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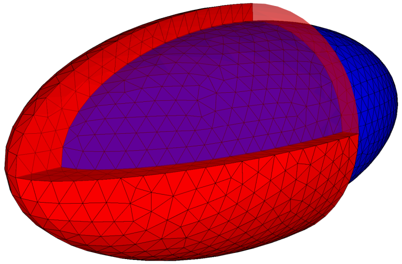

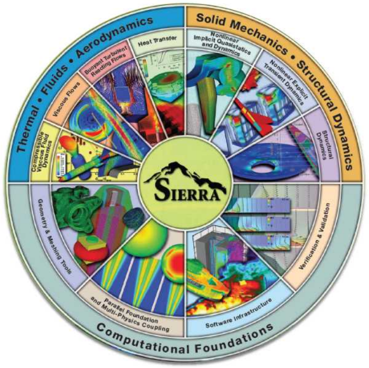

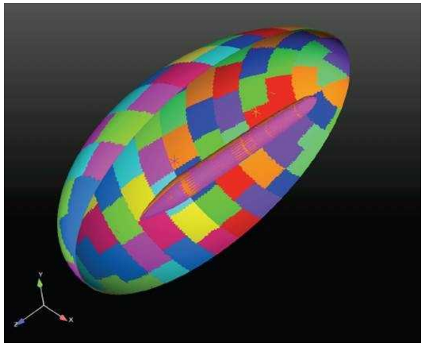

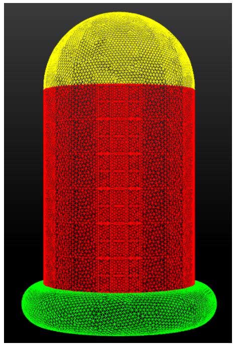

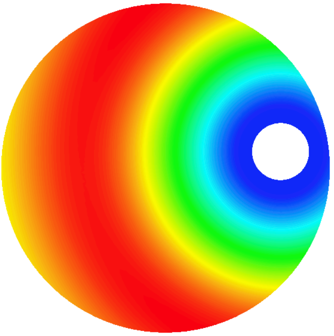

Massively Parallel Structural Acoustics for Forward and Inverse Problems

Timothy Walsh, Wilkins Aquino

173rd Meeting of the Acoustical Society of America
and the 8th Forum Acusticum

Boston, Massachusetts

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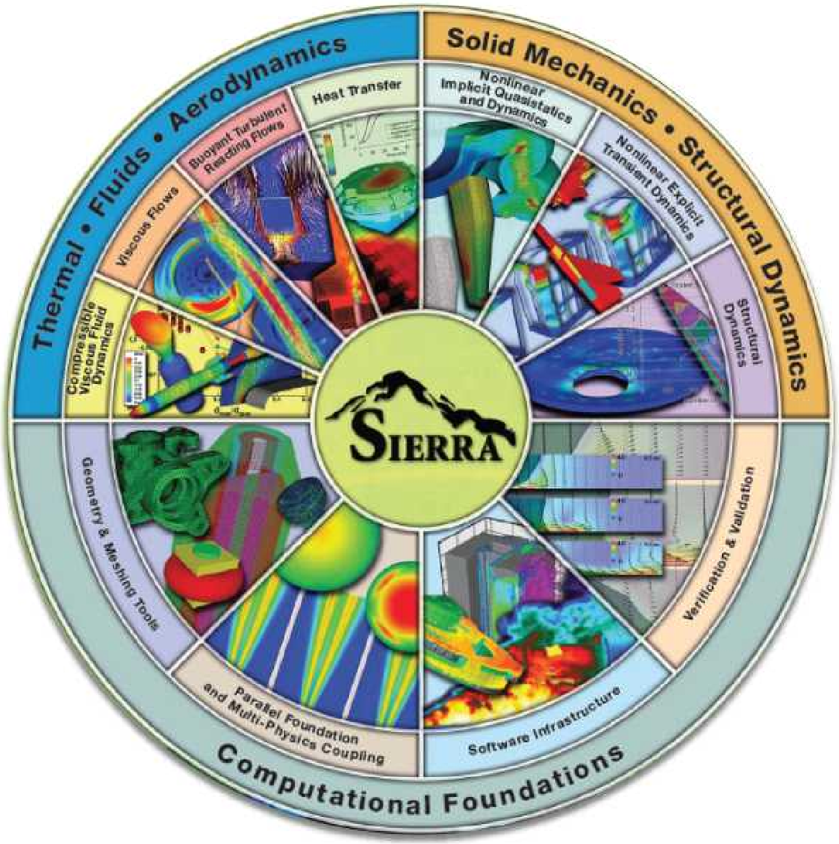
Outline

- Overview of Sierra Mechanics
- Overview of Sierra-SD(Salinas)
- Some acoustics research areas in Sierra-SD
- Example applications of Sierra-SD for acoustics, inverse problems

Overview of Sierra Mechanics

- Goal: massively parallel coupled multiphysics calculations
- Modules for structural dynamics, solid mechanics, fluids, thermal, etc



Sierra-SD: A Brief History

- Sierra-SD was created in the 1990's at Sandia National Laboratories for large-scale structural analysis
- Intended for extremely complex structural and structural acoustics models
  - Routinely used to solve models with 100's of millions of degrees of freedom
- Scalability is the key
  - Sierra-SD can solve n-times larger problem using n-times many more compute processors, in nearly constant CPU time

