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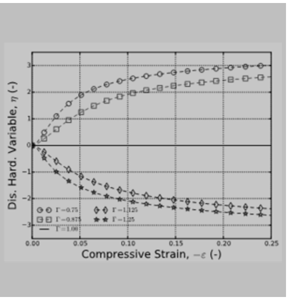

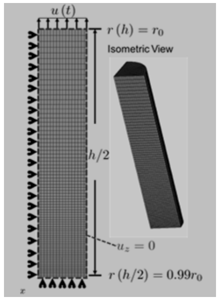

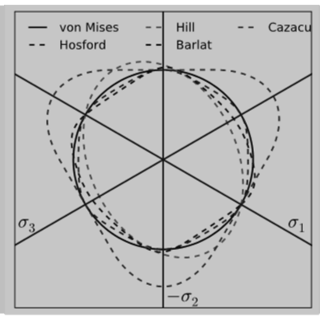

A New Approach to Distortional Hardening

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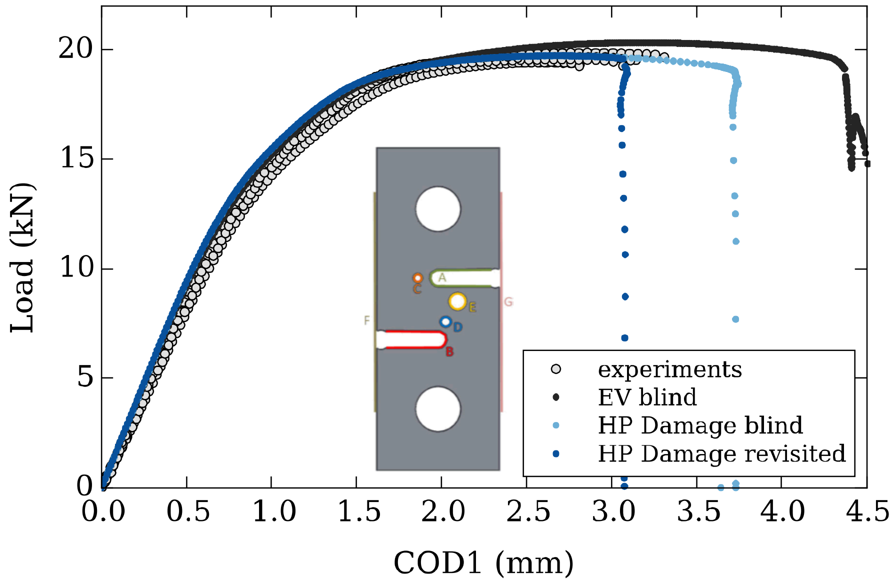
Anisotropic Plasticity

- Plastic anisotropy needed for complex, multiaxial loadings of structures
  - Manufacturing processes (e.g. sheet metal forming)
  - Ductile failure

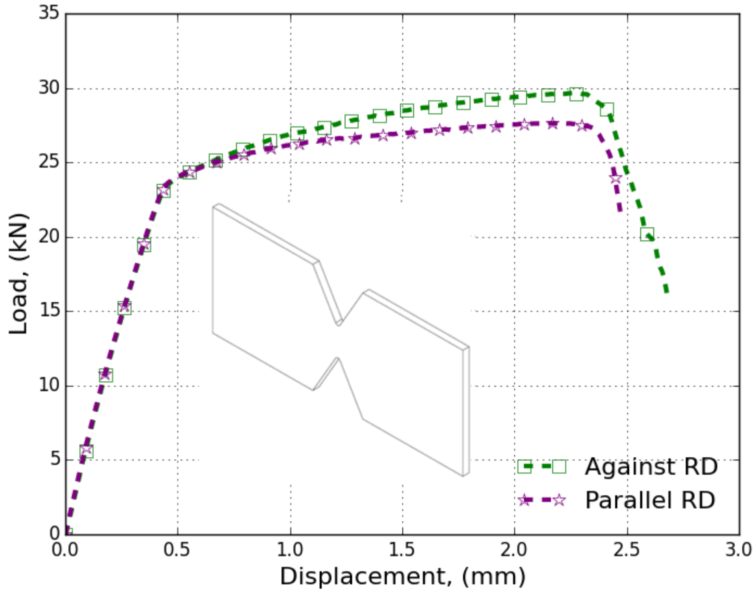
2nd Sandia Fracture Challenge (SFC2) (Ti-6Al-4V)

Isotropic (EV) and anisotropic (HP – Hill) failure predictions



Karlson et al., 2016, *Int. Jrnl. Frac.*, 198: 179-195

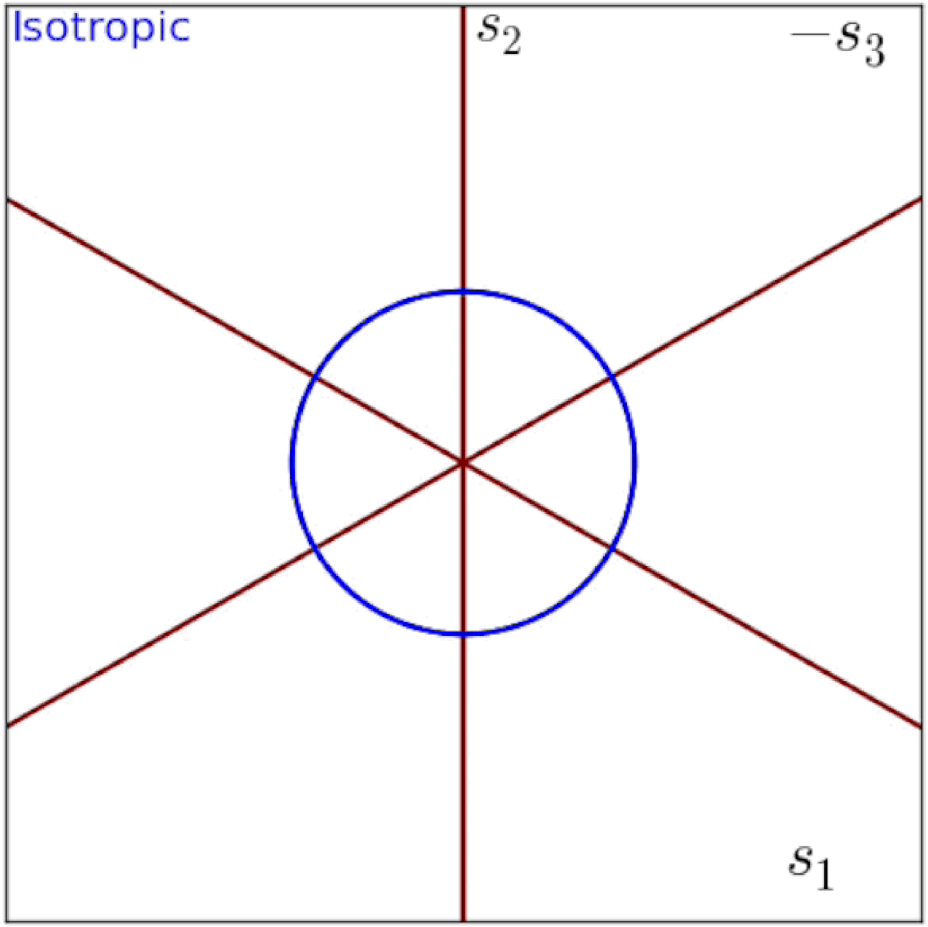
Notched Shear Calibration Data for SFC2



Boyce et al., 2016, *Int. Jrnl. Frac.*, 198: 5-100

Plastic Hardening

- Capturing multiaxial, history dependent response requires description of anisotropic yield and *hardening*



Distortional Hardening

- Need computationally efficient, flexible distortional hardening model
- Existing implementations:
  - Expensive and/or difficult to implement – HAH (e.g. Barlat et al.), Projection Tensor (e.g. Feigenbaum and Dafalias, Shi and Mosler)
  - Thermodynamic issues/calibration specific -- "Isotropic Distortion" (e.g. Aretz, Plunkett et al.)
- Current objective: Development of new distortional hardening model
  - Simplified way of introducing distortional effects
  - Thermodynamically consistent
  - *Amenable to 3D numerical implementation*

MODELING

Free Energy

State Variables

Traditional: $\varepsilon_{ij}^{\mathrm{el}}, \quad \kappa$ New Distortional ISV: η

Free Energy

$$\psi = \hat{\psi}\left(\varepsilon_{ij}^{\mathrm{el}}, \kappa, \eta\right) = \psi^{\mathrm{el}}\left(\varepsilon_{ij}^{\mathrm{el}}\right) + \psi^{\mathrm{iso}}(\kappa) + \psi^{\mathrm{dis}}(\eta)$$

$$\psi^{\mathrm{el}}\left(\varepsilon_{ij}^{\mathrm{el}}\right) = \frac{1}{2\rho}\varepsilon_{ij}^{\mathrm{el}}\mathbb{C}_{ijkl}\varepsilon_{kl}^{\mathrm{el}} \quad \psi^{\mathrm{iso}}(\kappa) = \frac{1}{\rho}g(\kappa) \quad \psi^{\mathrm{dis}}(\eta) = \frac{1}{\rho}h(\eta)$$

- "Traditional" state variables
  - Elastic strain tensor, $\varepsilon_{ij}^{\mathrm{el}}$
  - Isotropic hardening variable (IHV), κ
- Introduce single scalar ISV for distortional hardening, η
  - Encapsulates all microstructural effects of distortional hardening
  - Likely multiple mechanisms
- Assume isotropic and distortional energetic effects are independent and separable

Constitutive Behavior

$$\sigma_{ij} = \mathbb{C}_{ijkl}\varepsilon_{kl}^{\mathrm{el}} \quad \varepsilon_{ij} = \varepsilon_{ij}^{\mathrm{el}} + \varepsilon_{ij}^{\mathrm{p}}$$

