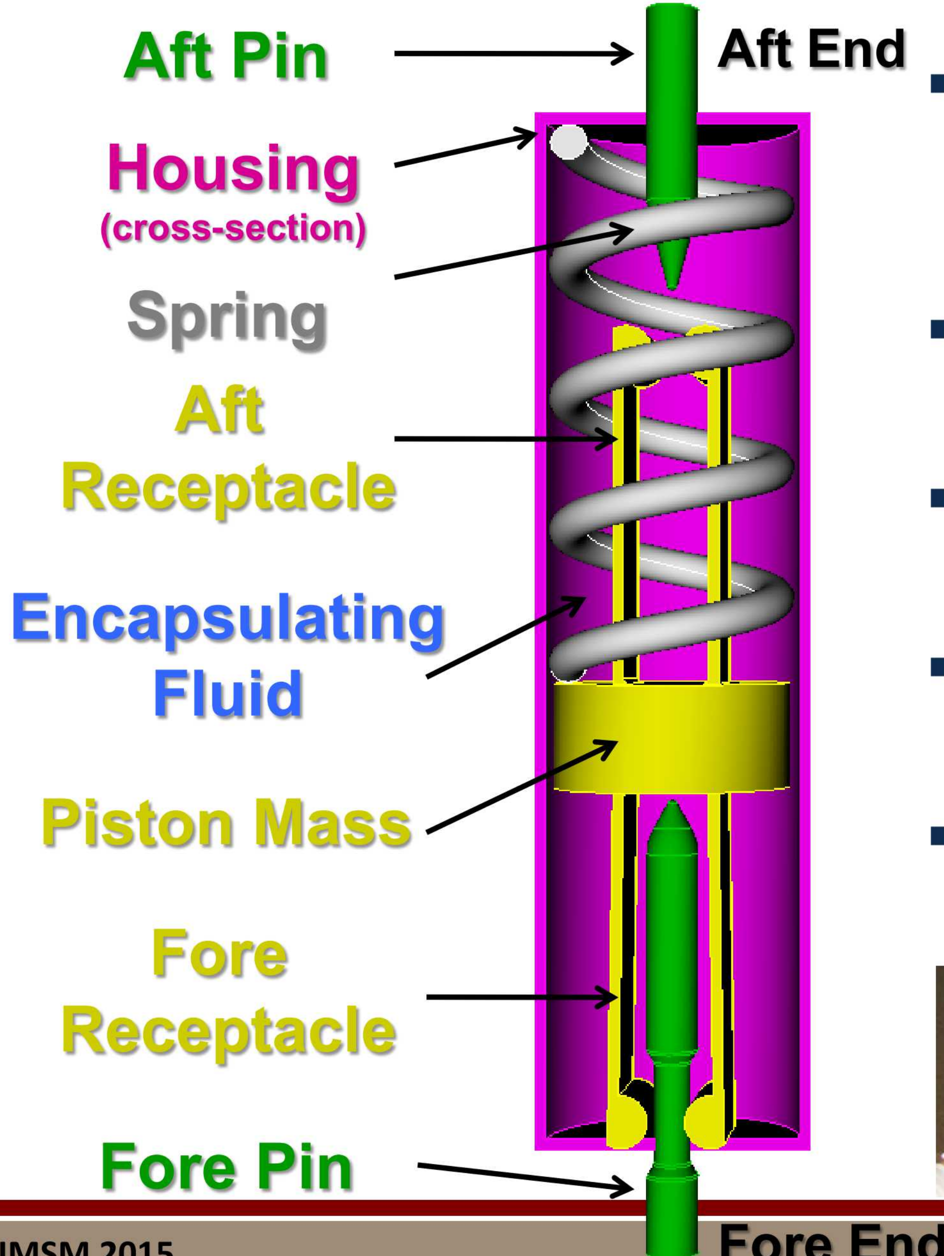
## Research Disciplines

# Unparalleled Test Capabilities

- **From nanometers to kilometers**
- **From nanowatts to gigawatts**
- **..and beyond.**
- **Extreme environments testing.**