Overview of Sierra-SD Structural Acoustic Capabilities

- Massively parallel
- Hex, wedge, tet acoustic elements (up to order p=6), coupled with both 3D and 2D (shell) structural elements
- Linear and nonlinear acoustics
- Allows for mismatched acoustic/solid meshes
  - Mortar or multi-point constraints (MPC)'s
- Infinite elements and Perfectly Matched Layers (PML)
- Solution procedures:
  - Frequency response (frequency-domain)
  - Transient response (time-domain)
  - Eigenvalue (modal) analysis
    - Linear and quadratic (complex modes)

Structural Acoustic Equations of Motion

Sandia National Laboratories

Acoustics

$$\nabla^2 \phi = \frac{1}{c^2}\ddot{\phi}, \quad \text{in } \Omega_f \times (0,T)$$

$$\nabla \phi \cdot \boldsymbol{n}_f = -\rho_f \ddot{u}_n, \quad \text{on } \partial\Omega_f^N \times [0,T]$$

$$\phi = 0, \quad \text{on } \partial\Omega_f^D \times [0,T]$$

$$\phi(0,T) = 0, \quad \text{in } \Omega_f$$

$$\dot{\phi}(0,T) = 0, \quad \text{in } \Omega_f$$

Solid Mechanics

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\boldsymbol{u}}, \quad \text{in } \Omega \times (0,T)$$

$$\boldsymbol{\sigma n} = \boldsymbol{h}, \text{ on } \partial\Omega^N \times [0,T]$$

$$\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\nabla u}, \quad \text{in } \Omega \times [0,T]$$

$$\boldsymbol{u} = \boldsymbol{0}, \quad \text{on } \partial\Omega^D \times [0,T]$$

$$\boldsymbol{u}(0,T) = \boldsymbol{0}, \quad \text{in } \Omega$$

$$\dot{\boldsymbol{u}}(0,T) = \boldsymbol{0}, \quad \text{in } \Omega$$

Time domain

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

Frequency domain (Helmholtz)

$$[H(\omega)]\boldsymbol{z}(\omega) = \boldsymbol{F}(\omega)$$

$$[H(\omega)] = -\omega^2[M] + i\omega[C] + [K]$$

Discretized Equations of Motion

- Fully coupled time domain formulation

$$\begin{bmatrix} M_s & 0 \\ 0 & -M_a \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_s & L^T \\ L & -C_a \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ -f_a \end{bmatrix}$$

- Fully coupled eigenanalysis formulation

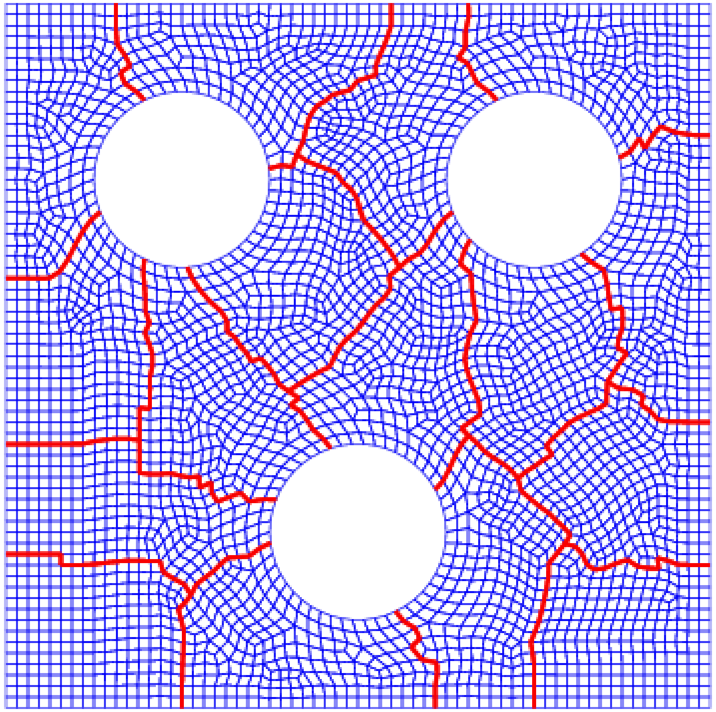
$$\lambda^2 \begin{bmatrix} M_s & 0 \\ 0 & -M_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} + \lambda \begin{bmatrix} C_s & L^T \\ L & -C_a \end{bmatrix} \begin{bmatrix} u \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- Fully coupled frequency-domain formulation

$$-\omega^2 \begin{bmatrix} M_s & 0 \\ 0 & -M_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} + i\omega \begin{bmatrix} C_s & L^T \\ L & -C_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ -f_a \end{bmatrix}$$

