Ordinary, Isotropic Constitutive Models for Peridynamics

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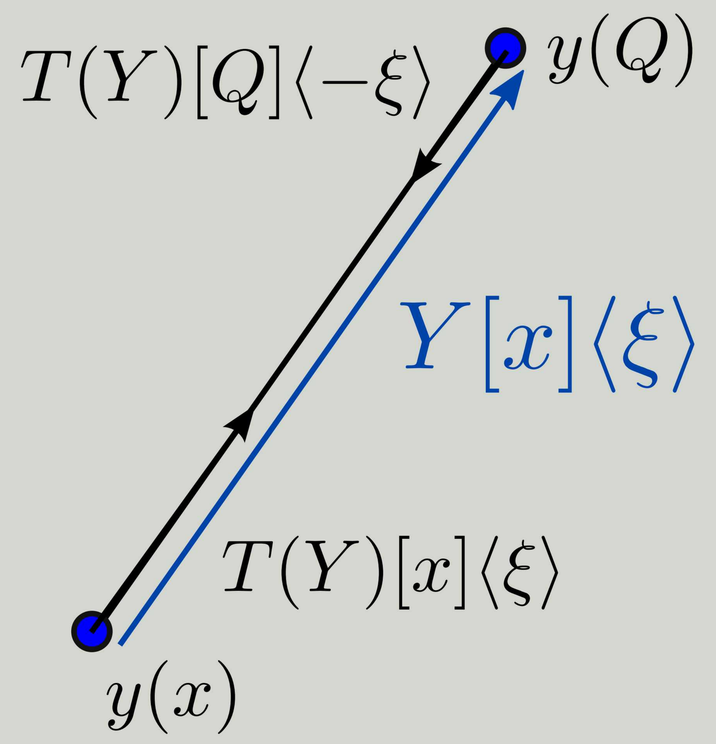
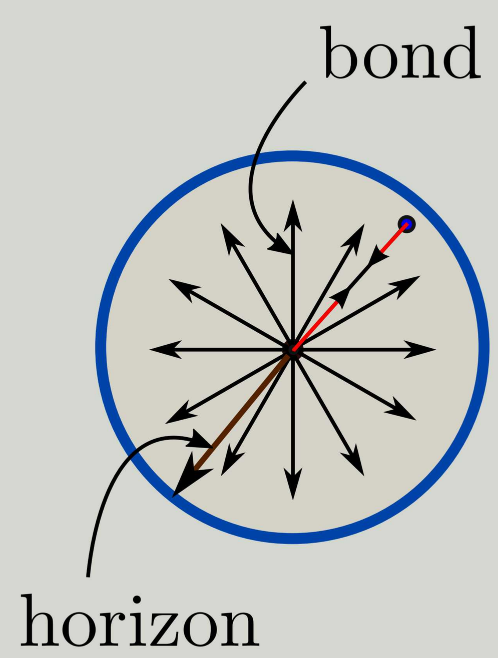
Introduction[1]

Peridynamics and momentum equation

$$\rho(x)\ddot{u}(x,t) = \int_H \{T(Y)[x]\langle\xi\rangle - T(Y)[Q]\langle -\xi\rangle\}\, dV_Q + b(x,t)$$



Deformed configuration



Infinitely many bonds ξ within neighborhood H

Ordinary material models

$$T(Y) = \underline{t}(Y)\frac{Y}{|Y|}, \text{ scalar force state } \underline{t}(Y)$$

Kinematics

Scalar extension state $\underline{e}\langle\xi\rangle = |Y| - |\xi|$

Dilatation $\theta = (\underline{\omega}|\xi|) \bullet \underline{e} = \int_H \underline{\omega}|\xi|\underline{e}\langle\xi\rangle\, dV_Q$

Deviatoric extension state $\underline{\varepsilon} = \underline{e} - \frac{\theta|\xi|}{3}$

Position aware models[2]

Scalar force state obtained from elastic energy density functional

$$W(\theta, \underline{\varepsilon}) = \frac{\kappa\theta^2}{2} + \mu(\underline{\sigma}\underline{\varepsilon}) \bullet \underline{\varepsilon}$$