Dissipation Inequality

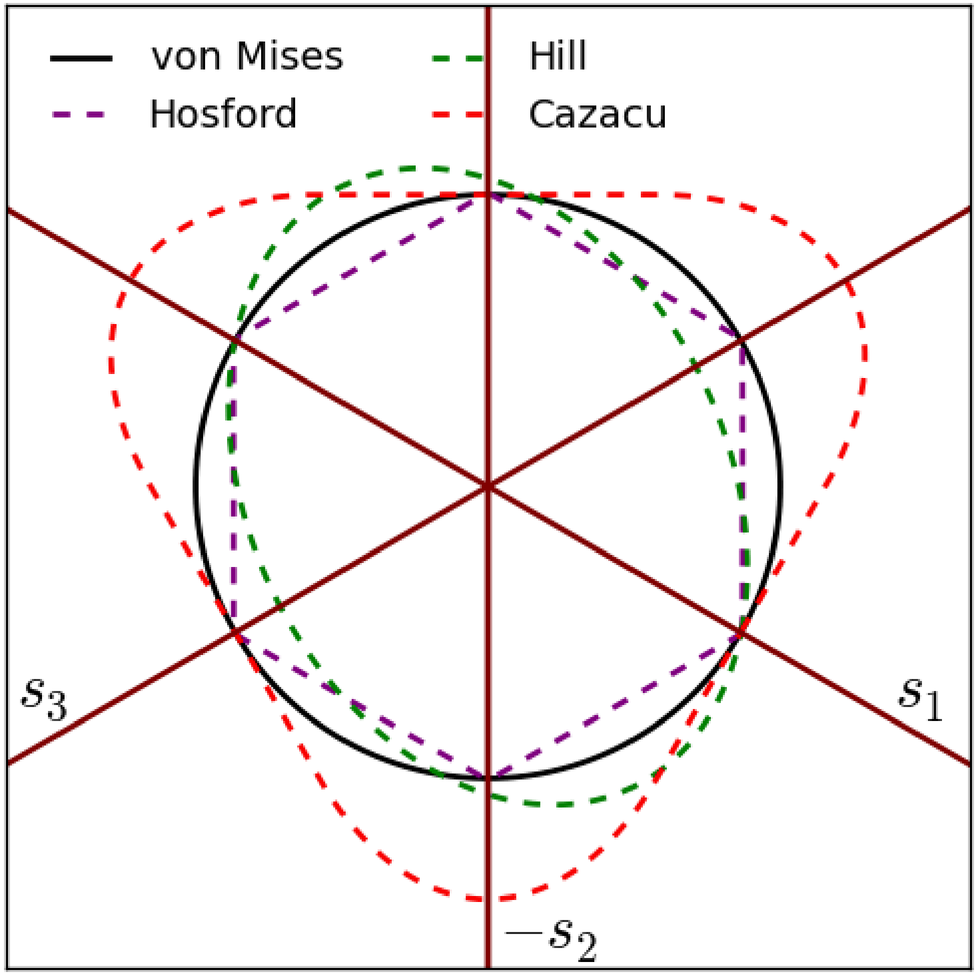
$$\mathcal{D} = \sigma_{ij}\dot{\varepsilon}_{ij}^{\mathrm{p}} - K\dot{\kappa} - N\dot{\eta} \geq 0$$

$$K := \rho\frac{\partial\psi}{\partial\kappa} = \frac{\partial g}{\partial\kappa} \qquad N := \rho\frac{\partial\psi}{\partial\eta} = \frac{\partial h}{\partial\eta}$$

Traditional Yield Functions

$$f = f\left(\sigma_{ij}, K\right) = \phi\left(\sigma_{ij}\right) - \sigma_y\left(K\right)$$

$\phi\left(\sigma_{ij}\right)$ Effective Stress $\quad \sigma_y\left(K\right) = \sigma_y^0 + K$ Flow Stress



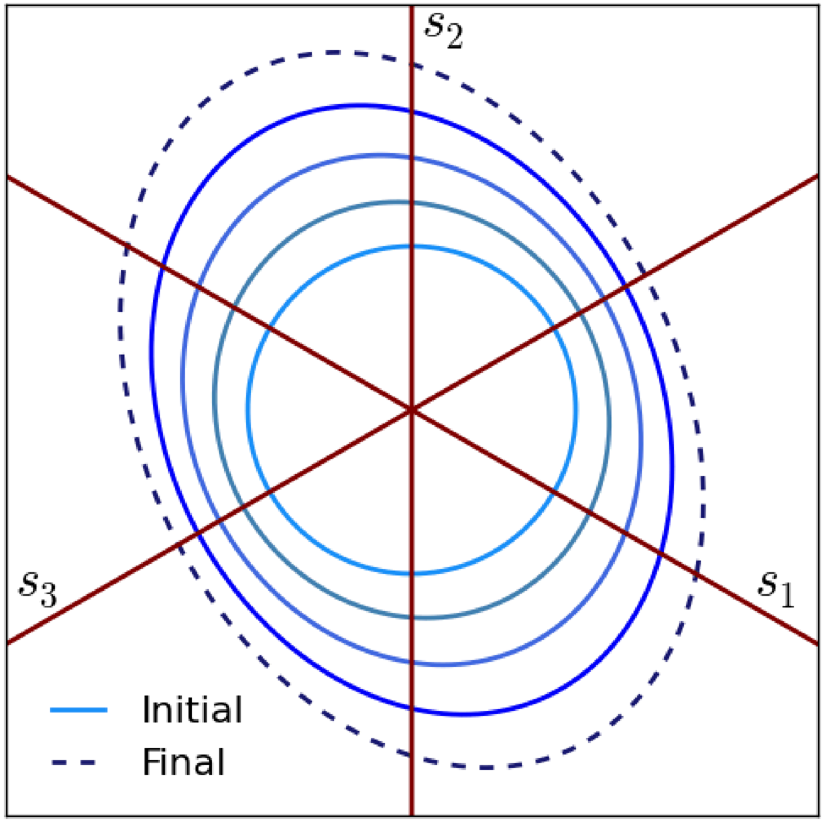
- Many existing effective stress definitions:
  - Non-quadratic
  - Strength-differential
  - Anisotropic
- Can they be leveraged for distortional capabilities?

New Yield Function

$$f = f(\sigma_{ij}, K, N) = \phi(\sigma_{ij}, N) - \sigma_y(K)$$

$$\phi(\sigma_{ij}, N) = \sum_{k=1}^{n_{\mathrm{es}}} \zeta^{(k)}(N)\phi^{(k)}(\sigma_{ij}) \qquad \sum_{k=1}^{n_{\mathrm{es}}} \zeta^{(k)} = 1\ ;\ \zeta^{(k)} \geq 0$$

- Introduce a new ”Evolving Effective Stress” (EES)
- “Mix and Match” effective stress combinations
- Weighted sum of different definitions for desired features
- Use/Evolve distortional variable to change weights



Evolution Equations

- Evolution equations found by trying to maximize dissipation
- Flow rules correspond to Karush-Kuhn-Tucker conditions:

$$\dot{\kappa} = \lambda$$

$$\dot{\varepsilon}^{\mathrm{p}}_{ij} = \lambda \frac{\partial \phi}{\partial \sigma_{ij}} \qquad \lambda f\left(\sigma_{ij}, K, N\right) = 0$$

$$\dot{\eta} = -\lambda \frac{\partial \phi}{\partial N}$$

- Leads to rate of dissipation density of

$$\mathcal{D} = \left(\sigma_y^0 + N \frac{\partial \phi}{\partial N}\right) \dot{\kappa}$$

Can be positive or negative

Weighting Function Definition

- For current cases consider a two effective stress definition

$$\phi\left(\sigma_{ij}, N\right) = \zeta\left(N\right)\phi^{(1)}\left(\sigma_{ij}\right) + \left(1 - \zeta\left(N\right)\right)\phi^{(2)}\left(\sigma_{ij}\right)$$

$$\frac{\partial \phi}{\partial N} = \frac{\partial \zeta}{\partial N}\left(\phi^{(1)} - \phi^{(2)}\right)$$

- For weighting functions want functions
  - Have non-zero initial derivatives
  - Satisfy previous constraints
  - Eventually saturate
  - Continuous

$$\zeta = \exp\left(-kN\right) \qquad N\left(\eta\right) = \frac{1}{2}P^{\text{mod}}\eta^2$$

$k,\ P^{\text{mod}}$ Fitting constants

Numerical Implementation

Sandia National Laboratories

- Use Line –Search Augmented Newton Raphson (LS-NR) approach

Minimize $$\psi = \frac{1}{2}\left[\left(\frac{E}{\sigma_y^0}\right)^2 r^{\varepsilon}_{ij} r^{\varepsilon}_{ij} + \left(\frac{r^f}{\sigma_y^0}\right)^2 + \left(\frac{P^{\text{mod}} r^{\eta}}{\sigma_y^0}\right)^2\right]$$

Residuals

System of Linearized Residuals

Consistency
$$r^{f(k)} = f(\sigma_{ij}, \kappa, \eta)$$
$$-r^{f(k)} = \frac{\partial \phi}{\partial \sigma_{ij}} \Delta\sigma_{ij} - \frac{\partial \sigma_y}{\partial \kappa}\Delta\kappa + \frac{\partial \phi}{\partial \eta}\Delta\eta$$

Plastic Strain Flow Rule
$$-r^{\varepsilon(k)}_{ij} = \mathscr{L}^{-1}_{ijkl}\Delta\sigma_{kl} + \Delta\kappa \frac{\partial \phi}{\partial \sigma_{ij}} + d\kappa \frac{\partial^2 \phi}{\partial \sigma_{ij}\partial \eta}\Delta\eta$$

DHV Flow Rule
$$r^{\eta(k)} = d\eta - d\kappa \frac{\partial \phi}{\partial N}$$
$$-r^{\eta(k)} = d\kappa \frac{\partial^2 \phi}{\partial N \partial \sigma_{ij}}\Delta\sigma_{ij} + \frac{\partial \phi}{\partial N}\Delta\kappa - \left(1 - d\kappa \frac{\partial^2 \phi}{\partial N \partial \eta}\right)\Delta\eta$$

"Classical" solution for isotropic hardening plasticity

$$\Delta\kappa = \frac{-\frac{\partial \phi}{\partial \sigma_{ij}}\mathscr{L}_{ijkl} r^{\varepsilon}_{kl} + r^f}{\frac{\partial \phi}{\partial \sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial \phi}{\partial \sigma_{kl}} + \frac{\partial \sigma_y}{\partial \kappa}}$$

$$\Delta\kappa = \frac{-\frac{\partial \phi}{\partial \sigma_{ij}}\mathscr{L}_{ijkl} r^{\varepsilon}_{kl} + r^f + \frac{1}{\omega}\left(\frac{\partial \phi}{\partial \eta} - d\kappa \frac{\partial \phi}{\partial \sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial^2 \phi}{\partial \sigma_{kl}\partial \eta}\right)\left(r^{\eta} - d\kappa \frac{\partial^2 \phi}{\partial N \partial \sigma_{ij}}\mathscr{L}_{ijkl} r^{\varepsilon}_{kl}\right)}{\frac{\partial \phi}{\partial \sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial \phi}{\partial \sigma_{kl}} + \frac{\partial \sigma_y}{\partial \kappa} + \frac{1}{\omega}\left(\frac{\partial \phi}{\partial \eta} - d\kappa \frac{\partial \phi}{\partial \sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial^2 \phi}{\partial \sigma_{kl}\partial \eta}\right)\left(\frac{\partial \phi}{\partial N} - d\kappa \frac{\partial \phi}{\partial \sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial^2 \phi}{\partial \sigma_{kl}\partial N}\right)}$$