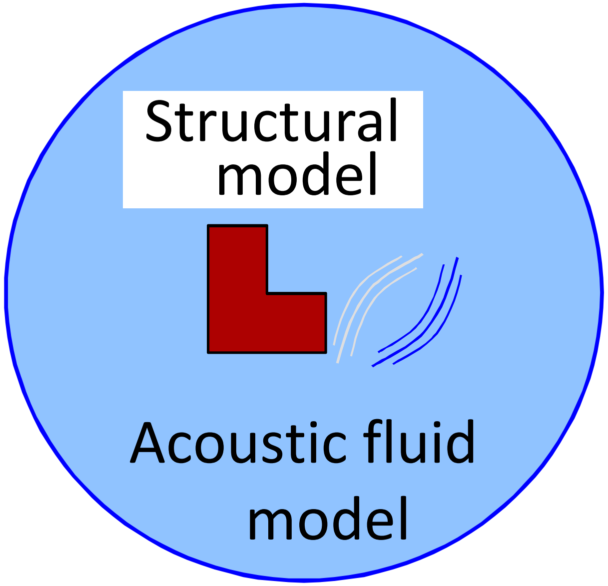
Parallel Helmholtz Solver in Sierra-SD

- Overlapping Schwarz domain decomposition approach
  - Effectively handles large numbers of constraint equations for mismatched fluid/solid meshes
- Elements partitioned into subdomains
- Solve local problems on each overlapping subdomain with Dirichlet BCs on boundary
- Massively parallel implementation based on GDSW solver



Why Nonlinear Acoustics?

- Linear acoustics is inadequate for many applications:
  - Resonating cavities
  - Large-amplitude sources
  - Far-field explosions
  - Aeroacoustic noise



<u>Assumptions of Linear Acoustic Theory</u>

- Small amplitude waves
- Linear constitutive fluid model
- No fluid convection

<u>Consequences</u>

- Resonance leads to infinite amplitude waves
- "Sine wave remains a sine wave"
- No wave distortion
- Wavespeed independent of stress state in fluid

Eulerian Formulations for Nonlinear Acoustics

- The linear acoustic wave equation

$$\frac{1}{c^2}\phi_{tt} - \Delta\phi = 0$$

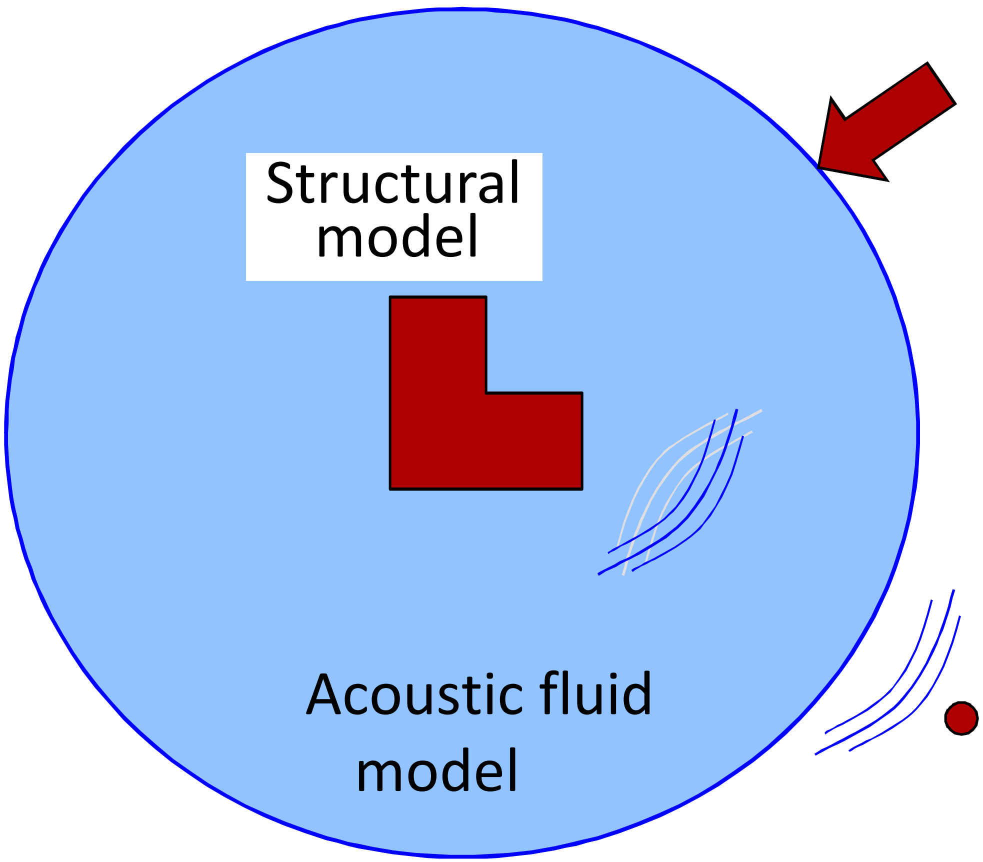
- The 2nd order Kuznetsov Equation

$$\frac{1}{c^2}\phi_{tt} - \Delta\phi + \frac{1}{c^2}\frac{\partial}{\partial t}\left[(\nabla\phi)^2 + \frac{B/A}{2c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 + b\nabla^2\phi\right] = 0$$

- High order nonlinear acoustic equation

$$\frac{1}{c^2}\phi_{tt} - \Delta\phi + \frac{1}{c^2}\frac{\partial}{\partial t}\left[(\nabla\phi)^2 + b\nabla^2\phi\right] + \frac{1}{2c^2}\nabla\psi \bullet \nabla(\nabla\phi)^2 + \frac{\gamma - 1}{c^2}\left(\frac{\partial\psi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right)\Delta\psi = 0$$