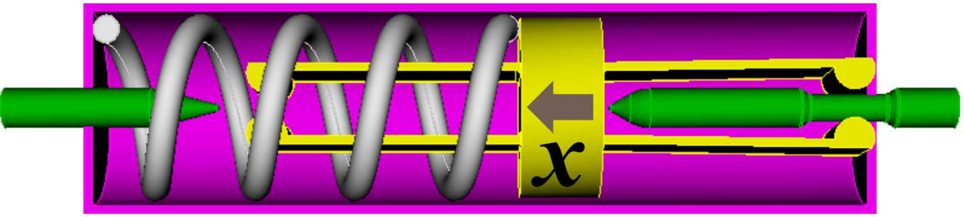
# An Acceleration Switch




- Designed to close during axial acceleration along the track during a test.
- Receptacles contact pins to close and activate instrumentation.
- Housing is filled with damping liquid.
- **Preloaded spring** works against axial movement.
- Our awesome IMSM 2010 team analyzed this switch.

# Model Switch Operation

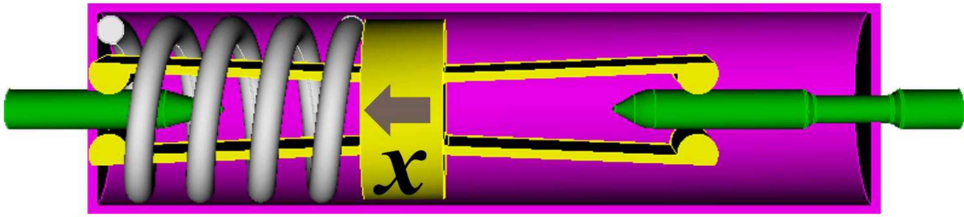
**Open**




**Aft End** **Fore End**

**Closed**

**Sled Acceleration**



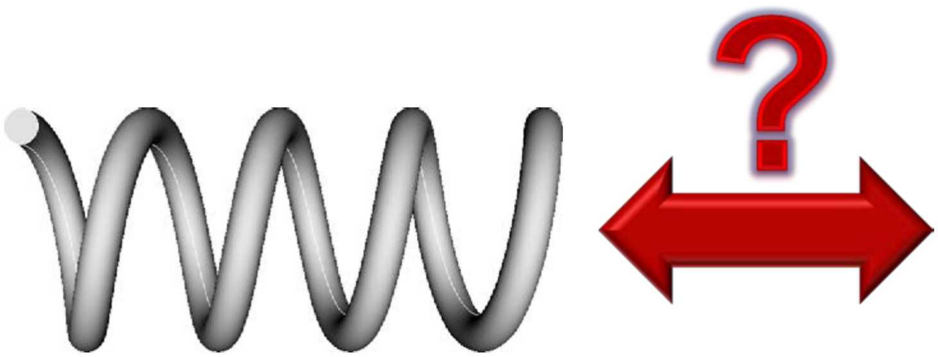

$$m\ddot{x} = F_{sled} - \left( F_{spring} + F_{friction} + F_{contact} + F_{drag} \right)$$

$$x(0) = 0, \dot{x}(0) = 0$$

# Critical Parameter: Spring Force

$$m\ddot{x} = F_{sled} - \left( F_{spring} + F_{friction} + F_{contact} + F_{drag} \right)$$