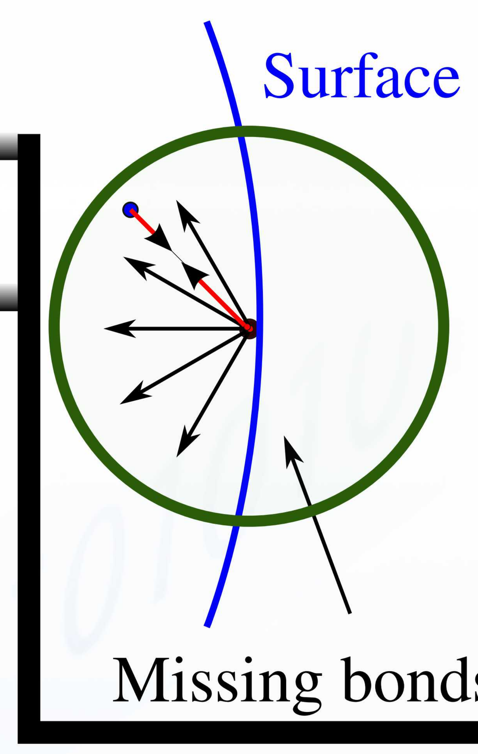
$$\underline{t}(Y) = p\underline{\omega}\underline{x} + 2\mu\underline{\sigma}\underline{\varepsilon}$$

Inelastic extension states[3,4]

$$\underline{t}(Y) = p\underline{\omega}\underline{x} + 2\mu\underline{\sigma}\underbrace{(\underline{\varepsilon} - \underline{\varepsilon}^p)}_{elastic}$$

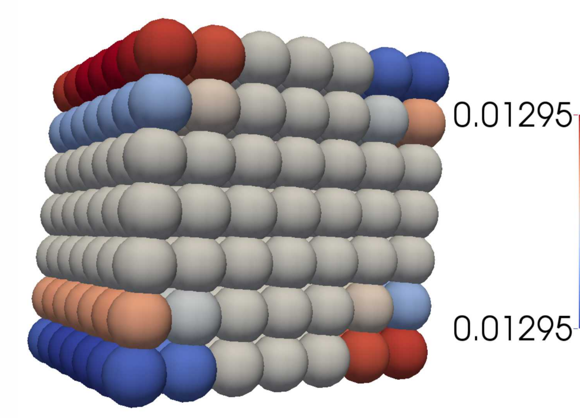
Construction of influence functions

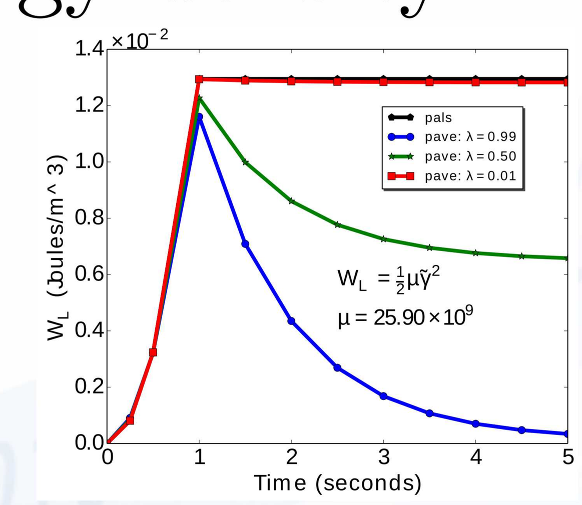
- $\underline{\omega}$, $\underline{\sigma}$ computed at each mesh point
- Initial influence functions $\underline{\omega}^0$, $\underline{\sigma}^0$ given
- Select $\underline{\omega}$, $\underline{\sigma}$ as best approximations to $\underline{\omega}^0$,$\underline{\sigma}^0$ subject to constraints: *matching deformations* $\underline{e}^k\langle\xi\rangle = \frac{\xi \cdot H^k \xi}{|\xi|}$
- $\underline{\omega}$, $\underline{\sigma}$ compensate for surface effects due to missing bonds