Far-Field Acoustics

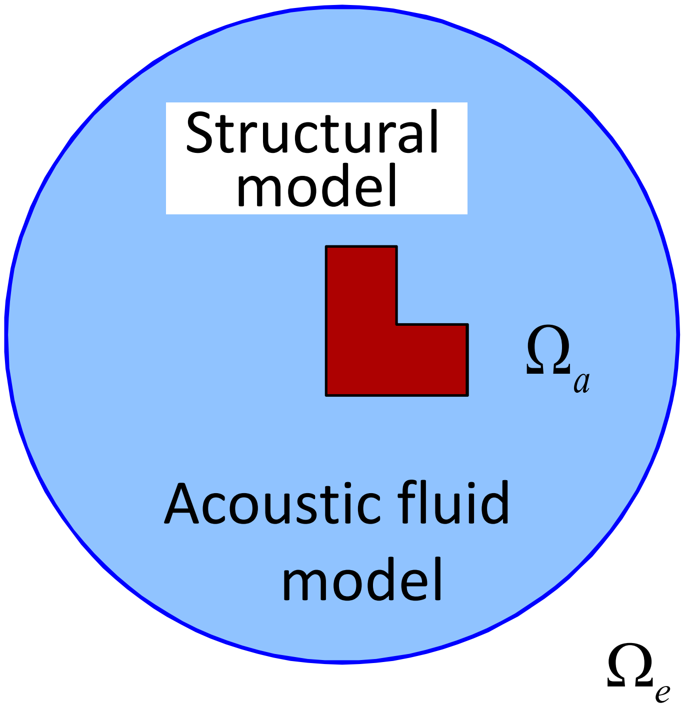


Common Requirement: far-field boundary conditions for finite element analysis

- Infinite Elements
- Perfectly Matched Layers (PML)

Microphone: compute far-field response

Infinite Element Formulation



$$\Omega = \Omega_a + \Omega_e$$

Acoustic wave equation for fluid

$$\frac{1}{c^2} p_{tt} - \Delta p = 0 \qquad \Omega x [0, T]$$

$$\frac{\partial p}{\partial n} = g(x,t) \qquad \Gamma x [0, T]$$

Weak formulation on exterior domain

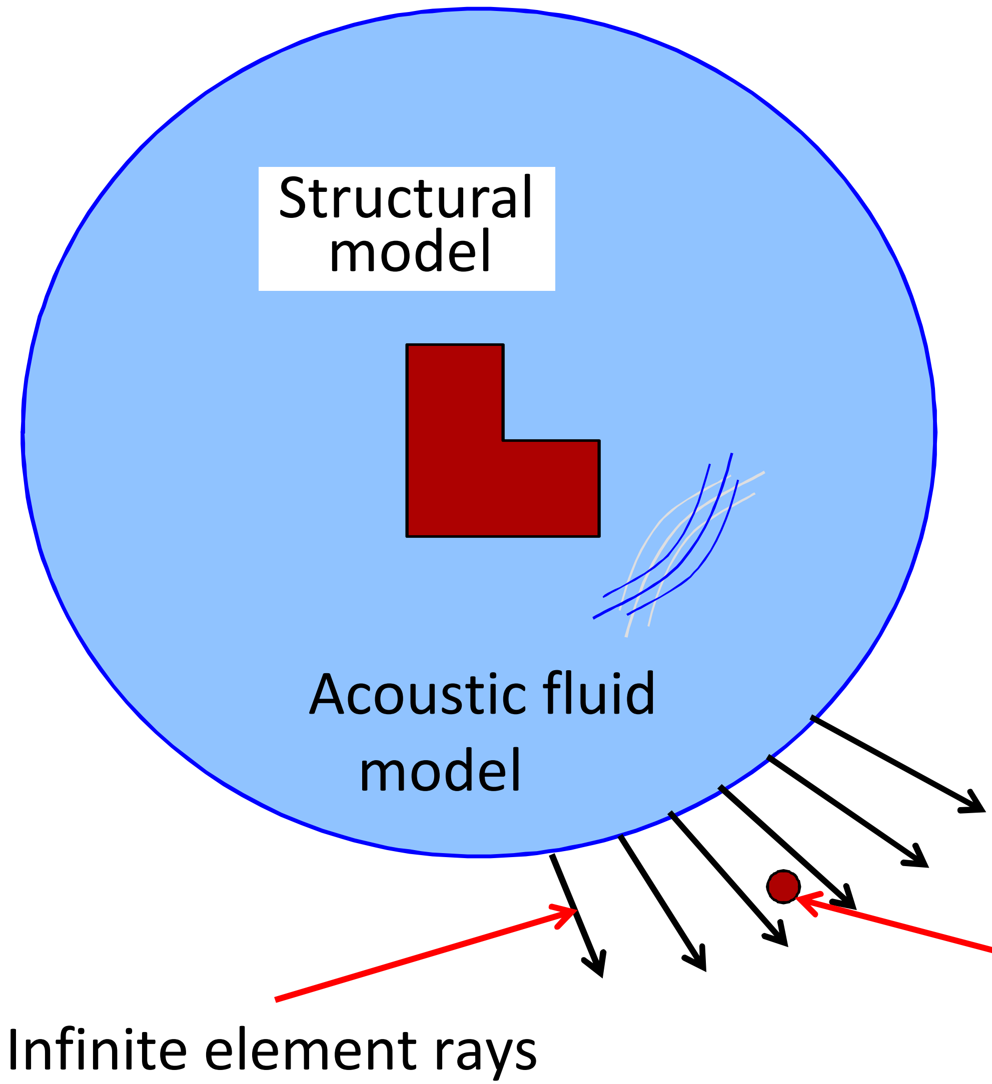
$$\int_{\Omega} \frac{1}{c^2} \ddot{p}\, q\, dV + \int_{\Omega} \nabla p \bullet \nabla q\, dV = \int_{\Gamma} g q\, dS$$

