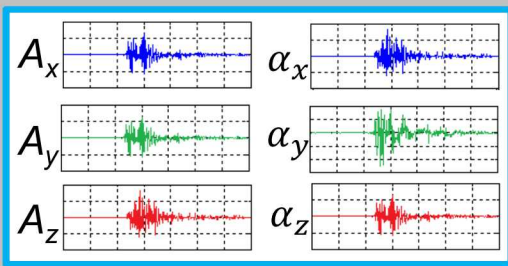


# 6 DOF Shock and Vibration: Testing and Analysis

Dr. Brian Owens, Dr. Greg Tipton, & Matthew McDowell  
*Analytical Structural Dynamics*

*Shock and Vibration Exchange  
Orlando, FL*

*October 7, 2015*



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND NO. 2011-XXXXP

# Motivation

- 6 Degree of Freedom (DOF)
  - Inputs applied to a test article/model with 6 DOFs
  - 3 translational DOF, 3 rotational DOF
- Experimental
  - 6 DOF shakers have potential to provide a more representative test of an actual environment than a single-axis test specification
  - Off-axis inputs arise from single-axis testing on single-axis shaker
  - 6 DOF shakers allow off-axis inputs to be controlled/minimized during a single-axis test specification
- Analytical
  - Apply 6 DOF input to models in shock and vibration analysis
  - More accurate post-test analysis and calibration
    - Cross-axis inputs can be significant
  - Investigate unmeasured quantities from testing (i.e. stress)
  - Tool for understanding and developing better 6 DOF testing

# Outline

- 6 DOF Analysis Procedure
  - Derivation of 6 DOF inputs
  - Considerations
- Model Configuration
- Vibration
  - 1 DOF vs. 6 DOF Testing
  - Impact on stresses
- Shock
  - Single-axis test spec vs. 6 DOF environment
  - Impact on stresses

# Derivation of 6 DOF Inputs

- 6 DOF input is the effective rigid body motion of the structure about seismic mass.
  - Should be relatively non-compliant to avoid capturing deformable effects in the calculation of rigid body motion.
  - Typically use accelerometers on a fixture.
- Consider a measurement by tri-axial accel “i”:  $\vec{a}^{(i)}(t)$
- Let the coordinate of this measurement be:  $\vec{x}^{(i)}$
- Let the reference/ input location coordinate be:  $\vec{x}^{(0)}$
- The accel has a position vector:  $\vec{r}^{(i)} = \vec{x}^{(i)} - \vec{x}^{(0)}$

# Derivation of 6 DOF Inputs (2)

- Let the effective rigid body acceleration of the structure be:

- $\vec{A}(t)$  for translational acceleration
  - $\vec{\alpha}(t)$  for angular acceleration

- The accelerometer acceleration can be expressed as:

- $\vec{a}^{(i)}(t) = [I_{3 \times 3}] \vec{A}(t) + \vec{\alpha}(t) \times \vec{r}^{(i)}$

- $\vec{a}^{(i)}(t) = [I_{3 \times 3}] \vec{A}(t) + [\widetilde{\vec{r}^{(i)}}]^T \vec{\alpha}(t)$

$$[\widetilde{\vec{r}}] = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}$$

- We can develop a system of equations for multiple measurement points:

$$\begin{bmatrix} \vec{a}^{(1)}(t) \\ \vec{a}^{(2)}(t) \\ \vdots \\ \vec{a}^{(n)}(t) \end{bmatrix} = \begin{bmatrix} [I_{3 \times 3}] & [\widetilde{\vec{r}^{(1)}}]^T \\ [I_{3 \times 3}] & [\widetilde{\vec{r}^{(2)}}]^T \\ \vdots & \vdots \\ [I_{3 \times 3}] & [\widetilde{\vec{r}^{(3)}}]^T \end{bmatrix} \begin{bmatrix} \vec{A}(t) \\ \vec{\alpha}(t) \end{bmatrix}$$

**Solve for RB Accelerations**



# Considerations

- Methodology to derive 6 DOF inputs relies heavily on:
  - Accelerometers mounted to non-compliant points
  - Sufficient measurements to characterize 6 DOF motion
  - Accurate measurement of accelerometer:
    - Position
    - Orientation
    - Polarity (signs do matter)
- Requires measurement of time histories from vibration testing
- Thorough documentation of test setup/instrumentation & accurate channel tables are very helpful
- We've developed some tools to troubleshoot some of these issues