$$F_{spring} = kx + F_{preload}$$

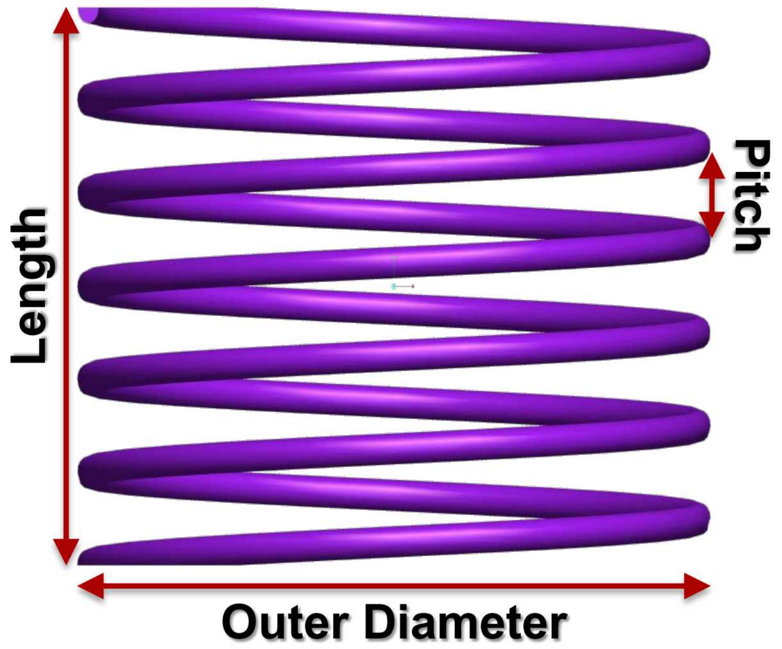


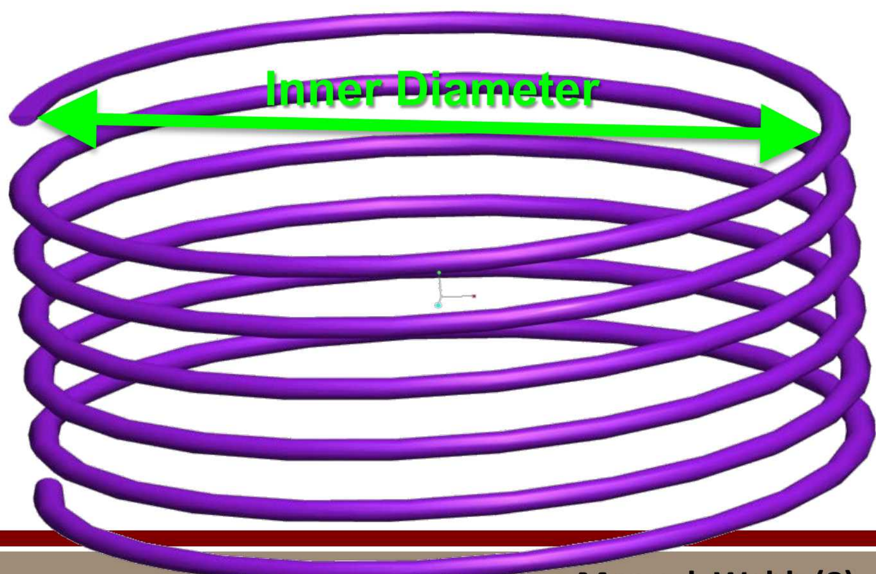
$$k, F_{preload}$$

# Helical Spring Anatomy 101

*Assuming typical compression spring made of round wire.*

- **Free Length**: spring height under no compression.
- **Solid Height**: spring length at full compression (all coils touch).
- **Pitch**: distance between wire centers at free length.
- **Diameters**: wire, spring outer/inner
- **Total Coils**: 1 coil = 360° turn in-plane.
- **Ground Ends**: top/bottom coils are ground flat.
- **Closed/Open Ends**: top/bottom coils attached to adjacent coil.



# Key Spring Properties

### Spring Rate

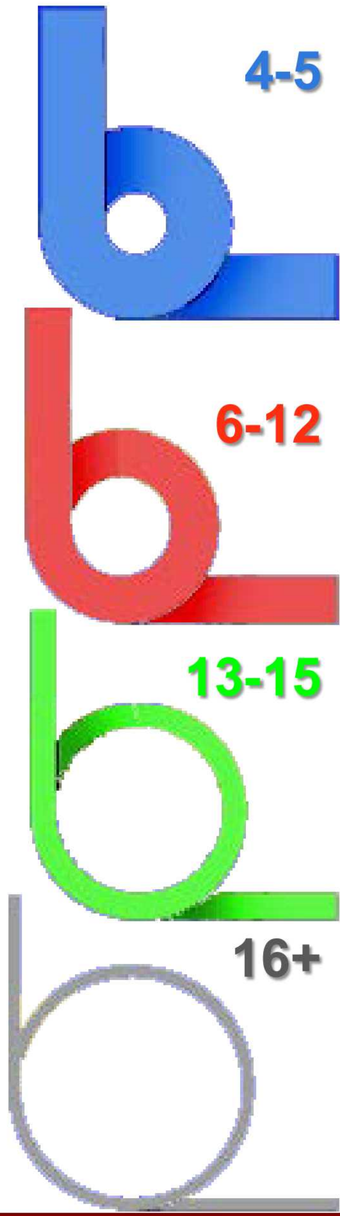
$$K = \frac{G}{8N_a(ec)} \frac{{d_w}^4}{(d_i + d_w)^3}$$

- Effective stiffness of spring in compression.
- Force typically varies linearly with displacement in operating range.
- **Optimal Spring**: specified force when needed, **low Spring Rate** otherwise.