Simple demonstrations

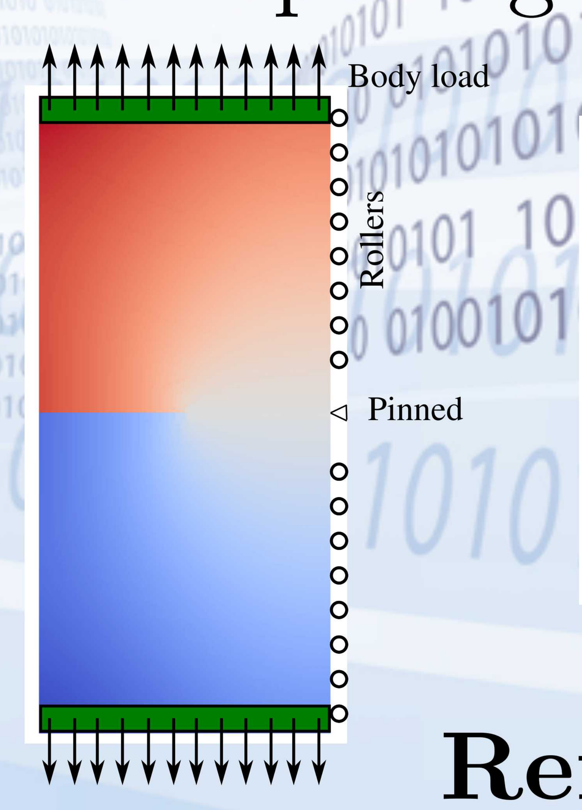
Stored elastic energy density

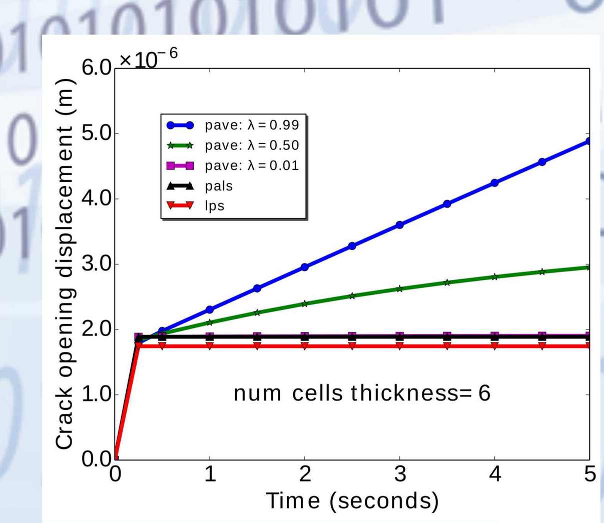


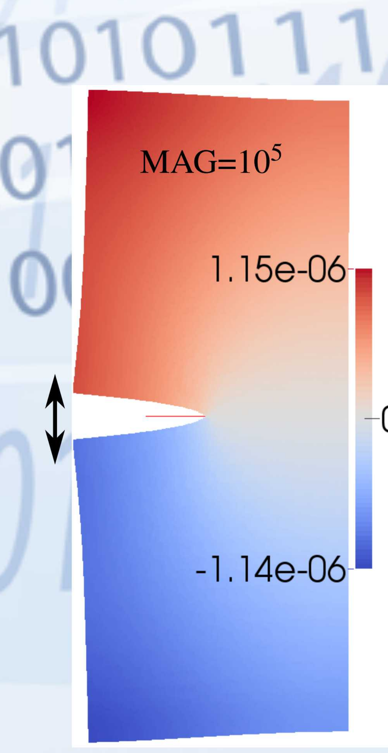


- LPS: linear peridynamic solid
- PALS: position aware linear solid
- PAVÉ: position aware viscoelastic

Crack opening displacement







References

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2. J.A. Mitchell, S.A. Silling and D.J. Littlewood, A Position-Aware Linear Solid Constitutive Model for Peridynamics, *Journal of Mechanics of Materials and Structures* (to appear 2016).
3. John A. Mitchell, A nonlocal, ordinary, state-based plasticity model for peridynamics, Sandia National Laboratories tech report, SAND2011-3166,(May 2011).
4. John A. Mitchell, A nonlocal, ordinary, state-based viscoelasticity model for peridynamics, Sandia National Laboratories tech report, SAND2011-8064,(October 2011).

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