# Analysis Work Flow

- Toolkit automates deriving 6 DOF inputs from test data
- Written for interface to Sandia's Sierra-SD FEA Software

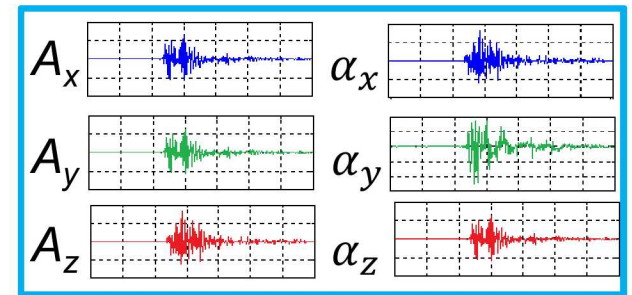
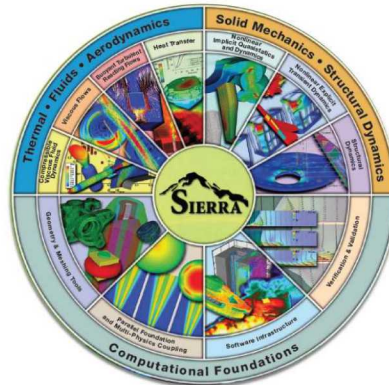
## Inputs:

- Accel coordinates
- Seismic mass coordinate
- Accel time histories

6 DOF Toolkit

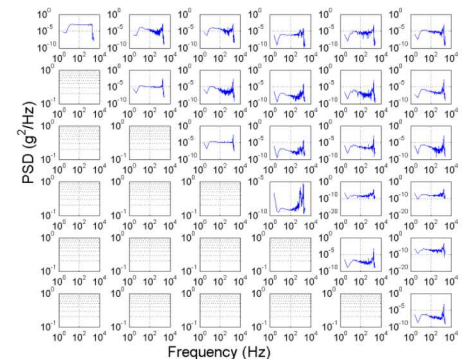
## Outputs:

- Sierra function files
- PSD/SRS plots
- Troubleshooting plots



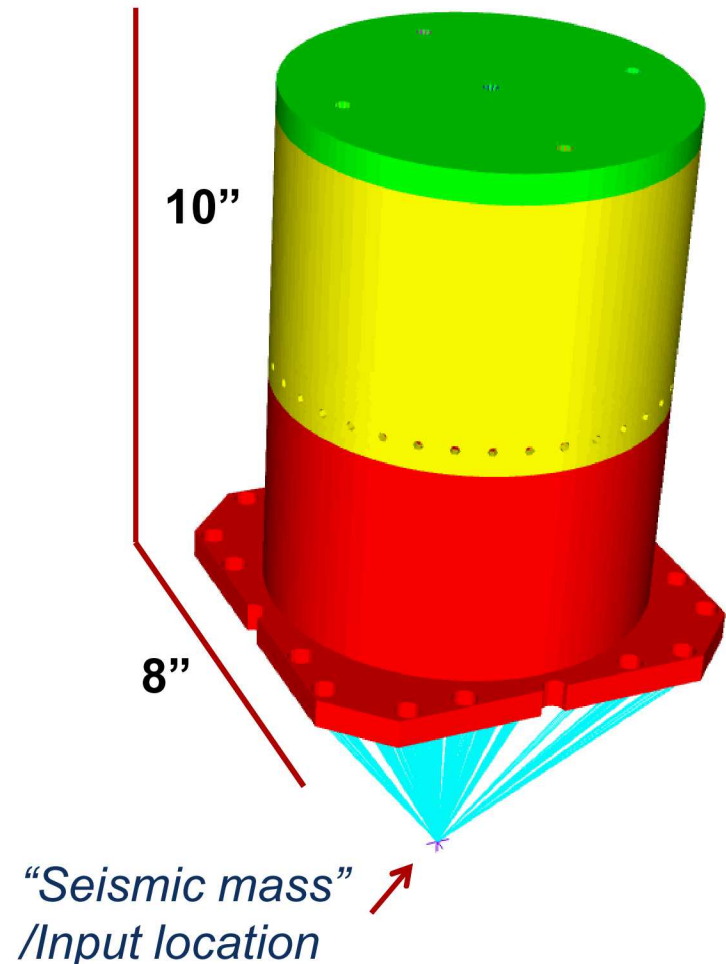
Shock

Vibration



# Model Configuration

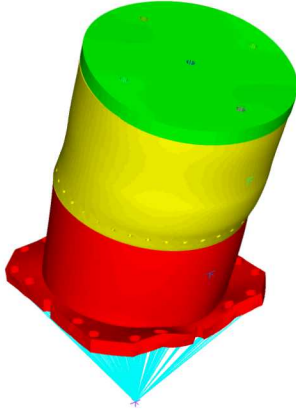
- 3 piece bolted “can” structure
  - *Bottom can/flange*
  - *Upper can*
  - *Top plate/mass*
  - *Aluminum*
- Test-bed with reasonable complexity
- Rigid elements & concentrated mass model test stand/connection
- Employed in physical 1 DOF and 6 DOF testing
- 6 DOF input will be derived from accelerometers on corners of flange



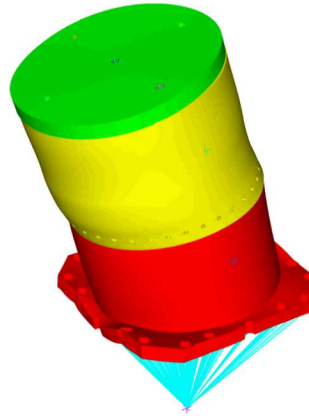


# Analytical Modes

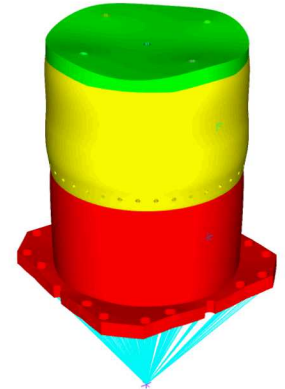
Frequency = 709 Hz



Frequency = 709 Hz



Frequency = 1816 Hz



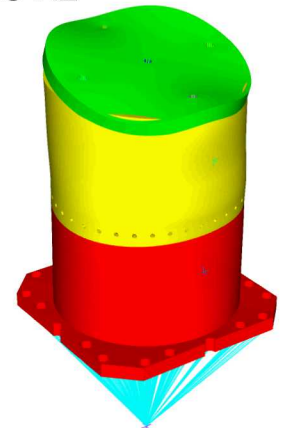
Frequency = 2761 Hz



Frequency = 2764 Hz

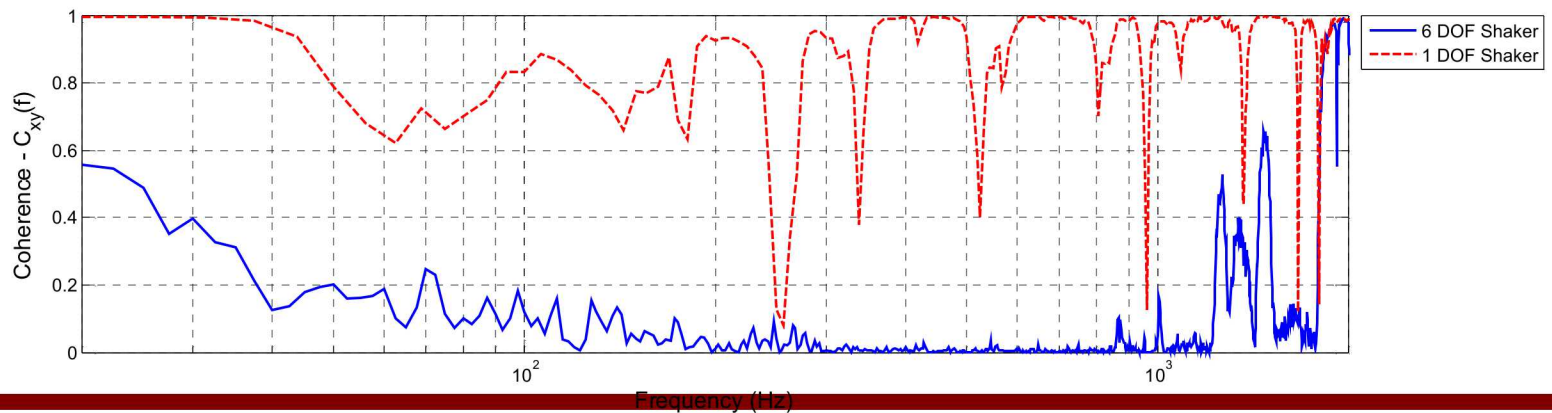
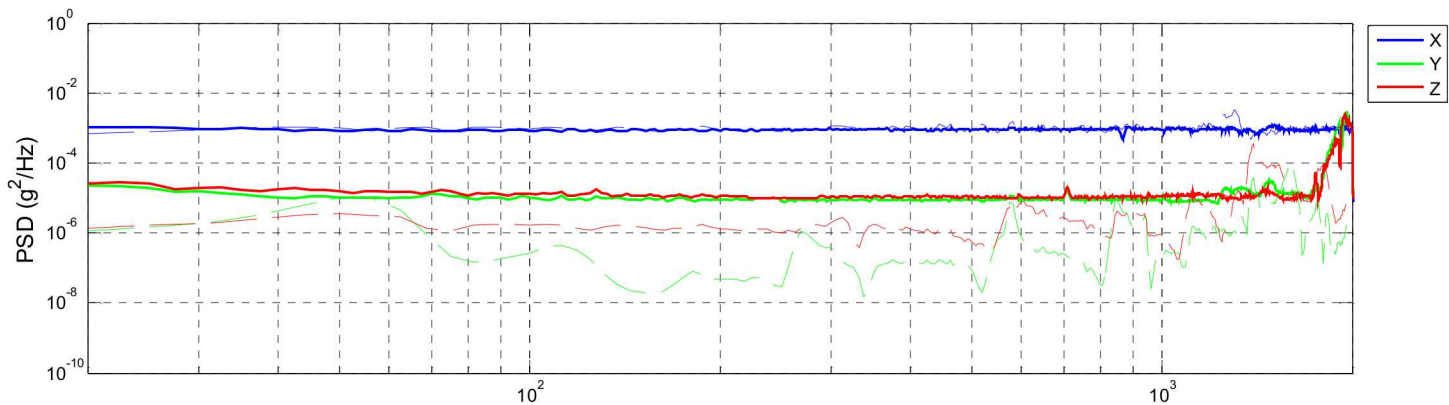