### Spring Index

$$C = \frac{d_i}{d_w} + 1$$

- Determines stress distribution and magnitude, and manufacturability and tolerancing.
- **Optimal Spring**: **low Index**.



# Can we optimize the main spring?

- Low Rate (K) = High Index
- Low Index (C) = High Rate

$$K = \frac{G}{8N_a}\frac{{d_w}^4}{\left(d_i + d_w\right)^3}$$

$$C = \frac{d_i}{d_w} + 1$$

Strategy to keep both $C$ and $K$ low:

- minimize $d_i$ to lower $C$,
- then increase $d_w$ to reach largest "low" $K$ that is acceptable.
- Also, consider more active coils and lower $G$ (new material).

***Simple enough, but…***

# Design Constraints (*just some!*)

**Inner/Outer Diameter Bounds**

$$d_i > d_i^{min}$$

$$d_i < d_o$$

$$d_i + 2d_w < d_o^{max}$$

**Diametral Expansion**

$$d_{expand}\left(d_i, d_w, L_{free}, N_a; \mathrm{ec}\right) < d_o^{max}$$

**Maximum Spring Rate**

$$\frac{G}{8N_a}\frac{d_w^{\,4}}{\left(d_i + d_w\right)^3} \le K_{max}$$

**Maximum Spring Index**

$$\frac{d_i}{d_w} + 1 \le C_{max}$$

**Force Requirement**

$$\left(L_{free} - L_{reset}\right)\frac{G}{8N_a}\frac{d_w^{\,4}}{\left(d_i + d_w\right)^3} = F_{reset}$$