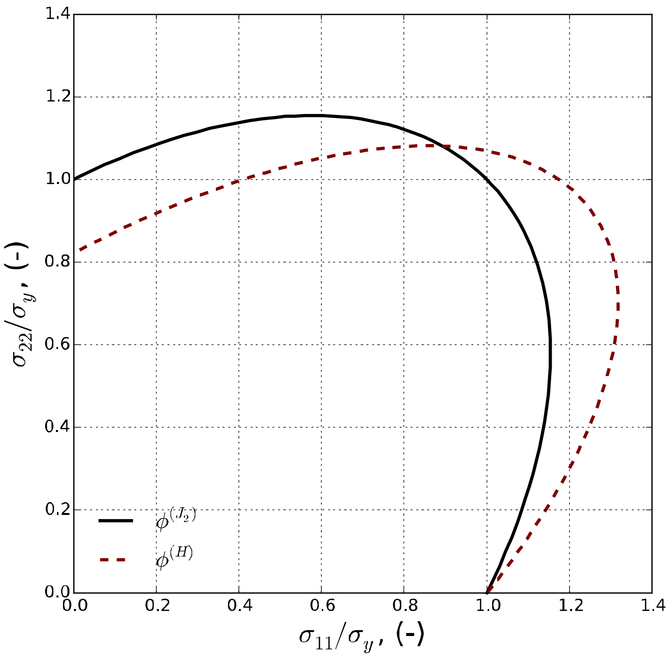
RESULTS

Anisotropy Evolution

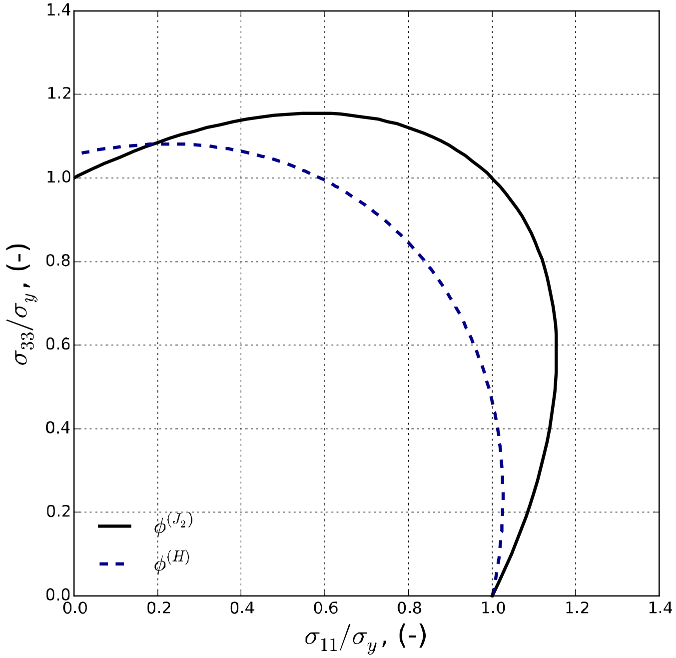
- Want to investigate evolution of anisotropy
- Consider case of von Mises evolving to anisotropic Hill ('48)

$$\left(\phi^{(\mathrm{H})}\left(\sigma_{ij}\right)\right)^2 = F\left(\hat{\sigma}_{22}-\hat{\sigma}_{33}\right)^2 + G\left(\hat{\sigma}_{33}-\hat{\sigma}_{11}\right)^2 + H\left(\hat{\sigma}_{11}-\hat{\sigma}_{22}\right)^2$$
$$+2L\hat{\sigma}_{23}^2 + 2M\hat{\sigma}_{31}^2 + 2N\hat{\sigma}_{12}^2$$

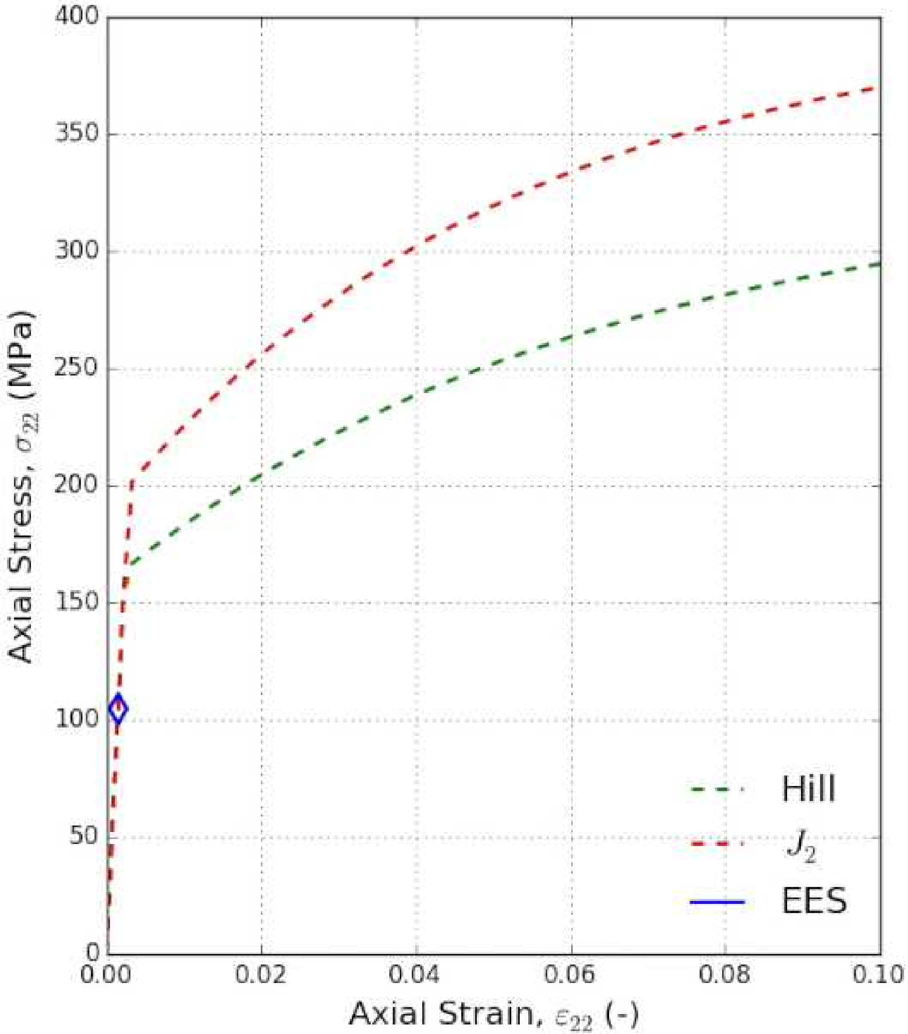
$\sigma_{11}-\sigma_{22}$ Yield surfaces