Frequency = 2765 Hz



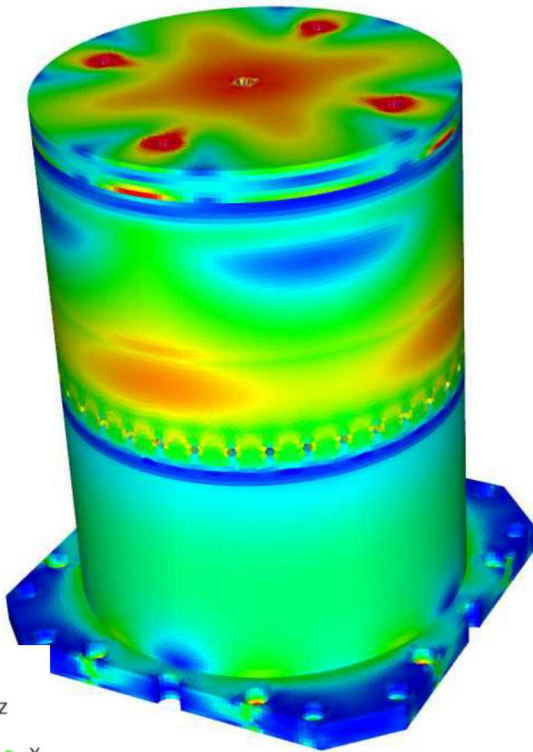
# 1 DOF vs. 6 DOF Testing

- Single-axis (axial) low level random vibration input on 1 and 6 DOF shakers
  - Off-axis inputs controlled 2 orders of magnitude lower than in-axis
  - Coherence is minimized on 6 DOF shaker