Trial and weight functions

$$\phi(x,\omega) = P(x) e^{-ik\mu(x)} \qquad q = D(x) P(x) e^{ik\mu(x)}$$

$$\left(-\omega^2 M + i\omega C + K\right) p = f$$

Kirchhoff Integral vs Infinite Elements



Kirchhoff Integral:

1. Store entire time history of pressure and velocity on entire exterior surface
2. Evaluate Kirchhoff integral

Infinite Elements:

1. Determine which infinite element owns microphone location
2. Element-level summation

Microphone: compute far-field response

General Formulation for PML

Complex coordinate stretching $$\tilde{x} = x - \frac{i}{\omega}\int_x^a \sigma(\xi)d\xi \quad a < x < \bar{a}$$

Helmholtz equation over complex coordinates $$-\tilde{\Delta}p - k^2 p = 0$$

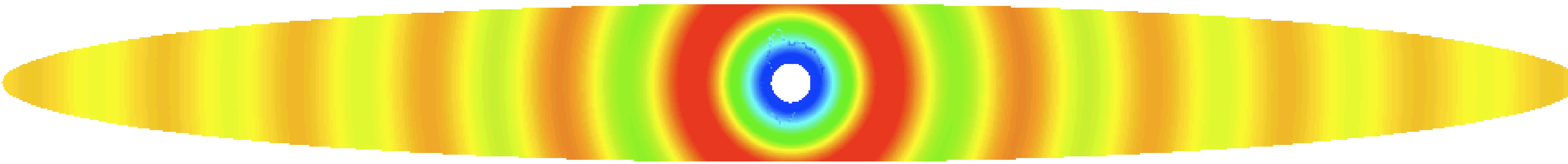
Weak form over complex coordinates $$\int_{\tilde{\Omega}_I} \langle \tilde{\nabla} p, \tilde{\nabla} q \rangle - k^2 pq \; d\Omega_I = \int_{\tilde{\Gamma}_S} gq dS$$

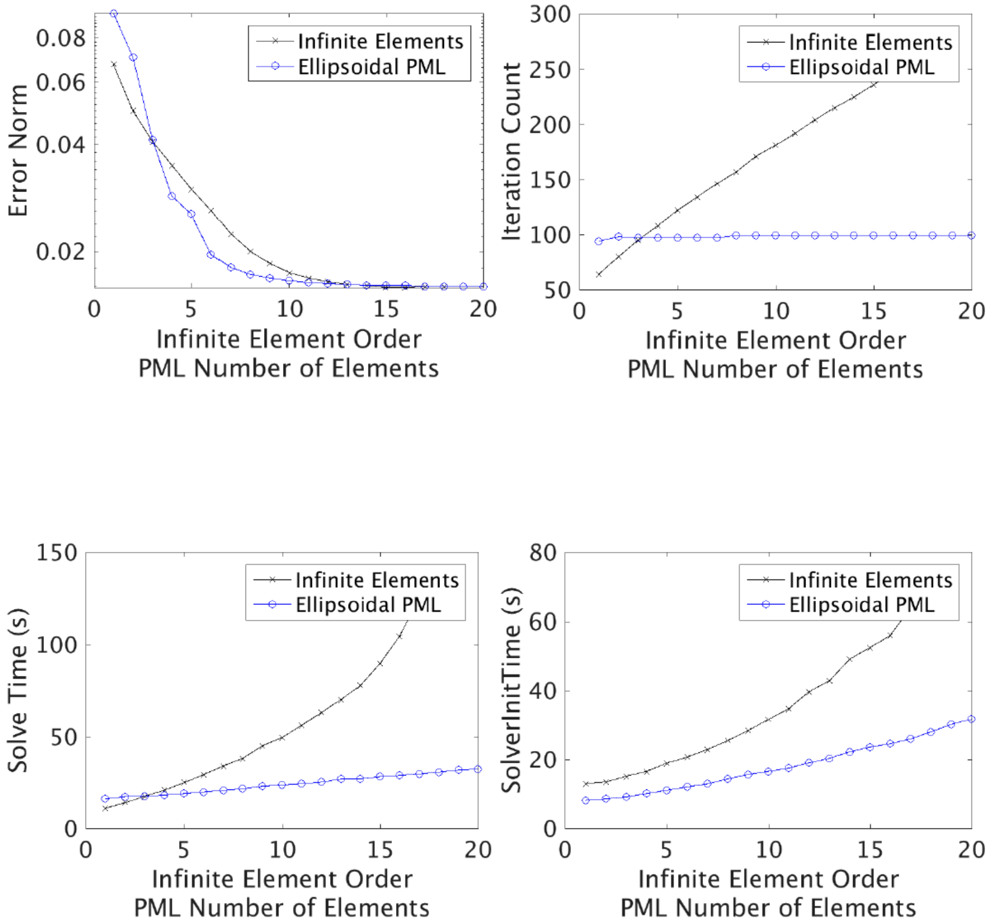
Mapped weak form back to real coordinates $$\int_{\Omega_I} \left[(\boldsymbol{J}^{-1}\nabla p)\cdot(\boldsymbol{J}^{-1}\nabla q) - k^2 pq \right] J(x,y,z) d\Omega_I = \int_{\Gamma_S} gq dS$$