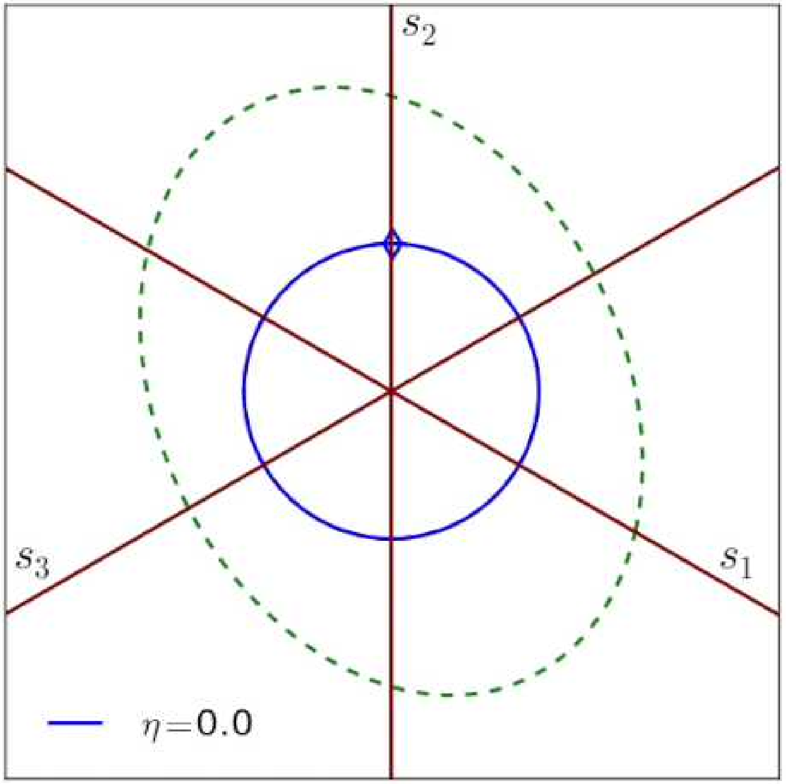


$\sigma_{11}-\sigma_{33}$ Yield surfaces



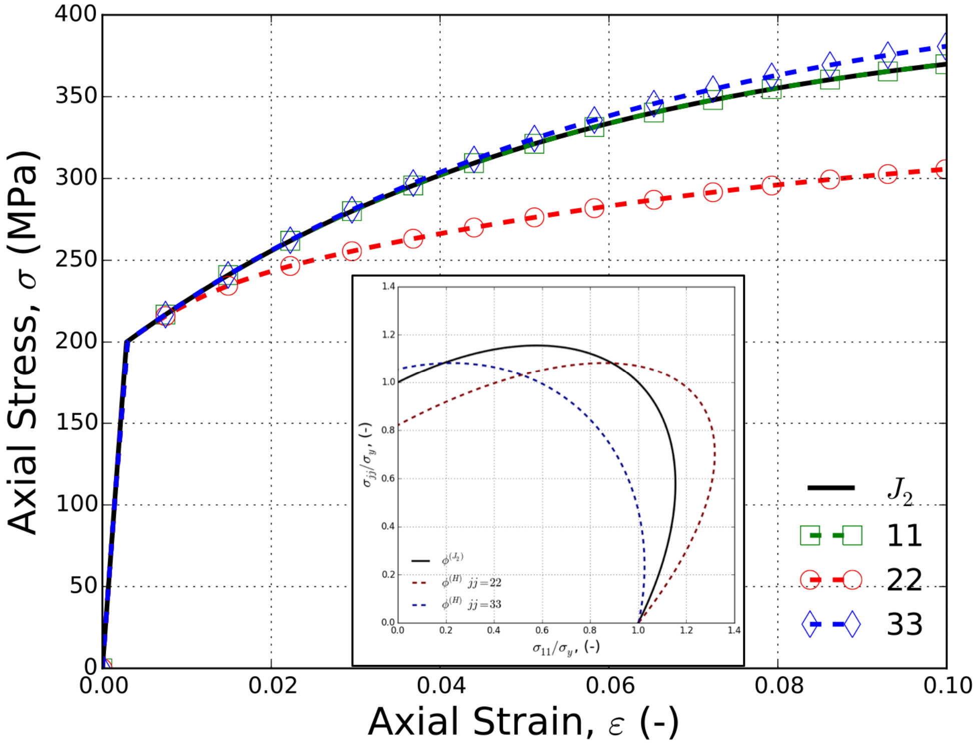
Evolving Effective Stress

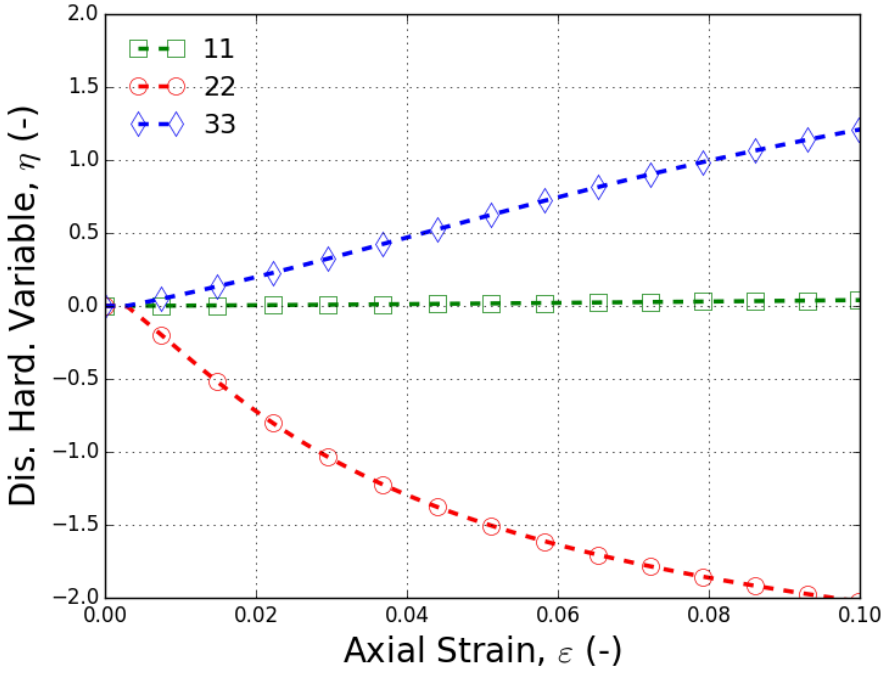




- New EES model able to capture distortional hardening

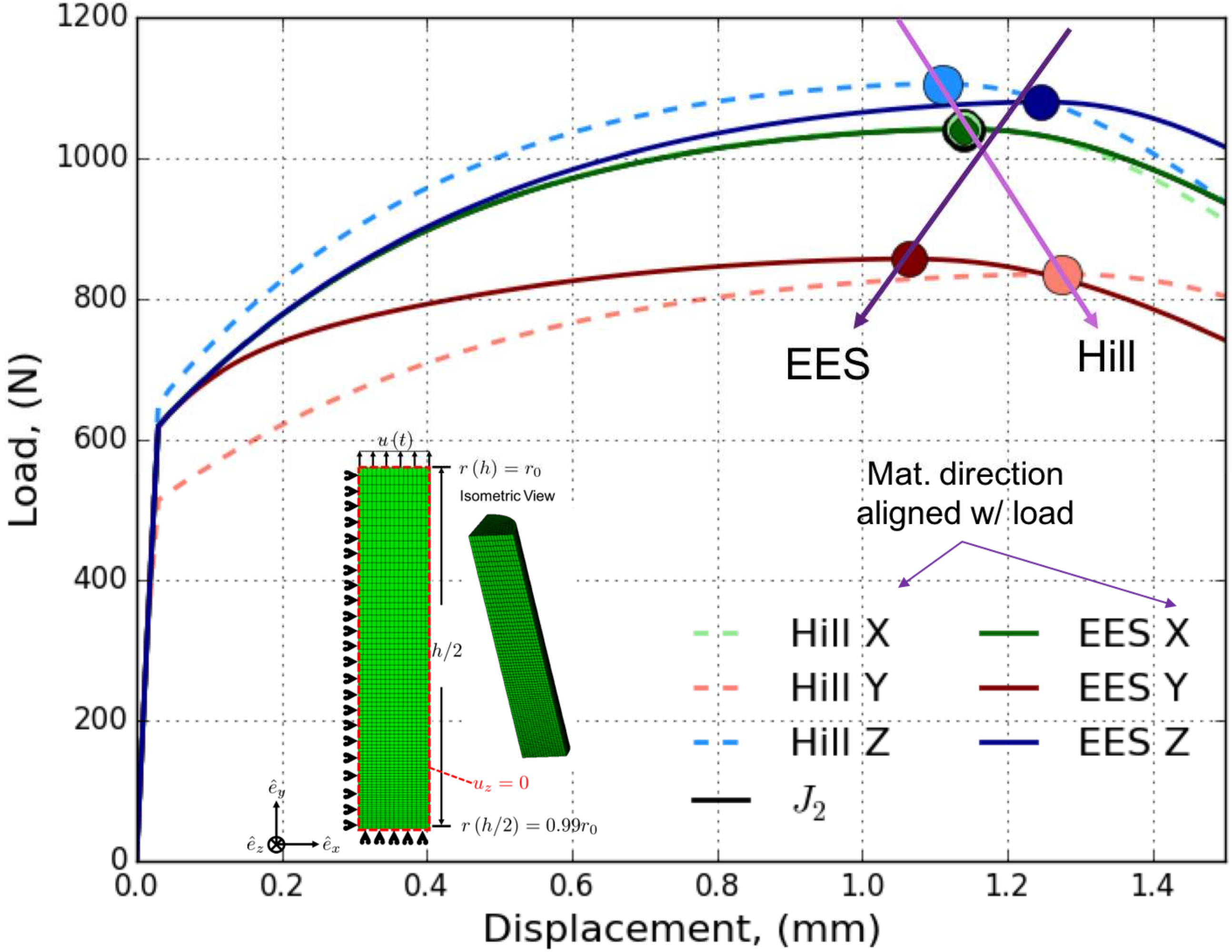
Constitutive Behavior - Hill





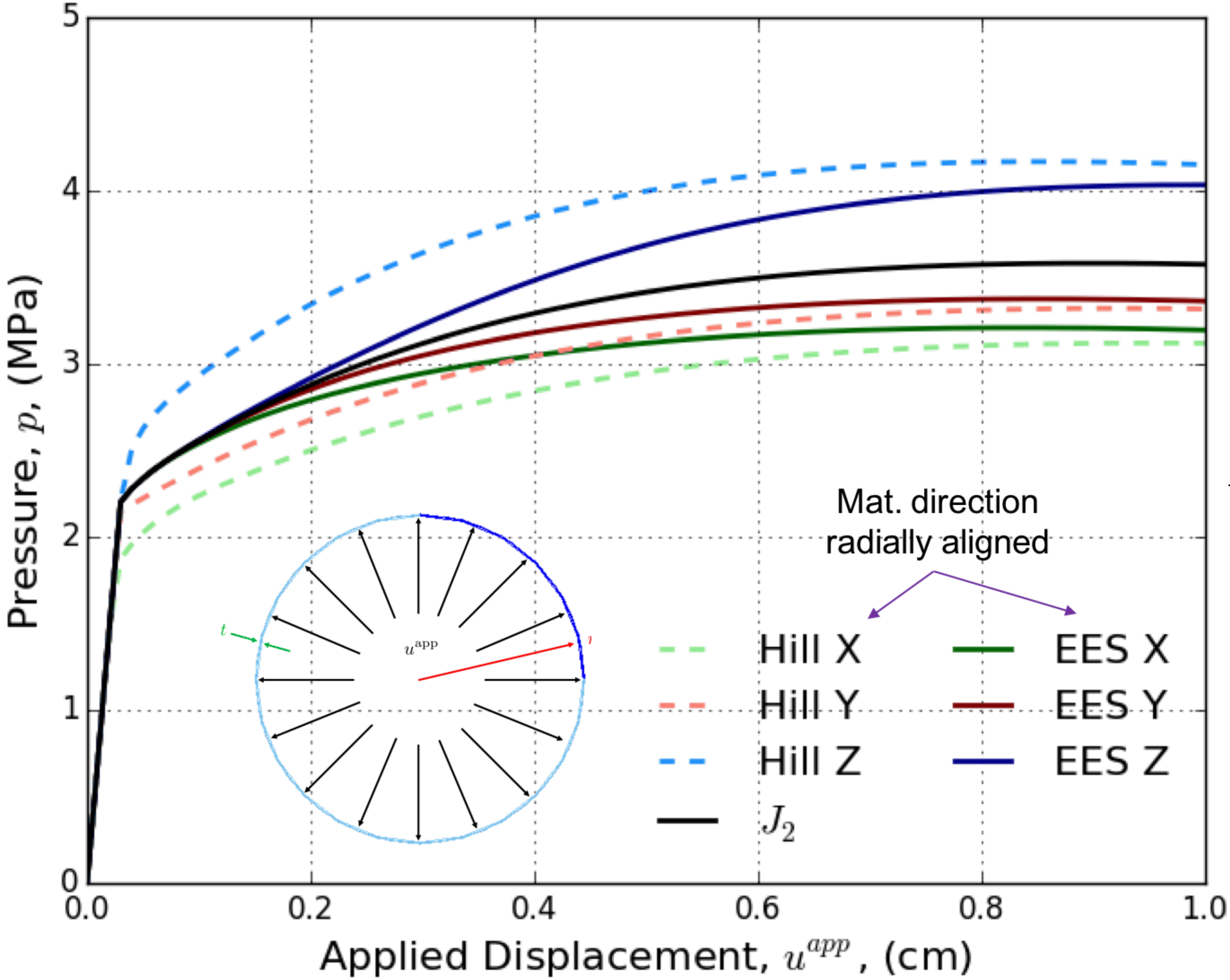
- Distortional hardening is anisotropic

Tensile Cylinder



- Distortional hardening impacts structural response (e.g. necking strain)