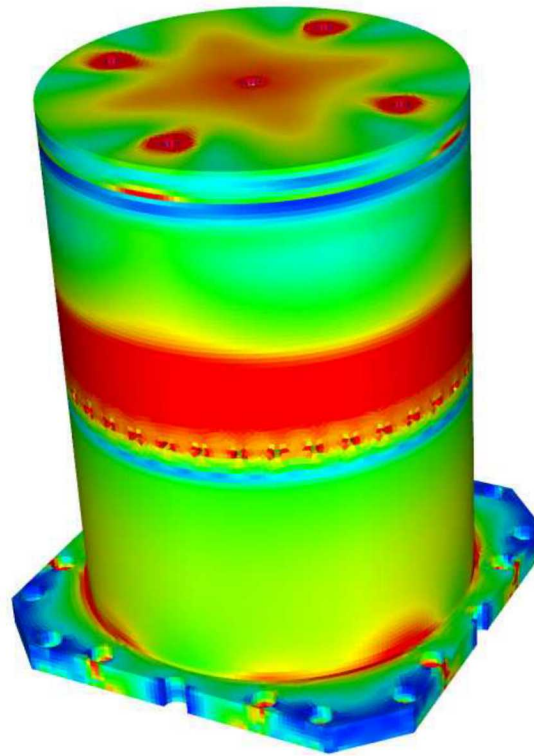


# Vibration: Stress Contours

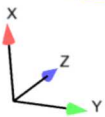
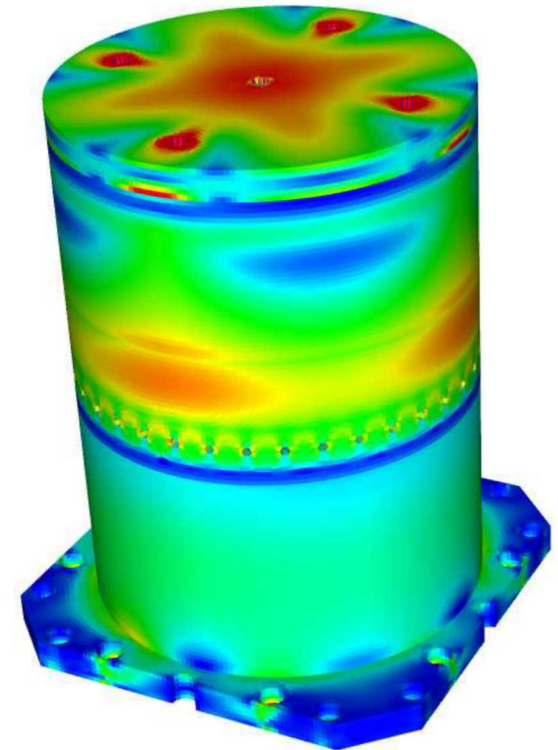
*Ideal Single-Axis:*



*6 DOF Shaker:*



*1 DOF Shaker:*

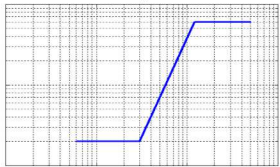


# 6 DOF Shock Input

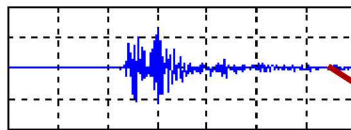
- Comparison of single-axis test spec and 6 DOF input recovered from environment
- Traditionally environmental data is broken out into single-axis test specs
- 6 DOF testing affords new opportunities

## Single-Axis Test Spec:

SRS Spec



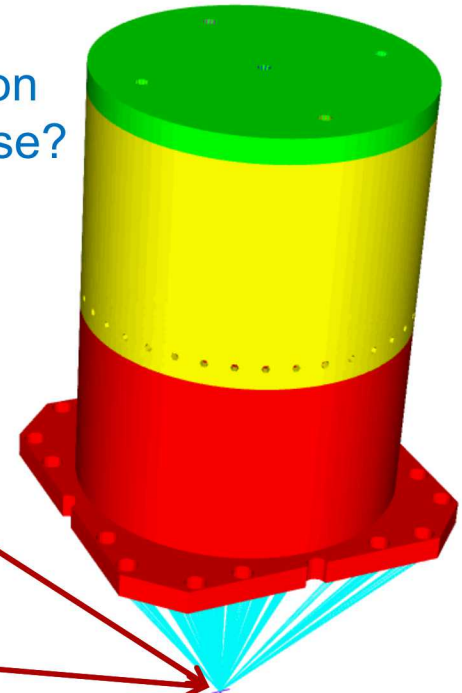
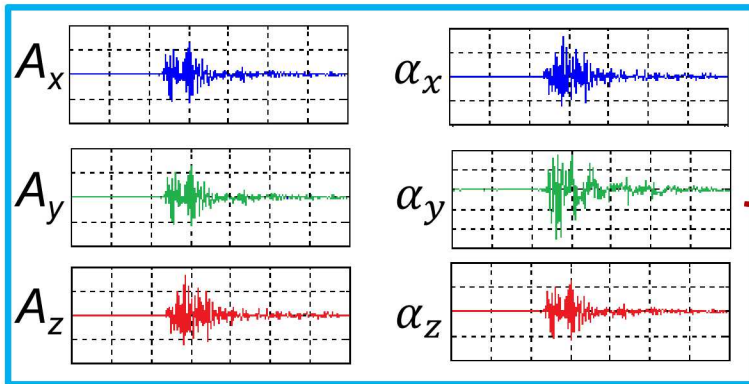
Decayed Sine Series



What's the impact on component response?  
Stresses?