**Coil Binding Gap**

$$\frac{L_{hard} - L_{solid}\left(d_w, N_a; ec\right)}{N_t - 1} \ge g_{min}$$

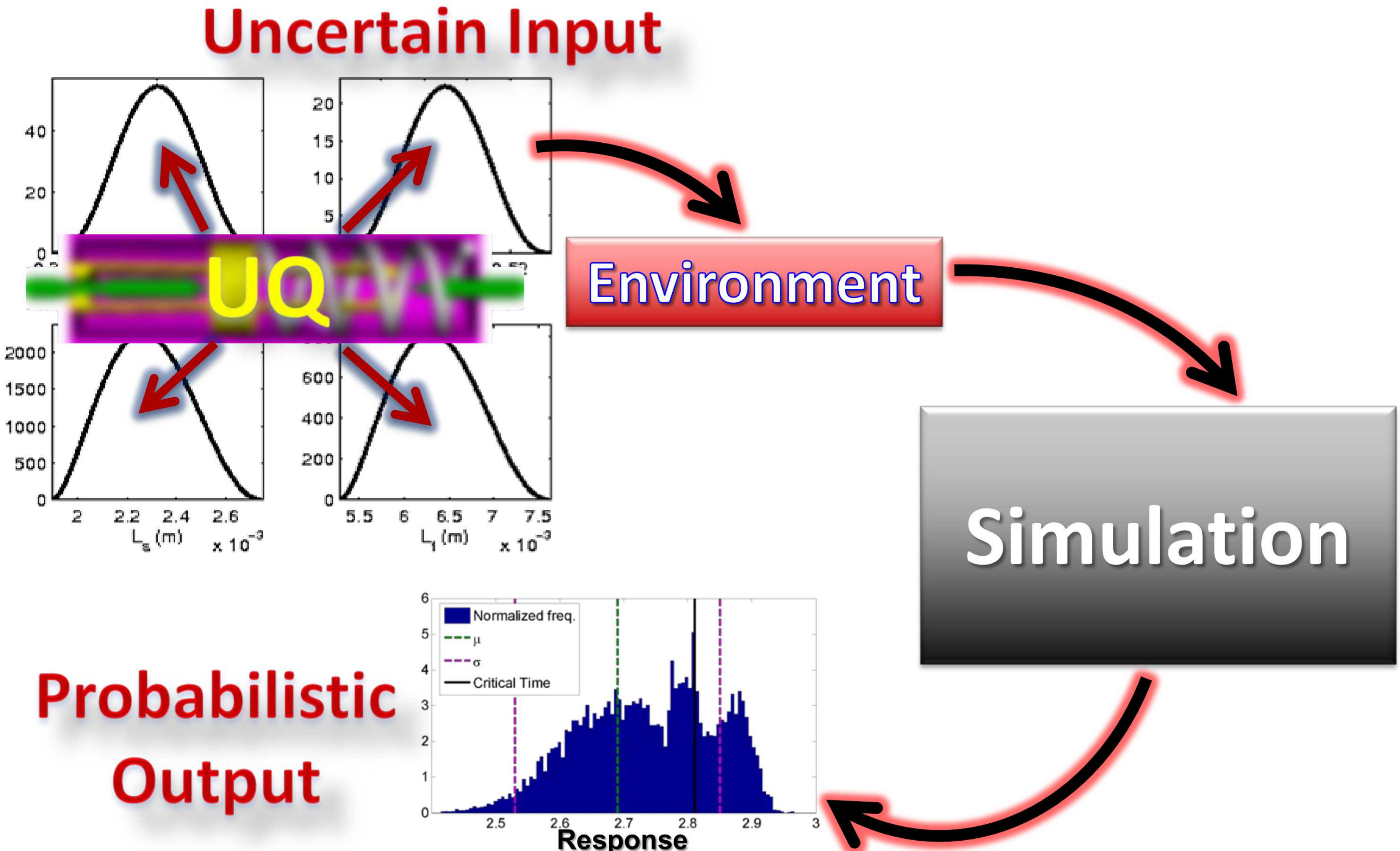
**Buckling Slenderness**

$$\frac{L_{free}}{d_i + d_w} < \pi\sqrt{\frac{2\left(2\nu + 1\right)}{\nu + 2}}$$

**Maximum Shear Stress**

$$\mathrm{UTS} > \frac{G\left(L_{free} - L_{hard}\right)}{4\pi N_a}\left[\frac{d_w\left(4d_i^{\,2} + 9.46 d_i d_w + 3d_w^{\,2}\right)}{d_i\left(d_i + d_w\right)^3}\right]$$

# Uncertainty Analysis



# What Sandia Wants to Know

**Given a helical compression spring**
**in a spring-mass-damper system:**
***What are optimal springs?***

- Questions to address include:
  - What is an algorithm that optimizes springs with interchangeable objectives and constraints?
  - Can you quantify and incorporate spring force relaxation?

***Addressing this problem can help us develop a screening analysis for spring designs.***

- *Seattle's Coffee: are there spring designs that maximize the probability of **spring** performance under uncertain conditions?*
- *Seattle's Best Coffee: are there spring designs that maximize the probability of the **switch** performing under uncertain conditions?*

# Other Class Notes

**Shear Modulus (Modulus of Rigidity)**

$$G = \frac{E}{2(1+\nu)}$$

*homogeneous isotropic materials*

$E$ Young's Modulus
$\nu$ Poisson Ratio

| Property | 302 Stainless Steel |
|---|---|
| Young's Modulus (GPa) | 193 |
| Poisson's Ratio | 0.27-0.30 |
| Shear Modulus (GPa) | 77 |
| Ultimate Tensile Strength (GPa)* | 2.0 |
| CTE (ppm/K) | 17.6 |
| Density (g/cm$^3$) | 8.0 |

*Ultimate Torsional/Shear Strength is assumed to be 35-50% of Ultimate Tensile Strength

# Spring Index Relations

- Defined as ratio of mean diameter to wire diameter.
- Mean diameter is average of inner and outer diameters.
- Wire diameter is related to inner and outer diameters.
- Index can be written in terms of inner and wire diameters.

$$C = \frac{d_m}{d_w}$$

$$d_m = \left(d_o + d_i\right)/2$$

$$d_w = \left(d_o - d_i\right)/2$$

$$d_m = d_w + d_i$$

$$\Rightarrow C = \frac{d_i}{d_w} + 1$$

# Spring End Conditions (ec)

**Closed = End Coils Welded | Open = End Coils Not Welded | Ground = End Coils Flattened**

| Dependent Parameter | Closed-Ground | Open-Ground |
|---|---|---|
| Number of Total Coils $N_t(N_a)$ | $N_a + 2$ | $N_a + 1$ |
| Pitch $p_{ec}(d_w, L_{free}, N_a)$ | $\dfrac{L_{free} - 2d_w}{N_a}$ | $\dfrac{L_{free}}{N_a + 1}$ |
| Solid Height $L_{solid}(d_w, N_a)$ | $(N_a + 2)d_w$ | $(N_a + 1)d_w$ |
| Diametral Expansion $d_{expand}(d_i, d_w, L_{free}, N_a)$ | Formula Closed | Formula Open |

- $N_a$: "average" number of active coils estimated over spring deflection; can be chosen, but typically calculated given $N_t$.
- **Pitch**: used to calculate Diametral Expansion.
- **Solid Height**: used to calculate Coil Binding Gap at compressed state.
- **Diametral Expansion**: spring OD increases when spring is compressed.