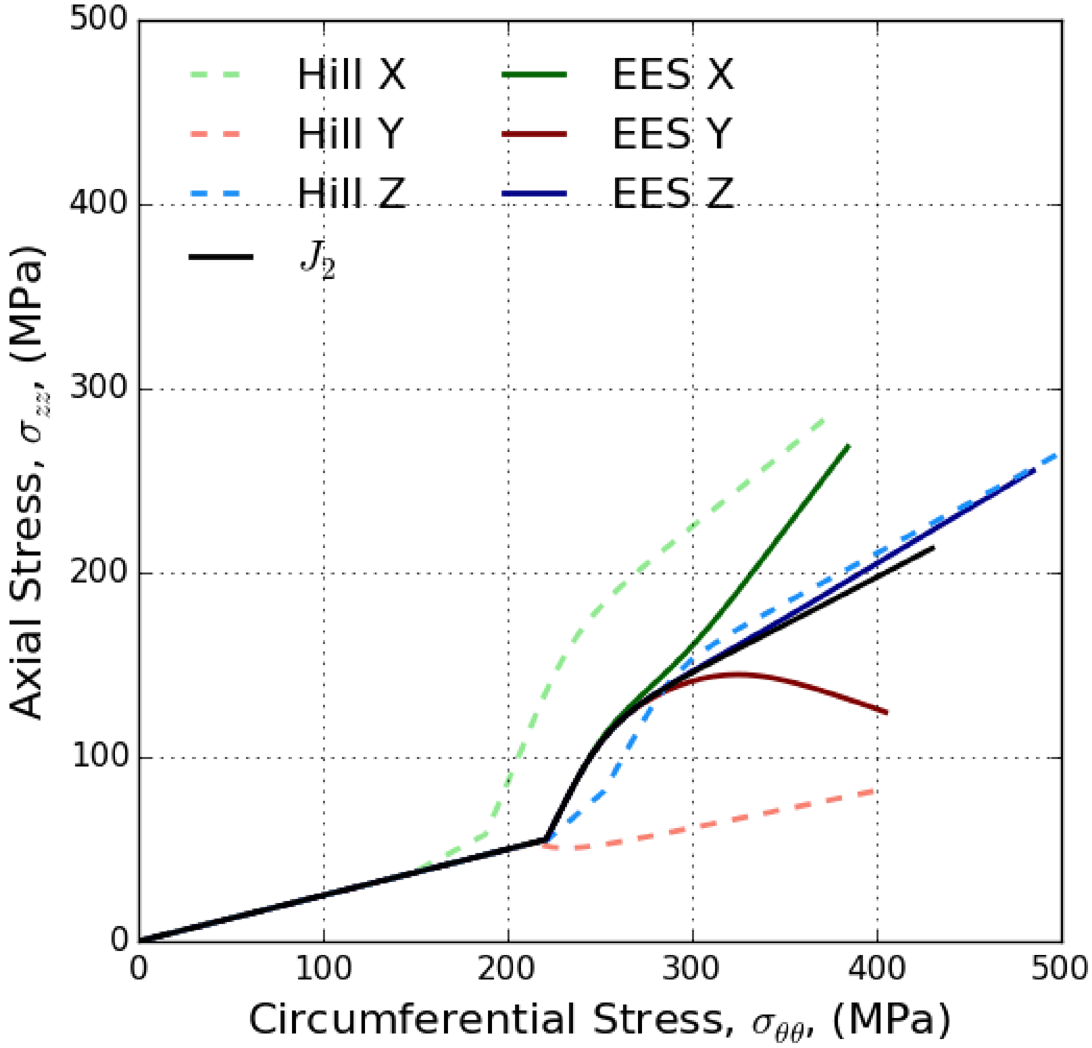
Pressurized Cylinder



$$p = \frac{F}{(r + \bar{u}_r)\, h}$$

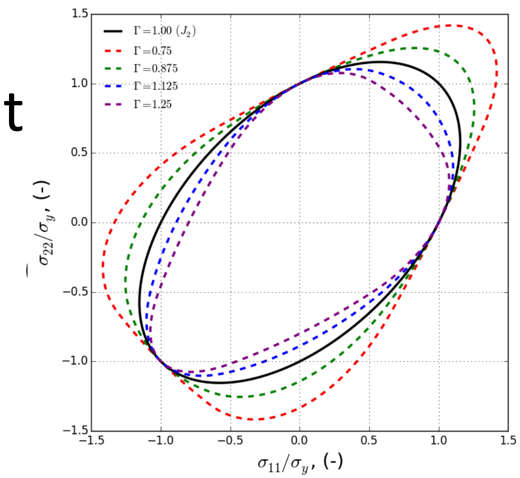
Pressurized Cylinder – Stress Path

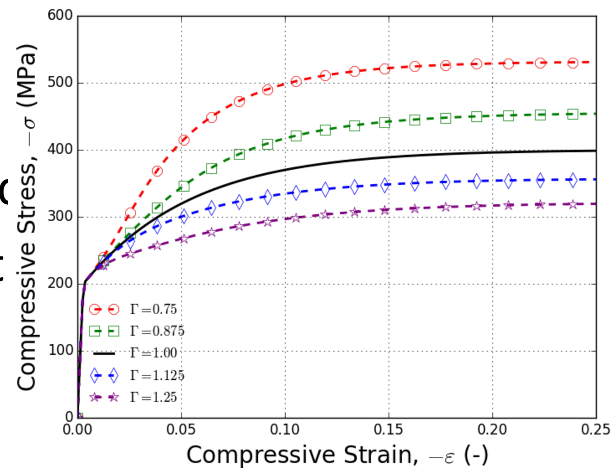
- Loading case results in complex, multiaxial, non-proportional loadings
- New EES model sufficiently robust to handle such deformations



Conclusion

- Developed theory and numerical implementation for evolving effective stress (EES) distortional hardening model
  - Introduce additional scalar internal state variable (η) associated specifically with distortional hardening
  - Evolution equations derived in a thermodynamically consistent fashion producing associative flow rules
  - Numerical implementation via fully implicit, closest point projection line-search augmented Newton-Raphson return mapping algorithm
  - Demonstrated capability to solve structural problems





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Acknowledgements

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Appendix

Model Timings

Round Cylinder Run Times

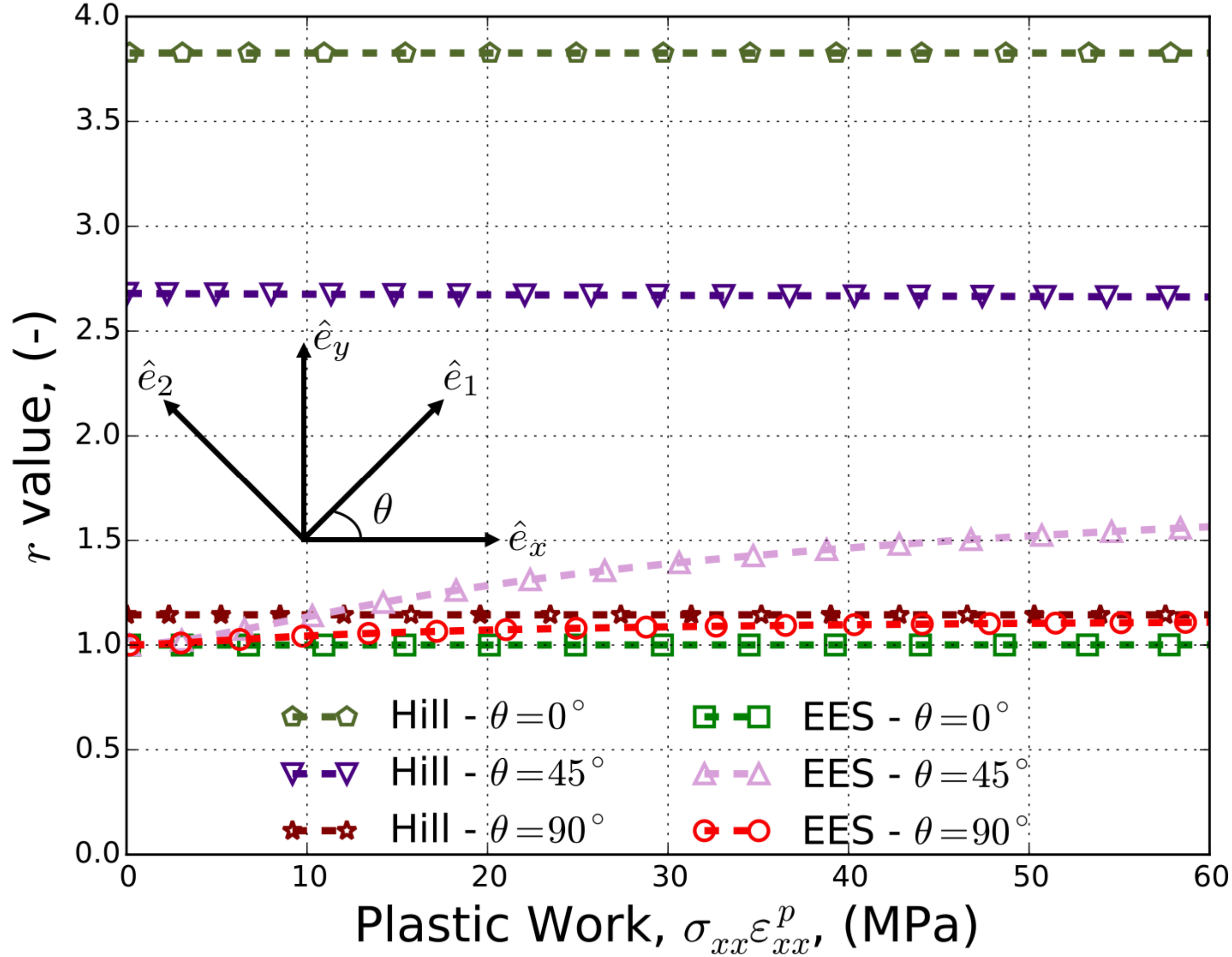
	Run Time/(Run Time)$_{J_2}$	
Case	Hill	EES
X	2.416	1.390
Y	1.420	1.850
Z	2.539	1.645

Pressurized Cylinder Run Times

	Run Time/(Run Time)$_{J_2}$	
Case	Hill	EES
X	1.127	1.555
Y	1.154	1.192
Z	0.972	1.085
Γ	Cazacu	EES
0.75	1.118	1.114
0.875	0.945	1.135
1.125	0.979	1.255
1.25	1.159	1.385