VS.

## Recovered 6 DOF Input:

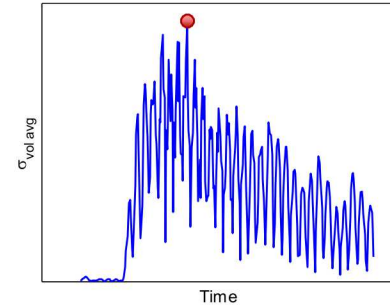




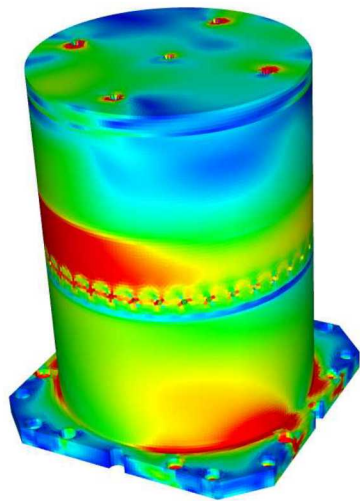
# 1 & 6 DOF Shock: Stresses

- Compared Von Mises stress distributions at instance of maximum volume averaged stress
- Noticeably different levels and distributions between 6 DOF and 1 DOF inputs

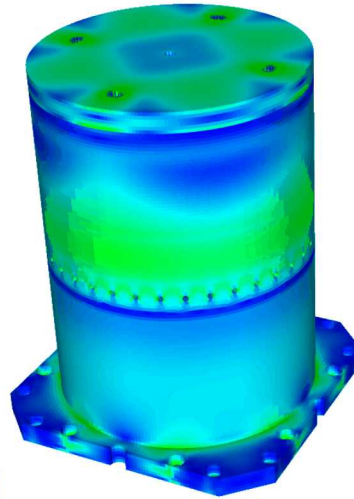
$$\sigma(t)_{vol\ avg} = \frac{\sum_{i=1}^{Nel} \sigma_i(t) v_i}{\sum_{i=1}^{Nel} v_i}$$



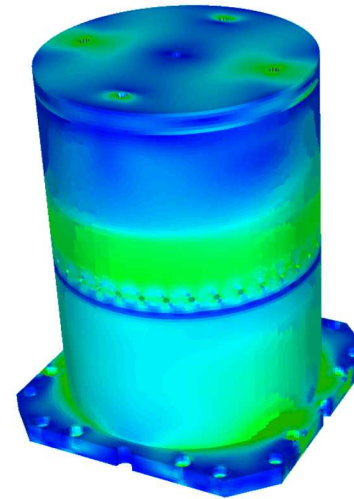
**6 DOF:**



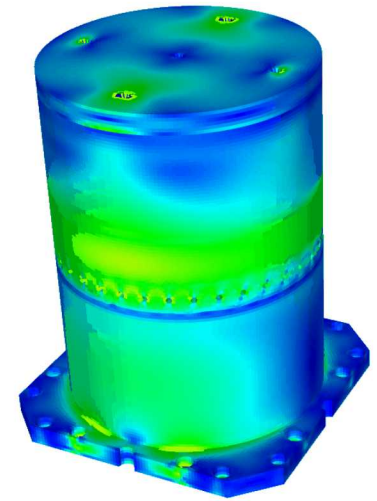
**1 DOF X:**



**1 DOF Y:**



**1 DOF Z:**

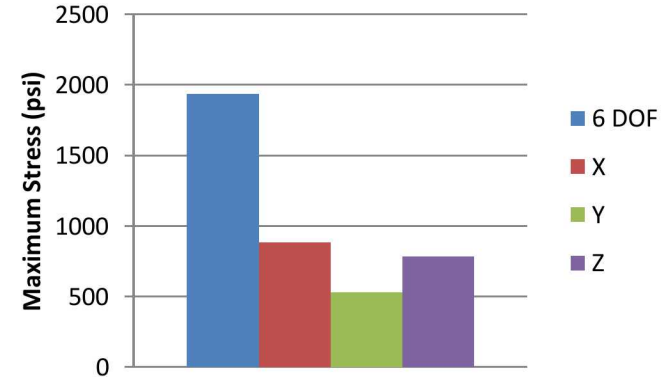
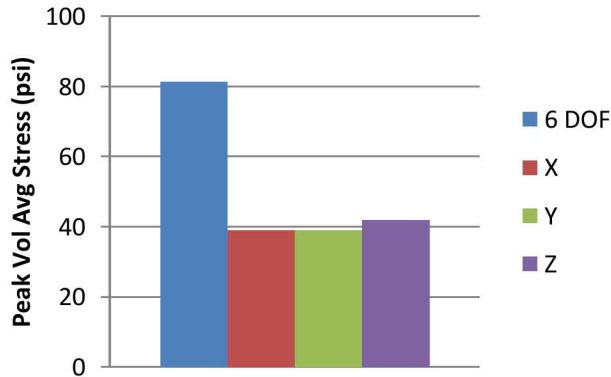