Re-write as Helmholtz equation with variable coefficients $$\int_{\Omega_I} \tilde{\boldsymbol{A}}\langle \nabla p, \nabla \bar{q} \rangle - k^2 \tilde{J} p \bar{q} \; d\Omega_I = \int_{\Gamma_S} g\bar{q} d\Gamma_S$$

$$\tilde{\boldsymbol{A}} = \tilde{J}\tilde{\boldsymbol{J}}^{-1}\tilde{\boldsymbol{J}}^{-T}$$

Results: 10-to-1 Prolate Spheroid





For a fixed level of accuracy

- PML required many less iterations than infinite elements
- PML solution times were much faster
- In frequency domain, PML is clear winner over infinite elements

What is an Inverse Problem?

- Inverse problems arise when we have partial information and indirect observations of a system and need to infer (hidden) quantities of interest of the system.

- An inverse problem can be viewed as a quest for information that is not directly available from observations or measurements.

- The pursuit of a solution to an inverse problem calls for a balance synergy between analysis and experimentation.

Inverse Problems - Motivation

Forward Solver

Material properties

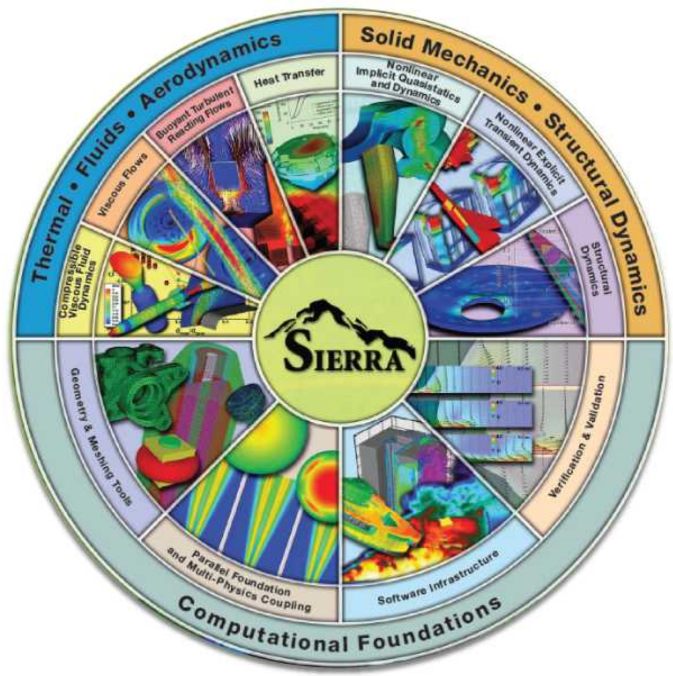
Geometry

Boundary conditions

Loads

Residual stresses

etc



Displacement

Pressure

Temperature

Flow field

etc

System parameters

State Variables (outputs)

Experimental data + inverse solution = missing link!

Abstract Optimization Formulation

Abstract optimization formulation

$$\underset{u,p}{\text{minimize}} \quad J(u,p)$$

Objective function

$$\text{subject to} \quad g(u,p) = 0$$

PDE constraint

$$\mathcal{L}(u,p,w) := J + w^T g$$

Lagrangian

$$\begin{Bmatrix} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{Bmatrix} = \begin{Bmatrix} J_u + g_u^T w \\ J_p + g_p^T w \\ g \end{Bmatrix} = \{0\}$$

First order optimality conditions

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & g_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & g_p^T \\ g_u & g_p & 0 \end{bmatrix} \begin{Bmatrix} \delta u \\ \delta p \\ w^\star \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ g \end{Bmatrix}$$

Newton iteration

$$W \Delta p = -\hat{J}',$$

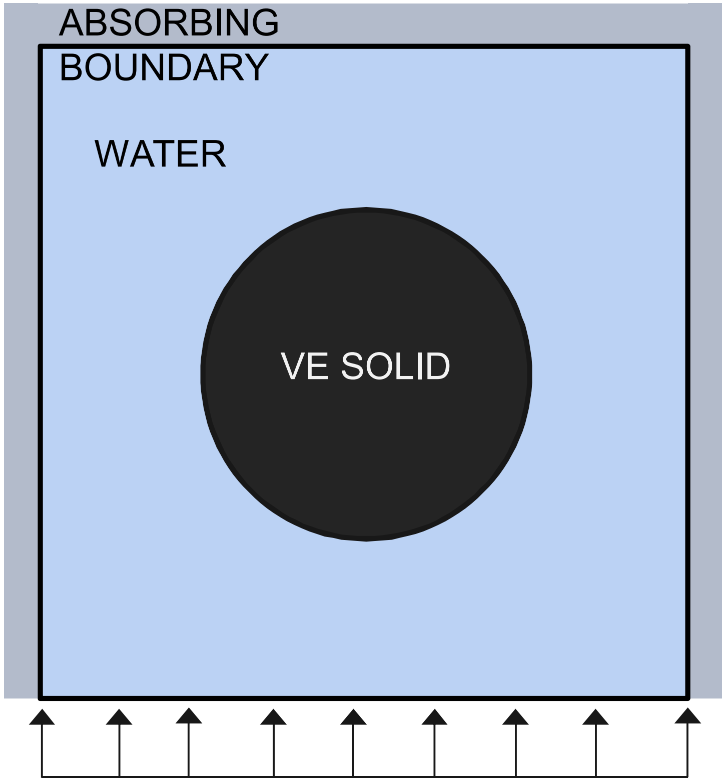
$$W = g_p^T g_u^{-T} (\mathcal{L}_{uu} g_u^{-1} g_p - \mathcal{L}_{up}) - \mathcal{L}_{pu} g_u^{-1} g_p + \mathcal{L}_{pp}$$