- EES Model run times comparable to analogous isotropic hardening forms

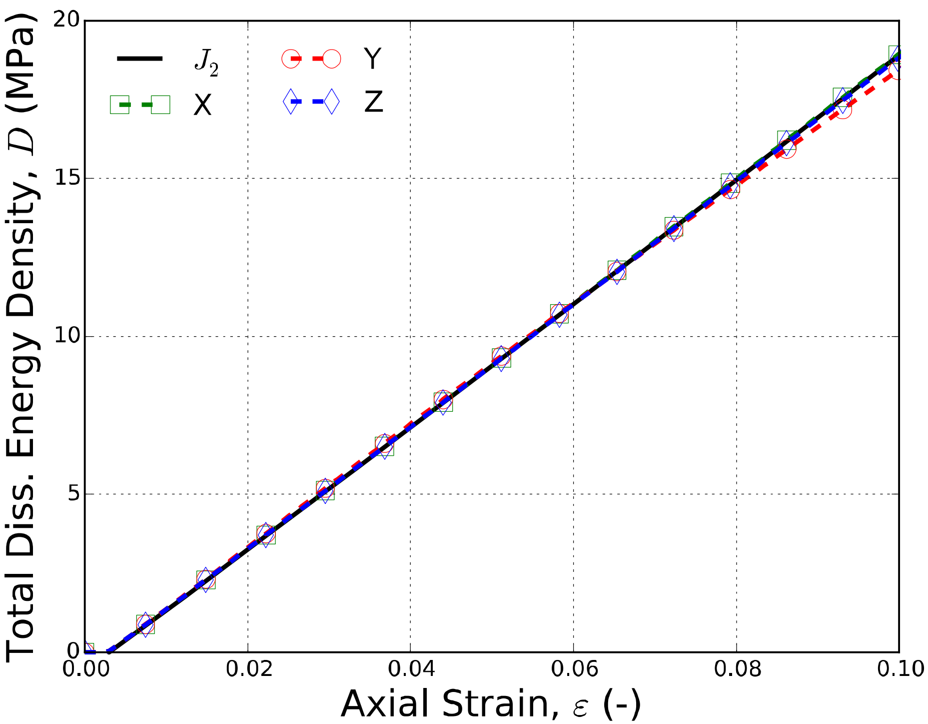
Lankford Ratio Evolution

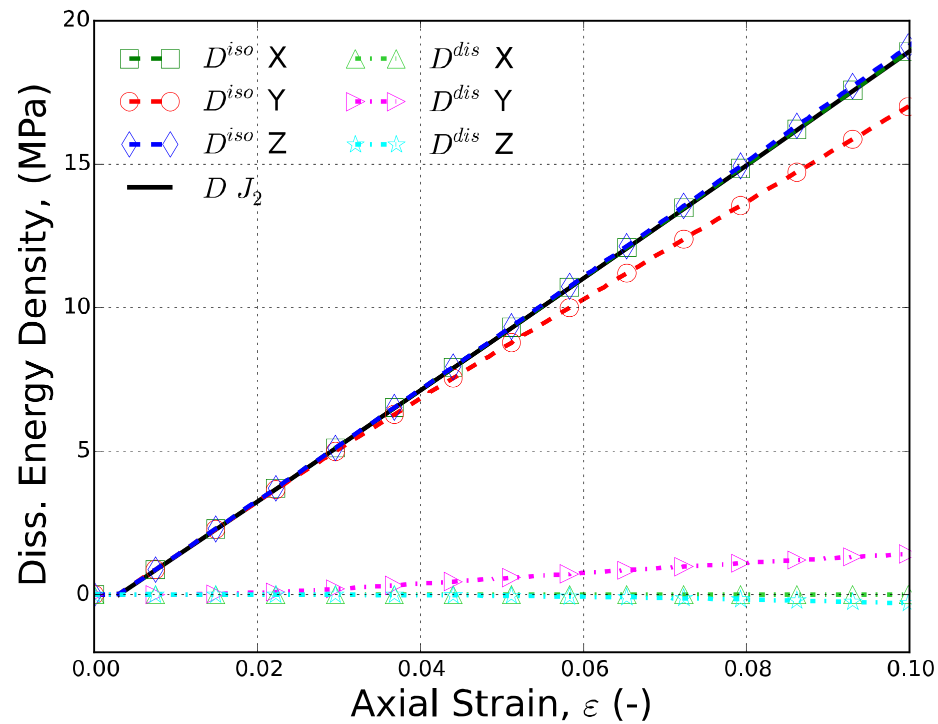


Dissipation - Constitutive

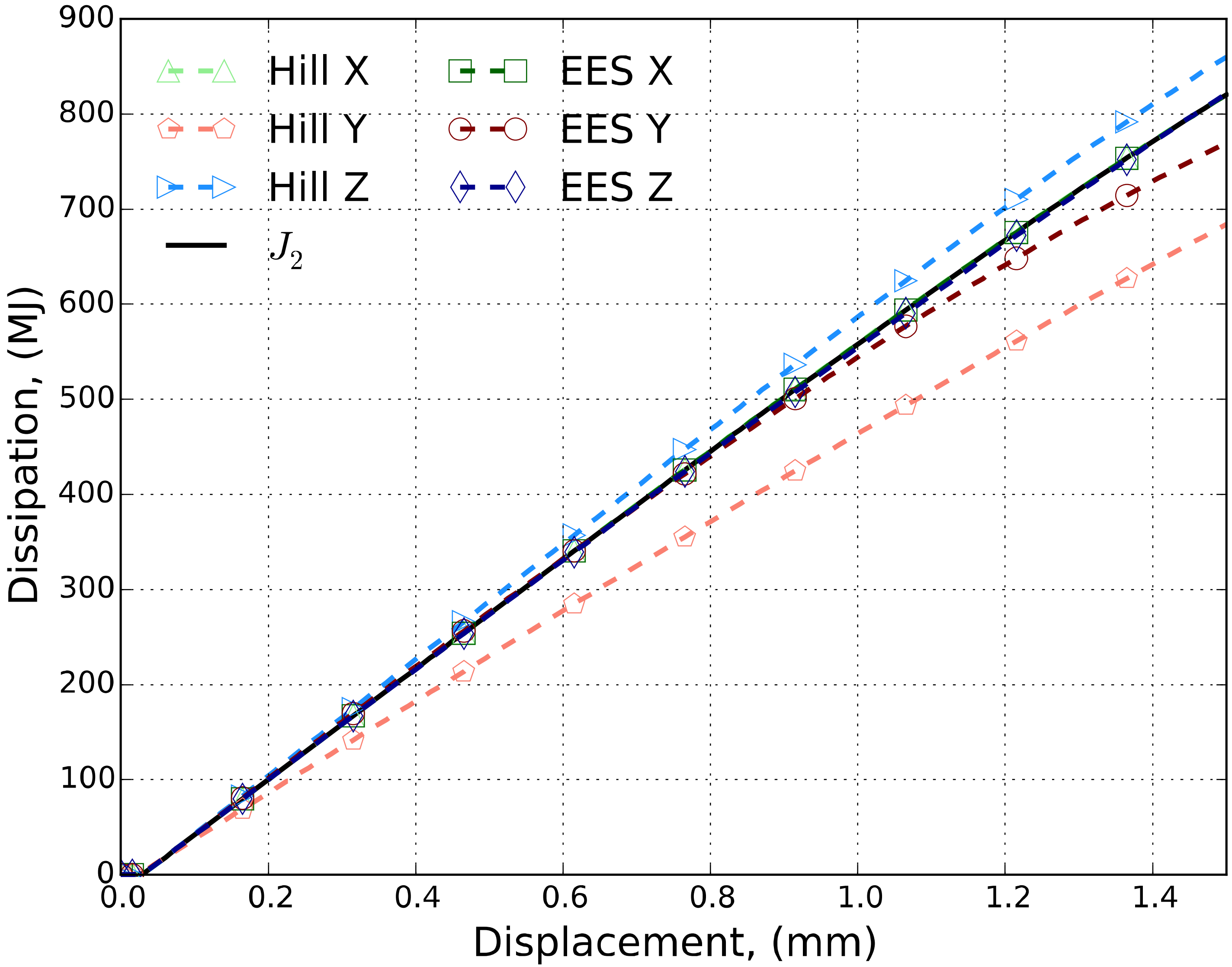
$$\mathcal{D} = \left(\sigma_y^0 + N \frac{\partial \phi}{\partial N} \right) \dot{\kappa}$$

$$\mathcal{D}^{\text{iso}} = \sigma_y^0 \dot{\kappa} \qquad \mathcal{D}^{\text{dis}} = N \frac{\partial \phi}{\partial N} \dot{\kappa}$$

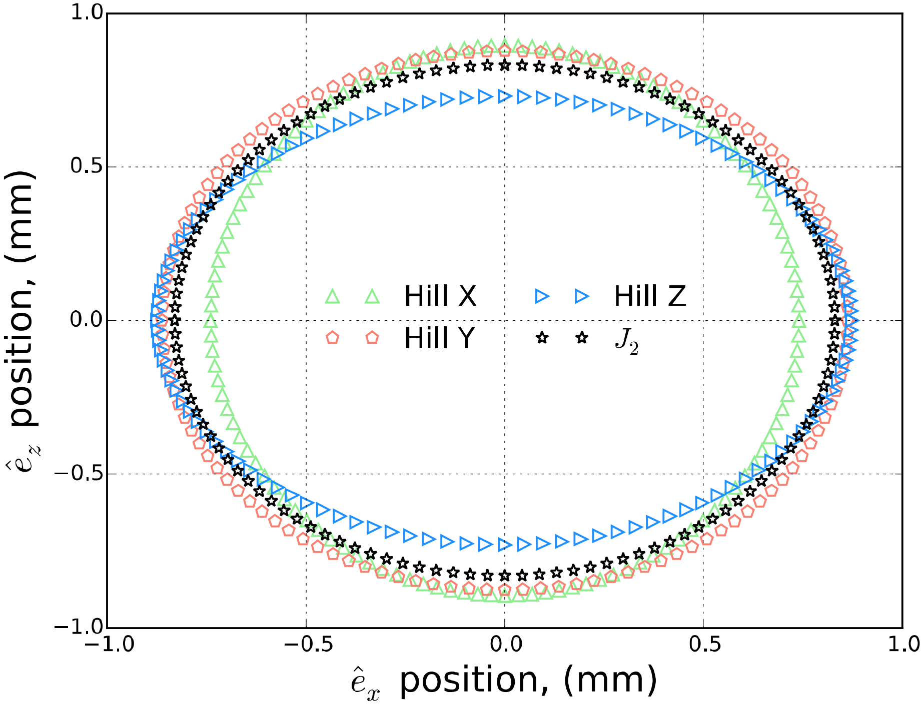


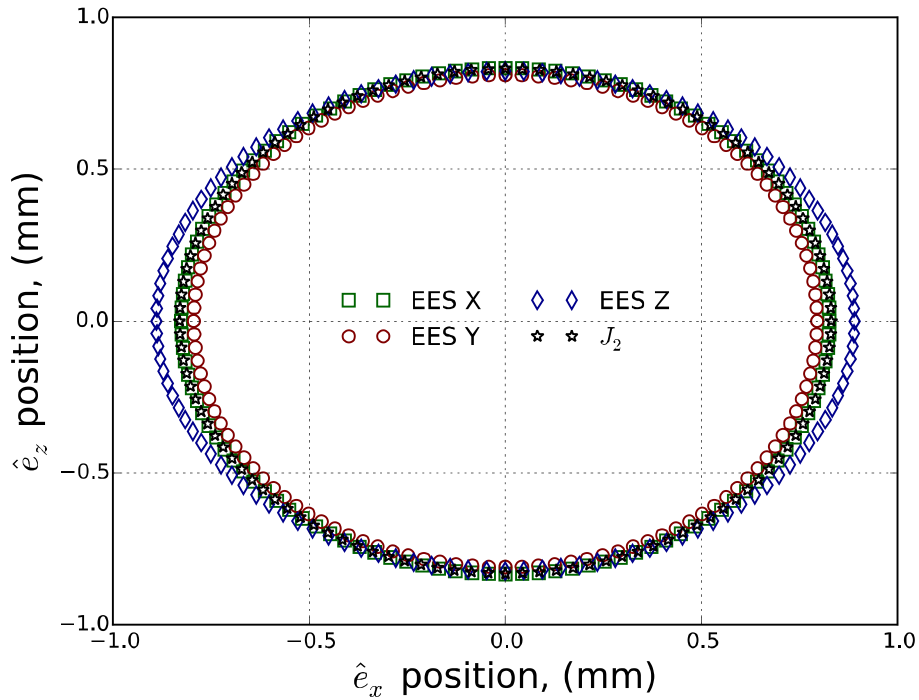


Dissipation – Tensile Bar



Tensile Bar – Final Shape

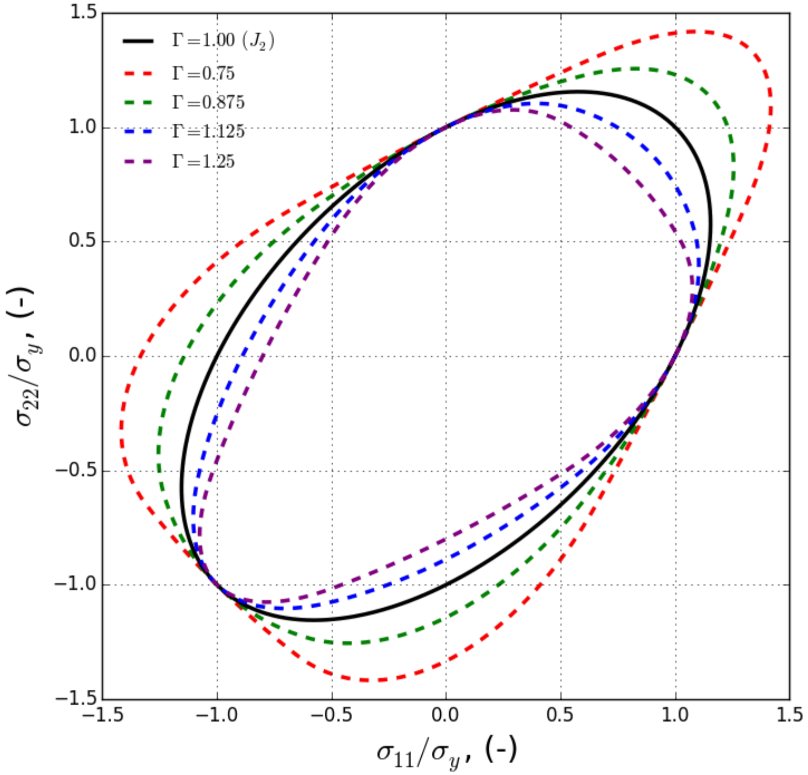




Strength-Differential Evolution

- Want to look at the effect of developing strength-differential
  - Consider isotropic form of Cazacu *et al.* effective stress

$$\phi^{(C)} = \{[|s_1| - ks_1]^a + [|s_2| - ks_2]^a + [|s_3| - ks_3]^a\}^{1/a}$$