\_VonMises

2.000e+02  
1.500e+02  
1.000e+02  
5.000e+01  
0.000e+00



# 1 & 6 DOF Shock: Stresses (2)

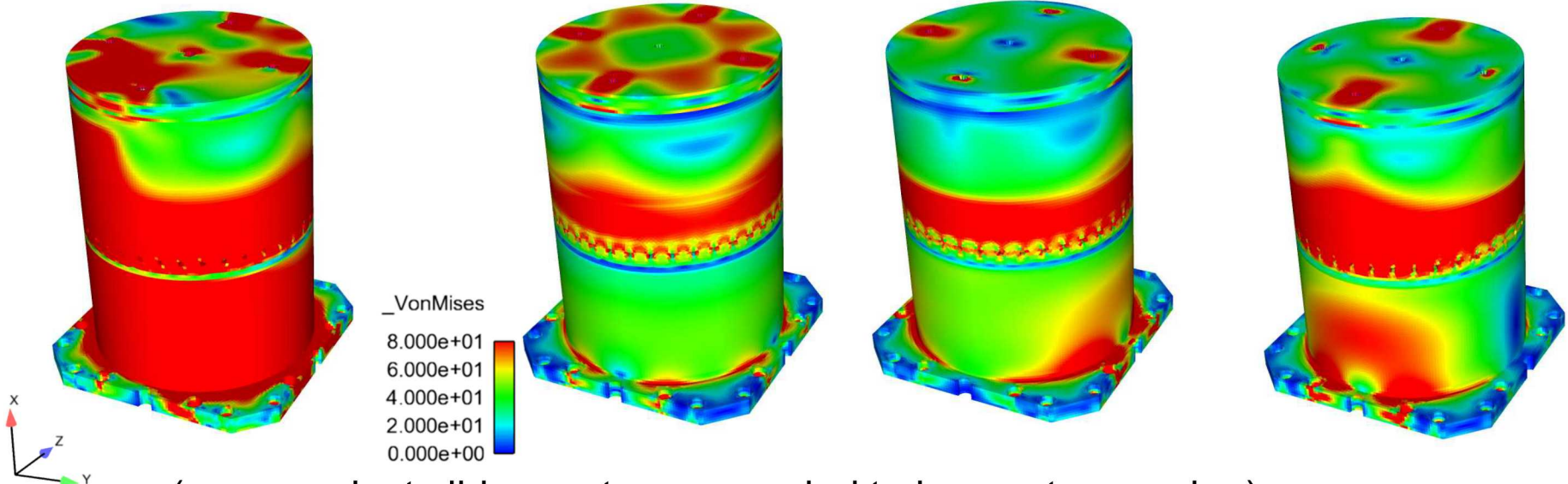


**6 DOF:**

**1 DOF X:**

**1 DOF Y:**

**1 DOF Z:**



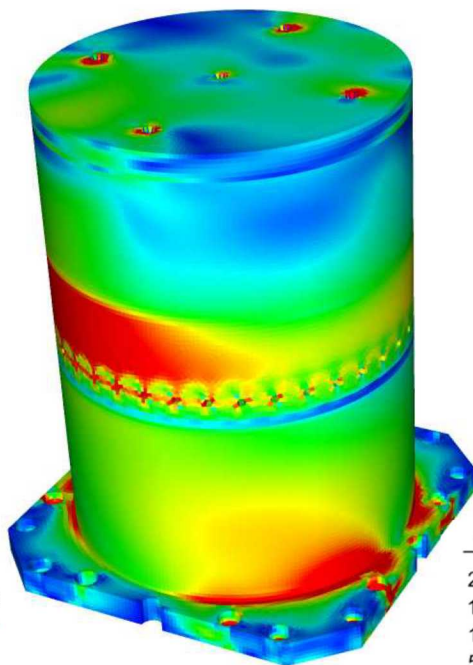
(same as last slide, contours rescaled to lower stress value)