Hessian calculation

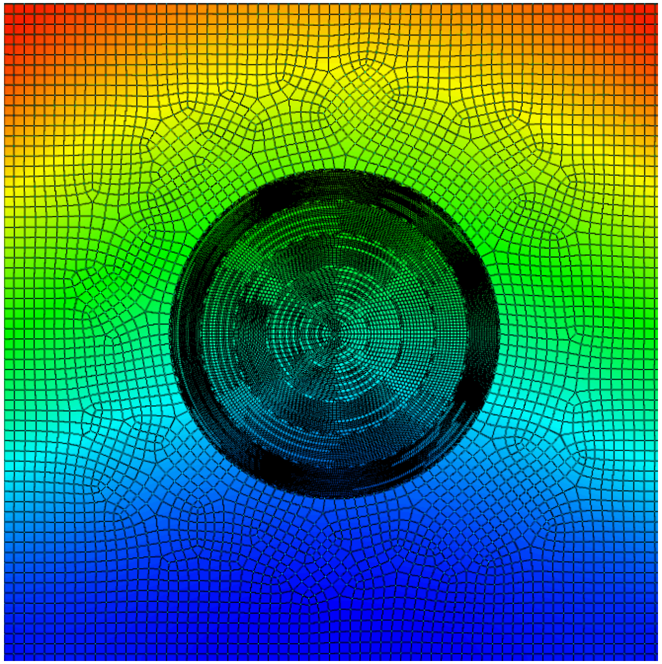
Inverse Problems: *Acoustic Cloaking*

- 2-D fluid region with circular VE solid inclusion
- Inclusion consists of concentric rings with distinct material properties
- Periodic acoustic load applied to end
- Match forward problem pressure distribution by adjusting **VE material parameters**