# Diametral Expansion (Closed Springs)

- Formula for compression spring with ends fixed against rotation about helix axis (Wahl, 1953):

$$d_{expand}\left(d_i, d_w, L_{free}, N_a\right) = d_w + \sqrt{\left(d_i + d_w\right)^2 + \frac{p_{closed}\left(d_w, L_{free}, N_a\right)^2 - d_w^{\ 2}}{\pi^2}}$$

- Assumes compression from free length to solid height (maximum diameter expansion expected).
- Most commonly used relation.
- May substantially underestimate true diameter expansion (Bruns, 2012).

# Diametral Expansion (Open Springs)

- Wahl (1953) has a derivation for Open springs that additionally includes Poisson ratio.
- Few (if any) use it because his equations are difficult to solve without several simplifications.
- Maintaining fewer assumptions, solution for $d_{expand}$ reduces to roots of a cubic polynomial.
- Predicts larger expansion compared to Closed case.

$$A_3 d_{expand}{}^3 + A_2 d_{expand}{}^2 + A_1 d_{expand} + A_0 = 0$$

$$A_3 = -\pi^2 (1+\nu) d_m$$

$$A_2 = \pi^4 (1+\nu) d_m{}^2 + 3\pi^4 (1+\nu) d_w d_m + \pi^2 p_{open}(d_w, L_{free}, N_a)\left[(1+\nu) p_{open}(d_w, L_{free}, N_a) - d_w\right]$$

$$A_1 = -2\pi^4 (1+\nu) d_w d_m{}^2 - \pi^2 (1+3\pi^2)(1+\nu) d_w{}^2 d_m - 2\pi^2 d_w p_{open}(d_w, L_{free}, N_a)\left[(1+\nu) p_{open}(d_w, L_{free}, N_a) - d_w\right];$$

$$A_0 = \pi^2\left[\pi^2(1+\nu)+1\right] d_w{}^2 d_m{}^2 + \pi^2(\pi^2+1)(1+\nu) d_w{}^3 d_m + d_w{}^2 p_{open}(d_w, L_{free}, N_a)\left[(\pi^2+1)\left(p_{open}(d_w, L_{free}, N_a) - d_w\right) + \pi^2 \nu\, p_{open}(d_w, L_{free}, N_a)\right]$$

$$d_m = d_i + d_w$$

# Quantify Optimization

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- ***Minimize*** $J$ over chosen design variables, subject to all constraints.
- $J$ is a weighted sum of normalized Spring Rate and Index.
- Normalization factors are chosen by the customer.
- Choose weights to tailor optimized designs.

**Objective Function**

$$J = a_K \frac{K\left(d_i, d_w, N_a, G\right)}{K_{max}} + a_C \frac{C\left(d_i, d_w\right)}{C_{max}}$$

**Normalization Factors**

$$K_{max}, C_{max}$$

**Weights**

$$a_K, a_C$$

$a_k$ is 0 or 1

$a_c$ is ~4

# Design Optimization Parameters

- There are many design variables, but problem is highly constrained: optimization problem reduces to 2 essential design variables and $N_a$:
  - Choose $d_w$ since wire is manufactured at specifiable diameters.
  - Choose $d_i$ as other optimization variable.
- $L_{free}$ is uniquely determined given $F_{reset}$ and $L_{reset}$.
- Resulting Parameters: $N_a$, **2 design**, **5 objective**, **9 constraint**.

**Main Spring Design Optimization Problem**

$$\min_{d_i, d_w} J\left(N_a; G, K_{max}, C_{max}, a_K, a_C; F_{reset}, L_{reset}, L_{hard}, g_{min}, d_i^{min}, d_o^{max}, \nu, \mathrm{UTS}, \mathrm{ec}\right)$$



$G$ is also a constraint parameter.