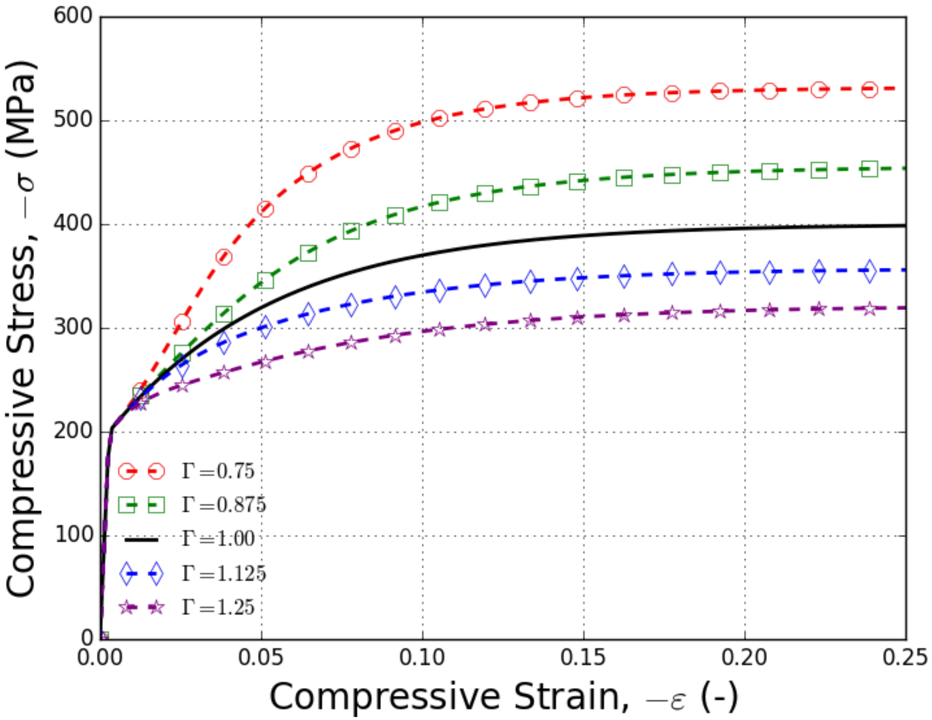


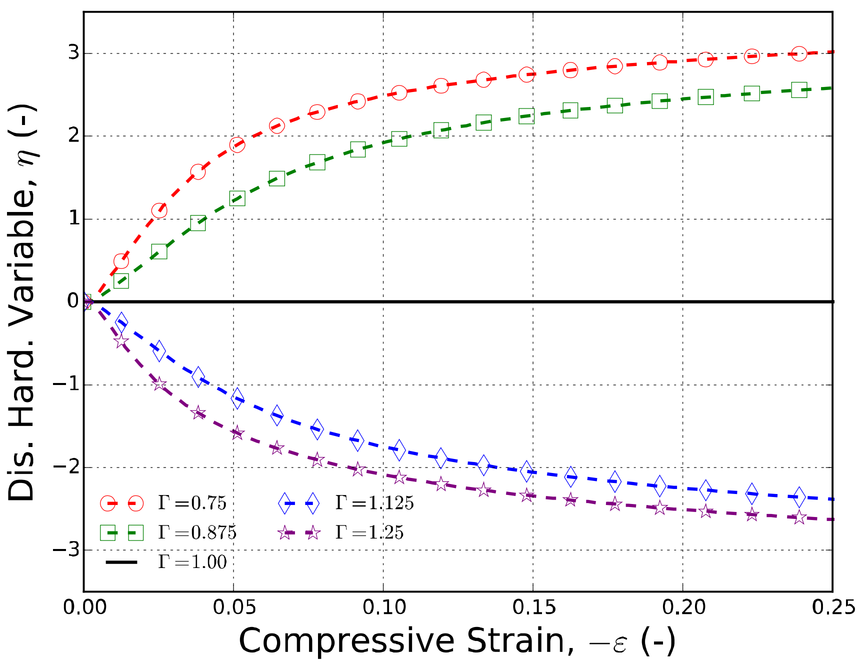
$$\Gamma = \frac{\sigma_y^{0(t)}}{\sigma_y^{0(c)}}$$

$$k = \frac{1 - h(\Gamma)}{1 + h(\Gamma)}$$

$$h(\Gamma) = \left[\frac{2^a - 2\Gamma^a}{(2\Gamma)^a - 2}\right]^{\frac{1}{a}}$$

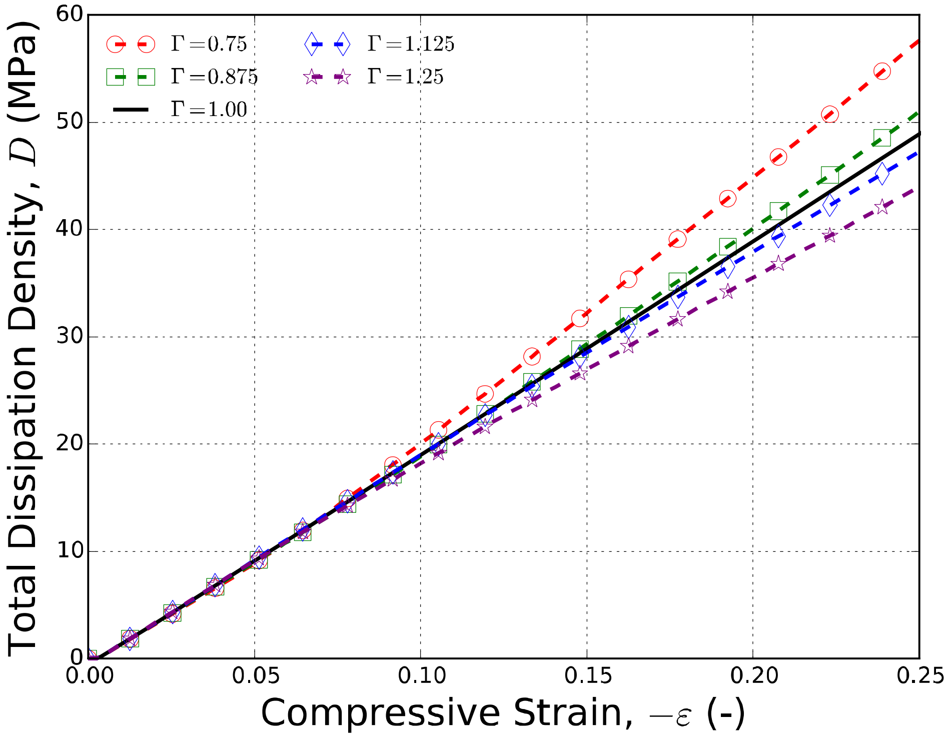
Constitutive Behavior - Cazacu

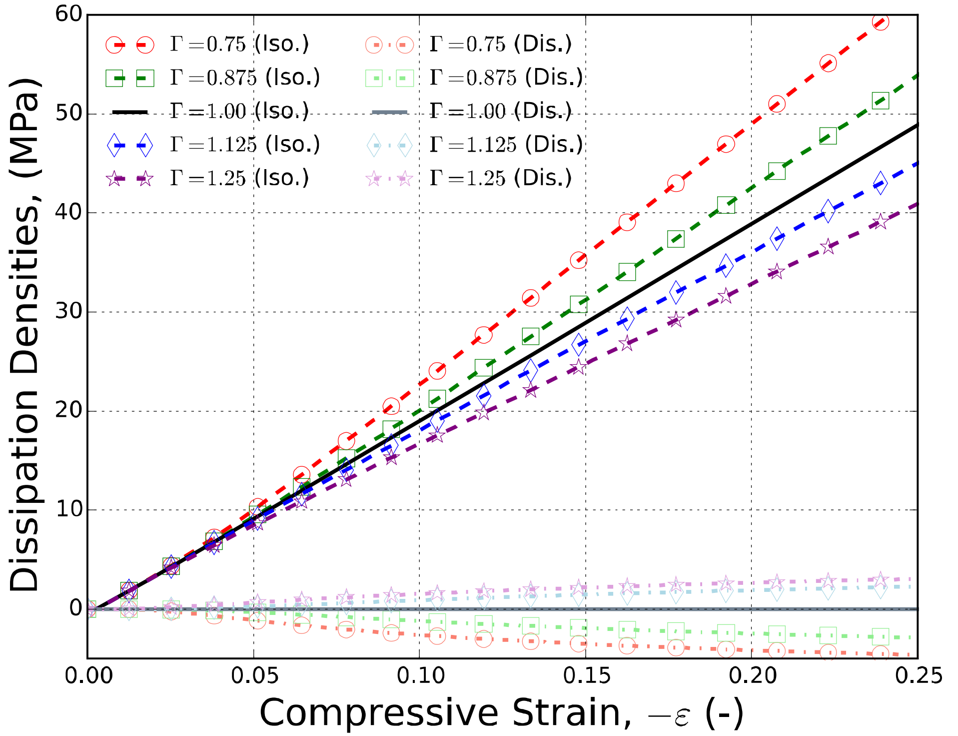




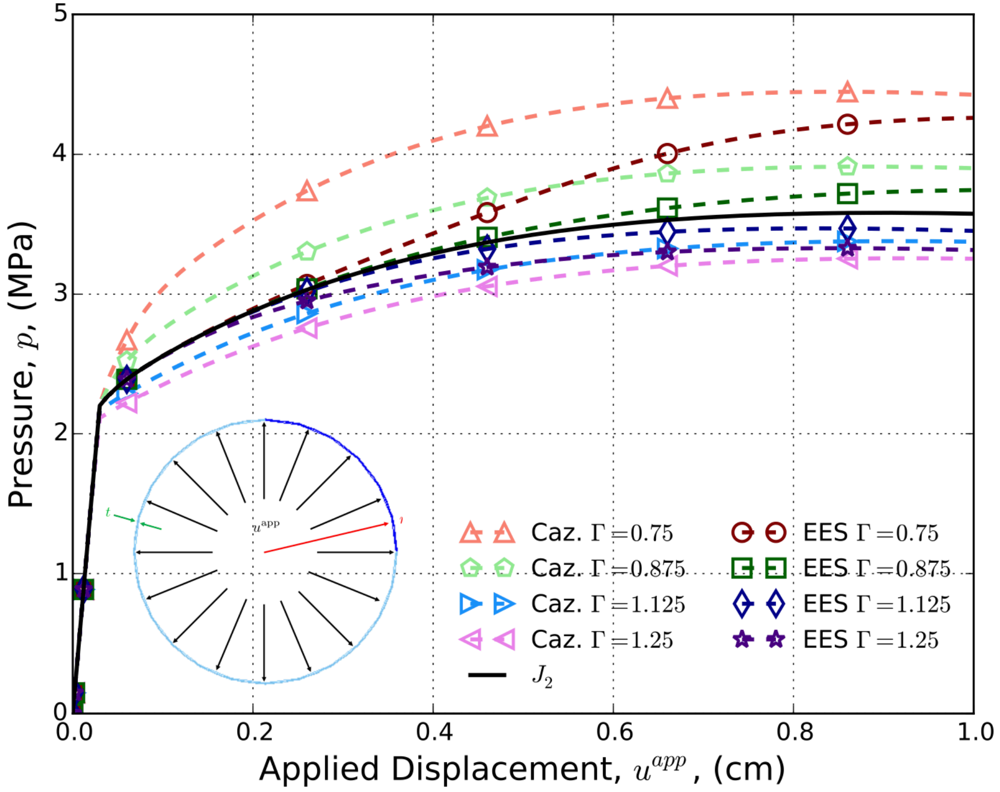
- EES approach captures development of tension-compression asymmetry