Model setup

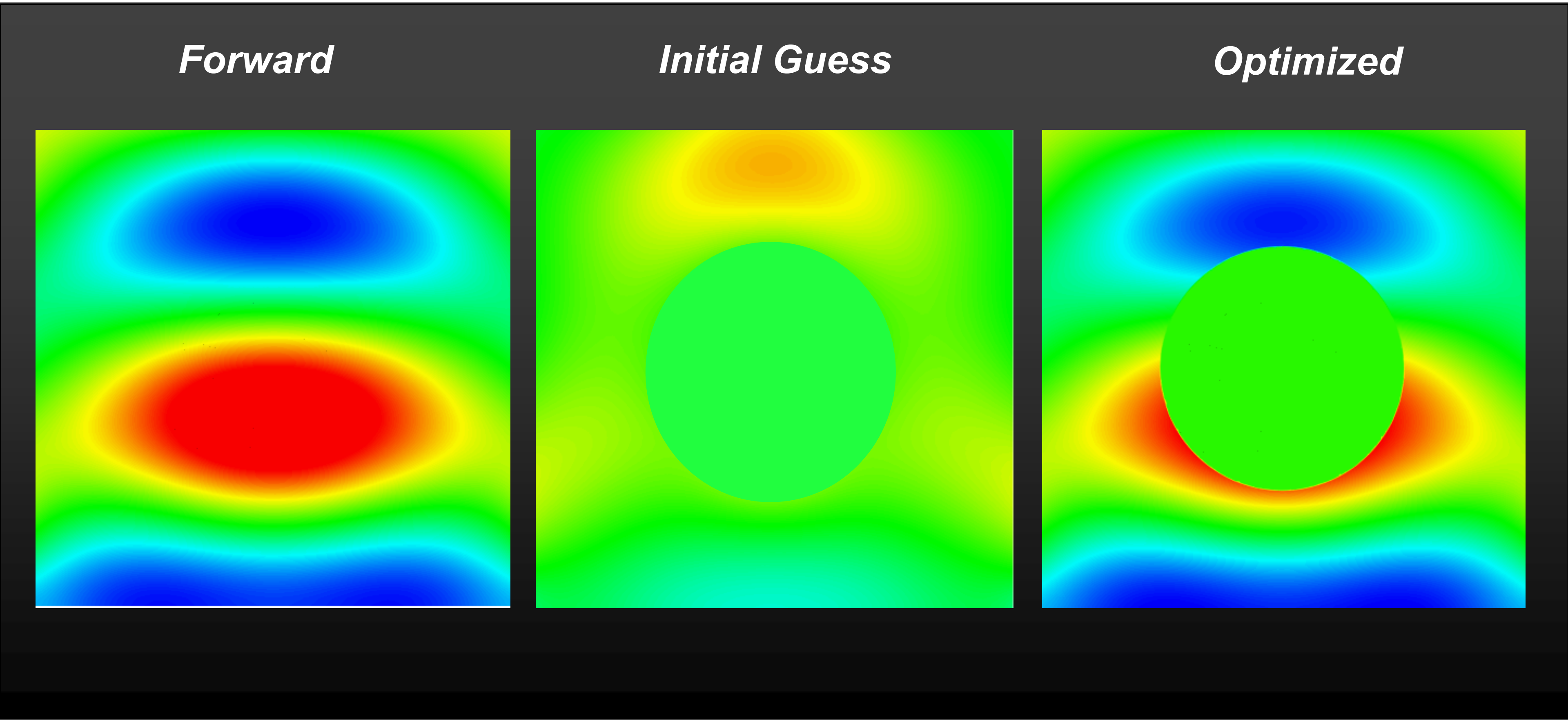


Forward problem pressure distribution (500 Hz loading) in model with 50 layers



Acoustic Cloaking

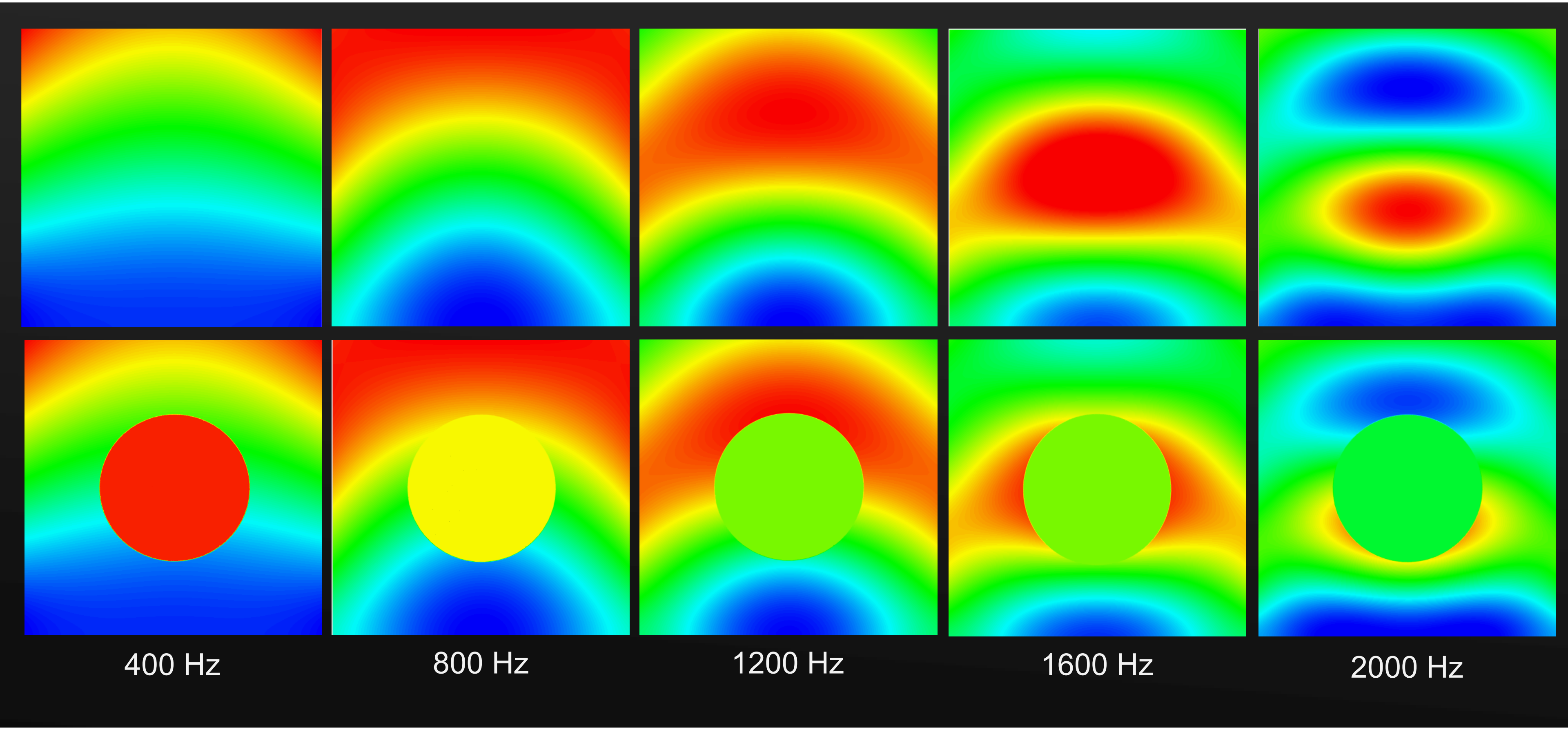
- Optimized VE foams allow recovery of desired pressure distribution



Left: Target acoustic pressure distribution, from forward problem
Center: Acoustic pressure distribution with initial material guess (2 kHz Loading)
Right: Pressure distribution after convergence to optimized design

Acoustic Cloaking

- Optimized VE foams allow recovery of desired forward pressure distribution
  - **Top:** Acoustic pressure from forward analysis
  - **Bottom:** Acoustic pressure with optimized solid inclusion



Conclusions

- Massively parallel finite element structural acoustics capability Sierra-SD has been developed for large-scale analysis
- Applicable to large-scale models with many degrees of freedom
- Sierra-SD and optimization code (ROL) have been loosely coupled for the solution of source and material inversion problems
- Capability has been applied to a variety of problems inside and outside of Sandia