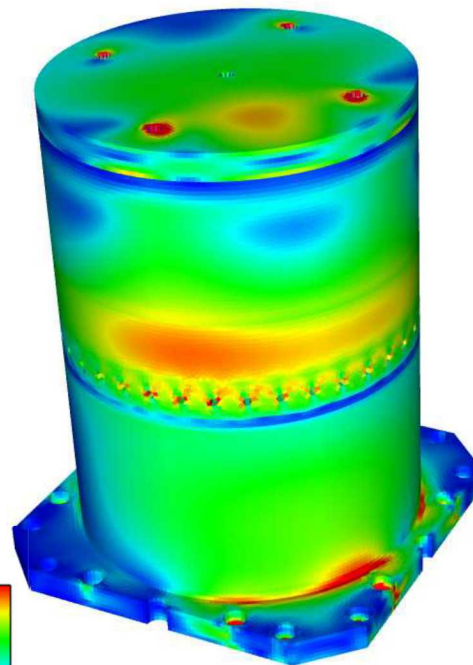
# 3 & 6 DOF Shock: Stresses

- Can we capture the fundamental input & response with 3 DOF inputs?
- How important are rotations? ... They can be very important

**6 DOF:**

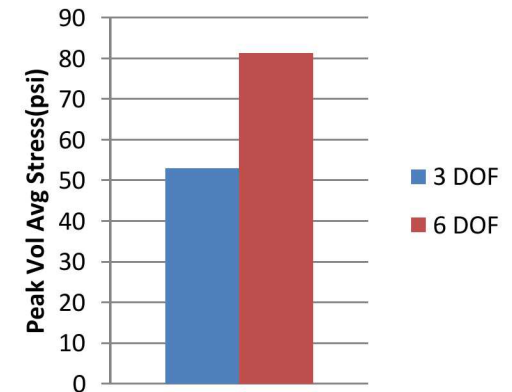


**3 DOF:**  
(translational)



\_VonMises

2.000e+02  
1.500e+02  
1.000e+02  
5.000e+01  
0.000e+00



$$\begin{bmatrix} N^i \\ V_2^i \\ V_3^i \\ T^i \\ M_2^i \\ M_3^i \\ N^j \\ V_2^j \\ V_3^j \\ T^j \\ M_2^j \\ M_3^j \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & 0 & 0 & -\frac{12EI_3}{L^3} & 0 \\ 0 & 0 & \frac{12EI_2}{L^3} & 0 & 0 & -\frac{12EI_2}{L^3} \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_3}{L^3} & 0 & 0 & \frac{12EI_3}{L^3} & 0 \\ 0 & 0 & -\frac{12EI_2}{L^3} & 0 & 0 & \frac{12EI_2}{L^3} \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_3}{L^3} & 0 & 0 & -\frac{12EI_3}{L^3} & 0 \\ 0 & 0 & \frac{12EI_2}{L^3} & 0 & 0 & -\frac{12EI_2}{L^3} \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_3}{L^3} & 0 & 0 & \frac{12EI_3}{L^3} & 0 \\ 0 & 0 & -\frac{12EI_2}{L^3} & 0 & 0 & \frac{12EI_2}{L^3} \end{bmatrix} \begin{bmatrix} u_1^i \\ u_2^i \\ u_3^i \\ \theta_1^i \\ \theta_2^i \\ \theta_3^i \\ u_1^j \\ u_2^j \\ u_3^j \\ \theta_1^j \\ \theta_2^j \\ \theta_3^j \end{bmatrix}$$

# Conclusions

- 6 DOF testing affords new opportunities for testing
  - More representative, multi-axis test environments
  - More control over single-axis test inputs and associated cross-axis inputs
- 6 DOF analysis allows for greater insight to be obtained
  - Designing better 6 DOF tests, understanding relationships of input and stress
  - Better model validation through accurate characterization of inputs
  - Development of tools to integrate 6 DOF into analysis workflow
- Vibration
  - Compared cross-axis input levels and coherence of tests on 1 & 6 DOF shakers
  - Ongoing work to understand impact of off-axis control levels and coherence
- Shock
  - Compared a 6 DOF test spec to single-axis test specs
  - Examined impact of testing on stresses in a component
  - Rotational input can be very important