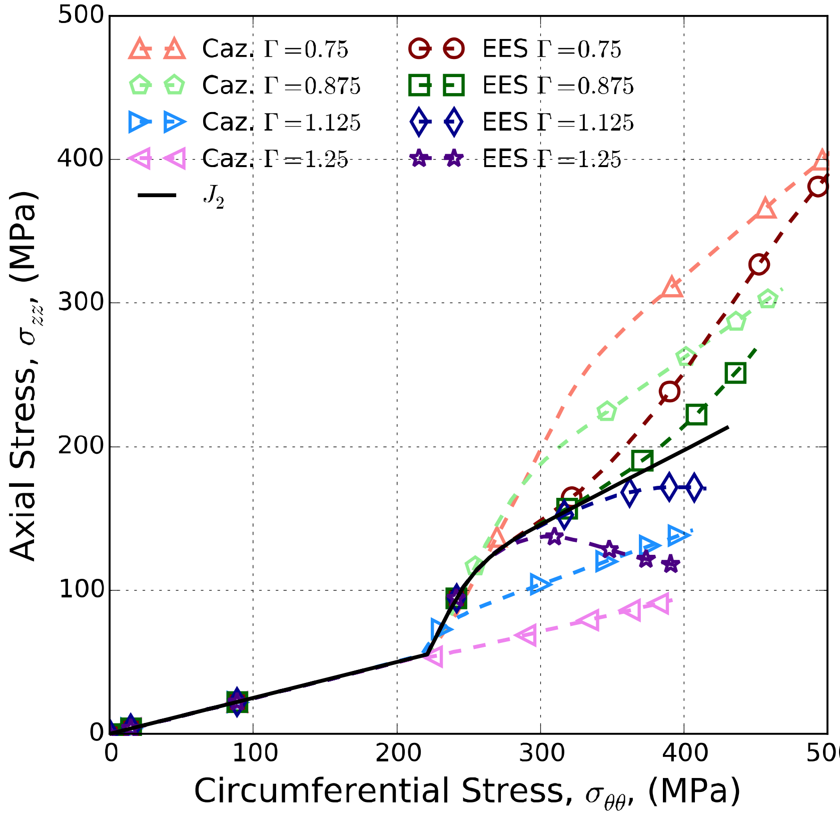
Cazacu Dissipation





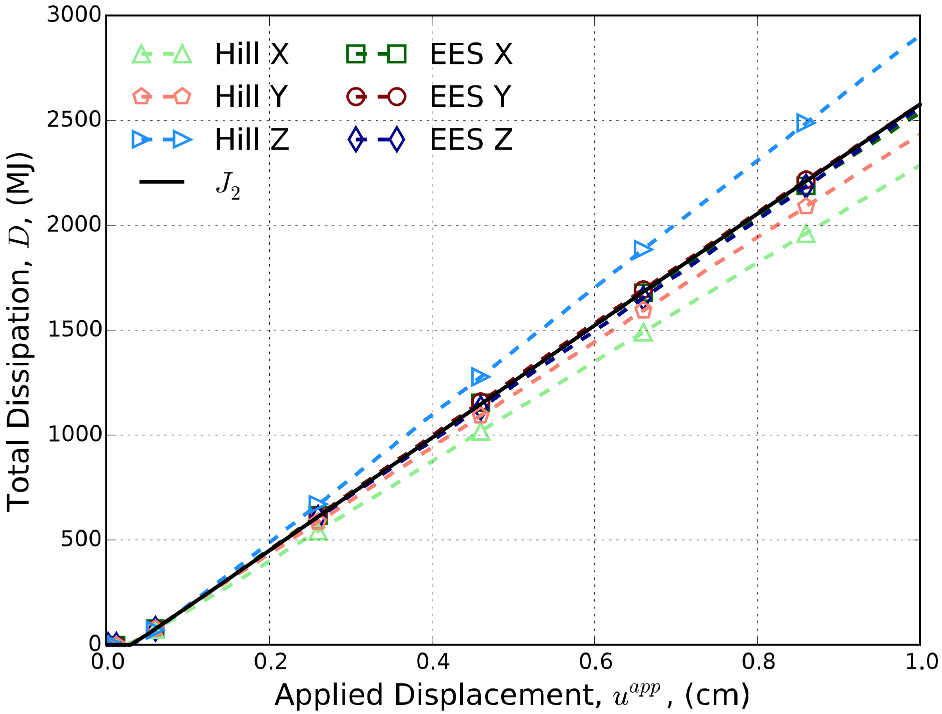
Pressurized Cylinder - Cazacu

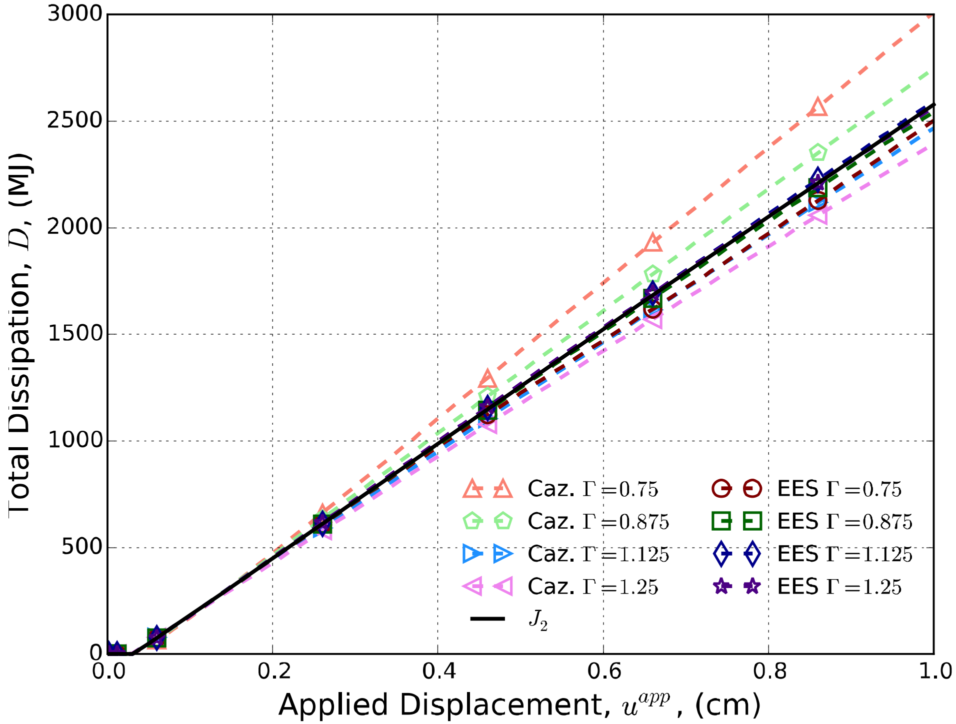




- Implementation robust under complex, non-proportional, multiaxial load paths

Cylinder Dissipation





Numerical Solution

$$\Delta\kappa = \frac{-\frac{\partial\phi}{\partial\sigma_{ij}}\mathscr{L}_{ijkl}r^{\varepsilon}_{kl} + r^{f} - \frac{1}{\omega}\left(\frac{\partial\phi}{\partial\eta} - d\kappa\frac{\partial\phi}{\partial\sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial^2\phi}{\partial\sigma_{kl}\partial\eta}\right)\left(r^{\eta} - d\kappa\frac{\partial^2\phi}{\partial N\partial\sigma_{ij}}\mathscr{L}_{ijkl}r^{\varepsilon}_{kl}\right)}{\frac{\partial\phi}{\partial\sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial\phi}{\partial\sigma_{kl}} + \frac{\partial\sigma_y}{\partial\kappa} + \frac{1}{\omega}\left(\frac{\partial\phi}{\partial\eta} - d\kappa\frac{\partial\phi}{\partial\sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial^2\phi}{\partial\sigma_{kl}\partial\eta}\right)\left(\frac{\partial\phi}{\partial N} - d\kappa\frac{\partial\phi}{\partial\sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial^2\phi}{\partial\sigma_{kl}\partial N}\right)}$$

$$\omega = 1 + d\kappa\left(\frac{\partial^2\phi}{\partial N\partial\eta} - d\kappa\frac{\partial^2\phi}{\partial N\partial\sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial^2\phi}{\partial\sigma_{kl}\partial\eta}\right)$$

$$\Delta\eta = \frac{1}{\omega}\left[-r^{\eta} + d\kappa\frac{\partial^2\phi}{\partial N\partial\sigma_{ij}}\mathscr{L}_{ijkl}r^{\varepsilon}_{kl} - \left(\frac{\partial\phi}{\partial N} - d\kappa\frac{\partial^2\phi}{\partial N\partial\sigma_{ij}}\mathscr{L}_{ijkl}\frac{\partial\phi}{\partial\sigma_{kl}}\right)\Delta\kappa\right]$$

$$\Delta\sigma_{ij} = -\mathscr{L}_{ijkl}\left(r^{\varepsilon}_{kl} + \frac{\partial\phi}{\partial\sigma_{kl}}\Delta\kappa + d\kappa\frac{\partial^2\phi}{\partial\sigma_{kl}\partial\eta}\Delta\eta\right)$$

$$\mathscr{L}_{ijkl} = \left[\mathbb{C}^{-1}_{ijkl} + d\kappa\frac{\partial^2\phi}{\partial\sigma_{ij}\partial\sigma_{kl}}\right]^{-1}$$

Convexity

- To maximize dissipation, minimize constrained Lagrangian

$$\mathcal{L}\left(\sigma_{ij}, K, N, \lambda\right) = -\mathcal{D}\left(\sigma_{ij}, K, N\right) + \lambda f\left(\sigma_{ij}, K, N\right)$$

$$\mathcal{D} = \sigma_{ij}\dot{\varepsilon}^{\mathrm{p}}_{ij} - K\dot{\kappa} - N\dot{\eta} \geq 0$$

- Second-order necessary and sufficient conditions for relative minimum satisfied if

$$y \cdot \nabla^2 \mathcal{L} y = \lambda y \cdot \nabla^2 f y \geq= 0 \qquad \forall y \quad \text{s.t.} \quad y \cdot \nabla f = 0$$

$$\hat{\sigma}_{ij} \frac{\partial^2 \phi^*}{\partial \sigma_{ij} \partial \sigma_{kl}} \hat{\sigma}_{kl} + 2\hat{N} \frac{\partial^2 \phi^*}{\partial \sigma_{ij} \partial N} \hat{\sigma}_{ij} + \hat{N}^2 \frac{\partial^2 \phi^*}{\partial N^2}$$

- Some issues need to be addressed for general convexity of distortional hardening

Luenberger and Ye, 2008, “Linear and Non-linear Programming”